An Analysis of Cutting Stock Problems Generated by CUTGEN1

Peter A. Huegler
Bethlehem Steel Corporation

Joseph C. Hartman
Lehigh University

Louis A. Martin-Vega
National Science Foundation &
Lehigh University

Report No. 00T-005

An Analysis of Cutting Stock Problems Generated by CUTGEN1

Peter A. Huegler
Homer Research Laboratories
Bethlehem Steel Corporation
and
I&MSE Department
Lehigh University

Joseph C. Hartman
I&MSE Department
Lehigh University

Louis A. Martin-Vega
National Science Foundation
and
I&MSE Department
Lehigh University

Abstract

CUTGEN1, presented in Gau and Wäscher (1995), is a problem generator for the one-dimensional cutting stock problem. Gau and Wäscher also present an analysis of problems generated by CUTGEN1 using a column generation algorithm. This analysis is used to propose a set of benchmark problems for the one-dimensional cutting stock problem. This paper provides a further analysis of the CUTGEN1 problems using the First-Fit Decreasing (FFD) algorithm. The FFD analysis is used to propose a different set of benchmark problems for the one-dimensional cutting stock problem.

1 Introduction

This paper provides an analysis of one-dimensional cutting stock problems generated by the problem generator CUTGEN1 (Gau and Wäscher (1995)) using the First-Fit Decreasing (FFD) algorithm. The remainder of this section reviews the cutting stock problem and the FFD algorithm. Section 2 examines the CUTGEN1 problem generator. Section 3 presents the FFD analysis of the CUTGEN1 problems and section 4 presents the conclusions of the analysis and proposes a different set of benchmark problems for the one-dimensional cutting stock problem.

1.1 The Cutting Stock Problem

The one-dimensional cutting stock problem (CSP) can be stated as follows:

> Given a set of order requirements, $\boldsymbol{O}$, and for each requirement i in $\boldsymbol{O}$, a demand, d_i, and a length, l_i, and given a set of inventory items, $\boldsymbol{I}$, each of length L, with a sufficient quantity of items in $\boldsymbol{I}$ to meet all of the demand in $\boldsymbol{O}$ (guaranteeing a feasible solution). What is the minimum number of inventory items in $\boldsymbol{I}$ required to meet all of the demand in $\boldsymbol{O}$?

The one-dimensional cutting stock problem can be formulated in many ways. The classical formulation involves the application of cutting patterns to the inventory items (Gilmore and Gomory (1961, 1963)). A cutting pattern is defined as an m-tuple $(a_{1j}, a_{2j}, \ldots, a_{mj})$ where m is the number of order requirements in $\boldsymbol{O}$ and a_{ij} represents the amount of demand for order i being met by cutting pattern j. A valid cutting pattern is defined by constraints 1 through 3:

$$\sum_{i \in O} l_i \cdot a_{ij} \le L \qquad (1)$$

$$a_{ij} \ge 0 \qquad (2)$$

$$a_{ij} \text{ an integer} \qquad (3)$$

By introducing the decision variable x_j representing the number of cutting patterns j to be cut and the set $\boldsymbol{P}$ representing the valid cutting patterns from $\boldsymbol{O}$ onto $\boldsymbol{I}$, the formulation is:

$$\text{CSP:} \quad \text{Min} \quad \sum_{j \in P} x_j \qquad (4)$$

$$\text{s.t.} \quad \sum_{j \in P} a_{ij} \cdot x_j \geq d_i, \forall i \in O \tag{5}$$

$$x_j \geq 0 \tag{6}$$

$$x_j \text{ an integer} \tag{7}$$

The cutting stock and the bin-packing problems are equivalent (Coffman et al., 1996).

1.2 First Fit Decreasing Heuristic

The FFD algorithm first sorts the order requirements into non-increasing order such that $l_1 \geq l_2 \geq \ldots \geq l_n$. The algorithm then cuts the first order requirement from the first inventory item with at least l_i remaining length. This is repeated until no order requirements remain.

