Contracting in Electronic Market-Driven Supply Chains: Models and Analysis

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Report No. 00T-007

CONTRACTING IN ELECTRONIC MARKET-DRIVEN SUPPLY CHAINS: MODELS AND ANALYSIS*

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Abstract. A signicant part of the supply chain literature considers the coordination of one-supplier, one-buyer pair, while assuming the supplier is the market leader who designs the contract. In emerging electronic market transactions, situations often arise where it is more appropriate to model the system as multiple suppliers and one buyer. In addition, it is necessary to consider the role of a third party: the Internet aggregator, who develops coordination mechanisms for the different parties involved. Although the information exchange is fast, information control still tends to be distributed and asymmetric. Auction is a common mechanism to reveal private information. The relationship among the suppliers may be competitive, while at the same time cooperative. This research addresses both types of relationship using game theory and its extension. We first investigate existing mechanisms and evaluate their impacts on the suppliers and buyers. We then propose new auction mechanism to increase protfit for the buyer and the supply chain. In this paper, we examine the analytical implications of this new environment to supply chain contracting.

Key words. Supply Chain Contracts, Electronic Markets, Game Theory, Cooridination Mechanisms

1. Introduction. Supply chain contracting has been used as a means to coordinating buyers and suppliers in the supply channel to achieve higher efficiency. Through various contract structures, researchers investigate possible mechanisms to provide incentives for the self-interested buyers and suppliers to achieve channel coordination (a recent survey is given by Tsay et al. [1999]). The literature on supply chain contracts has its roots in economics, game theory, and more recently, supply chain management. Two basic issues in supply chain contracting are that of *incentive compatibility* and *information symmetry*. Since the buyer and the supplier represent different decision entities from different firms, they face different decision problems to that of the system's. Without a deliberate coordination scheme, their collective decision could be far from system efficiency. On the other hand, each party owns private information without necessarily the incentives to share, which makes the assumption on information symmetry an important one. Corbett and Tang [1999] investigate contracts provided by the supplier under a one- supplier and one-buyer setting, considering the cases of complete or asymmetric information. One-part contract, two-part linear contract, and two-part nonlinear contract are studies. Weng [1995], Rosennblatt and Lee [1985] and Monahan[1984] consider quantity discounting contract for channel coordination. Ha [1998] consider the issues of incentives, and asymmetric cost information with stochastic and price-sensitive demand. Bassok and Anupindi[1997] used total minimum commitment in supply contracts. As traditional supply chains are sequential and vertical, most research focus their attention on one-supplier one -buyer pair. Very few study examine the parallel interaction among multiple suppliers, or multiple buyers. Another general assumption is that the supplier is the Stackelberg leader (the buyer is the follower), who has the right to design the contract. The aim for these research is to achieve channel coordination, and to increase the supplier's profit. Due to these factors, channel coordination often scarifies the buyer interest.

Motivated by emerging applications in electronic markets, we consider the interaction between buyers and suppliers in electronic market-driven supply chains. In this

*Working Paper, Department of Industrial and Mfg. Systems Engineering, Lehigh University.

environment, not only vertical interactions between any pair of supplier and buyer, but horizontal interactions among multiple suppliers need to be considered. We consider, in specific, the case where multiple suppliers must compete with one-another to form contracts with the buyer. While this competition undoubtedly exists in traditional supply chains, in the emerging world of eCommerce, the competition/contracting process is no longer paced over time, but are likely to occur simultaneously. The nature of these electronic supply chain transactions can be observed in current eCommerce industry. *FreeMarkets.com [6]*, one of the leading business-to-business eCommerce company, reported that in the first quarter of 2000 the number of the buyers participating in their procurement markets was 47, with some 4000 suppliers. All auctions were triggered by the demands from the buyers. FreeMarkets estimated that the total saving for the buyers participating in the market is $300 million with the total auction value of $1.4 billion (in this quarter). A given auction involves one buyer and multiple suppliers. Other leading eCommerce companies such as *Ariba[2]* and *CommerceOne[5]* also report similar phenomenon.

Due to the enormous potential market for business-to-business eCommerce, the competition among eCommerce firms is fierce. To achieve a larger market share, these companies must compete for more customers (especially more buyers) to their network. In this research, we will consider these eCommerce companies as market intermediaries in the supply chain, it is their goal to achieve channel coordination while providing savings for the buyers. Channel coordination increase overall efficiency for all parties involved, increased buyer savings improves the competitiveness of the market maker themselves. The role of market intermediary is different from that of the central agent or system optimizer, because the intermediary does not have access to private information owned by the market participants, and the participant still reserve all the rights to make their own decision. We argue that an eCommerce company should serve the dual roles of a market maker and market intermediary, the former creates efficient and stable market mechanisms for the suppliers and buyers, while the latter maintains efficient market transactions by providing proper incentives and information. A main contribution of this paper is to provide insights that allow us to compare alternative contracting mechanisms offered by the market maker, and to evaluate their impact on the suppliers, the buyer and the system.

1.1. Related Literature in Supply Chain Management. The quantitative analysis of supply chain originates from the multi-echelon inventory theory, first introduced by Clark and Scarf in 1960. They propose the coordination of inventory over multiple echelons of the system so as to achieve a certain system optimum solution. They propose optimal coordination policies, making use the notion of echelon inventory, the sum of local inventory and the inventory downstream in the chain. During the 1980s, researchers began to consider the environment where there are multiple decision makers, who each has a different objective and owns private information. A main issue is that locally optimized decision is usually not optimal from the system's perspective. This introduces channel inefficiency due to conflicting interests among distributed decision makers, and asymmetric information. A main issue in supply chain coordination is known as *double maginalization.* Spengler [1950] specify two entities in the channel that would cause *double marginalization:* (1) the buyer and the supplier only receive a portion of the total contributing margin, and (2) the buyer and the supplier solve their own utility optimizing problem based on their own margin hence the resulting decision is suboptimal for the channel. For the purpose of eliminating double marginalization and achieve channel coordination, several contracts have been studied. Under complete information, the supplier could implement a two-part tariff contract to reach channel coordination while maximizing his profit (Katz 1989). Essentially, the buyer pays the supplier a fixed fee (tariff) then purchases the goods at the supplier's cost.

