A Bargaining Game for Supply Chain Contracting in Electronic Markets

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Abstract

This paper examines a bargaining theoretic approach to supply chain coordination. We first propose a one-buyer one-supplier non-coopertive bargaining game for supply chain contracting, where the buyer negotiates with a sourcing supplier the order quantity and wholesale price. We show that in *subgame perfect equilibrium*, the channel coordinated optimal quantity is also optimal for the players, but the players must negotiate the surplus generated by the contract in a bargaining game. The model allows us to predict the negotiation outcome between a pair of buyer and supplier considering their outside options, the breakdown probability, and random proposers. Motivated by emerging applications in electronic marketplaces, we then propose a one-buyer multiple-supplier *negotiation sequencing problem* where the buyer could determine an optimal subset of sourcing suppliers to negotiate with, and the sequence to carry out the negotiations so as to maximize his expected gain. We show that the one-supplier version bargaining game serves as a building block and the negotiation sequencing problem can be solved as a network flow problem.

1. Introduction

Planning and coordination in the supply chain has attracted a great deal of attention in the last two decades due to the general trend in the industry toward strategic sourcing and alliances. This need for broader coordination can be partially attributed to the ever-increasing integration of the supply chain and the emergence of business-to-business electronic commerce, which not only promote quick and reliable transaction platforms between business partners, but also bring in diverse new players into the marketplace. A significant part of the research in *supply chain coordination* has been focusing on *supply contracting* (c.f., Tsay et al., 1999). Much of this research focuses on multi-echelon inventory decisions, while game theoretic models are used to analyze the incentives of the parties (supplier, buyer, and channel coordinator) involved. A general research question is whether the non-coordinated solutions from each party coincide with the channel coordinated system optimal. A coordination scheme typically involves contract-imposed transfer pricing where the players are incentimized to optimize the system's marginal profit function.

A supply contract determines the quantity, the timing, and the flexibility that the buyer commits to purchase, and the prices that supplier would charge. Most common forms of contract encountered both in practice and literature are quantity discounts (Parlar and Wang, 1994), quantity flexibility (Tsay, 1999), minimum commitment (Bassok and Anupindi, 1997), and buy-back contracts. Jin and Wu (2000) consider the impact of electronic market competition to various supply contracts. A related body of research investigates the buyer-supplier relationship from the viewpoint of their inventory decisions. Among these are Lee and Wang, (1999), Chen, (1999) and Cachon and Zipkin (1999). Typically, a coordination scheme would suggest a particular contract form through which a channel coordinated system optimal solution could be achieved at equilibrium. It is also assumed that the incentives provided is sufficient for independent players to accept the contractual terms. But in a broader context, there is little guarantee that the players involved in the negotiation should necessarily accept a "channel coordinated" contract, especially when additional outside options are easily accessible. Our research is motivated by the rapid development of electronic marketplaces, and in particular supply chain exchanges such as Covisint (automotive), WWRE (retail), Exostar (aerospace), Converge (electronics), Pantellos (utility), and Trade Ranger (energy), where outside options are both plentiful and easily accessible. We believe that the dynamics of supplier-buyer interaction changes in this environment and the basic premise of the supply chain contract research deserves re-examination.

This paper is set out to examine the following research questions: With the presence of outside options, *why* and *when* would the supplier and buyer in a negotiation accept a coordination contract? Even when the coordination mechanism guarantees a greater overall gain, how does the buyer and supplier split the surpluses? How does a player weights the deal on hand against other opportunities present on the market? Given proper information, could the buyer (the supplier) uses the insights from the bargaining game to form a negotiation strategy in more general settings? To analyze the theoretic underpinnings of these questions we propose a bargaining model. We will examine these issues using the simple form of one-part linear contract, but the concepts can be generalized to other forms of contracts as well.

Bargaining theory deals with resolving bargaining situations between two parties. In the seminal work of Nash (1950), he defines the bargaining problem as "two individuals who have the opportunity to collaborate for mutual benefits in more than one way. (p. 155)." In our case, the two parties are the buyer and the sourcing supplier who are to negotiate a contract that would distribute a surplus (e.g., profit) generated from mutual efforts. With the presence of outside options, even with one-part linear contract there are large number of ways where the contract could be set.

There has been two main stream of research on bargaining theory: 1) axiomatic (cooperative game) models, and 2) strategic (non-cooperative game) models. Nash (1950 and 1953) lays down the framework for the axiomatic *Nash Bargaining Solution* where he first defines the basic axioms of which any bargaining solution should "naturally" satisfy, he then shows that the solution of the so called *Nash product* is a unique solution satisfying the axioms. Kalai and Smorodinsky (1975) replace a controversial axiom from the original Nash proposal and revise the unique solution. Binmore (1987) summarizes the efforts over the years that either relaxes or add to the Nash axioms and gives further analysis of the Nash's bargaining model. An important characteristic of the axiomatic approach is that they leave out the actual process of negotiations while focusing on the expected outcome based on pre-sepecified solution properties. Rubinstein (1982) lays out the framework for noncooperative (strategic) bargaining models. He suggests an *alternating offer* bargaining procedure where the players take turns in making offers and counter offers to one another until an agreement is reached. The players face time-discounted gain (a "shrinking pie") which provide them the incentive to compromise. In each iteration, a player must decides to either (1) accept the opponent's offer (in which case the bargaining stops), or (2) propose a counter offer. Binmore et al. (1988) propose a third option where a player may decide to leave the current negotiation and opt for her "outside options" (e.g., previously quoted deals). Ponsati and Sacovics (1998) also consider outside options as part of the Rubinstein model. Muthoo (1995) considers outside options in the form of a search in a bargaining search game. An important aspect of the extended bargaining model is to allow the possibility for the negotiation to breakdown. Binmore et. al (1986) study a version of the alternating offer model with *breakdown probability*. In this model, there is no time pressure (time-discounted gain) exists, but there is a probability that a rejected offer is the *last offer* made in the game, meaning that the negotiation breaks down. An intuitive comparison between the axiomatic and strategic bargaining theory can be found in Sutton (1986).

