A Bargaining Game for Supply Chain Contracting

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Abstract

This paper examines a bargaining theoretic approach to supply chain coordination. We first propose a one-supplier, one-buyer infinite horizon bargaining game for supply chain contracting, where the buyer negotiates the order quantity and wholesale price with a sourcing supplier. We show that in subgame perfect equilibrium, the channel coordinated optimal quantity is also optimal for the players, but the players must negotiate the surplus generated by the contract in a bargaining game. The model allows us to predict the negotiation outcome between a buyer and a supplier considering their outside options and the breakdown probability. Motivated by emerging applications in electronic marketplaces, we then propose a one-buyer multiple-supplier negotiation sequencing model where the buyer could determine an optimal subset of sourcing suppliers to negotiate with, and the sequence to carry out the negotiations so as to maximize her expected gain. We show that the one-supplier bargaining game serves as a building block and the negotiation-sequencing problem can be solved as a network flow problem.

1. Introduction

Planning and coordination in the supply chain has attracted a great deal of attention in the last two decades due to the general trend in the industry toward strategic sourcing and alliances. A significant part of the research in supply chain coordination has been model-based approach on supply chain contracting (c.f., Tsay et al., 1999 for a comprehensive review). This line of research focuses on sourcing decisions to be made by the supplier and the buyer in the supply chain. A general research goal has been to devise coordination mechanisms (e.g., contracts) that provide each player the incentive to implement the collectively optimum, "channel-coordinated" solution. For instance, the coordination contract may stipulate a transfer payment between the supplier and the buyer, providing each of them the incentive to optimize the system's marginal profit function. Game theoretic models are often used to characterize the player's incentives.

A supply chain contract may include clauses specifying quantity, pricing, minimum purchase commitments, buyback or return policies, lead time, and allocation rule, etc., to be negotiated between the buyer and the supplier. Some popular forms of supply chain contracts in the literature include quantity discounts (Lal and Staelin 1984, Parlar and Wang, 1994), quantity flexibility (Tsay, 1999), minimum commitment (Bassok and Anupindi, 1997), and buy-back contracts (Donohue, 1996). A majority of the research focuses on the price and quantity clauses in the supply chain contract. A related body of research investigates supplier-buyer coordination from the viewpoints of multi-echelon inventory decisions. Among these are work by Lal and Staelin (1984), Lee and Wang, (1999), Chen (1999), and Cachon and Zipkin (1999). An inventory coordination scheme may suggest a particular contract form through which the jointly optimized system solution could be achieved at equilibrium. A commonly used approach is to assume a minimum acceptable profit by the opposing party, then optimize the contract terms from one player's point of view. Supply chain coordination has been examined in many other contexts as well, for instance, Jin and Wu (2000) consider supply chain contracting in the context of on-line auctions and electronic markets. Jin and Wu (2001) extend the notion of supply chain contracting to consider capacity reservation contracts in the high-tech industry.

While the supply contract literature provides deeper understanding of the incentives between any given pair of supplier and buyer, the broader aspects of contracting are often overlooked. First, the competition from other suppliers (or buyers) in the marketplace is often ignored, i.e., most analysis starts from the point where the buyer has already chosen the sourcing supplier, and concentrates on finalizing the detailed contract terms. Second, it is typically assumed that the surplus generated from the channel-coordinated contract will be split between the players using some pre-determined static scheme. For instance, in the well-known two-part linear contract (Katz, 1989), the supplier receives a side payment equal to the extra surplus generated by the system-optimum contract. In this paper, we consider the possibility that additional outside options are easily accessible, and the players involved in the negotiation may at some point choose to abort the current negotiation. In this context, the process of contract negotiation can be more accurately captured by a bargaining process where the players dynamically exchange offers while weighing their current option against potential future options. Thus, the bargaining process captures an important, dynamic dimension of contracting that was not there before. This extension is particularly important in the emerging environment of electronic marketplaces, where outside options are both plentiful and easily accessible. We believe that in this environment the dynamics of supplier-buyer interaction will change fundamentally, and the basic premise of supply contract research deserves re-examination. This forms the motivation for this research.

