Forming Supplier Coalitions in eCommerce Auctions: Validity Requirements and a Profit Distribution Mechanism

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Abstract: We study the formation of supplier coalitions in the context of Business-to-business eCommerce. Considered under a second-price seal-bid auction environment, we propose a profit distribution scheme that allows suppliers to form coalitions with one another for the purpose of enhancing profitability. We identify basic requirements for a valid coalition mechanism including characteristics such as individual rationality, society welfare compatibility, maintaining competition, and financial balancedness. We verify that the proposed profit distribution mechanism satisfies all validity requirements and will increase the market efficiency as a whole. We also investigate the stable coalition structure under this mechanism and prove that under symmetric information there exists one unique strongly stable coalition structure.

Keywords: Supply chain economics, Systems architecture for Internet auctions, Coalition Structures, Mechanism Design

1. Introduction

Business-to-business electronic commerce, the use of electronic means (e.g., the Internet) to conduct business transactions within or across business entities, has been adopted by companies of all sizes. It has been predicted that, by the year 2003, over 90 percent of all Fortune 500 companies will conduct their core business using some form of eCommerce. According to Giga Information Group [1], savings due to the use of business-to-business eCommerce for US businesses will soar from $15 billion in 1998 to $600 billion in 2002. Forrester Research predicts that by 2004, 53 percent or $1.4 trillion of the goods and services will be traded through electronic markets of corporate buyers and sellers. The transition of conventional supply chain management functions to the realm of electronic commerce is among the most critical elements of economic activities in the decades to come. This transition is highlighted by the recent (February, 2000) announcement by the big-three automakers to create a $250 Billion/year electronic market for the procurement of nearly all automotive parts and services.

From the viewpoint of supply chain management, business-to-business eCommerce includes three main parties: buyers, suppliers and market makers. The role of the three parties can be simply stated as follows: the buyers try to find the most economical ways to fill their orders. The suppliers are interested in improving their market share while increasing the overall profit. The market makers are eCommerce firms such as Ariba and Free Markets, who provide third-party electronic market services. Their short-term goal maybe to establish credibility in niche commerce areas, and in the long-run increase market participations, therefore transactional income. Of interest to all three parties is market efficiency. In current electronic markets, the buyers tend to be a few large firms. The suppliers, on the other hand, are much more numerous and the competition among them more fierce. For example, Free Markets [6], report in the first quarter of year 2000 that a total of 47 buyers traded in their auction market, with some 4,000 suppliers. All auctions during this quarter were triggered by buyer's demands, and they claimed a total saving of $300 million (for the buyers), with the total auction value of $1.4 billion. There were one buyer and many more suppliers in a given auction. Similarly, Ariba [2] reports some 20,000 suppliers in its network with much fewer number of buyers. Due to this trend, market makers are likely to compete with each other for the big buyers (and their interests), which do not necessarily translate into market efficiency, or the suppliers' profit.

Many would attribute the fast development of electronic procurement to the apparent savings it brings the buyers. Through electronic markets, the buyers have direct access to a large number suppliers and therefore lower prices. They would urge their current supply partners to join this electronic network, who have little choice but to join the market as otherwise they would severely damage their customer base. Once joining the electronic market, the supplier may quickly lose previous benefits from location proximity and long-term business relations, while facing fierce competition in a global scale. In the short-term, these suppliers may suffer from undercut profit margins, and facing the dilemma of whether joining the electronic network or not.

In order for electronic market transactions to prosper and to grow in the long run, competition among market makers must increase, and markets that are attractive to *both* buyers and suppliers will eventually prevail. We believe that market efficiency and stability are the keys to its long-term success. In this paper, we examine a new mechanism for eCommerce auction where suppliers form coalitions with one another so as to increase their profits. We will show that such mechanism could be constructed so that the buyers do not loose the advantage from supplier competition, and that a stable coalition structure could be formed.

In traditional supply chain operations, where each buyer tends to deal with a limited number of suppliers via long-term, off-line negotiations, the interaction among the suppliers is limited (and sometimes considered collusion). From a modeling viewpoint, it is sufficient to consider supply chain coordination via one-buyer one-supplier negotiation pairs, as is the case in a majority of the supply chain literature. However, the emergence of business-to-business eCommerce makes inter-supplier interactions more direct and much more critical. It is of importance to consider not only supplier-buyer interactions but also the interactions among competing suppliers who may be at the same time collaborating. This research intents to provide some insights to the nature of this interaction in an electronic auction environment.

2. Background Literature

Two lines of research are directly related to the current work: auction mechanisms of various forms, and coalition formation in non-corporative games. Auction is a popular form of price determination in eCommerce, perhaps due to the fact that it provides an efficient means of coordination assuming only distributed and privately owned information. The basic form of auction has been in the human economy for a long time, but the research literature on auction theory exploded after the seminal paper by Vickrey[1961], which proposes the concept of the *second-price sealed-bid auction*.