In this paper, we model supply chain contracting and coordination as a bargaining game. The basic model allows us to predict the negotiation outcome between a pair of buyer and supplier. We incorporate outside options, breakdown probability, and "random proposers" into the model so as to capture the main essence of a supply contract negotiation. In an electronic market environment, the buyer may want to consider in a systematic fashion a set of potential sourcing suppliers before reaching a final agreement. Using the bargaining game as a building block, we propose the *negotiation sequencing problem* from the buyer's perspective, where the buyer must determine the sequence of which she would negotiate with a set of sourcing suppliers so as to maximize her potential gain. The main question here is (1) which subset of suppliers to negotiate with, and (2) in what sequence should the negotiation take place. We will show that the negotiation sequence indeed lead to different final gains for the buyer, and we propose a solution methodology to optimize the buyer's decision.

2. The Bargaining Game for Supply Chain Contracting

We first consider the basic bargaining game where a buyer and a single sourcing supplier enter the negotiation for a *one-part linear contract*. The contract specifies the *order quantity* and *unit wholesale price* for the supply of some future period. We assume that both players are rational, self-interested, and risk neutral (expected value maximizers). The buyer is subject to price sensitive market demand, while both parties have recallable outside options (e.g., a previously quoted deal with another supplier/buyer) with known net profits when they enter the negotiation. The *order quantity* and the *wholesale price* determine the *total surplus* for the trade, while the players must negotiate the split the total surplus. We now summarize the notations as follows :

q : Contractual quantity to be transacted between the buyer and the supplier.

w : Unit wholesale price to be charged by the supplier.

$P(q)$: Unit market price given quantity q, defined as a linear function as follows:

$$P(q) = \frac{a-q}{b}$$

c : Unit cost for the buyer.

s : Unit cost for the supplier.

$\Pi_B(q, w)$: Profit function of the buyer

$\Pi_S(q, w)$: Profit function of the supplier

W_B : Recallable outside option for the buyer in net profit.

W_S : Recallable outside option for the supplier in net profit.

The profit functions of the buyer and supplier in one-part linear contract are as follows:

$$\Pi_B(q, w) = (P(q) - w - c)q \tag{1}$$

$$\Pi_S(q, w) = (w - s)q \tag{2}$$

Based on the definition of the unit market price, $P(q)$, a is the maximum market demand and b is the slope of the market demand line. It is straightforward to see that in the system optimal or "channel coordinated" solution, the contract parameters would be as follows

$$w^* = s, \quad q^* = \tfrac{1}{2}[a - b(s + c)] \tag{3}$$

and the maximum system surplus from this trade is

$$\begin{aligned} \pi &= [P(q^*) - w^* - c]\, q^* \\ &= \frac{1}{4} \frac{[a - b(s + c)]^2}{b} \end{aligned} \tag{4}$$

It should be clear that in the system optimal contract, the buyer receives all the profit and the supplier receives no profit (setting her wholesale price at cost). Two-part contracts have been proposed in the literature (Tsay et al. 1999), where the contract offers a lump sum side payment to the supplier as incentives. We propose a different approach to this problem which resembles the real-life situation where the buyer and the supplier operate independently without a channel coordinator, and both sides expect to make profit through contract negotiation. We model this negotiation process through a bargaining game. Inspired by Rubinstein's *alternating offer* model, our bargaining game considers three main factors which influence the bargaining process: (1) the two players are equally likely to make the next offer, (2) the probability that the negotiation will break down after a given offer, and (3) the effect of either player's outside options. The first factor allow us to generalize the bargaining processes in that regardless of who makes the previous offer, either player could make a new offer. This is by assuming that the two players have equal probability of making the next offer after a given offer so long as the negotiation continues. The second factor, *breakdown probability,* allows us to capture the situation when the parties are not perfectly rational, when either player anticipates a more attractive future deal, and other considerations that can not be measured by monetary gains (e.g., trust and goodwill). The third factor, or the outside options of the players is important since the player with better outside options is in a better strategically position to negotiate, and is more likely to receive a bigger share from the total surplus.

As common in the bargaining literature, we will assume that the total maximum surplus from the current trade is greater than or equal to the sum of the outside options, i.e., $\pi \geq W_B + W_S$. This is reasonable since otherwise at least one of the players will receive a deal worse than his outside option, and would have no incentive to enter the negotiation in the first place. We also assume that when a player is *indifferent* between accepting the current offer or waiting for future offers, he will prefer accepting the offer.