Asymmetric information is another main issue in the supply chain. This information may includes the cost structure of each party and the knowledge owned by each party. Cachon and Lariviere [1999] propose contracts that promote information sharing in a supply chain. Corbett and Tang [1999] and Ha [1998] propose two part tariff pricing policies, and evaluate the value of the information for the supplier under various contracts. Monahan [1984], Lee and Rosenblatt [1986] and Weng [1995] consider the use of quantity discount to increase the supplier's profit. Pasternack [1985] and Donohue [1996] used return policy to improve the channel's performance. Researchers also study the effect of asymmetric information in the supply chain. For instance, the well known *bullwhip* effect in a supply chain is believed to be the results of information asymmetry between the retailers and his upstream suppliers . While the retailers know well the market demand, operating under their own best interest they may place orders that has larger variances than the actual demands. This effect propagates and magnifies toward the upstream direction of the chain. The phenomenon is documented by Baganha and Cohen [1995], Kahn [1987], and Kaminsky and Simchi-Levi [1996]. Lee, Padmanabhan and Whang [1999] and Chen, et al. [1998] propose models that analyze its possible causes.

2. One-supplier, One-buyer Supply Chain Contract Under Complete Information. To gain some insights for further analysis, we first consider supply contract negotiation between a single supplier-buyer pair under complete information. We will follow the general setting in the supply contract literature. Consider a market where the price and quantity relationship can be characterized by a linear function $p = a - bq$, where q is the order quantity, p is the market price, a and b are non-negative market-specific coefficients. The supplier has a marginal cost s for producing his products and the buyer is responsible for the unit handling cost c. The supplier announces a unit wholesale price w for his product and the buyer in turn set an order quantity q. Without a contract to coordinate their activities, the buyer's optimal order quantity is $q=\frac{a-c-w}{2b}$ with profit $\pi_b=\frac{(a-c-w)^2}{4b}$, under complete information (case c1), the supplier will set the wholesale price $w = \frac{a-c+s}{2}$ with profit $\pi_{s,c1}=\frac{(a-c-s)^2}{8b}$. The resulting profit for the buyer will be $\pi_{b,c1}=\frac{(a-c-s)^2}{16b}$. If there is a central agent overseeing overall channel efficiency, the total profit would be $\pi^*_{T,c1}=\frac{(a-c-s)^2}{4b}$ with order quantity $q^*_{c1}=\frac{a-c-s}{2b}$. Since the supplier always set his wholesale price above cost ($w > s$) we may conclude that without channel coordination the buyer always orders less than the optimal quantity, and the system end up less efficient (with less profit). The above results is known as *Double Marginalization* in the literature.

Without coordination, the system's marginal cost is s and the marginal revenue is $a - c - 2bq$. The buy's marginal cost and marginal revenue are w and $a - c - 2bq$ respectively. For the supplier, the marginal cost is s and marginal revenue is $a - c - 4bq$. The usual explanation of Double Marginalization is that the buyers face a different marginal cost (w) then that of the system's (s) which leads to a suboptimal decision. Under the assumption that the supplier is the leader in the market, several forms of supply contract have been proposed in the literature which lead to channel coordination. An example is the two-part linear contract, where the supplier charges a constant unit wholesale price w but asks for a fixed lump sum side payment to the buyer. The supplier will transfer $w = s$ as the wholesale price to the buyer and extract all possible profit by asking the side payment L_s as

$$L_s = \frac{(a-c-s)^2}{4b} - \pi_b^- \tag{2.1}$$

Here π_b^- is the minimal profit required by the buyer to stay in this transaction. The reason why this two-part contract would lead to channel coordination is that the buyer (the follower), now faces the same marginal cost s as the system's. However, this result assumes complete information while in reality the supplier does not have access to π_b^- and c,which is private information owned by the buyer. Thus, if this contract form is adopted, the supplier faces the risks of charging too much to lose the buyer, or charging too little to hurt his own profit.

The above discussion summarizes main previous results which assume that the supplier is the leader during the negotiation. Now, consider the case where the buyer is the market leader who has the right to design the contract. This is the case in buyer-centric supply markets organized in the realm of electronic commerce. In this situation, we could explain *Double Marginalization* with the perspective that the supplier's marginal revenue is different from the system's. To resolve the problem, we propose a *buyer-initiated two-part contract* as follows.

DEFINITION 2.1. *In a* ***Buyer-Initiated Two-Part Contract*** *the buyer proposes a contract to the supplier with an order function $q = g(w)$ while requesting a side payment L_b. The supplier in turn chooses the wholesale price w, and the final order quantity is determined by the announced buyer function $g(w)$.*

In the following, we show that this two-part contract could achieve channel coordination while maximizing the buyer's profit.

THEOREM 2.2. *Under complete information the buyer-centric two-part contract (a) maximizes the buyer's profit, and (b) achieves channel coordination.*

Proof. Under the buyer-initiated two-part contract, the buyer announces order function $q = g(w)$ and requests a side payment L_b from the supplier. The supplier chooses the wholesale price w, and the final order quantity is given by $g(w)$. The final transfer from the buyer to the supplier is $wg(w) - L_b$. Under this contract, the best strategy for the buyer is to adhere to the linear market function $p = a - bq$ but offsetting his unit handling cost c, thus defining the order function by $w = a - c - bq$. Given the order function, the supplier will choose the whole sale price $w = \frac{a-c+s}{2b}$ and the final order quantity will be $q = \frac{a-c-s}{2b}$. Note that this is equivalent to the channel coordination solution. The buyer could maximize his profit by requesting a side payment considering the minimum required profit of the supplier, π_s^-.

$$L_b = \frac{(a-c-s)^2}{4b} - \pi_s^- \tag{2.2}$$

The system achieves channel coordination because now the supplier faces the same optimization problem as that of the system. □

In general, with the use of side payments, the leader in the market can design a contract such that the follower would face the same optimization problem as the system, thus achieving channel coordination. Under complete information, the leader

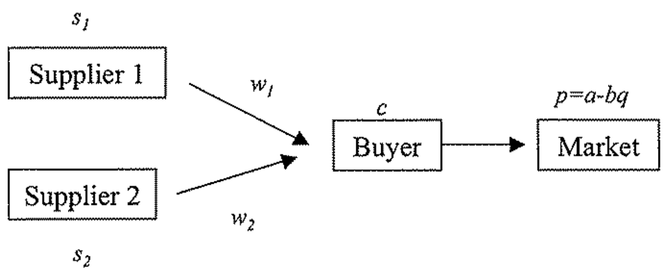

FIG. 3.1. *The Two-Supplier One-Buyer Contracting Environment*

could extract maximum possible profits while the follower can only specify his minimum required profit . Without competition or the introduction of additional assumptions, it is difficult to find a profit sharing scheme that is acceptable for all parties. We will further explore this issue.

3. Two-Supplier, One-Buyer Supply Chain Contract with Complete Information. In the following discussion, we consider a contracting environment with two suppliers and one buyer as depicted in Figure 2-1. An important feature is that the suppliers are now competing to get the buyer's order, as is the case in electronic markets.