Some well-known forms of auctions in one-buyer and multiple-seller environments are listed in Rasmusen et al [1992]. In the *English auction* (first-price open-cry), each supplier is free to decrease the price in each round. The auction ends when no further bids are offered. When the seller's cost is private information, his best strategy is to reduce the current price with a small amount (if he is not the current bidder), and stop when the current price drops below his true cost. In the *first-price sealed-bid auction*, each seller gives a bid simultaneously without knowing other sellers' decision. The one with the lowest price will win the bid with his offered price. Every seller will make the decision based on his own cost, and his prior beliefs of others' costs. According to Vickrey [1961], to increase the expected gains in this auction, there is no incentive to tell the truth. Vickrey proposes the *second-price sealed-bid auction* as a means to encourage all sellers to bid at true cost. This is accomplished by awarding the lowest bidder the second lowest bid price. He prove analytically that this scheme is *incentive compatible* because neither bidding above nor under the true cost would make the bidder better off. Sandholm [1996] point out some limitation of the Vickrey auction in computational multi-agent systems. Finally, in the *Dutch auction* (ascending), the buyer continuously increase the price for this order until one of the bidders take the order at the current price, which is similar to the first-price sealed-bid auction.

With the sale of licenses to the radio frequency spectrum for personal communication service presided by the Federal Communications Commission, *simultaneous ascending auction* was developed. McMillan [1994] discusses the mechanism design for the spectrum auction and compared open vs. sealed-bid auction, simultaneous vs. sequential auction, and several stopping rules. Milgrom [1998] discusses the simultaneous ascending auction for radio frequency spectrum in some detailed.

Current eCommerce auction has three basic forms: *forward, reversed* and *double* auction. In *forward auction*, the seller posts what he has and the buyers place the bids. In *reversed auction*, a buyer posts what he wants and the sellers place bids on the offered price. In *double auction*, both the buyers and the sellers post the bids with price and quantity, and the system matches the possible transactions. This is similar to the stock market in NYSE. As mentioned earlier, most electronic markets have more suppliers than buyers; therefore, *reversed auction* is the most popular in these markets, and will be our focus in this paper.

Coalition formation has been studied intensively by game theorists. A primary motivation for players in a game to form coalitions is to get more profit. There are three basic problems in coalition formation in a given game: whether a *stable coalition* exists, how to *share profit* among the participants, and who should be in which coalition (*coalition generation*). Among various theoretic developments, the *core* introduced by Gillies [1953] is the earliest and most well accepted for *coalition formation* problems. The *core* is the set of all feasible payoffs to the players that no player or group of players could improve upon by acting for themselves. The existence of a nonempty core implies that the game will have some stable solution and the players have the incentive to form the grand coalition. For the detailed discussion on *core*, please refer to Kannai[1992], Peleg[1992], Anderson[1992] and Gabszewicz[1992]. The literature on *core game* typically considers the stability of the grand coalition in a game, and does not offer practical solution for coalition formation.

3. Coalition in an Auction Market

3.1 Setting of the Auction Market

We consider an electronic market where the buyers post their orders electronically, the suppliers respond to one order at a time via an auction mechanism. The buyers must announce *a priori* all order parameters before the auction starts. Each supplier computes pricing based on his own cost structure relative to the announced specifications. The suppliers may form alliance with one another under a specific coalition mechanism. The coalition mechanism must satisfy a number of basic properties so that it not only improve the suppliers' profitability but also improve overall efficiency of the market. This will be the focus of the following analysis. While we adopt a buyer-centric view for the electronic market, our analysis can be generalized to other settings.

The market maker uses auction as a pricing mechanism to identify the lowest possible supply price. The buyer may specify a reserved price, and is entitled to reject the auction result if the final price exceeds the reserved price. Several different auction mechanisms are possible in this environment as described in Section 2. For our analysis, we assume that the market uses *second-price, seal-bid simultaneous descending auction* similar to that of Milgrom [1998]. In each round of the auction, the auctioneer decreases the price by one unit so long as the number of bidders is more than one. The auction stops when there is only one supplier left in the system. To ensure that each supplier bid at his true cost, we assume a second-price Vickrey auction, which identifies the supplier with the lowest cost but reward the lowest bidder the second lowest price. Consequently, the auction mechanism results in a supply cost that is the second lowest among all suppliers participating.