We now defined the sequence of events in our bargaining game as follows:

1. With equal probability, one of the two players proposes a contract with parameters (q, w).
2. The other player either (a) accepts the offer (the negotiation ends), (b) rejects the offer and takes his outside option, or (c) rejects the offer and waits for the next round offer.
3. With certain probability, the negotiation breaks down and the two players are forced to take their outside options.
4. If the negotiation continues, the game restarts from step 1.

3. Subgame Perfect Equilibrium of the Bargaining Game

The subgame perfect equilibrium (SPE) strategies are the ones that constitute the *Nash equilibrium* in every iteration of the game (the subgame). In a perfect equilibrium, a player will accept a proposal if it offers at least as much as what she expect to gain in the future, given the strategy set of the other player. In the following, we show that in SPE the players would agree on the order quantity that maximizes the total surplus (which achieves channel coordination), and then negotiate the split of this surplus by bargaining. In other words in the SPE, the order quantity is the quantity given in the system optimal solution and the bargaining can be reduced to that on the total surplus. This result has obvious appeal from the practical point of view since the channel coordination can be achieved while the players may negotiate for a wholesale price that matches their profit expectations.

Proposition 1 : In SPE, the system optimal quantity is also optimal for the buyer and the suppliers, i.e., $q^*_{SPE} = \frac{1}{2}[a - b(s + c)]$ (5)

Proof : Let q_B, w_B be the quantity and wholesale price offers made by the buyer, and α_S, α_B, be the least amount of share that supplier and buyer can accept in SPE respectively. Then buyer has the following maximization problem ;

$Max\ \Pi_B(q_B, w_B) = (P(q_B) - w_B - c)q_B$

$s.t.$

$(w_B - s)q_B \geq \alpha_s$

The constraint of the model will be binding. If we Lagrangean relax the constraint, we get

$$Max\, L(q_B, w_B, \lambda) \;=\; (P(q_B) - w_B - c)q_B \;+ \lambda(\,(w_B - s)q_B - \alpha_s)$$

The first order optimality conditions are ;

$$\frac{\partial L(q_B, w_B, \lambda)}{\partial q_B} = -\frac{q_B}{b} + \frac{(a - q_B)}{b} - w_B - c - \lambda\,(w_B - s) = 0$$

$$\frac{\partial L(q_B, w_B, \lambda)}{\partial w_B} = -\,q_B + \lambda\, q_B = 0$$

$$\frac{\partial L(q_B, w_B, \lambda)}{\partial\, \lambda} = (w_B - s)q_B - \alpha_S = 0$$

The solution that satisfies the first order conditions is ;

$$q_B^* = \frac{1}{2}[a - b(s + c)], \quad \lambda^* = 1, \quad w_B^* = s + \frac{2\alpha_S}{a - b(s + c)}$$

The hessian and the second order sufficiency conditions are ;

$$H = \begin{bmatrix} -\frac{2}{b} & \lambda - 1 \\ \lambda - 1 & 0 \end{bmatrix}$$

$$-\frac{2}{b} \leq 0, \text{ and, } \; (\lambda - 1)^2 \leq 0$$

If the supplier is the offering party then we have the following model and analysis ;

$Max\;\; \Pi_S(q_S, w_S) = (w_S - c)q_S$

$s.t.$

$(P(q_S) - w_S - c)q_S \geq \alpha_B$

The constraint of the model will again be binding. If we Lagrangean relax the constraint, we get

$$Max\, L(q_S, w_S, \lambda) \;=\; (w_S - c)q_S \;+ \lambda(\,(P(q_S) - w_S - c)q_S - \alpha_B)$$

The first order optimality conditions are ;

$$\frac{\partial L(q_S, w_S, \lambda)}{\partial q_S} = w_S - s + \lambda\,(-\frac{q_S}{b} + \frac{(a - q_S)}{b} - w_S - c)$$

$$\frac{\partial L(q_S, w_S, \lambda)}{\partial w_S} = q_S - \lambda\, q_S = 0$$

$$\frac{\partial L(q_B, w_B, \lambda)}{\partial\, \lambda} = \left[\frac{(a - q_S)}{b} - w - c\right] q_S - \alpha_B = 0$$

A solution that satisfies the first order conditions is ;

$$q_S^* = \frac{1}{2}[a - b(s + c)], \quad \lambda^* = 1, \quad w_S^* = \frac{1}{2}\frac{-(a - cb)^2 + s^2b^2 + 4\,b\alpha_B}{b\,(-a + cb + sb)}$$

The hessian and the second order sufficiency conditions are ;

$$H = \begin{bmatrix} -\frac{2\lambda}{b} & 1-\lambda \\ 1-\lambda & 0 \end{bmatrix}$$

$$-\frac{2\lambda}{b} \leq 0, \text{ and, } (1-\lambda)^2 \leq 0$$

Since the second order sufficiency conditions are satisfied by the solution, the solution is an optimal solution. Thus, $q^*_{SPE} = q^*_B = q^*_S = \frac{1}{2}[a - b(s+c)]$.