We assume that supplier i has a fixed marginal cost s_i and the buyer face unit handling cost c. Demand in the market is price sensitive and satisfies the market function: $p = a - bq$. To ensure market competitiveness, we impose an additional condition that supplier i's cost must satisfy $a - c - s_i \geq 0$ and $s_i \leq \frac{a-c+\min_i s_i}{2}$,otherwise the supplier will be eliminated from the market.

Without lose of generality, we assume that supplier 1 has lower cost throughout our discussion. As before, the channel coordinated system optimal has order quantity $q_c^* = \frac{a-c-s_1}{2b}$ and profit $\pi_{T,c}^* = \frac{(a-c-s_1)^2}{4b}$.

3.1. Scenario c1: No Contract Coordination. Without contract coordination, each supplier announces his wholesale price w_i, the buyer chooses the supplier with the lowest wholesale price and place the order with quantity q based on lower wholesale price and the market function. Thus, the buyer's problem is as follows:

$$\pi_{b,c1}(q) = \max_q (a - c - bq - w_{c1})q \tag{3.1}$$

which yield $q_{c1}^* = \frac{a-c-w_{c2}}{2b}$.

Due to the competition from supplier 2, supplier 1 will set the whole price as s_2 (i.e., supplier 1 has no incentive to set the price below s_2, while setting the price above s_2 would prompt the buyer to choose supplier 2). Thus the profit for supplier 1 will be

$$\pi_{s_{1,c1}}^* = \frac{(a - c - s_2) \cdot (s_2 - s_1)}{2b} \tag{3.2}$$

The maximal profit for the buyer is

$$\pi^*_{b,c1} = \frac{(a-c-s_2)^2}{4b} \tag{3.3}$$

with the quantity $q^*_{c1} = \frac{a-c-s_2}{2b}$.

Since $s_2 > s_1$, the system can not reach channel coordination with lower quantity, which is consistent with the result of Double Marginalization in the one-supplier, one-buyer case. However, with the competitiveness assumption $s_i \leq \frac{a-c+\min_i s_i}{2}$, the system with two suppliers is closer to the channel coordinated system, comparing to the one-supplier system.

THEOREM 3.1. *By introducing competition among the suppliers, the effect of Double Marginalization will decrease and the profit for the buyer will increase.*

Without the need of a contract, the buyer could gain more profit by introducing competition among the suppliers even if he still buys from the same supplier. Perhaps this explains the rapid growth of electronic procurement and why big buyers urge their existing suppliers to join the eProcurement market.

3.2. Scenario c2: Two-Part Contract Initiated by the Suppliers. We will now show that even when the suppliers have the right to design the contract, and when two-part contracts are allowed, the buyer could still gain more profit by introducing competition. Suppose Suppliers 1 and 2 each propose a two-part contract with the whole sale price and side payment, and the buyer chooses between the two contracts. Under complete information, the suppliers know the unit handling cost of the buyer (c) and the market demand line defined by a and b. After solving his profit maximizing problem, the optimal contract given by supplier 1 will be as follows:

$$L_{s,c2} = \frac{(a-c-s_1)^2}{4b} - \frac{(a-c-s_2)^2}{4b} \text{ and } w = s_1$$

The system could still achieve channel coordination, but the supplier can not extract all possible profit from the buyer. Obviously now the buyer can get $\frac{(a-c-s_2)^2}{4b}$ if $\pi^-_b \leq \frac{(a-c-s_2)^2}{4b}$.

3.3. Scenario c3: Two-Part Contract Initiated by the Buyer. This situation is no different from the one-supplier, one-buyer case. The buyer can still use the side payment given in 2.2 to extract all profit from Supplier 1. Again, under the more realistic assumption of asymmetric information, the buyer may not know what is Supplier 1's minimal required profit $\pi^-_{s_1}$ and the side payment in 2.2 maybe difficult to determined. We will address this issue in the following section.

4. Two-Supplier, One-Buyer Contract with Asymmetric Information.

4.1. Model Description. In this section, we consider the case where the suppliers and the buyer hold asymmetric cost information. Each supplier i knows only his own unit cost s_i and the buyer knows only his own unit handling cost c. The market still face a price sensitive demand line $p = a - bq$, where a is only known to the buyer while b is public information. We further assume that there is a intermediary agent in this market, who represents the market maker such as a third-party eCommerce firm. The buyer holds a prior probability density function $f(s)$ in the interval $[\underline{s}, \bar{s}]$ over the suppliers' marginal cost s_i, and he believes that $s_1$and s_2 are independent from each other. Supplier i holds the same prior probability density function $f(s)$

over the *other* supplier's marginal cost s_j. This prior distribution function might be offered by the market intermediary, who has access to historic supplier information and who is willing to share this information for the interest of attracting more customers into the market. We assume that these functions are correct and all players believe its correctness. To streamline our analysis, we make additional assumptions on the density function as follows. Define random variable $t = s - \underline{s}$, and random variable T as the *larger* number of any two samples of random variable t. We assume that $f(s)$ satisfies the inequalities $E[t^2] \geq E[T] \cdot E[t]$ and $2E[T] \leq 3E[t] \leq 2.5E[T]$. These assumptions are satisfied by a variety of distribution functions, for example Weilbull and its special cases uniform and exponential. We assume all players in this market to be risk-neutral, so they are only concerned with the expected profit. Furthermore, we again assume that the market imposes a competitiveness requirement, $\bar{s} \leq \frac{a-c+\underline{s}}{2}$.

Since neither the players nor the intemediary are assumed to know the true costs of all players, the market needs a truth revealing mechanisms that would elicit the true costs. As is now common practice in electronic markets, we make use of an auction mechanism for this purpose. In the following, we first derive the channel coordinated system optimal solution. We then propose three forms of supply contracts for the environment of electronic markets: (1) *wholesale price auction,* where suppliers bid wholesale prices for an announced quantity, (2) *catalog,* where the buyer determines the order quantity based on catalog-posted wholesale prices, and (3) *two-part contract auction,* where sidepayments are introduced for channel coordination.

4.2. The System Optimum Solution. Suppose there is a central agent in the system, and the suppliers and the buyer submit their true costs voluntarily, we may define a channel coordinated system optimal solution that determines which supplier would process the current order with what quantity so as to optimize system profit. The wholesale price is merely an internal transfer between the suppliers and the buyer, which does not influence the system profit.