To explain the motivation for suppliers to form alliances during the auction, consider the cost scenario for a two-supplier example depicted in Figure 1. The buyers announce in the market an order with specification q, which could be any attribute describing the order, such as quantity or quality. For all orders to be considered in the market, q is a random variable described by density function $f(q)$. $s_1(q)$ and $s_2(q)$ represent the cost structures of supplier s_1 and s_2 in q, respectively. Orders in the electronic market has Assuming the reserved price for the buyers is $r(q)$, define $s_m(q)$ as the collective cost structure of all *other* suppliers in the market S, i.e., $s_m(q)$ represents integrated information of the market. The *second-price, seal-bid simultaneous descending auction* described above is used to determine which supplier(s) wins a given order in the market. As shown in the example, when the order attribute q falls into region A, supplier s_1 would win the order and awarded the bid $min\{s_2(q),s_m(q)\}$; when the order fall into region B, supplier s_2 would win the order and awarded $min\{s_1(q),s_m(q)\}$. Thus, the expected profit for s_1 is defined by area a, and the expected profit for s_2 is b. If suppliers s_1 and s_2 form a coalition, (i.e., they still bid at their true costs, but now they win the bid for the coalition) they will win the orders fall in either region A or B, *and* gain additional expected profit represented by area c, which they may share. This provides the basic incentive for suppliers to form coalition in the auction market.

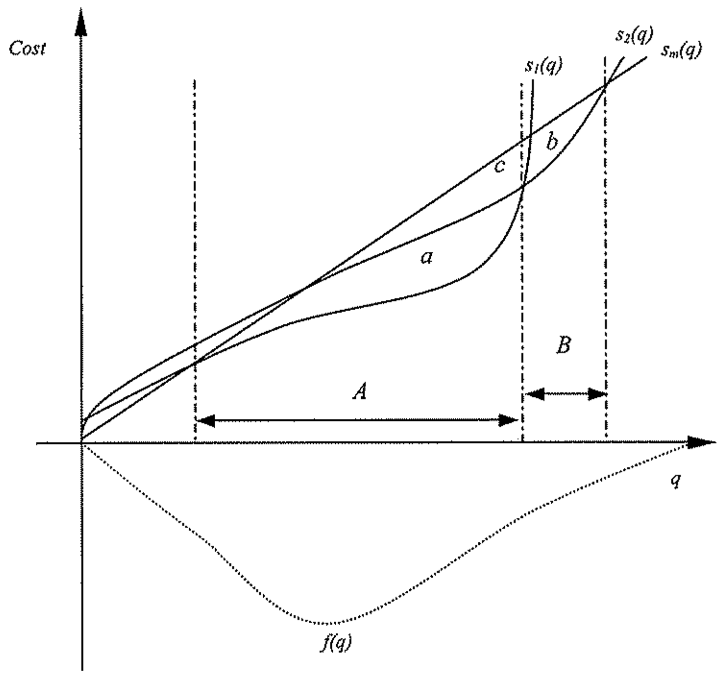

Figure 1. An Example Cost Structure for Suppliers in the Market

3.2 Previous Results on Coalition Formation: The Core Game

Our proposed specification for supplier coalition mechanism rooted from the principles of classical *core game*. A detailed discussion on various aspects of the *core* can be found in [Kannai, 1992]. Let $N=\{1,2,...n\}$ be the set of all players. A subset α of N is called a *coalition*. The *characteristic function* is a real-valued function $v(\cdot)$ defined on the coalition α which represents the outcome for α and $v(\phi)=0$. An outcome of the game is an n-dimensional vector $x=\{x_1,...,x_n\}$, which represents how much each player $i\in N$ receives as payoff. Almost all existing coalition theory requires the assumption that $v(\alpha)$ is *well defined*, i.e., the value of a specific coalition is independent from the other coalitions.

Gillies [1953] proposed the concept of *core game* for the *grand coalition*. A *core* is the set of all feasible outcomes that no player or group of players could improve upon by forming a subset coalition. An outcome is in the core when it satisfies the following:

$$\sum_{i\in\alpha} x_i \geq v(\alpha) \qquad \forall \alpha \subset N \qquad (1)$$

Much of the theoretical development in the *core game* sets out to answer the question: "Whether the core is empty for a the set of players N in a given game?" If the core is nonempty, the *grand coalition* has one or more stable outcome solutions. An important concept in the core game is that of a *balanced collection of coalitions*. A collection $\{\alpha_1, ..., \alpha_k\}$ of coalitions is *balanced* if there exists positive numbers (called *balancing weights*) $\lambda_1, \ldots, \lambda_k$ such that for *every* $i \in N$, $\sum_{j:i \in \alpha_j} \lambda_j = 1$. Shapley [1967] and Bondareva [1963] give the following theorem for this classical problem:

Theorem *The core of the game v is non-empty iff for every balanced collection* $\{\alpha_1, \ldots, \alpha_k\}$ *with balancing weights* $\{\lambda_1, \ldots, \lambda_k\}$, *the following inequality holds:*

$$\sum_{j=1}^{k} \lambda_j v(\alpha_j) \leq v(N) . \qquad (2)$$

Clearly, the core for a given set of players N could be empty. Shapley and Shubik [1973] suggest constraints for coalition formation such that non-empty coalitions could be formed.