□

Given the channel coordinated order quantity, the players then negotiate to split the total surplus, $\pi = \frac{1}{4}\frac{[a-b(s+c)]^2}{b}$, corresponding the system optimal profit. The *proposition* shows that in *subgame perfect equilibrium* the channel coordinated optimal quantity is also optimal for the players. Note that the total system profit is not a function of the wholesale price, as the latter is merely an *internal* transfer between the buyer and the supplier. Once the order quantity is determined, we can reduce the negotiation to that on the *total surplus* (π). We thus revise step 1 of our bargaining game as follows:

$1'$. With equal probability, one of the two players proposes a split of the surplus π

The remainder of the game is the same. In this bargaining game, each subgame starting with a certain player's offer has the same structure. Thus, the perfect equilibrium strategies of the players are the same in each subgame. We will analyze our game in a time line of offers to find the subgame perfect equilibrium, similar to the approach taken in Saked and Suton (1984) and Suton (1986). We introduce the following additional notation:

$M_B\,(M_S)$: The largest share buyer (supplier) can get as a subgame perfect equilibrium in any subgame starting with buyer's (supplier's) offer.

$m_B\,(m_S)$: The smallest share buyer (supplier) can get as a subgame perfect equilibrium in any subgame starting with buyer's (supplier's) offer.

p : The probability that negotiations continue to the next round.

In Figure 1 we illustrate a subgame beginning with the buyer's offer in a tree where the buyer gets the largest subgame perfect equilibrium share. The nodes show the party making the offer or the breakdown event, while the figures on the arcs represent the probability that the associated event occurs. The expressions underneath the nodes correspond to the share the supplier gets at that node. The tree represents the *least* share the supplier would get in perfect equilibrium and the maximum share the buyer would gain. A tree symmetrical to this from the suppliers perspective can be generated as well.

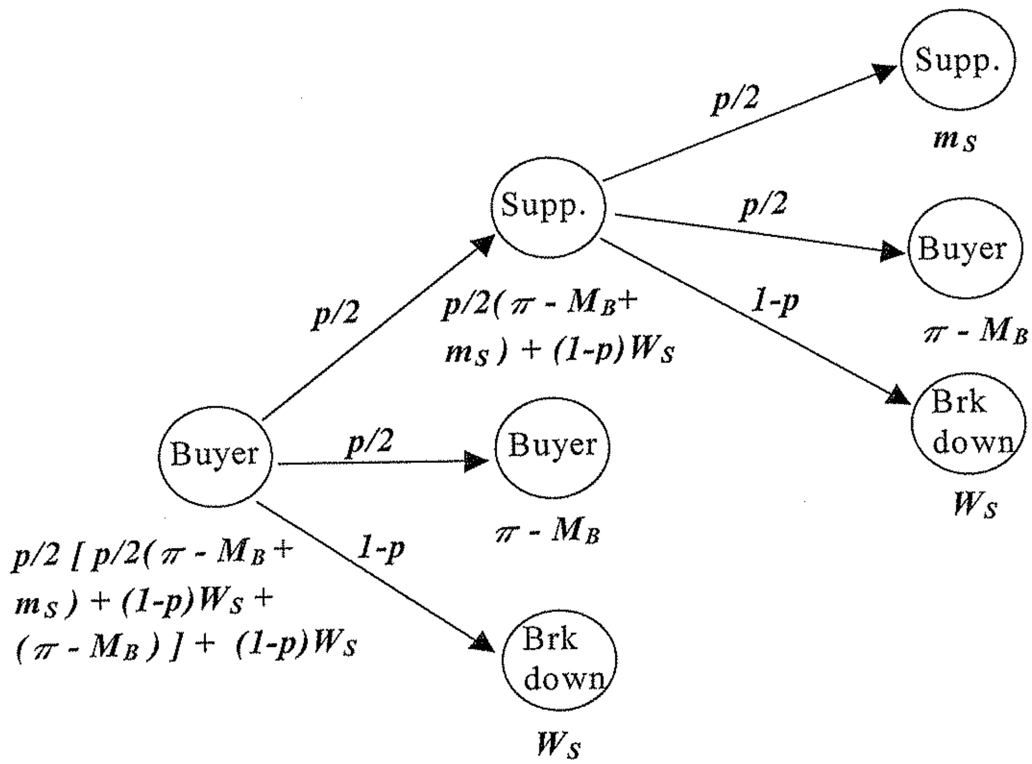

Figure 1. A tree defining the largest share the buyer could obtain in a subgame perfect equilibrium

To derive the SPE condition we evaluate the tree backward from the leave nodes. Since we are analyzing the case where the buyer gets the largest possible perfect equilibrium share, when the buyer makes the offer, he asks for the *largest* share possible and leaves $\pi - M_B$ to the supplier. If the supplier is the one making the offer, he asks for the *least* perfect equilibrium share he would get, which is m_S. In case of breakdown the supplier gets his outside option W_S. The offers at the level 2 follows the same reasoning. When the buyer makes the offer, he leaves $\pi - M_B$ to supplier. If the supplier makes the offer, he asks for the least amount that he expect to get in the future, which is equal to

$$(1-p)W_S + \frac{p}{2}(\pi - M_B + m_S)$$

Going back one offer to the root node, we can see that the supplier would expect to gain in the perfect equilibrium a minimum share of

$$\frac{p}{2}\left[(1-p)W_S + \frac{p}{2}(\pi - M_B + m_S) + \pi - M_B\right] + (1-p)W_S$$