The FFD algorithm has been extensively studied in the context of cutting and packing problems. In general, as the average size of the order requirement decreases, the worst-case performance of FFD improves. The worst-case behavior for FFD is:

$$FFD(L) \leq \frac{11}{9} \cdot OPT(L) + 3 \tag{8}$$

Where ***L*** is any list of ordered items, ***FFD(L)*** is the FFD solution and ***OPT(L)*** is the optimal packing (Baker (1983)). Coffman et al. (1996) provides an excellent discussion of the worst-case and average-case FFD performance.

2 CUTGEN1 Problem Generator

2.1 Background

Gau and Wäscher (1995) present a problem generator for the one-dimensional cutting stock problem. Five parameters are identified that define problem classes. The parameters are listed in Table 1.

m	Number of order requirements
L	Inventory length
v1	Minimum order requirement relative to inventory length
v2	Maximum order requirement relative to inventory length
dbar	Average order demand

Table 1 – CUTGEN1 Parameters

The parameter *m* is the number of order requirements (|***O***|) and *L* is the length of the stock inventory pieces. The order lengths, l_i, are generated from a uniform distribution between *v1*L* and *v2*L*. The order demands, d_i, are generated such that the average demand for all order requirements is *dbar*. The outcome of this is that each problem generated for a specific *m* and *dbar* will have the same total number of demands (Σd_i).

2.2 Benchmark Problems

Gau and Wäscher (1995) propose a set of benchmark problems for the one-dimensional cutting stock problem. These problems are generated using a specific set of values for the above parameters. Table 2 shows the possible values.

m	25, 50, 75
L	10,000
v1	.0001
v2	.25, .50, .75, 1.00
dbar	5, 10, 20

Table 2 - Benchmark Problem Parameters

With all of the combinations of these values, 36 classes of problems are generated. A column generation algorithm is used to analyze the benchmark problems. The algorithm is based on Gilmore and Gomory's seminal work (Gilmore and Gomory (1961, 1963)). The algorithm solves a relaxed problem with the integer constraint relaxed (constraint 7). Difficult problems are identified by long solution times because, according to Gau and Wäscher (1995), most cutting stock solution methods embed column generation. Gau and Wäscher propose that six problem classes as the hardest problems. The six problem classes are in Table 3.

m	*v2*	*Dbar*
50	.50	5
50	.50	10
50	.50	20
75	.50	5
75	.50	10
75	.50	20

Table 3 - Hardest Benchmark Problem Classes

The hardest classes of problems are defined as those classes, which on average, have the longest solution time.

3 First Fit Decreasing Analysis

3.1 Lower Bounds

A critical difference between Gau and Wäscher's analysis and this analysis is the use of the FFD heuristic. The FFD analysis compares the FFD solution to a lower bound for the CSP. To calculate a lower bound for the CSP, the integer constraint for valid cutting patterns is relaxed (constraint 3). By relaxing this constraint, a solution to the cutting stock problem can be constructed by inspection. The equation to calculate the lower bound for this relaxed problem (without constructing a solution) is:

$$LB_1 = \left\lceil \frac{\sum_{i \in O} l_i \cdot d_i}{L} \right\rceil \tag{9}$$

The lower bound equation 8 can also be rewritten as a function of the average order requirements. Calculating the average as follows:

$$A = \frac{\sum_{i \in O} l_i \cdot d_i}{\sum_{i \in O} d_i} \tag{10}$$

And substituting equation (10) into equation (9), the result is:

$$LB_1 = \left\lceil \left(\frac{\sum_{i \in O} d_i}{L} \right) \cdot A \right\rceil \tag{11}$$

A better lower bound for CSP uses the number of large order requirements. A large order requirement is defined as an order requirement such that l_i>.50*L. In other words, an order requirement that is larger than half of the size of the inventory pieces. Two or more of these large pieces cannot be cut from the same inventory piece. If the number of the large pieces dominates the solution, the better lower bound is:

$$LB_2 = \sum_{i \in B} d_i \tag{12}$$

where $\boldsymbol{B}$ is the set of large order requirements. An improved lower bound for the CSP is a combination of $\boldsymbol{LB_1}$ and $\boldsymbol{LB_2}$:

$$LB_3 = \max\left(\sum_{i \in B} d_i , \left\lceil \left(\frac{\sum_{i \in O} d_i}{L} \right) \cdot A \right\rceil \right) \tag{13}$$

3.2 Analysis Results

Table 4 shows the analysis of the CUTGEN1 problem classes using the FFD algorithm. The Gap Average is the average difference between the FFD solution and the lower bound (equation 13). The right-hand columns display the frequency of the solutions for each problem class. The maximum gap for the FFD algorithm is for these problem sets is approximately 22% using equation (8). The highlighted rows are the difficult classes identified by Gau and Wäscher.