The core game assumes that the coalition maximizes some measure of system's welfare, and studies whether the players in the system have proper incentives to form such coalition. However, answering the question "whether the *core* exist" may not be sufficient to model coalition formation in an electronic market. Additional insights might be gained by modeling the players' choice based on the *profit distribution* scheme within the coalition. Later in Section 4, we propose such a mechanism. Note that the classical core game does not model this aspect of coalition formation. In the following, we first specify requirements for a proper coalition mechanism in the electronic market.

3.3 Required Properties of the Coalition Mechanism

In this section, we define basic requirements for a *supplier coalition mechanism* in the electronic market specified above. These requirements form the criteria through which we may evaluate the design of a coalition mechanism. We first define as follows the expected profit of supplier $s_i \in S$ before he joins any coalition:

$$E[p_i(q)] = \int_0^\infty [\min\{r(q), s_j(q) \mid j \in S - \{i\}\} - s_i(q)]^+ f(q) dq \qquad (3)$$

A coalition mechanism Ω defines the basic rules that allow suppliers to form efficient alliances with one another. Specifically, we define a coalition mechanism as two basic components:

(1) *coalitions generation*, where the membership of each coalition is indentified,

(2) *profits distribution* (among coalition members) once the coalitions are formed.

We argue that *profit distribution* is the key to a coalition mechanism. Once the *profit distribution* is properly defined, *coalition generation* follows by considering each player's incentive via computed expectations of the player's profits. Thus, our discussion in the rest of the paper will focus on the properties of profit distribution. Following the convention of the core game, if under mechanism Ω the total profit for any given coalition is independent from all other coalitions, we say mechanism Ω is *well defined*. Denote a specific supplier collation α_j, where $\alpha_j \subseteq S$. A *coalition structure* is a set of coalitions $\{\alpha_j\}$ that are mutually exclusive and exhaustive, i.e., $\bigcup_j \alpha_j = S$ and $\alpha_j \cap \alpha_k = \phi$. Clearly, the possible number of coalition among n suppliers is $2^n - 1$. Under a specific coalition mechanism Ω, we denote supplier s_i's profit as $p_{\Omega,i}(\alpha, q)$. We now list requirements for a *valid* coalition mechanism:

1. ***Individual Rationality:*** A fundamental requirement for a coalition membership is that all members in the coalition have an expected profit higher than his own expected profit by acting alone. In other words, the coalition mechanism should satisfy individual rationality, which is defined as follows:

$$E[p_{\Omega,i}(\alpha, q)] \geq E[p_i(q)] \quad \forall i \in \alpha, \qquad (4)$$

where $E[p_{\Omega,i}(\alpha, q)]$ is the expected profit of supplier s_i after joining coalition α under mechanism Ω.

2. ***Private and Distributed Information:*** The coalition mechanism should *not* require the members to reveal their cost structure or other private information. Although information could be communicated rapidly and conveniently in the electronic market, information essential for a firm's competitiveness must be kept private. As in normal game theoretic assumptions, even if an agent is asked to provide pricing information, there is no guarantee that the information provided is truthful.

*3. **Observability and Controllability***: Since each player maintains his private information in a distributed fashion, only information observable by all could be utilized by the coalition mechanism, i.e., the mechanism must be implemented solely on *observable* information. Further, all actions to be taken by the mechanism should be *controllable* by the market, e.g., a meaningful punishment could be imposed on the members who violate the market rules.
*4. **Social Welfare Compatibility:*** From the viewpoint of the buyers, an efficient market is one where every order is processed with the lowest achievable cost in regard to the order. A *cartel* in the tradition sense doesn't satisfy this condition as it might intentionally assign some orders to members with higher cost for the interest of increasing profit. Such coalition is unlikely to survive in a competitive market.
*5. **Maintaining Competition:*** Most countries prevent monopoly by anti-thrust regulations, as monopoly diminishes the incentive for true competition. Thus, the coalition forming mechanism must maintain competition among its members, i.e., regardless of which coalition they belong; every player in the market should have the incentive to reduce cost in order to compete with other players.
*6. **Financially Blancedness:*** The mechanism should guarantee that the formed coalition is financially balanced, i.e., the market maker doesn't need to provide subsidy to any coalition under the mechanism.

$$\sum_{i \in \alpha} p_{\Omega,i}(\alpha, q) = v_{\Omega}(\alpha, q) \quad \textit{for any } \alpha \textit{ and } q \qquad (5)$$

4. An Auction Mechanism with Supplier Coalitions

In the following, we describe an auction mechanism that facilitates supplier coalitions. We propose a *profit distribution* scheme, which we will show providing the proper incentives for suppliers to form *valid* coalitions, i.e., a coalition mechanism defined based on this profit distribution scheme is *well defined* and satisfies the six basic requirements specified above.