The marginal cost for suppliers 1 and 2 are s_1 and s_2 , respectively. The central agent will assign the order to the supplier with less marginal cost, and the maximal profit $\pi_T^*(s_1, s_2)$ will be

$$\pi_{T,a}^*(s_1, s_2) = \frac{(a - c - \min(s_1, s_2))^2}{4b} \tag{4.1}$$

and the optimal quantity $q^*(s_1, s_2)$ will be

$$q_a^*(s_1, s_2) = \frac{a - c - \min(s_1, s_2)}{2b} \tag{4.2}$$

The expected maximal profit for the system will be

$$E[\pi_{T,a}^*] = 2 \int_{\underline{s}}^{\bar{s}} \frac{(a - c - s)^2}{4b} f(s)[1 - F(s)]ds \tag{4.3}$$

and the expected optimal quantity with complete information will be

$$E[q_a^*] = \frac{a - c - 2E(s) + 2\eta}{2b} \tag{4.4}$$

Here $\eta = \int_{\underline{s}}^{\bar{s}} sf(s)F(s)ds$ and $E(s) = \int_{\underline{s}}^{\bar{s}} sf(s)ds$. If the distribution function is uniform, η will be $\frac{2\bar{s}+\underline{s}}{6}$.

LEMMA 4.1. *For any distribution function f(s) defined in a bounded interval, we have* $2\eta \geq E(s)$.

Because

$$\begin{aligned} 2\eta - E(s) &= \int_{\underline{s}}^{\bar{s}} sf(s)(2F(s)-1)ds \\ &= \frac{1}{2}\int_{\underline{s}}^{\bar{s}} s(2F(s)-1)d(2F(s)-1) \\ &= \frac{\bar{s}-\underline{s}}{2} - \int_{\underline{s}}^{\bar{s}} sf(s)(2F(s)-1)ds - \frac{\int_{\underline{s}}^{\bar{s}}(2F(s)-1)^2 ds}{2} \\ \text{and} \quad & 0 \leq (2F(s)-1)^2 \leq 1 \end{aligned}$$

4.3. Supply Contracts Based on Wholesale Price Auction. In a buyer-centric market, the most straightforward and widely adopted form of electronic market auction would be *wholesale price auction*. In this scheme, the buyer announces his order quantity and the suppliers bid on the wholesale price w through an auction. The market intermediary guarantees that the order will be transacted between the buyer and the winning supplier following the wholesale price and quantity determined by the auction. Suppose the English (Simultaneous Descending Bid) Auction is used, we can find the resulting wholesale price as $w(s_1, s_2) = max(s_1, s_2)$ with the given s_1 and s_2. The problem for the buyer is thus

$$\begin{aligned} B_{a1} \qquad & \max_q E[\pi_{b,a1}(q)] \qquad (4.5) \\ Here \qquad & \pi_{b,a1}(q, s_1, s_2) = (p(q) - c - w)q \\ & = (a - c - bq - max(s_1, s_2)))q \end{aligned}$$

Solving this problem we know that the optimal quantity q_{a1} for the buyer will be

$$q_{a1} = \frac{a - c - 2\eta}{2b} \qquad (4.6)$$

Given (s_1, s_2), the profit for the buyer will be

$$\pi_{b,a1}(s_1, s_2) = \frac{[a - c + 2\eta - 2\max(s_1, s_2)](a - c - 2\eta)}{4b} \qquad (4.7)$$

and the optimal expected profit for the buyer will be

$$E[\pi^*_{b,a1}(a, b, c)] = \frac{(a - c - 2\eta)^2}{4b} \qquad (4.8)$$

We can find the prior distribution function influencing the optimal expected profit of the buyer through η, which equals to the half of the expected cost of the higher-cost supplier s_2 (see Lemma 4.1).

The profit for the higher-cost supplier s_2 is 0, because he will lose the auction. However, before the auction, supplier i only knows his own marginal cost s and the prior distribution of the other supplier's marginal cost. Thus, the expected profit for a specific supplier with marginal cost s is as follows:

$$E[\pi_{s,a1}(s)] = \frac{a-c-2\eta}{2b} \int_{s}^{\bar{s}} (x-s) f(x) dx \tag{4.9}$$

Given (s_1, s_2) and $s_1 < s_2$, the system's profit will be

$$\pi_{T,a1}(s_1, s_2) = \frac{(a-c-2\eta)(a-c+2\eta-2s_1)}{4b} \tag{4.10}$$

The expected value for the system's profit is

$$E[\pi_{T,a1}] = \frac{(a-c-2\eta)(a-c+6\eta-4E[s])}{4b} \tag{4.11}$$

THEOREM 4.2. *Regardless of prior distribution on the supplier's cost, supply contracts based on wholesale price auction results in (a) lower expected total profit, and (b) lower expected order quantity, when compared to the system optimum.*

The result concerning a suboptimal (lower) expected order quantity follows directly from 4.4 , 4.6 and Lemma 4.1. The result concerning less expected profit should follow from intuition. In Appendix 1, we provide a more formal proof for the theorem. While the above result seems similar to the complete information case with no contract coordination, it should be clear that the earlier result is due to double marginalization, but the result here is due to asymmetric information.

4.4. Supply Contracts Based on Catalog Auction. Although *wholesale price auction* is the most common in current business-to-business eCommerce, its performance could be improved from the viewpoints of the buyer and from the system. We propose such an improvement called the *catalog auction.* Each supplier announces their wholesale price for general product categories and quantity on an electronic catalog. In addition, the suppliers may revise his posted wholesale price in real-time if (1) there are other suppliers who offer a lower price, and (2) this price is higher than his own cost. The buyer has direct access and will compare wholesale prices listed in the catalogs, he will choose the supplier offering the lowest wholesale price and *then* determine his order quantity. This catalog auction scheme can be viewed as a variant of the wholesale price auction as follows:

1. The market intermediary administers a wholesale price auction for general product categories and quantity. Using simultaneous descending bid auction, the intermediary determines the market wholesale price for all products.
2. The buyer chooses his order quantity based on the announced wholesale price for his product.