Class	m	v2	dbar	Gap Average	Gap Standard Deviation	0%	1%	2%	3%	4%	5%	6%	7%	8%	9%	10%	11%	12%	13%	14%	15%	16%	17%	18%	19%	20%	21%	22%	23%	24%	25%
1	25	0.25	5	0.64%	1.87%	89	-	-	-	-	2	7	1	-	-	1	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
2	25	0.25	10	0.65%	1.25%	78	-	1	10	10	1	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
3	25	0.25	20	0.78%	0.86%	53	-	42	4	1	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
4	25	0.50	5	1.86%	1.73%	43	-	-	29	21	3	2	2	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
5	25	0.50	10	2.09%	1.36%	15	-	47	17	12	6	3	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
6	25	0.50	20	2.20%	1.60%	4	14	42	19	9	6	2	1	2	-	1	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
7	25	0.75	5	3.95%	3.54%	20	-	6	27	10	6	4	12	4	1	3	1	1	1	2	1	1	-	-	-	-	-	-	-	-	-
8	25	0.75	10	3.92%	3.57%	17	1	16	13	15	10	4	7	4	2	5	1	1	2	1	-	-	-	-	-	1	-	-	-	-	-
9	25	0.75	20	3.44%	2.99%	12	4	18	25	16	5	5	3	-	3	4	2	2	-	1	-	-	-	-	-	-	-	-	-	-	-
10	25	1.00	5	5.55%	4.05%	13	-	10	6	16	6	6	10	5	6	7	5	3	2	2	2	-	-	-	1	-	-	-	-	-	-
11	25	1.00	10	5.59%	4.21%	14	2	3	10	9	15	7	7	7	6	6	4	3	1	4	-	-	-	-	-	1	1	-	-	-	-
12	25	1.00	20	5.52%	4.56%	16	4	5	6	13	9	7	7	4	6	8	3	2	3	1	2	1	-	2	1	-	-	-	-	-	-
13	50	0.25	5	0.26%	0.89%	92	-	-	2	6	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
14	50	0.25	10	0.34%	0.66%	78	1	21	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
15	50	0.25	20	0.43%	0.42%	48	50	2	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
16	50	0.50	5	1.15%	0.67%	36	-	54	7	3	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
17	50	0.50	10	1.10%	0.62%	8	54	28	9	1	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
18	50	0.50	20	1.18%	0.65%	-	48	36	16	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
19	50	0.75	5	2.30%	2.14%	10	4	40	24	5	9	1	3	-	1	1	-	2	-	-	-	-	-	-	-	-	-	-	-	-	-
20	50	0.75	10	2.49%	2.36%	9	5	41	24	7	3	3	1	1	3	1	1	-	-	1	-	-	-	-	-	-	-	-	-	-	-
21	50	0.75	20	2.15%	1.92%	9	17	37	14	10	5	3	4	-	-	-	-	-	-	1	-	-	-	-	-	-	-	-	-	-	-
22	50	1.00	5	4.32%	2.96%	10	4	10	16	9	13	10	8	8	6	1	3	1	-	1	-	-	-	-	-	-	-	-	-	-	-
23	50	1.00	10	4.17%	2.78%	8	3	7	15	24	16	9	2	7	4	-	3	1	-	-	-	1	-	-	-	-	-	-	-	-	-
24	50	1.00	20	4.35%	3.07%	13	4	7	10	17	10	9	7	7	9	5	-	1	1	-	-	-	-	-	-	-	-	-	-	-	-
25	75	0.25	5	0.26%	0.68%	87	-	9	4	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
26	75	0.25	10	0.39%	0.50%	62	14	24	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
27	75	0.25	20	0.26%	0.28%	53	47	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
28	75	0.50	5	0.77%	0.61%	33	13	52	2	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
29	75	0.50	10	0.78%	0.44%	4	72	23	-	1	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
30	75	0.50	20	0.80%	0.48%	-	80	17	2	1	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
31	75	0.75	5	1.65%	1.56%	5	32	43	11	2	3	-	1	2	-	1	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
32	75	0.75	10	1.85%	1.27%	1	10	64	14	6	1	2	1	-	1	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
33	75	0.75	20	1.71%	1.36%	4	23	53	9	3	2	3	2	1	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
34	75	1.00	5	3.53%	2.56%	9	8	13	16	21	8	9	6	4	1	3	1	1	-	-	-	-	-	-	-	-	-	-	-	-	-
35	75	1.00	10	3.73%	2.19%	8	5	6	17	18	19	17	3	3	1	3	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
36	75	1.00	20	4.25%	2.82%	7	8	8	11	21	9	10	6	8	3	5	-	3	-	-	-	-	-	-	-	-	-	-	-	-	-