We first define the meaning of *profit distribution* for supplier coalitions.

Definition 1. *Profit distribution within a supplier coalition refers to the splitting of the portion of the overall profit exceeding what any individual suppliers could achieve by acting alone.*

We now describe an auction mechanism in conjunction with a profit distribution scheme given a *coalition structure* $\{\alpha_i\}$. The mechanism consists of three main phases that includes two sub-auctions both supervised by the market maker.

Auction Mechanism with Supplier Coalitions (AMSC)

Phase 1. All suppliers compute their bids based on their own cost structure for a posted order. A supplier coalition submits the bid that is the lowest among its members, and will be viewed as a single supplier during the auction. The *second-price, seal-bid simultaneous descending auction* takes place, i.e., in each round of the auction; the auctioneer decreases the price by one unit if the number of bidders is more than one. The auction stops when there is only one supplier left in the system. This is the lowest-cost supplier who will be reward the second lowest bid price (outside his coalition). Note that suppliers belonging to the same coalition do not compete against each other in the view of the auctioneer.
***Phase* 2**. Suppose supplier s_i wins the order with price p_1 and $s_i \in \alpha_j$, we say that coalition α_j wins this order with price p_1 (since supplier s_i's bid is not compared against other member of his coalition, the second lowest bid price p_1 is offered by a supplier outside coalition α_j). The market maker starts a *second-price sealed bid auction* within coalition α_j for this order and the supplier with the lowest bid wins the order with the second lowest price p_2.
***Phase* 3.** The coalition shares the *addition* profit $(p_1 - p_2)$ evenly among the members, i.e., each receives $(p_1 - p_2)/|\alpha_j|$.

Again, consider the example in Figure 1. Suppose an order described by q falls into region A. The bid would be won by the s_1-s_2 coalition (due to supplier s_1) with reward $p_1 = s_m(q)$ (since s_2 does not compete against s_1) given to the coalition. In the *Phase 2* auction within the coalition, supplier s_1 win the bid with reward $p_2=s_2(q)$. The two suppliers split the additional profit ($p_1 - p_2$) evenly. As a result, supplier s_1 receives profit $s_2(q)-s_1(q)+(s_m(q)-s_2(q))/2$, supplier s_2 receives $(s_m(q)-s_2(q))/2$.

The reason that the Phase 1 and Phase 2 auctions are both needed in the mechanism is due to the truth revealing property of second-price seal-bid auction. An implicit but important property here is that the second-price auction will always induce the suppliers *participating in the auction* to reveal their true costs. Suppose we eliminate *Phase 2* auction but simply record in *Phase 1* the lowest bid and the second lowest bid in each coalition, say p_{min} and p_{mis}, respectively. At the end of the auction, the supplier with bid p_{min} would win a bid at p_1 for the coalition. Since the second lowest bid in the coalition is already recorded, (p_{mis} $=p_2$), we would have no problem computing the additional profit ($p_1 - p_2$) brought by the coalition. This makes the Phase 2 auction seemly unnecessary. Nevertheless, without the Phase 2 auction supplier s_{mis} who has a true cost of p_{mis} would not actually participate in the auction since the supplier with p_{min} would represent the coalition. Consequently, there is no proper incentive that would insure supplier s_{mis} to reveal his true cost. For example, in the case of equal profit sharing described in Phase 3, supplier s_{mis} actually has the incentive to lower his bid below his true cost in order to increase his share of the coalition profit $(p_1 - p_2)/|\alpha_j|$.

4.1 Incentive Compatibility

In the following, we prove that the above auction and profit distribution mechanism provide proper incentives such that (1) non-competition with other suppliers in the same coalition, and (2) revealing the true cost, are the best strategies for all suppliers. We assume that all suppliers, with or without a coalition, are self-interested and rational, and they will try to maximize their *own* profits. Given the auction mechanism described above, and a coalition structure $\{\alpha_j\}$, the suppliers know precisely how profits would be distributed among the members when the auction terminates. Under these assumptions, we state the following proposition:

Proposition 1. *In a second-price, seal-bid simultaneous descending auction participated by all suppliers, if profit distribution (Definition 1) is used among coalition members, the best strategy for a supplier s_i in coalition α_j is not to compete with other members in the same coalition α_j during the auction.*

Proof: Suppose the first and second lowest cost suppliers are s_{min} and s_{mis}, respectively. Coalition α_j may only win the bid if the lowest cost supplier s_{min} is in the collation. When coalition α_j wins the order characterized by q, the reward p_1 will be

$$p_1(\alpha_j, q) = \min\{r(q), s_i(q) \mid \forall i \notin \alpha_j\} \qquad (6)$$