We will analyze the performance of *catalog auction* in the following. Given the suppliers' marginal cost are s_1 and s_2 respectively, the buyer's problem becomes

$$B_2 \qquad \max_{q} \ \pi_{b,a2}(q, s_1, s_2) \tag{4.12}$$

$$\textit{Here} \qquad \pi_{b,a2}(q, s_1, s_2) = (p(q) - c - \max(s_1, s_2))q$$

The optimal quantity $q_{a2}(s_1, s_2)$ for the buyer is

$$q_{a2}(s_1, s_2) = \frac{a - c - \max(s_1, s_2)}{2b} \tag{4.13}$$

and the optimal profit for the buyer is

$$\pi_{b,a2}(s_1, s_2) = \frac{(a - c - \max(s_1, s_2))^2}{4b} \tag{4.14}$$

The expected profit for the buyer will be

$$E[\pi_{b,a2}] = \frac{(a - c)^2 - 4\eta(a - c) + 2\int_{\underline{s}}^{\bar{s}} s^2 F(s)f(s)ds}{4b} \tag{4.15}$$

for supplier i with cost s, the expected profit with given a, b, c will be

$$E[\pi_{s,a2}(s)] = \frac{(a - c)\int_s^{\bar{s}}(x - s)f(x)dx - \int_s^{\bar{s}}(x - s)xf(x)dx}{2b} \tag{4.16}$$

THEOREM 4.3. *When comparing to supply contracts based on wholesale price auction, contracts based on catalog auction result in (a) higher expected profit for the buyer i.e,* $E[\pi_{b,a2}(a, b, c)\ ] > E[\pi_{b,a1}(a, b, c)]$ *given any a,b,c, and (b) lower expected profit for the supplier, i.e.,* $E[\pi_{s,a2}(a, b, c, s)\ ] \leq E[\pi_{s,a1}(a, b, c, s)\ ]$*, for supplier with cost s.*

The formal proof of this theorem can be found in Appendix 2.

If we again assume $s_1 < s_2$,the profit for the system is

$$\pi_{T,a2}(s_1, s_2) = \frac{(a - c + s_2 - 2s_1)(a - c - s_2)}{4b} \tag{4.17}$$

The expected quantity is

$$E[q_{a2}] = \frac{a - c - 2\eta}{2b} \tag{4.18}$$

The expected value for the system's profit is

$$\begin{aligned} E[\pi_{T,2}(s_1, s_2)] &= 2\int_{\underline{s}}^{\bar{s}}[\int_{\underline{s}}^{s_2} \frac{(a - c + s_2 - 2s_1)(a - c - s_2)}{4b} f(s_2)ds_2]f(s_1)ds_1 \\ &= \frac{(a - c)^2 + (a - c)(4\eta - 4E(s)) + 2E^2(s) - 2\int_{\underline{s}}^{\bar{s}} s^2 f(s)F(s)ds}{4b} \end{aligned} \tag{4.19}$$

THEOREM 4.4. *When comparing to supply contracts based on wholesale price auction, contracts based on catalog auction yields (a) the same expected order quantity, but (b) higher expected system profit iff* $2E[(M - E[s])^2] \geq 3E[(M - E[M])^2]$.

Here we define M as $max(s_1, s_2)$. and $E[M] = 2\eta$. This condition can be easily obtained by comparing 4.11 and 4.19. Given a distribution function, the market intermediary may choose between the wholesale price auction and catalog auction based on expected system's profit. Unfortunately, neither scheme will achieve channel coordination and maximize the system profit.

THEOREM 4.5. *Regardless of prior distribution on the supplier's cost, supply contracts based on catalog auction results in (a) lower expected total profit, and (b) lower expected order quantity, when compared to the system optimum.*

Because the wholesale price auction and the catalog auction result in the same expected order quantity (Theorem 4.4), this first part of theorem is trivial, while the proof for the second part is similar to that of Theorem 4.2.

4.5. Supply Contracts Based on Two-Part Contract Auction. Neither of the above schemes result in channel coordination according to Theorems 4.2 and 4.5. We now propose a two-part contract scheme that would achieve channel coordination. The two parts of the contract includes the order charge $w \cdot q$, and a side payment L, from the supplier to the buyer (e.g., rebate). The sequence of events in this scheme is as follows:

(Two-Part Contract Auction)

1. The buyer announces an order function in wholesale price w : $q = \frac{(\hat{k}-w)}{b}$
2. The suppliers attend a English auction on the side payment L, the supplier offering the highest L wins the auction.
3. The winning supplier set the wholesale price w^* based on the order function and his own cost structure.
4. The final order quantity is determined by the buyer's order function and the wholesale price w^*.

Comparing to the wholesale price auction, now the buyer's order quantity is based on the wholesale price, which give the buyer more flexibility. With the third-party market intermediary, we can guarantee that the final order quantity follows the order function. Thus, the suppliers can trust this function and make their decisions L and w^* by solving a deterministic optimization problem. Here we separate the decision making of L and w^* such that the suppliers would reveal their true cost and eventually achieve channel coordination.

To analyze this scheme further, we consider two scenarios. In the first scenario, the buyer's retail price p and the unit handling cost c are both observable by the market intermediary. In the second scenario, the retail price and unit handling cost are private information owned by the buyer.

4.5.1. Scenario 1: The Retail Price and Handling Cost are Observable.

THEOREM 4.6. *If the retail price and the unit handling cost of the buyer are both observable, the two-part contract auction yields channel coordination.*

To proof this theorem, we should have a look at the problems for all of the players. For the supplier with a specific cost margin s, he may calculate his maximal obtainable profit before the auction given the announced order function. This profit will be $\frac{(\hat{k}-w-s)^2}{4b}$. Since the supplier who lose in the sidepayment auction will gain nothing, the resulting sidepayment from the auction will be

$$L(s_1, s_2, \hat{k}, b) = \frac{(\hat{k} - w - \max(s_1, s_2))^2}{4b} \tag{4.20}$$

In step 3, the winning supplier will solve her own model to determine the wholesale price w^*. Suppose Supplier 1 wins the auction, he will be solving the problem:

$$S_3 \qquad \max_{w} \ \pi_{s,a3}(w, s_1) \tag{4.21}$$

$$Here \qquad \pi_{s,a3}(q, s_1) = \frac{(w - s_1)(\hat{k} - w - s_1)}{4b}$$

The solution for this problem will be

$$w_{a3} = \frac{\hat{k} + s_1}{2} \qquad \text{and} \quad q_{a3} = \frac{\hat{k} - s_1}{2b} \tag{4.22}$$

If we set retail price $p = w + c$, the profit maximizing problem for the buyer would be

$$b_{a3} \qquad \max_{(\hat{k},\hat{b})} \ \pi_{b,a3}(\hat{k}, b, s_1, s_2) \tag{4.23}$$

$$Here\ \pi_{b,a3}(\hat{k}, b, s_1, s_2) = \begin{cases} \frac{(\hat{k}-w-\max(s_1,s_2))^2}{4b} & \text{if } \hat{k} \le a - c \\ \frac{(\hat{k}-w-\max(s_1,s_2))^2}{4b} - \frac{(\hat{k}\ -a+c)}{b} \cdot \frac{\hat{k}+s_1}{2} & \text{if } \hat{k} \ge a - c \end{cases}$$

When $\hat{k} \ge a - c$, the buyer will order $\frac{\hat{k}-s_1}{2b}$, but he can only sell $\frac{2a-2c-\hat{k}-s_1}{2b}$ with wholesale price $\frac{\hat{k}+s_1}{2} + c$ and he has to face the lost of surplus of the order. The solution for the buyer will be $\hat{k} = a - c$. In other words, the buyer will transfer the market demand function to the suppliers truthfully. Thus, we get $q_{a3} = \frac{\hat{k}-s_1}{2b} = q^*$, in other words, the system reaches channel coordination. In this scheme, the buyer will get profit

$$\pi_{b,a3}(s_1, s_2) = \frac{(a - c - \max(s_1, s_2))^2}{4b} \tag{4.24}$$

This is the same as the case in catalog auction.