Table 4 – FFD to Lower Bound Average Gap

An alternate presentation of the data in Table 4 is in Table 5. The data in Table 5 is sorted by gap standard deviation. A definite pattern appears in this table. In general, as the *v2* parameter increases, the average gap and the gap standard deviation increase. Note that the difficult problems identified by Gau and Wäscher are near the top of Table 5 and the FFD algorithm consistently solves these problems to within 4% of the lower bound and on average, to within 1% of the lower bound.

Class	m	v2	dbar	Gap Average	Gap Standard Deviation	0%	1%	2%	3%	4%	5%	6%	7%	8%	9%	10%	11%	12%	13%	14%	15%	16%	17%	18%	19%	20%	21%	22%	23%	24%	25%
27	75	0.25	20	0.26%	0.28%	53	47	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
15	50	0.25	20	0.43%	0.42%	48	50	2	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
29	75	0.50	10	0.78%	0.44%	4	72	23	-	1	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
30	75	0.50	20	0.60%	0.45%	-	80	17	2	1	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
26	75	0.25	10	0.39%	0.50%	62	14	24	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
28	75	0.50	5	0.27%	0.61%	33	13	52	2	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
17	50	0.50	10	1.10%	0.62%	6	54	26	9	1	4	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
18	50	0.50	20	1.18%	0.65%	-	48	36	16	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
14	50	0.25	10	0.34%	0.66%	78	1	21	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
25	75	0.25	5	0.26%	0.68%	87	-	9	4	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
3	25	0.25	20	0.78%	0.86%	53	-	42	4	1	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
13	50	0.25	5	0.26%	0.89%	92	-	-	2	6	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
16	50	0.50	5	1.15%	0.67%	36	-	54	7	3	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
2	25	0.25	10	0.65%	1.25%	78	-	1	10	10	1	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
32	75	0.75	10	1.85%	1.27%	1	10	64	14	6	1	2	1	-	1	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
33	75	0.75	20	1.71%	1.36%	4	23	53	9	3	2	3	2	1	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
5	25	0.50	10	2.09%	1.36%	15	-	47	17	12	6	3	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
31	75	0.75	5	1.65%	1.56%	5	32	43	11	2	3	-	1	2	-	1	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
6	25	0.50	20	2.20%	1.60%	4	14	42	19	9	6	2	1	2	-	1	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
4	25	0.50	5	1.86%	1.73%	43	-	-	29	21	3	2	2	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
1	25	0.25	5	0.64%	1.87%	89	-	-	-	-	2	7	1	-	-	1	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
21	50	0.75	20	2.15%	1.92%	9	17	37	14	10	5	3	4	-	-	-	-	-	-	1	-	-	-	-	-	-	-	-	-	-	-
19	50	0.75	5	2.30%	2.14%	10	4	40	24	5	9	1	3	-	1	1	-	2	-	-	-	-	-	-	-	-	-	-	-	-	-
35	75	1.00	10	3.73%	2.19%	8	5	6	17	18	19	17	3	3	1	3	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
20	50	0.75	10	2.49%	2.36%	9	5	41	24	7	3	3	1	1	3	1	1	-	-	1	-	-	-	-	-	-	-	-	-	-	-
34	75	1.00	5	3.53%	2.56%	9	8	13	16	21	8	9	6	4	1	3	1	1	-	-	-	-	-	-	-	-	-	-	-	-	-
23	50	1.00	10	4.17%	2.78%	8	3	7	15	24	16	9	2	7	4	-	3	1	-	-	-	1	-	-	-	-	-	-	-	-	-
36	75	1.00	20	4.25%	2.82%	7	8	8	11	21	9	10	6	8	3	5	-	3	-	-	-	-	-	-	-	-	-	-	-	-	-
22	50	1.00	5	4.32%	2.96%	10	4	10	16	9	13	10	8	8	6	1	3	1	-	1	-	-	-	-	-	-	-	-	-	-	-
9	25	0.75	20	3.44%	2.99%	12	4	18	25	16	5	5	3	-	3	4	2	2	-	1	-	-	-	-	-	-	-	-	-	-	-
24	50	1.00	20	4.35%	3.07%	13	4	7	10	17	10	9	7	7	9	5	-	1	1	-	-	-	-	-	-	-	-	-	-	-	-
7	25	0.75	5	3.95%	3.54%	20	-	6	27	10	6	4	12	4	1	3	1	1	1	2	1	1	-	-	-	-	-	-	-	-	-
8	25	0.75	10	3.92%	3.57%	17	1	16	13	15	10	4	7	4	2	5	1	1	2	1	-	-	-	-	-	1	-	-	-	-	-
10	25	1.00	5	5.55%	4.05%	13	-	10	6	16	6	6	10	5	6	7	5	3	2	2	2	-	-	-	1	-	-	-	-	-	-
11	25	1.00	10	5.59%	4.21%	14	2	3	10	9	15	7	7	7	6	6	4	3	1	4	-	-	-	-	-	1	1	-	-	-	-
12	25	1.00	20	5.52%	4.56%	16	4	5	6	13	9	7	7	4	6	8	3	2	3	1	2	1	-	2	1	-	-	-	-	-	-