This paper is set out to examine the following research questions: With the presence of outside options, why and when would the supplier and buyer in a negotiation accept a coordination contract? When the coordination mechanism guarantees a greater overall gain, how do the buyer and supplier split the extra surplus? How does a player weigh the deal on hand against other opportunities present in the market? Given proper information, could the buyer (the supplier) use the insights from simple bargaining situation to form a negotiation strategy in more general settings? To answer these questions we propose a bargaining theoretic model for supply chain contracting. To streamline our analysis, we will focus on the simple one-part linear contracts, but many of the insights can be generalized to more complex contract forms.

As defined in Muthoo (1999), "...a bargaining situation is a situation in which two players have a common interest to cooperate, but have conflicting interest over exactly how to cooperate (p. 1)." Bargaining theory is a branch of game theory that deal with the bargaining situations between two parties. In particular, if the bargaining game is single-shot, one may characterize its Nash equilibria. If the game is repetitive, as is the case in our analysis, one may characterize its subgame perfect equilibrium (SPE). In our case, the two parties are the buyer and the sourcing supplier who are to negotiate a contract that would distribute a surplus (e.g., profit) generated from mutual efforts. There has been two main stream of research on bargaining theory: axiomatic, and strategic models. Nash (1950 and 1953) lays down the framework for the axiomatic Nash Bargaining Solution where he first defines the basic axioms that any bargaining solution should "naturally" satisfy, and then shows that the Nash product is a unique solution satisfying the axioms. Kalai and Smorodinsky (1975) replace a controversial axiom from the original Nash proposal and revise the unique solution. Binmore (1987) summarizes the efforts over the years that either relaxes or add to the Nash axioms and gives further analysis of the Nash's bargaining model. An important characteristic of the axiomatic approach is that they leave out the actual process of negotiations while focusing on the expected outcome based on pre-specified solution properties. Rubinstein (1982) lays out the framework for strategic bargaining models. He suggests an alternating offer bargaining procedure where the players take turns in making offers and counter offers to one another until an agreement is reached. The players face time-discounted gain (a "shrinking pie") which provide them the incentive to compromise. In each iteration, a player must decides to either (1) accept the opponent's offer (in which case the bargaining stops), or (2) propose a counter offer. Binmore et al. (1988) propose a third option where a player may decide to leave the current negotiation and opt for her "outside options" (e.g., previously quoted deals). Ponsati and Sacovics (1998) also consider outside options as part of the Rubinstein model. Muthoo (1995) considers outside options in the form of a search in a bargaining search game. An important aspect of the extended bargaining model is to allow the possibility for the negotiation to breakdown. Binmore et. al (1986) study a version of the alternating offer model with breakdown probability. In this model, there is no time pressure (time-discounted gain) exists, but there is a probability that a rejected offer is the last offer made in the game, meaning that the negotiation breaks down. An intuitive comparison between the axiomatic and strategic bargaining theory can be found in Sutton (1986).

In this paper, we model the process of supply chain contracting as an alternating offer repetitive bargaining game under complete information. To capture the main essence of supply contract negotiation, we examine a bargaining model between a pair of buyer and supplier, taking into consideration the effects of outside options, breakdown probability, and "random proposers." We establish the subgame perfect equilibrium result for this dynamic game. Using this bargaining model and the SPE results as the basic building block, we consider a more complex contracting situation as follows: a buyer is to negotiate with a set of potential sourcing suppliers in an open market, the buyer must determine (1) which subset of suppliers to negotiate with, and (2) in what sequence should the negotiation take place. We formulate a *negotiation sequencing problem* from the buyer's perspective. We show that both decisions lead to different payoffs for the buyer, and we propose an efficient solution methodology that optimizes the buyer's decision.

2. A Bargaining Game for Supply Chain Contracting

2.1. The General Setting

We first consider a basic bargaining game where a buyer and a single sourcing supplier enter the negotiation for a supply chain contract. We assume that both players are rational, self-interested, and risk neutral (expected value maximizers). The buyer is subject to a price-sensitive market demand. Before entering the negotiation, both players have recallable outside options (e.g., a previously quoted deal with another supplier/buyer) with known net profits.