THEOREM 4.7. *Comparing to the catalog auction, the two-part contract auction under scenario 1 yields (a) the same expected profit for the buyer, and (b) higher expected profit for the supplier.*

Proof. Part (a) of the theorem is trivial to proof. To proof the part (b), suppose suppliers 1 and 2 have marginal costs, s_1,and s_2, respectively, and $s_1 < s_2$. The profit for supplier 1 would be

$$\pi_{s1,a3}(s_1, s_2) = \frac{(a - c - s_1)^2}{4b} - \frac{(a - c - s_2)^2}{4b}$$

For the supplier with cost s, the expected profit is

$$E[\pi_{s,a3}(s)] = \frac{2(a - c)\int_s^{\bar{s}}(x - s)f(x)dx - \int_s^{\bar{s}}(x^2 - s^2)f(x)dx}{4b} \tag{4.25}$$

Compared with the expected supplier profit $E[\pi_{s,a2}(s)]$ under *catalog auction* , we can proof (b) with the fact that $\int_s^{\bar{s}}(x - s)^2 f(x)dx \ge 0$. □

THEOREM 4.8. *Comparing to the wholesale price auction, the two-part contract auction under scenario 1 yields higher expected profit for supplier with marginal cost* $s \le 4\eta - \bar{s}$.

The proof of this theorem is given in Appendix 3 . Although we can't guarantee all suppliers to get higher expected profit, the suppliers with cost below the stated threshhold can get more.

4.5.2. Scenario 2: The Retail Price and Handling Cost are Buyer's Private Information. When the unit handling cost and the market demand function are both private information, the buyer has the incentive to lie about the order function. We assume b is a known information and a, c is the private information owned by the buyer and assume $s_1 < s_2$.

THEOREM 4.9. *In two-part contract auction under scenario 2, the buyer has the incentive to report a larger unit handling cost or a lower market price in order to get more profit.*

$$b_{a4} \qquad \max_{(\hat{k})} E[\pi_{b,a4}(\hat{k}, s_1, s_2)] \tag{4.26}$$

$$\text{Here} \qquad \pi_{b,a4}(\hat{k}4, s_1, s_2) = \frac{(2(a-c) - \hat{k} - s_1)(\hat{k} - s_1)}{4b} - \frac{(\hat{k} - s_1)^2 - (\hat{k} - s_2)^2}{4b}$$

Solve problem b_4, we get

$$\hat{k}_{a4} = a - c + 2E(s) - 4\eta \tag{4.27}$$

Because $2\eta - E(s) \geq 0$, the reported $\hat{k}_{a4}$ is less than $a - c$.

THEOREM 4.10. *Comparing to wholesale price auction and catalog auction, the two-part contract auction under scenarios 2 yields (a) higher system profit , but (b) equivalent expected order quantity.*

The expected order quantity is

$$E[q_{a4}] = E[\frac{a - c + 2E(s) - 4\eta - s_1}{2b}] = \frac{a - c - 2\eta}{2b} \tag{4.28}$$

The profit for the system in s_1 is

$$\pi_{T,a4}(s_1) = \frac{(2a - 2c(a - c + 2E(s) - 4\eta - s_1)(a - c + 2E(s) - 4\eta - s_1)}{4b} \tag{4.29}$$

The expected profit for the system is

$$E[\pi_{T,a4}] = \frac{(a-c)^2 - 4E^2(s) - 16\eta^2 + 16\eta E(s) + (4\eta - 4E(s))(a-c) + 2E(s^2) - 2\int_{\underline{s}}^{\bar{s}} s^2 F(s) f(s) ds}{4b} \tag{4.30}$$

Compared with 4.11 and 4.19, we can find $E(\pi_{T,a4}) \geq E(\pi_{T,a1})$ and $E(\pi_{T,4}) \geq E(\pi_{T,a2})$. Further details are given in Appendix 4.

THEOREM 4.11. *Comparing to wholesale price auction and catalog auction, the two-part contract auction under scenarios 2 yields higher buyer profit*

The buyer could get the same profit by telling the truth as in the case of the catalog auction. The buyer could get more profit by telling a lie, which can be obtained by Theorems 5 and 11.

For the supplier with marginal cost s, the expected profit is

$$E[\pi_{s,a4}(s)] = \frac{2(a - c + 2E(s) - 4\eta)\int_{s}^{\bar{s}} (x - s) f(x) dx - \int_{s}^{\bar{s}} (x^2 - s^2) f(x) dx}{4b} \tag{4.31}$$

The comparison for the supplier's expected profit depends on the distribution function and the marginal cost of the supplier and it is hard to get some general conclusions.

5. An Uniform Distribution Example. Here, we give an example with uniform distribution. We assume $f(s) = \frac{1}{\bar{s}-\underline{s}}$, then $F(s) = \frac{x-\underline{s}}{\bar{s}-\underline{s}}$. $E(s) = \frac{\bar{s}+\underline{s}}{2}$, and $\eta = \frac{2\bar{s}+\underline{s}}{6}$. We can also obtain $\int_{\underline{s}}^{\bar{s}} s^2 f(s)ds = \frac{\bar{s}^2+\bar{s}\underline{s}+\underline{s}^2}{3}$ and $\int_{\underline{s}}^{\bar{s}} s^2 f(s)F(s)ds = \frac{3\bar{s}^2+2\bar{s}\underline{s}+\underline{s}^2}{12}$. We summarize all of the profits as the follows. For the buyer with unit handling cost c, the expected profit under the four cases ($a1$: wholesale price auction, $a2$: catalog auction, $a3$: two-part contract auction under Scenario 1, $a4$: two-part contract auction under Scenario 2).