If $p_1(\alpha_j, q) < s_i(q)\ \forall i \in \alpha_j$, coalition α_j will not win this order in the first place. If coalition α_j does win the order, it is sufficient to consider two cases:(1) the second lowest cost supplier s_{mis} (where $s_{mis}(q) < p_1$) is also in α_j, and (2) otherwise

When (1) is the case, it is the best interest for the α_j suppliers other than s_{min} (including s_{mis}) not to participate in the auction since the profit to be *distributed* in the coalition is the difference between the second lowest cost outside the coalition, $p_1(\alpha_j, q)$ and the second lowest cost inside the coalition $s_{mis}(q)$, *i.e.*, $p_1(\alpha_j, q) - s_{mis}(q)$. If any supplier s_k in coalition α_j with cost $s_k(q) < p_1(\alpha_j, q)$ competes in the auction, $s_k(q)$ would replace $p_1(\alpha_j, q)$ and his profit would decreases. On the other hand, for a supplier s_l in coalition α_j with cost $s_l(q) \geq p_1(\alpha_j, q)$, competing in the auction will not increase his profits.

When (2) is the case, it is again the best interest for the α_j suppliers other than s_{min} not to participate in the auction since all profits $p_1(\alpha_j, q) - s_{min}(q)$ go to supplier s_{min} in any case.

The above proposition answers a fundamental question: "If all suppliers in a coalition participate directly in the auction, do they have the incentive to act collaboratively as a member of the coalition?" The answer is yes. Thus, the rule of participation in the above auction mechanism is indeed incentive compatible. In the following, we show that giving the true cost during the auction is the best strategy for the suppliers.

Proposition 2. *Given the profit distribution scheme defined by AMSC, the best strategy for a supplier s_i in coalition α is to reveal his true cost during the Phase 1 and Phase 2 auctions.*

Proof. If all other members in the coalition reveal their true costs, the *expected profit* for supplier s_i with the placed price $s_i'(q)$ is as follows:

$$(\min\{s_j(q)|j \in s-\{i\}\} - s_i(q) + \frac{p_1 - \min\{s_j(q)|j \in s-\{i\}\}}{|\alpha|}) \cdot \Pr[\min\{s_j(q)|j \in s-\{i\}\} > s'_i(q)]$$
$$+(\frac{p_q - s'_i(q)}{|\alpha|}) \cdot \Pr[\min\{s_j(q)|j \in s-\{i\}\} \le s'_i(q) \text{ and } mis\{s_j(q)|j \in s-\{i\}\} > s'_i(q)]$$
$$+\frac{p_1 - mis\{s_j(q)|j \in s-\{i\}\}}{|\alpha|} \cdot \Pr[mis\{s_j(q)|j \in s-\{i\}\} \le s'_i(q)] \qquad (7)$$

Where function mis{} return the second lowest values in the set. It is easy to verify that the best choice (to maximize the expected profit) for supplier i is to let $s_i'(q) = s_i(q)$. Thus, revealing the truth corresponds to *Nash Equilibrium* for the members of coalition. In fact, this is the unique equilibrium for this game.

This result is consistent with that of the *second-price sealed-bid* auction, implying that the profit distribution scheme within the coalition does not change the basic properties of the second-price auction. The above results verify two important properties for the auction mechanism with supplier coalitions. In the following, we will evaluate this mechanism against basic requirements for coalition formation.

4.2 Properties of AMSC

As pointed out earlier, a coalition mechanism includes *coalitions generation*, and *profits distribution*. While we do not address the issue of *coalition generation* in this paper, the profit distribution scheme described by *AMSC* is sufficient to evaluate the basic properties of the coalition mechanism. In the following, we will verify all the mentioned requirements for *AMSC*.

Proposition 3. *AMSC mechanism is a well-defined mechanism.*

Proof. Under mechanism *AMSC*, the expected profit for a coalition α is as follows:

$$E[v(\alpha,q)] = \int_0^\infty [\min\{r(q), s_j(q) \mid j \notin \alpha\} - mis\{s_i(q) \mid i \in \alpha\}]^+ f(q)dq \qquad (8)$$

Obviously, $v(\alpha,q)$ is independent from other supplier coalition structures, however, it relies on aggregated market information on second lowest price defined by $\min\{r(q), s_j(q) \mid j \notin \alpha\}$.

This feature is critical to simplify the *coalition generation* problem.