Therefore, the largest share buyer could obtain in a SPE is as follows:

$$M_B = \pi - \left[\frac{p}{2}\left[(1-p)W_S + \frac{p}{2}(\pi - M_B + m_S) + \pi - M_B\right] + (1-p)W_S\right] \quad (6)$$

With slight modification, we can also find the *least* subgame perfect equilibrium share that the buyer could get in a subgame starting with the buyer's offer. In specific, we only need to replace M_B with m_B, and m_S with M_S in the above equation. Thus, the least share the buyer would get in the subgame is as follows:

$$m_B = \pi - \left[\frac{p}{2}\left[(1-p)W_S + \frac{p}{2}(\pi - m_B + M_S) + \pi - m_B\right] + (1-p)W_S\right] \quad (7)$$

Since the roles of the supplier and buyer are symmetrical in the game, we can easily write the expressions for M_S and m_S by changing the indices. We end up with four linear equations with four unknowns. The following proposition summarizes the subgame perfect equilibrium descriptions.

Proposition 2: The following system of equations defines the subgame perfect equilibrium for the buyer and supplier.

$$\begin{aligned}
M_B &= \pi - \left[\tfrac{p}{2}\left[(1-p)W_S + \tfrac{p}{2}(\pi - M_B + m_S) + \pi - M_B\right] + (1-p)W_S\right] \\
m_B &= \pi - \left[\frac{p}{2}\left[(1-p)W_S + \frac{p}{2}(\pi - m_B + M_S) + \pi - m_B\right] + (1-p)W_S\right] \\
M_S &= \pi - \left[\frac{p}{2}\left[(1-p)W_B + \frac{p}{2}(\pi - M_S + m_B) + \pi - M_S\right] + (1-p)W_B\right] \\
m_S &= \pi - \left[\frac{p}{2}\left[(1-p)W_B + \frac{p}{2}(\pi - m_S + M_B) + \pi - m_S\right] + (1-p)W_B\right]
\end{aligned}$$

One can solve these linear equations and finds the exact expressions for M_B, m_B, M_S, and M_S. We can now specify the subgame perfect equilibrium strategies of the buyer and supplier as follows.

Proposition 3: The unique subgame perfect equilibrium strategies of the buyer and the supplier are given as follows;

- The buyer always asks X_B share of the surplus if he offers and accepts an offer giving him a share of at least $\pi - X_S$.

- The supplier always asks X_S share of the surplus if he offers and accepts an offer giving him a share of at least $\pi - X_B$.

where X_B and X_S are defined as ;

$$X_B = \frac{1}{2}\left[(2-p)(\pi - W_S) + \frac{p^2}{(2-p)}W_B\right]$$
$$X_S = \frac{1}{2}\left[(2-p)(\pi - W_B) + \frac{p^2}{(2-p)}W_S\right]$$

Proof : If we solve the system of equations describing the SPE for the players given in *Proposition 1*, we get that

$$M_B = m_B = X_B\,,\ and\ \ M_S = m_S = X_S$$

Since we showed that the largest and the least SPE share of the players are equal, these will be the unique SPE offers that players will make when they make an offer. □

4. Analysis of the Bargaining Game

Based on *Proposition 3,* the bargaining game should end in one iteration when one of the player initiate by making the SPE offer, and the other player would accept the offer so long as the offer is no worse than his outside options. This is true since the SPE offer makes him indifferent between accepting the current offer or waiting for future offers. One important issue remains is whether there exists a first mover advantage in the game. We attend to this matter in the following proposition.

Proposition 4: There exits a first mover advantage in the game. It diminishes as the probability of breakdown decreases, and goes to zero if the probability of breakdown is zero.

Proof : If we take the difference between the SPE shares of the players we get ;

$$X_B - (\pi - X_S) = X_S - (\pi - X_B) = \frac{(\pi - W_B - W_S)(2 - p^2 - p)}{2-p}$$

Since $2 \geq p^2 + p$ and $(\pi - W_B - W_S) \geq 0$, the expression always yield a value greater that or equal to zero. It becomes zero when $p = 1$, or equivalently when the probability of breakdown is zero.□

As stated above that in SPE a player would only accept the offer when it is no less than his outside options. This should be intuitive by considering a simple strategy for the players as follows: always reject the offer that is less attractive than the outside option, and ask for a share equal to the best outside option. This strategy would guarantee that the player will get a share that is greater than or equal to his outside options. The following proposition shows this formally.

Proposition 5: In the SPE, both the offering party and the opponent get a share that is greater than or equal to their respective outside options.

Proof : For the player who initiate the offer, we can find the difference between the SPE shares of the player and his outside options as follows:

$$X_B - W[B] = X_S - W[S] = \frac{(\pi - W_B - W_S)(p^2 - 2p - 4)}{p - 2}$$

The expression above is always positive, hence the player will get no less than his outside option when making an offer in SPE. For the player who is not initiating the offer, we can find the difference between his SPE share and his outside option as follows ;

$$\pi - X_B - W[B] = \pi - X_S - W[S] = \frac{-(\pi - W_B - W_S)p^2}{2(p - 2)}$$

This expression is always positive as well. Therefore, both players get at least their outside options in SPE. □

The relationship between the probability of breakdown and SPE share of the offering party is described in the following propositions.

Proposition 6: The SPE share of the offering party is non-decreasing and linearly increasing for p>0 in his outside option and linearly decreasesing in the opponent's outside option.