We define the *total surplus* of a coordination contract as the total profits generated from the system-optimal solution. Our main concern is about the splitting of this *total surplus* between the two parties. The main purpose of a supply chain contract is to overcome an inefficiency known as *double marginalization* (Spengler, 1950). This is because without coordination, the supplier and the buyer only have the incentive to optimize their own profit margin, and their collective decision is always less efficient than what could have been achieved by the system-optimal. Thus, the aim of a coordination contract is to provide the incentive for both players to implement the system-optimal solution, which results in higher total profits for the collective whole. As discussed earlier, we believe this particular aspect (surplus splitting) of the negotiation should be part of the contracting process. In the following, we will use a simple one-part linear contract to illustrate the analysis framework. Note that the exact form of the supply contract does not affect our analysis significantly so long as the coordination contract generates a non-negative *total surplus,* π. We now summarize the notations as follows:

q : Contractual quantity to be transacted between the buyer and the supplier.

w: Unit wholesale price to be charged by the supplier.

$P(q)$: Unit market price given quantity q, defined as a linear function as follows:

$P(q) = \dfrac{a-q}{b}$, where a is the maximum market demand and b is the slope

c : Unit cost for the buyer.

s : Unit cost for the supplier.

$\Pi_B(q)$: Profit function of the buyer based on the order quantity q

$\Pi_S(w)$: Profit function of the supplier based on the wholesale price w

W_B : Recallable outside option for the buyer in net profit.

W_S : Recallable outside option for the supplier in net profit.

The classical setting of a one-part linear contract can be described by the following sequence of events: (1) the supplier announces the wholesale price w, (2) the buyer chooses an order quantity q, (3) The supplier produces each unit at cost s, then deliver quantity q to the buyer. (4) The buyer assumes a unit handling cost c and sales q units of the product to the market at a price $P(q)$. The profit functions of the buyer and the supplier, Π_B and Π_S, respectively, are as follows:

$$\Pi_B(q) = (P(q) - w - c)q \tag{1}$$

$$\Pi_S(w) = (w - s)q$$

The natural bounds for q and w are;

$$0 \le q \le a, \quad s \le w \le \frac{a}{b} - c$$

Given an announced wholesale price w, from the first order condition, $\Pi'_B(q_B^*) = 0$ the buyer's optimal order quantity is

$$q_B^* = \tfrac{1}{2}[a - b(w + c)] \tag{2}$$

Now, suppose the supply chain is centrally controlled, the system's profit would be

$$\Pi_C(q) = (P(q) - c - s)q \tag{3}$$

Again, from the first-order condition, the system-optimum quantity q^* is as follows:

$$q^* = \tfrac{1}{2}[a - b(s + c)] \tag{4}$$

First, note that the system's profit is only a function of the order quantity, but not the wholesale price, as the latter is merely an *internal* transfer between the buyer and the supplier. Second, by comparing (2) and (4), we know that the buyer will only order the system optimal quantity q^* when the wholesale price w^* is set to the supplier's cost s. If the supplier is to make a profit by raising the wholesale price $w > s$, the buyer will order less, which results in sub-optimal system performance. This is the classical problem used to explain the effect of *double marginalization* (c.f., Spengler, 1950; Cachon, 1999; Tsay et al., 1999). Thus, we may define the maximum *total surplus* for this contract as follows:

$$\begin{aligned} \pi &= [P(q^*) - w^* - c]\, q^* \\ &= \frac{1}{4}\frac{[a - b(s + c)]^2}{b} \end{aligned} \tag{5}$$

Clearly, since the wholesale price $w^* = s$ the supplier would have received no profit. The two-part linear contract (Katz, 1989) discussed in Section 1 was used to address this issue, where the supplier would offer the wholesale price at cost, while receiving a side payment from the buyer with an amount equal to π. The two-part contract achieves system-optimal by transferring the system's marginal profit function to the supplier. Jeuland and Shugan (1983) propose a different scheme which split the *total surplus* between the supplier and the buyer by a pre-specified friction *f*. This eliminates the effect of *double marginalization* since the wholesale price is no longer necessary to define the player's profit, and the buyer would order q^* to optimize his own profit.