Table 5 – FFD to Lower Bound Average Gap by Standard Deviation

Table 6 shows the FFD results for each of the difficult problems identified by Gau and Wäscher for each problem class. Note that the most difficult problems for the difficult problem classes are solved to within 1.35% of the lower bound.

Class	Problem	m	v2	dbar	Gap	Gap Percent
1	1	25	0.25	5	-	0.00%
2	58	25	0.25	10	1	2.17%
3	6	25	0.25	20	-	0.00%
4	39	25	0.50	5	1	4.17%
5	24	25	0.50	10	-	0.00%
6	63	25	0.50	20	1	1.10%
7	89	25	0.75	5	-	0.00%
8	69	25	0.75	10	1	1.43%
9	2	25	0.75	20	2	1.53%
10	49	25	1.00	5	1	1.92%
11	20	25	1.00	10	2	2.25%
12	42	25	1.00	20	3	1.60%
13	44	50	0.25	5	-	0.00%
14	72	50	0.25	10	-	0.00%
15	9	50	0.25	20	1	0.66%
16	21	50	0.50	5	-	0.00%
17	40	50	0.50	10	1	0.85%
18	85	50	0.50	20	3	1.35%
19	44	50	0.75	5	1	1.30%
20	53	50	0.75	10	1	0.72%
21	17	50	0.75	20	3	1.01%
22	51	50	1.00	5	1	0.92%
23	65	50	1.00	10	7	3.04%
24	2	50	1.00	20	13	2.99%
25	36	75	0.25	5	1	1.85%
26	7	75	0.25	10	-	0.00%
27	37	75	0.25	20	1	0.48%
28	32	75	0.50	5	-	0.00%
29	7	75	0.50	10	2	1.02%
30	87	75	0.50	20	3	0.79%
31	23	75	0.75	5	2	1.61%
32	57	75	0.75	10	3	1.19%
33	17	75	0.75	20	4	0.84%
34	79	75	1.00	5	2	1.47%
35	98	75	1.00	10	12	3.67%
36	5	75	1.00	20	14	2.08%

Table 6 – Most Difficult Problem for each Class

Figure 1 shows the actual gap (the difference between the number of inventory items consumed) between the FFD solution and the lower bound. Note that all of the larger gaps occur at the larger average order requirements. The FFD algorithm solves problems with smaller order requirements more consistently than those problems with larger order requirements.