Proposition 4. *Any coalition under the AMSC mechanism satisfies the Individual Rationality requirement*

Proof. For a supplier s_i in coalition α, he can always get more profit than acting alone in the market. Because for any q, the profit for one specific member i in α is as follows:

$$p_i(\alpha,q) = p_i(q) + [\min\{r(q), s_j(q) \mid j \notin \alpha\} - mis\{s_i(q) \mid i \in \alpha\}]^+ / |\alpha| \qquad (9)$$

Where $p_i(q)$ is s_i's profit by acting alone. Define the *additional profit* for a given coalition α as

$$d(\alpha,q) = [\min\{r(q), s_j(q) \mid j \notin \alpha\} - mis\{s_i(q) \mid i \in \alpha\}]^+ \qquad (10)$$

Clearly, by acting alone, $d_i(\{i\},q)=0$.

The remaining requirements can be stated in the following propositions and can be verified in words.

Proposition 5. *A coalition operates under AMSC require only distributed information that are observable and controllable.*

In *AMSC*, there is no private information required from the suppliers. All information required for the mechanism can be obtained from the auctions and therefore observable by the market maker. As following the coalition rule is incentive compatible (Propositions 1 and 2), while *AMSC* satisfies individual rationality (Proposition 4), the behavior of the suppliers is consistent with the mechanism's intent, because such behavior can lead to the maximal benefit for the suppliers. Thus, the coalition structure is also controllable, i.e., exclusion from a coalition is a meaningful punishment, as the player will suffer from less profit.

Proposition 6. *Any coalition structure operates under AMSC satisfies Society Welfare*

AMSC does not avoid competition among members of the same coalition. Every supplier has the incentive to decrease cost. Only when a supplier has the lowest cost in the market and when this price is less than the reserved price of the buyer, can his coalition win the order in *phase1* of *AMSC*. In the *phase2* auction, only the supplier with the lowest cost for this specific order can win the order. Thus, the supplier with the lowest cost will process any given order, therefore *AMSC* satisfies the society welfare requirement.

Proposition 7. *Any coalition operates under AMSC is financially-balanced.*

It is easy to verify that coalition operates under *AMSC* is financially balanced. This is because the coalition only distributes *additional* profit from the current transaction to its members, thus, equation (5) is always satisfied. The coalition is balanced not only in a long-run, but also for each order. A potential benefit from *AMSC* is that the market maker such as an eCommerce company may potentially profit from the coalition's additional profit. All features of *AMSC* mechanism remain, if the eCommerce companies charge, say, one fixed percent of $d(\alpha,q)$.

5. Stable Coalition Structure

We verify in the previous section the *profit distribution* scheme for *AMSC* satisfy all requirements for a valid coalition mechanism. A related issue is that of *coalition generation*, the formation of the coalition. While we do not address the mechanism involved in *coalition generation*, we will establish that *AMSC* sustains a stable coalition structure. First, consider the following proposition.

Proposition 8. *Under AMSC, the additional profit for the coalition satisfies the additivity property.*

Proof. Given (10), we may write the additional profit for any member in the coalition α as

$$d_i(\alpha,q) = d(\alpha,q)/|\alpha| \qquad (11)$$

For any two coalitions α and β with $\alpha \cap \beta = \phi$, we have

$$d(\alpha \cup \beta, q) = \left[\min\{r(q), s_j(q) \mid j \notin \alpha \cup \beta\} - mis\{s_i(q) \mid i \in \alpha \cup \beta\}\right]^+ \quad and\text{, } furthermore$$
$$\min\{r(q), s_j(q) \mid j \notin \alpha \cup \beta\} \geq \max\{\min\{r(q), s_j(q) \mid j \notin \alpha\}, \min\{r(q), s_j(q) \mid j \notin \beta\}\} \quad and$$
$$mis\{s_i(q) \mid i \in \alpha \cup \beta\} \leq \min\{mis\{s_i(q) \mid i \in \alpha\}, mis\{s_i(q) \mid i \in \beta\}\}$$

thus, the following additivity property holds:

$$d(\alpha \cup \beta, q) \geq d(\alpha,q) + d(\beta,q) \quad \text{for any coalition } \alpha,\beta \text{ where } \alpha \cap \beta = \phi \qquad (12)$$

The *additivity property* seems to imply that all coalitions have the incentive to merge with other coalitions and eventually lead to a grand collation, i.e., all suppliers in N belong to one coalition. We will discuss show in the following that the grand coalition is usually not the result under *AMSC*. We first need to establish a few results from the *collation formation* literature.

Definition 2. *The set of coalition* $\{\alpha_i\}$ *is a coalition structure if* $\cup_{i\in c}\alpha_i = s$ *and* $\alpha_i \cap \alpha_j = \varnothing$ *for* $i\neq j$,

Coalition structure is defined based on the idea of set partitioning. For example, if we have three players *{1,2,3}*, the possible coalition will be *{1},{2},{3},{1,2},{1,3}, {2,3}* and *{1,2,3}* and the possible coalition structures will be *({1},{2},{3}), ({1},{2,3}), ({1,2},{3}), ({1,3},{2})* and *({1,2,3})*.