	$E(\pi_b)$
a1	$\frac{(3a-3c-2\bar{s}-\underline{s})^2}{36b}$
a2	$\frac{6(a-c)^2-4(a-c)(2\bar{s}+\underline{s})+(3\bar{s}^2+2\bar{s}\underline{s}+\underline{s}^2)}{24b}$
a3	$\frac{6(a-c)^2-4(a-c)(2\bar{s}+\underline{s})+(3\bar{s}^2+2\bar{s}\underline{s}+\underline{s}^2)}{24b}$
a4	$\frac{6(a-c)^2-4(a-c)(2\bar{s}+\underline{s})+(3\bar{s}^2+2\bar{s}\underline{s}+\underline{s}^2)+\frac{2(\bar{s}-s)^2}{3}}{24b}$

For the supplier with cost s, the expected profit under the four scenarios will be

	$E(\pi_s(s))$
a1	$\frac{(3a-3c-2\bar{s}-s)(\bar{s}-s)^2}{12b(\bar{s}-\underline{s})}$
a2	$\frac{((3a-3c)(\bar{s}-s)-2\bar{s}^2+\bar{s}s+s^2)(\bar{s}-s)}{12b(\bar{s}-\underline{s})}$
a3	$\frac{((3a-3c)(\bar{s}-s)-\bar{s}^2-\bar{s}s+2s^2)(\bar{s}-s)}{12b(\bar{s}-\underline{s})}$
a4	$\frac{((3a-3c-\bar{s}+\underline{s})(\bar{s}-s)-\bar{s}^2-\bar{s}s+2s^2)(\bar{s}-s)}{12b(\bar{s}-\underline{s})}$

The system's expected profit under the four scenarios will be

	$E(\pi_T)$
a1	$\frac{(3a-3c-2\bar{s}-\underline{s})(a-c-\underline{s})}{12b}$
a2	$\frac{6(a-c)^2-4(a-c)(\bar{s}+2\underline{s})+4\bar{s}\underline{s}+2\underline{s}^2}{24b}$
a3	$\frac{6(a-c)^2-4(a-c)(\bar{s}+2\underline{s})+\bar{s}^2+2\bar{s}\underline{s}+3\underline{s}^2}{24b}$
a4	$\frac{6(a-c)^2-4(a-c)(\bar{s}+2\underline{s})+\bar{s}^2+2\bar{s}\underline{s}+3\underline{s}^2-\frac{(\bar{s}-s)^2}{6}}{24b}$

It is easy to verify that the results listed in the table satisfy all of the theorems in this paper.

6. Discussion and Conclusion. One interesting point is that the expected order quantity remains the same under all cases with asymmetric information. However, the expected profits for different parties involved are different. According to their expected profits, the buyers' preference should be a4, a3, a2 and a1. Almost all proposed contracting mechanisms favor the buyer and the system. With the same expected order quantity, the order quantity for schemes 1,2 and 4 are fixed, related to s2 or related to s1 respectively. In the optimal solution, order quantity should be related to true s1. In theory, s2 is positively correlated to s1. This is the reason the system's profit is usually ranked as a4, a2 and a1.

Although in this paper we only consider the situation with two suppliers, we believe that similar results could be obtained for multiple suppliers. Further research could be done with other forms of supplier's cost, especially any convex cost function. The two-part contract with observable unit handling cost and retail price could work well for any cost structure of the suppliers. Here, we consider the demand as deterministic and price sensitive. If the demand is stochastic, the buyer will face different optimization problem and we need to investigate all contract schemes again, which could be another extension of the current research.

7. Appendixes. *Appendix 1*

To prove $E(\pi^*_{T,a}) > E(\pi_{T,a1})$

from (3.8) and (3.10)

$$
\begin{aligned}
[E(\pi^*_{T,a}) - E(\pi_{T,a1})] \cdot b &= \frac{1}{2}\int_{\underline{s}}^{\bar{s}} s^2 f(s)(1-F(s)ds + 3\eta^2 - 2E(s)\eta \\
&= \frac{1}{2}\int_{\underline{s}}^{\bar{s}} s^2 f(s)(1-F(s)ds + \int_{\underline{s}}^{\bar{s}} 2\eta s F(s) f(s) ds - 2\int_{\underline{s}}^{\bar{s}} \eta s f(s) ds + \eta^2 \\
&= \frac{\int_{\underline{s}}^{\bar{s}} f(s)(1-F(s)(s-2\eta)^2 ds}{2} \\
&> 0
\end{aligned}
$$

Appendix 2

To prove $E[\pi_{b,a2}(a,b,c)\ ] \geq E[\pi_{b,a1}(a,b,c)\ ]$

$$
\begin{aligned}
(E[\pi_{b,a2}(a,b,c)\ ] - E[\pi_{b,a1}(a,b,c)\ ]) \cdot 2b &= \int_{\underline{s}}^{\bar{s}} s^2 F(s) f(s) ds - 2\eta^2 \\
&= \int_{\underline{s}}^{\bar{s}} (s-2\eta) s F(s) f(s) ds
\end{aligned}
$$

Because $\int_{\underline{s}}^{\bar{s}}(s-2\eta)F(s)f(s)ds = 0$ and $(s-2\eta)F(s)f(s)$ change the sign from negative to positive just once between $[\underline{s}, \bar{s}]$, we can assume D=-$\int_{\underline{s}}^{2\eta}(s-2\eta)F(s)f(s)ds = \int_{2\eta}^{\bar{s}}(s-2\eta)F(s)f(s)ds \geq 0$. With Mean Value Theorem

$$
\begin{aligned}
(E[\pi_{b,a2}(a,b,c)\ ] - E[\pi_{b,a1}(a,b,c)\ ]) \cdot 2b &\geq \int_{\underline{s}}^{2\eta} s(s-2\eta)F(s)f(s)ds + \int_{2\eta}^{\bar{s}} s(s-2\eta)F(s)f(s)ds \\
&= D\alpha - D\beta \\
&= D(\alpha - \beta)
\end{aligned}
$$

Here α is one number between $[2\eta, \bar{s}]$ and β is one number between $[\underline{s}, 2\eta]$.

Then we get the result.