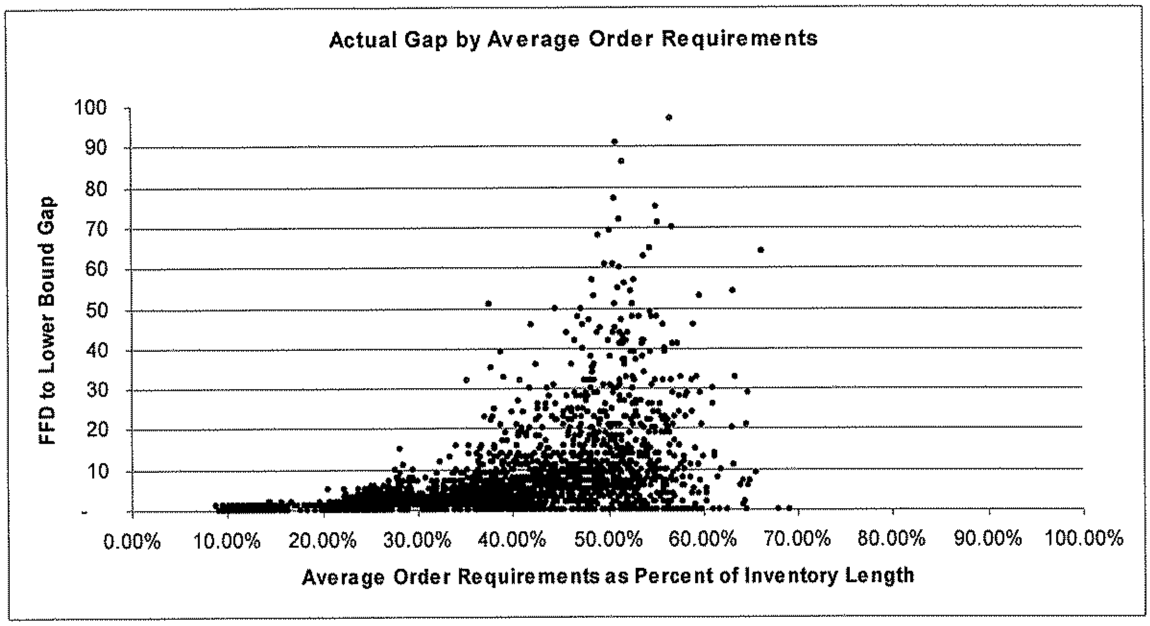

Figure 1 – Actual Gap by Order Requirements

3.3 *Identification of Difficult Problems*

The problem classes identified as difficult by Gau and Wäscher are consistently solved with small gaps by the FFD algorithm. Figure 2 displays a consistent pattern to the FFD solutions as plotted against the average order requirements. There are seven distinct lines in the chart. Each of these lines correspond to a value of *m*dbar*, for which there are seven values in the problem classes proposed by Gau and Wäscher.