Proposition 9. [Sandolhm 1999] *The number of coalition structure is* $O(n^n)$ *and* $\omega(n^{n/2})$

Clearly, to find the optimal coalition structure based on some performance measure is *NP-hard*. Sandolhm [1999] develops several *coalition generation* heuristics using both the centralized and distributed viewpoints. If we use x_k to represent the current expected additional profit for supplier *k*.

Definition 3. A *coalition structure c is stable in a n-supplier game if it satisfies the following condition.*

$$x_k \geq \max\{E[d(\alpha_i + \{k\}, q)]/|\alpha| + 1 \;\Big|\; E[d(\alpha_i + \{k\}, q)]/|\alpha| + 1 \geq E[d(\alpha_i, q)]/|\alpha|, \forall \alpha_i \in c \text{ and } k \notin \alpha_i\} \quad (13)$$

for any $k \in N$

This definition of *stable coalition* is similar to the notion of *Nash Equilibrium*. In a stable coalition, no one can get more profit by just one movement. In other words, no supplier could gain more profit by joining another coalition that is willing to accept him. Note that this definition is different from the *core* of the game. Given the cost structures of all suppliers, there will be more than one stable coalition structure, so we introduce an additional concept about *stability*.

Definition 4 *A coalition structure c is strongly stable in a n-supplier game if it satisfies the following condition.*

$$\forall \alpha \notin c, \exists k \in \alpha : E[d(\alpha, q),]/|\alpha| \leq x_k \quad (14)$$

If a market reaches the *strongly stable* coalition structure, no supplier could gain more profits with any number of movements. Obviously, a *strongly stable solution* must be a *stable solution*. This concept is similar to the *core* discussed earlier. However, the *core game* only considers the stability of the *grand* coalition, but here the strongly stable coalition structure may be any coalition structure.

If we allow the suppliers to freely make coalitions and all cost structure information is public, we can state the following.

Proposition 10. *Under AMSC there is one unique strongly stable coalition structure.*

Proof.
We may construct (in theory) the unique strongly stable coalition structure as follows:
(Assuming all cost structure of the suppliers, order distribution and the reserved price are known).
Step 0.. Set $c=\varnothing$ and $i=1$. Set *M* as the set of all possible coalition, i.e., *M* is the set of all subset of *N*.
Step 1. Find coalition α in *M* with the highest value of $E[d(\alpha,q)]/|\alpha|$ and call this coalition α_i
Step 2. Set $c \leftarrow c \cup \alpha_i$ and $i \leftarrow i+1$
Step 3. Delete from *M* all coalitions that share the same member of α_i in *M*
Step 4. If $M\neq\varnothing$, go to step 1. Otherwise, stop.

The coalition structure *c* resulting from the above procedure is the unique *strongly stable coalition* structure in this game. This is true because the coalitions in *c* would have no incentive to accept any additional suppliers: members of the coalition with a lower index would not have the incentive to join the coalitions with higher index. On the other hand, the coalition with a lower index has no incentive to accept suppliers from the coalition with a higher index. For any other coalition structure $c'\neq c$, suppliers in the coalition $\alpha'_i \in$

c' with the lowest coalition index i (which would not have the lowest index in c, by definition) would have the incentive to form a different coalition identical to $\alpha_i \in c$. Thus, the coalition structure c' cannot be a strongly stable coalition structure.

Since the grant coalition does not necessarily have the highest value of $E[d(\alpha,q)]/|\alpha|$, the strongly stable coalition structure under *AMSC* may not be the grand coalition. This result is important from the viewpoint of market design. A converse result would imply that regardless of the *coalition generation* mechanism, the market would gravitate toward one coalition, i.e., no collation at all. In reality, the cost structure of each supplier is private information not known to the system. The system is unlikely to reach the strongly stable coalition structure immediately. In this case, we shall assume that the suppliers are not allowed to switch from one coalition to another during the auction in order to avoid operational complexity.

6. Conclusion

In this paper, we consider the supplier's coalition problem in eCommerce auctions. We specify the requirements for a valid coalition mechanism as individual rationality, private and distributed information, observability and controllability, society welfare compatibility, maintaining competition, and financially balancedness. We propose an auction mechanism with supplier coalitions *(AMSC)* and we specify a profit distribution scheme under this auction mechanism. We verify that *AMSC* satisfies all requirements as a valid coalition mechanism.

We also show that, under symmetric information, a unique strongly stable coalition structure exists under *AMSC*. However, we do not address the issue of *coalition generation* neither do we discuss the issue of unique strongly stable coalition structure under asymmetric information. For further study, we will investigate the coalition generation mechanism and implementable rules that would force the suppliers to reach a stable coalition structure quickly.

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