To prove $E[\pi_{s,a1}(s)\ ] \geq E[\pi_{s,a2}(s)\ ]$

Define $G(s) = (E[\pi_{s,a1}(s)\ ] - E[\pi_{s,a2}(s)\ ]) \cdot 2b = \int_{\underline{s}}^{\bar{s}}(x-2\eta)(x-s)f(x)dx$

Obviously when s$\geq 2\eta$, G(s)$\geq$ 0

G(s)=$\int_{\underline{s}}^{\bar{s}}(x-2\eta)(x-s)f(x)dx - \int_{\underline{s}}^{s}(x-2\eta)(x-s)f(x)dx$

$$
\begin{aligned}
\frac{dG}{ds} &= 2\eta - E(s) - s(s-2\eta)f(s) + s(s-2\eta)f(s) + \int_{\underline{s}}^{s}(x-2\eta)f(x)dx \\
&= \int_{s}^{\bar{s}}(2\eta - x)f(x)dx
\end{aligned}
$$

We define ξ,which let $\int_{\xi}^{\bar{s}}(2\eta - x)f(x)dx = 0$, Because $(2\eta - x)f(x)$ just change the sign just once between$[\underline{s},\bar{s}]$, there must exist one and only one ξ and $\xi \in [\underline{s}, 2\eta]$. We know between $[\underline{s},\xi]$, $G(s)$ is increasing in s and between $[\xi, \bar{s}], G(s)$ in decreasing in s. Because G(s)$\geq$ 0 when s$\geq 2\eta$, we get G(s)$\geq$ 0 when s$\geq \xi$. With the assumption E[(s-$\underline{s}$)^2] $\geq E[M-\underline{s}] \cdot E[s-\underline{s}]$, we know G($\underline{s}$)$\geq$ 0. So, we get $G(s) \geq 0$. for any s.

Proof ends

Appendix 3

To prove $E[\pi_{s,a3}(s)] \geq E[\pi_{s,a1}(s)]$ when s$\leq 4\eta - \bar{s}$, obviously s$\leq 2\eta$

$$\begin{aligned}
\Delta(s) &= (E[\pi_{s,a3}(s)] - E[\pi_{s,a1}(s)]) \cdot 4b \\
&= \int_s^{\bar{s}} (x-s)(4\eta - x - s) f(x)dx \\
&= (\bar{s}-s)(4\eta - \bar{s} - s) - \int_s^{\bar{s}} (4\eta - 2x)F(x)dx \\
&= (\bar{s}-s)(4\eta - \bar{s} - s) - \int_{2\eta}^{\bar{s}} (4\eta - 2x)F(x)dx - \int_s^{2\eta} (4\eta - 2x)F(x)dx \\
&\geq (\bar{s}-s)(4\eta - \bar{s} - s) + F(2\eta)\int_{2\eta}^{\bar{s}} (2x - 4\eta)dx - F(2\eta)\int_s^{2\eta} (4\eta - 2x)dx \\
&= (\bar{s}-s)(4\eta - \bar{s} - s)(1 - F(2\eta)) \\
&\geq 0
\end{aligned}$$

Appendix 4

To rove $E[\pi_{T,a4}] \geq E[\pi_{T,a2}]$

From (3.36) and (3.22), we have

$$\begin{aligned}
(E[\pi_{T,a4}) - E(\pi_{T,a2}]) \cdot 2b &= 8\eta E(s) + E(s^2) - 8\eta^2 - 3E^2(s) \\
&= \int_{\underline{s}}^{\bar{s}} (s - 2\eta)(s + 4\eta - 3E(s))f(s)ds
\end{aligned}$$

$(s + 4\eta - 3E[s])f(s)$ changes the sign from negative to positive only once at $3E[s] - 4\eta$ between $[\underline{s}, \bar{s}]$ and $\int_{\underline{s}}^{\bar{s}}(s + 4\eta - 3E(s))f(s)ds = 4\eta - 2E(s) \geq 0$. Let $D = \int_{3E(s)-4\eta}^{\bar{s}}(s + 4\eta - 3E(s))f(s)ds \geq 0$ *and*

let $d = -\int_{\underline{s}}^{3E(s)-4\eta}(s + 4\eta - 3E(s))f(s)ds \geq 0$, we have $D \geq d$.

Because of Mean Value Theorem, $\int_{\underline{s}}^{\bar{s}}(s + E(s) - 4\eta)(s + 4\eta - 3E(s))f(s)ds = (a + E(s) - 4\eta) * D - (b + E(s) - 4\eta) * d$.

Here a is some number between $[3E(s) - 4\eta, \bar{s}]$ and b is some real number between $[\underline{s}, 3E(s) - 4\eta]$. We can find

$(b - 2\eta) \leq 0$. So,

$$\begin{aligned}
(a - 2\eta) * D - (b - 2\eta) * d &\geq (a + E(s) - 4\eta) * D - (b + E(s) - 4\eta) * D \\
&= (a - b) * D \\
&\geq 0
\end{aligned}$$

We got the result.

To prove$E[\pi_{T,a4}] \geq E[\pi_{T,a1}]$

¿From (3.36) and (3.16), we get

$$(E[\pi_{T,a4}] - E[\pi_{T,1}]) \cdot 2b = E(s^2) - \int_{\underline{s}}^{\bar{s}} s^2 F(s) f(s) ds + 4\eta E(s) - 2E^2(s) - 2\eta^2$$
$$= \int_{\underline{s}}^{\bar{s}} (s - 2\eta)(s + 2\eta - 2E(s))(1 - F(s)) f(s) ds$$

Because $\int_{\underline{s}}^{\bar{s}}(s + 2\eta - 2E(s))(1 - F(s))f(s)ds = 0$ and $(s + 2\eta - 2E(s))(1 - F(s))f(s)$ change sign only once from negative to positive at $2E(s) - 2\eta$ between $[\underline{s}, \bar{s}]$ and $\underline{s} \leq 2E(s) - 2\eta \leq \bar{s}$, we define D=$\int_{2E(s)-2\eta}^{\bar{s}}(s+2\eta-2E(s))(1-F(s))f(s)ds = -\int_{\underline{s}}^{2E(s)-2\eta}(s + 2\eta - 2E(s))(1 - F(s))f(s)ds \geq 0$

From the Mean Value Theorem,

$$\begin{aligned}
&\int_{\underline{s}}^{\bar{s}} (s - 2\eta)(s + 2\eta - 2E(s))(1 - F(s)) f(s) ds \\
&= (\alpha - 2\eta)) \cdot D - (\beta - 2\eta) \cdot D \\
&= (\alpha - \beta) \cdot D \\
&\geq 0
\end{aligned}$$

Here α is some real number between $[2E(s) - 2\eta, \bar{s}]$ and β is some real number between $[\underline{s}, 2E(s) - 2\eta]$.

Proof Ends.Mathematics and Text

Let H be a Hilbert space, C be a closed bounded convex subset of H, T a nonexpansive self map of C. Suppose that as $n \to \infty$, $a_{n,k} \to 0$ for each k, and $\gamma_n = \sum_{k=0}^{\infty} (a_{n,k+1} - a_{n,k})^+ \to 0$. Then for each x in C, $A_n x = \sum_{k=0}^{\infty} a_{n,k} T^k x$ converges weakly to a fixed point of T .

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