Proof : If we take the first and second derivative of the SPE offer of the buyer with respect to W_B, W_S, we can see the result given in the proposition;

$$\frac{\partial X_B}{\partial W_B} = \frac{p^2}{2(2-p)} \geq 0\,, \quad \frac{\partial X_B}{\partial^2 W_B} = 0$$

$$\frac{\partial X_S}{\partial W_S} = \frac{p-2}{2} < 0\,, \quad \frac{\partial X_S}{\partial^2 W_S} = 0$$

For the supplier, one can go through the same process to show the result.

□

One interesting aspect of the game is that, the offering party obtains the maximum share when the breakdown probability approaches 1 as described in the following proposition.

Proposition 7: The SPE share of the offering party is maximized when the probability of breakdown goes to 1 and is equal to total surplus less the outside option of the other party.

Proof : If we take the second derivative of the SPE offer of the buyer with respect to p:

$$\frac{\partial X_B}{\partial^2 p} = 4\frac{\pi - W_B - W_S}{(p-2)^3}$$

Here $\frac{\partial X_B}{\partial^2 p}$ is always less than or equal to zero since we have assumed that $\pi - W_B - W_S \geq 0$. Hence, X_B is a convex function in p. The maximizing values of p are 0, 4. Since p is a probability, 4 is not a feasible value. Therefore, $p = 0$ maximizes the SPE share of the offering party. For the supplier, we can go through the same reasoning from symmetry. □

Proposition 7 is intuitive; if the offering party knows that the negotiation is likely to breakdown, in which case the opponent will only get his outside option; knowing that the opponent is willing to accept any offer equivalent to the outside option, the offering party would have no reason to offer anything more than the opponent's outside option.

5. The Negotiation Sequencing Problem

In the previous section, we introduce a bargaining game which describes the splitting of the expected gains from a supply contract for a single buyer-supplier pair. We now describe the use of this bargaining model as a building block for multiple buyer-supplier environments such as the electronic markets. With efficient buyer-supplier matching mechanism offered in electronic marketplaces, the buyer could face a large number of potential sourcing suppliers at a particular point in time. *Freemarkets,* a leading eProcurement service provider, reports that in the first quarter of year 2000 the number of buyers participating in their procurement markets was less than 50, while the number of suppliers more than 4,000. While current electronic markets rarely offer negotiation services, semi-automated and off-line negotiation after the buyer-supplier matching is not unusual. In this environment, not only does the buyer benefit from negotiating with more than one supplier, but she could also optimize the sequence to negotiate with the suppliers. We will show that the negotiation sequence is important.

As the buyer negotiates with a list of suppliers in sequence, there are two alternative assumptions concerning previously negotiated deals: (1) all previously negotiated deals (with the suppliers) are recallable, and (2) the previous deals are not recallable. In the context of electronic marketplaces, one could assume that the negotiations occur in a relatively short time, and it would be reasonable to assume that all deals are recallable. This also applies to industries where the buyer has more bargaining power, and could ask for a time period during which a negotiated deal stay valid unless the buyer decides otherwise. Thus, in the following analysis we will assume that all previous deals are recallable. Outside options also play an important role. The better outside options a buyer has (from previous negotiations), the better deal he is likely to get in the current negotiation. After the negotiation with a particular supplier ends with an deal, this deal becomes an outside option for the buyer since all previous deals are recallable. To streamline the analysis, in our model, we consider the outside option as a deal at hand rather than a potential future deal. Another advantage is that when the outside option is a deal at hand, the credibility of this threat will not be under question.

In the above setting, the buyer will be in an increasingly better bargaining position as he continues with the negotiations so long as he gets increasingly better deals. In this regard, it is conceivable that certain sequence of negotiations is better than others for the buyer.

Suppose the supplier base is has n players, there will be $\sum_{j=1}^{n} P(n, j)$ possible negotiation sequences in total where $P(n, j)$ is the j-permutations of a set of size n and it is given by

$$P(n, j) = \frac{n!}{(n-j)!}$$

It will be in general not possible to enumerate all possible negotiation sequences.

5.1 The Negotiation Sequencing Model and Solution Methodology

Let S_i denote supplier i and E_i the expected gain of the buyer from negotiating with supplier i. Using the results from the bargaining game, we can write E_i as follows:

$$\begin{aligned} E_i &= \frac{1}{2}\Big[X_B + (\pi - X_{S_i})\Big] \\ &= \frac{1}{2}\Big[\pi + W_B - W_S\Big] \end{aligned} \tag{8}$$

As one can see, the expected gain of the buyer does not depend on the breakdown probability of the negotiation. This is due to the fact that both parties have the same probability of becoming the offering party in the game, and the effect of the breakdown probability cancel out as we take the expected value.

We now introduce the notation necessary to define the negotiation sequencing problem:

n : Number of suppliers in the supplier base.

x_{ij}^k : 1 if the buyer negotiates with supplier i in the jth position of a negotiation sequence of length k, 0 otherwise.