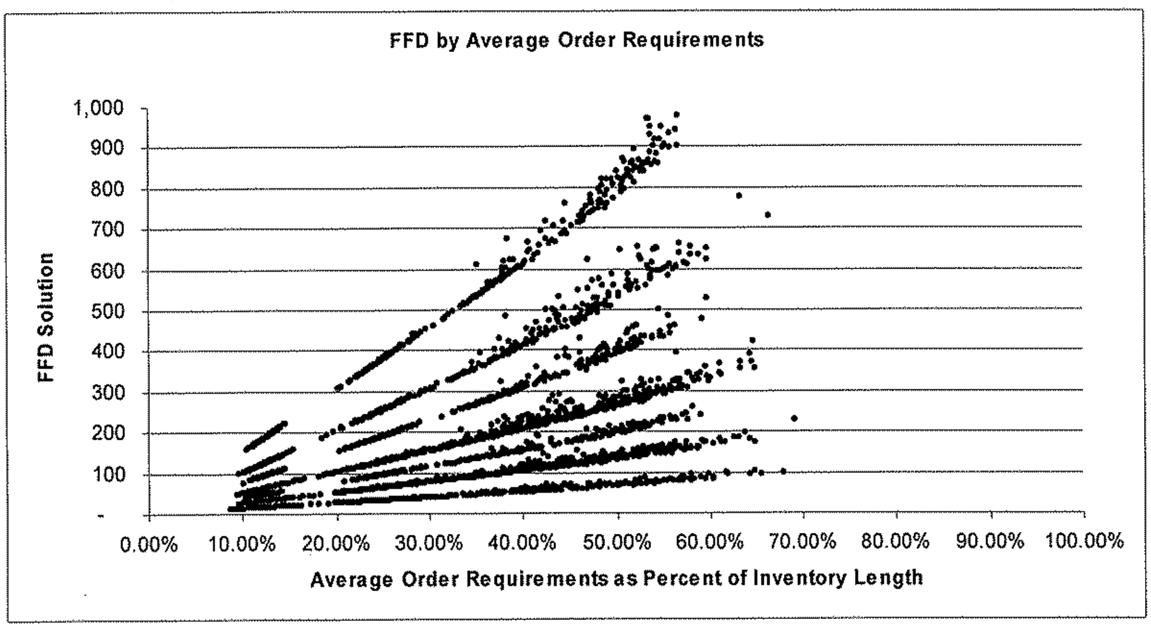

Figure 2 - FFD Solution by Order Requirements

LB_1 and LB_2 can be used to partition the problems into two sets. The first set contains problems where LB_1 is the larger of the two lower bounds. The second set contains the problems where LB_2 is the larger lower bound. The second set is defined by problems dominated by the large order requirements. In Figure 2, the large order requirement dominated problems appear furthest from the seven distinct lines.

Figure 3 shows only one group from Figure 2. The analysis on the other groups provides results similar to those presented in the remainder of this section. The group presented is for problems for which *m*dbar* is 500. These problems are generated in the classes where *(m,dbar)* is (25,20) and (50,10). Removed from Figure 3 are the problems that are dominated by the large order requirements. Added to Figure 3 is a line representing the lower bound as calculated using equation 8 (LB_1). Note how the FFD solutions diverge more from the lower bounds as the average order requirements increase.