W_b^0 : The outside options that the buyer has before starting the negotiation.

d_{ij}^k : The contribution to total gain when negotiating with supplier i in the jth position of a negotiation sequence of length k,

Δ_i : The difference between supplier i's total surplus and outside option i.e., $\Delta_i = \pi_i - W_{S_i}$

We define the negotiation sequencing problem as follows:

$$Max\,\{\, V_k,\, W_b^0 \;\mid k = 1, 2, .., n\,\}$$

where

$$V_k = Max\Big[\, \sum_{i=1}^{k}\sum_{j=1}^{k} d_{ij}^k x_{ij}^k + \tfrac{W_b^0}{2^k} \mid \sum_{i=1}^{k} x_{ij}^k = 1,\; \sum_{j=1}^{k} x_{ij}^k = 1,\; x_{ij}^k \in \{0,1\}\Big]$$

The second part of the objective is to take into account the contribution of the initial outside option which will be discussed later. We can see that the model for V_k is an assignment problem if we can compute the weight d_{ij}^k's *a priori*. This is addressed in the following proposition.

Proposition 8: The d_{ij}^k values are given by following:

$$d_{ij}^k = \frac{\Delta_i}{2^{k-j+1}} \tag{9}$$

Proof : Let W_j' be the outside option of the buyer after negotiating with jth supplier which can be expressed as;

$$W_j' = \frac{\Delta_{(j)} + W_{j-1}'}{2}$$

and let (j) represent the index of the supplier negotiated at jth place. The total gain of the buyer after the kth negotiation is;

$$\begin{aligned}
Total\ gain &= \frac{\Delta_{(k)} + W_{k-1}'}{2} = \frac{\Delta_{(k)}}{2} + \frac{\Delta_{(k-1)}}{2^2} + \frac{W_{k-1}'}{2^2} \\
&= \frac{\Delta_{(k)}}{2} + \frac{\Delta_{(k-1)}}{2^2} + \frac{\Delta_{(k-2)}}{2^3} + \frac{W_{k-2}'}{2^3} \\
&= \sum_{j=1}^{k} \frac{\Delta_{(j)}}{2^{k-j+1}} + \frac{W_b^0}{2^k}
\end{aligned}$$

Hence, the contribution of negotiating with supplier i at the jth place is the one defined in the proposition.

□

Now one can solve the negotiation sequencing problem by plugging in d_{ij}^k values and solve the models for V_k , $k = 1..n$. Since each V_k is an assignment problem, we may represent the k-supplier problem as a minimum-cost network flow problem in Figure 2.

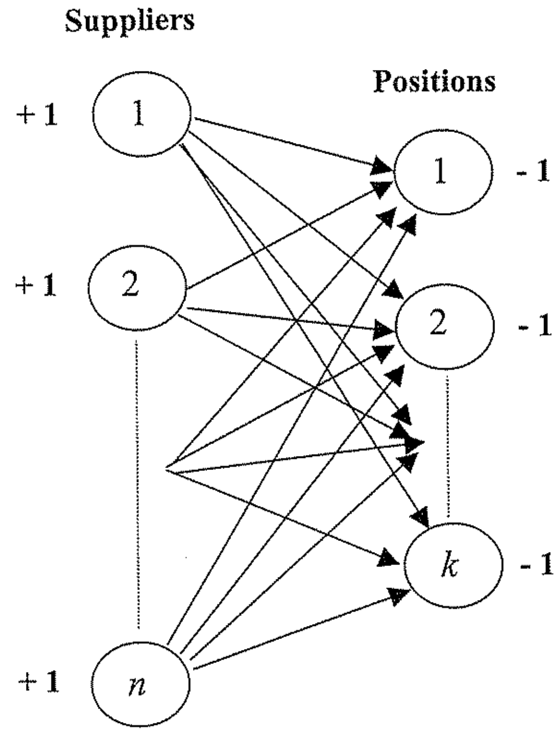

Figure 1 : Min-Cost Flow representation of k-supplier problem

The weight on the arc going from node i to node j is set to $-\,d^k_{ij}$. +1 and -1 values represent the surplus and demand, respectively, associated with the respective nodes. Solution of this min-cost network flow problem finds the assignment for each of the k suppliers to the k positions in the negotiation sequence that would maximizes the total gain for the buyer. Due to the special structure of the weight d^k_{ij} (marginal contribution to the total gain), the optimal sequence has the following properties.

Proposition 9: In the optimal sequence, the following condition is always satisfied

$$\Delta_{(j)} \leq \Delta_{(j+1)}$$

Proof : Consider the contributions of negotiating with supplier i at jth place and supplier l at $j+1th$ place in an optimal sequence. Since the sequence is optimal, it must hold that ;

$$\frac{\Delta_i}{2^{k-j+1}}+\frac{\Delta_l}{2^{k-(j+1)+1}} \geq \frac{\Delta_l}{2^{k-j+1}}+\frac{\Delta_i}{2^{k-(j+1)+1}}$$

$$\Rightarrow \frac{\Delta_i + 2\Delta_l}{2^{k-j+1}} \geq \frac{2\Delta_i + \Delta_l}{2^{k-j+1}} \Rightarrow \Delta_i \leq \Delta_l \text{ or } \Delta_{(j)} \leq \Delta_{(j+1)} \text{ in general.}$$

□

Thus, in the optimal sequence, the buyer would defer negotiating with the supplier who has a larger margins between potential surplus (π_i) and outside options (W_{S_i}) (i.e., potentially more fruitful for the buyer). This makes intuitive sense, since this represents the buyer's desire to get the best strategical position (strengthen his own outside options) before negotiating on more fruitful deals (in order to get the most out of them).

From the buyer's point of view, it may not be beneficial to negotiate with all suppliers in the market. Along with the negotiation sequencing problem is additional question of which subset of suppliers to negotiate with. In the following proposition, we specify the criteria for the buyer to continue negotiating with additional suppliers .

Proposition 10: In the optimal sequence of length k, the following relation always holds true

$$\Delta_{(k)} \geq \sum_{j=1}^{k-1} \frac{\Delta_{(j)}}{2^{k-1-j+1}} + \frac{W_b^0}{2^{k-1}}$$

Proof : If we consider negotiating with k suppliers in the optimal sequence versus negotiating with first $k-1$ suppliers in the optimal sequence, it is easy to write the following relation:

$$\frac{\Delta_{(k)}}{2} + \sum_{j=1}^{k-1} \frac{\Delta_{(j)}}{2^{k-j+1}} + \frac{W_b^0}{2^k} \geq \sum_{j=1}^{k-1} \frac{\Delta_{(j)}}{2^{k-1-j+1}} + \frac{W_b^0}{2^{k-1}}$$

$$\Rightarrow \frac{\Delta_{(k)}}{2} + \frac{1}{2}\left[\sum_{j=1}^{k-1} \frac{\Delta_{(j)}}{2^{k-1-j+1}} + \frac{W_b^0}{2^{k-1}}\right] \geq \sum_{j=1}^{k-1} \frac{\Delta_{(j)}}{2^{k-1-j+1}} + \frac{W_b^0}{2^{k-1}}$$

$$\Rightarrow \frac{\Delta_{(k)}}{2} \geq \frac{1}{2}\left[\sum_{j=1}^{k-1} \frac{\Delta_{(j)}}{2^{k-1-j+1}} + \frac{W_b^0}{2^{k-1}}\right]$$

$$\Rightarrow \Delta_{(k)} \geq \sum_{j=1}^{k-1} \frac{\Delta_{(j)}}{2^{k-1-j+1}} + \frac{W_b^0}{2^{k-1}}$$

□

Intuitively, the expected gain of negotiating with an additional supplier should be at least as good as the outside options at hand. The gain of the current deals will be degraded in the sense that it only indirectly affects the outcome of the next negotiation if the negotiations continue, or it goes into the final deal with half of its value, if the negotiations stop. Therefore, the expected gain of adding another supplier into the negotiation should be sufficient to counter the degradation on the current deal. In the following, we provide a numerical example for the negotiation sequencing problem.

5.2 An illustrative numerical example

Suppose there are 15 potential suppliers. We set the following parameter values for the buyer and market demand: c = 50, a = 1000, b = 2 . We have produced the s_i values, supplier unit costs, from a uniform distribution between 1 and 100 and rounded the values to nearest integer. Given the unit costs of the suppliers, we then find the system optimal solution and the corresponding surplus, π_i, for each supplier i using equation (4). Further, we assume that the outside option of each supplier is a random portion of the associated surplus, distributed uniformly from 1 to 100%. The problem data produced in this fashion is given in Table 1

Table 1 : Problem data

Supplier	s_i	π_i	W_i	Δ_i
1	10	940900.0	498677.0	442223.0
2	69	884540.3	778395.4	106144.8
3	88	866761.0	615400.3	251360.7
4	26	925444.0	629301.9	296142.1
5	20	931225.0	679794.3	251430.8
6	70	883600.0	229736.0	653864.0
7	56	896809.0	538085.4	358723.6
8	39	912980.3	766903.4	146076.8
9	8	942841.0	782558.0	160283.0
10	47	905352.3	18107.0	887245.2
11	1	949650.3	911664.2	37986.0
12	82	872356.0	235536.1	636819.9
13	32	919681.0	634579.9	285101.1
14	62	891136.0	828756.5	62379.5
15	51	901550.3	820410.7	81139.5

The number of potential solutions for this small example is

$$\sum_{j=1}^{15} \frac{15!}{(15-j)!} = 3.55463\text{x}10^{12}$$

We set the outside option of the supplier to 10,000 and construct and solve the k-supplier problems as we described in the previous section. The solution of the k-supplier problems for $k = 1..15$ is given in Table 2. From the table we can see that after negotiating with 5 suppliers, continuing with more negotiations would degrades the total gain that the buyer could obtain at the end of the negotiations. Thus the buyer should only negotiate with 5 out of 15 suppliers. The sequence of suppliers that corresponds to the solution of the 5-supplier is $\{4, 1, 12, 6, 10\}$

Table 2 : Solutions of k-supplier problems when k=1,..,15

k	Solution Value
1	481123
2	644589
3	714816
4	733080
5	735303
6	735070
7	734105
8	733329
9	732617
10	732174
11	731896
12	731732
13	731639
14	731587
15	731544

6. Conclusions

In this paper, we propose a bargaining theoretic approach to supply chain contracting and coordination. We analyzed the supply contracting with the focus of distributing the surplus in a non-cooperative fashion under a bargaining game. We have included the some real aspects of the process, such as the outside option effect, probability of breakdown of the negotiations, random proposer. Using the subgame perfect equilibrium results of the bargaining game we define the negotiation sequencing problem and show that the problem can be solved as a network flow problem.

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