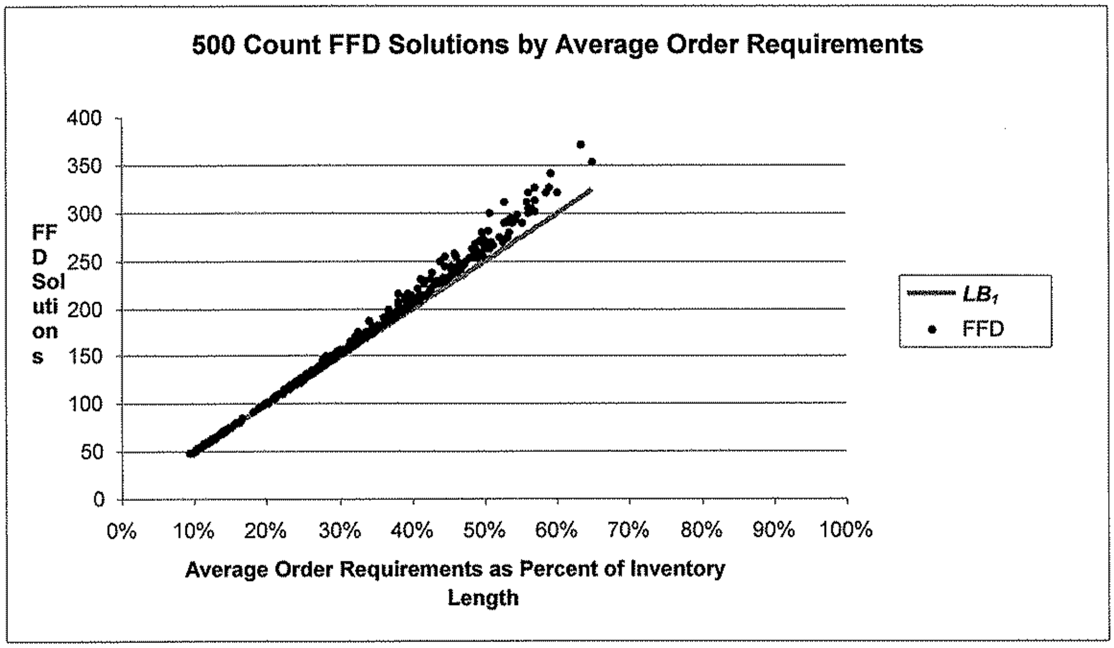

Figure 3 - 500 Count FFD Solutions by Average Order Requirements

Table 7 displays the analysis results for three different sets of 500 count problems. The first two sets, set A and set B, are problems where the lower bound is not dominated by large order requirements (the largest lower bound for the problem is LB_1). These problems are split into two subsets. Set A has problems where the average order requirements are less than 1/3 of the inventory length and set B has problems where the average order requirements are greater than 1/3 of the inventory length. The final category (set C) is made up of those problems with the lower bound dominated by large order requirements (the largest lower bound for the problem is LB_2). It should be noted that all of the problems dominated by large order requirements have an average order requirement of greater than 1/3 the inventory length.

	Set A	Set B	Set C
Average Order Requirements as Percent of L	19.60%	42.77%	47.90%
Percent FFD Optimal	32.25%	0.00%	31.16%
Average Gap	1.14%	4.74%	3.10%
Average Gap Standard Deviation	1.14%	3.58%	3.27%
Minimum Average Gap	0.00%	0.57%	0.00%
Maximum Average Gap	7.36%	18.18%	15.89%

Table 7 - 500 Count FFD Analysis Results

Generally, FFD can consistently generate good solutions for those problems that are not dominated by large order requirements (set A). FFD is good at generating optimal solutions for both the problems dominated by large order requirements and those problems not dominated with small average order requirements (set A and set C). FFD is most inconsistent generating solutions for problems with order requirements larger than 1/3 the inventory length (set B). The most difficult problems for FDD appear to be non-dominated large average order requirement problems.

Therefore, we suggest that a more comprehensive set of benchmark problems would include problems that are hard for both approaches (FFD and column generation). The last several problem classes in Table 5 present a wide spectrum of problems. It is proposed that these problem classes are used as the benchmark problems for the one-dimensional cutting stock problem. Table 8 presents the proposed benchmark problem classes.

m	*v2*	*Dbar*
25	1.00	5
25	1.00	10
25	1.00	20
50	1.00	5
50	1.00	10
50	1.00	20
75	1.00	5
75	1.00	10
75	1.00	20

Table 8 - Proposed Benchmark Problem Classes using CUTGEN1

4 Conclusions

CUTGEN1 is a problem generator for the one-dimensional cutting stock problem presented in Gau and Wäscher (1995). A column generation based algorithm is used to analyze the generated problems and a set of difficult benchmark problems are proposed. The difficult problems are identified by on average, long solution times of the column generation algorithm. In this paper, a different set of benchmark problems are identified using the FFD heuristic. The benchmark set encompasses problems that are difficult for the column generation algorithm and the FFD heuristic. Analysis shows that problems that are not dominated by large average order requirements are the most difficult for the FFD heuristic.

5 References

Baker, B. (1985) "A new proof for the first-fit decreasing bin-packing algorithm", *Journal of Algorithms* 6, 49-70.

Coffman, Jr., E., Garey, M. and Johnson, D. (1996) "Approximation algorithms for bin packing: a survey", *Approximation Algorithms for NP-Hard Problems*, PWS Publishing, Boston.

Gau, T., and Wäscher, G. (1995), "CUTGEN1: A problem generator for the Standard One-dimensional Cutting Stock Problem", *European Journal of Operational Research* 84, 572-579.

Gilmore, P. and Gomory, R. (1961), "A linear programming approach to the cutting stock problem", *Operations Research* 9, 948-959.

Gilmore, P. and Gomory, R. (1963), "A linear programming approach to the cutting stock problem – Part II", *Operations Research* 11, 863-888.