2.2 The Bargaining Game

Recognizing the fact that the system optimal is determined by the order quantity, while the wholesale price merely define the *split* of the *total surplus* (π), we propose a generalization of the above contracting schemes by modeling the process of *surplus splitting* as a bargaining game. Inspired by Rubinstein's *alternating offer* model, our bargaining game considers three main factors which influence the bargaining process: (1)

the probability that the negotiation breaks down after a given offer, (2) the presence of the players' outside options, and (3) the two players are equally likely to make an offer during a particular bargaining iteration. The first factor, *breakdown probability,* allows us to capture the situation when the parties are not perfectly rational, when the player anticipates a more attractive future deal, or other considerations that can not be measured by monetary gains (e.g., trust and goodwill). The second factor, players' outside options, is important since the player with better outside options is in a strategically better position to negotiate, and is more likely to receive a larger share from the total surplus. The third factor, random proposer, allows us to treat each iteration of the bargaining processes equally in that regardless of who makes the previous offer, either player could initiate the offer in an iteration. This is modeled by assuming that the two players have equal probability of making the next offer after a given offer so long as the negotiation continues.

As common in the bargaining literature, we will assume that the total maximum surplus from the current trade is greater than or equal to the sum of the outside options, i.e., $\pi \geq W_B + W_S$. This is reasonable since otherwise at least one of the players will receive a deal worse than her outside option, and would have no incentive to enter the negotiation to begin with. We also assume that when a player is *indifferent* between accepting the current offer or waiting for future offers, she will accept the current offer.

We now define the sequence of events in our bargaining game as follows:

1. With equal probability, one of the two players proposes a contract with parameters (q, w).
2. The other player may either
   (a) accept the offer (the negotiation ends), or
   (b) reject the offer and wait for the next round.
3. With a certain probability, the negotiation breaks down and the players take their corresponding outside options.
4. If the negotiation continues, the game restarts from step 1.

3. Subgame Perfect Equilibrium of the Bargaining Game

The subgame perfect equilibrium (SPE) strategies are the ones that constitute the *Nash equilibrium* in every iteration (the subgame) of a repeated game. The central idea underlying this concept is that equilibrium strategies should specify optimal behavior from any point in the game onward (Mas-Colell et al., 1995). As well-known in the dynamic game literature, every finite game of perfect information has a pure strategy subgame Nash equilibrium that can be derived through backward induction (Sorin, 1992). In the perfect equilibrium of our alternative offer bargaining game, a player will accept a proposal if it offers no less than what she expects to gain in the future, given the strategy set of the other player. In the following, we will show that in the SPE the players will agree on the order quantity that maximizes the *total surplus* (which achieves channel coordination). This reduces the bargaining game to the negotiation of the *split* of the total surplus. This result is important in that it reduces the complexity of the bargaining process, and it allows us to generalize our analysis to other contracting forms so long as channel coordination can be achieved at SPE, thus the bargaining process is used to negotiate the *internal* transfer (e.g., profit-sharing) between the buyer and the supplier.

Proposition 1: In SPE, the system optimal quantity is also optimal for the buyer and the suppliers, i.e., $q^*_{SPE} = \frac{1}{2}[a - b(s + c)]$

Proof: Let $(q_B,\ w_B)$ be the quantity, wholesale price offer made by the buyer, and $\alpha_S,\ \alpha_B$, be the least amount of share that supplier and buyer can accept in SPE respectively. Then buyer has the following maximization problem;

$$Max\ \Pi_B(q_B, w_B) = (P(q_B) - w_B - c)q_B$$

$$s.t. \qquad (w_B - s)q_B \geq \alpha_S$$

Since the constraint is binding, $(w_B - s)q_B{=}\alpha_S$, we may substitute w_B in turns of q_B and α_S. We thus have,

$$q^*_B = \frac{1}{2}[a - b(s + c)], \text{ and } w^*_B = s + \frac{2\alpha_S}{a - b(s + c)}$$

On the other hand, if the supplier is the offering party, she would have the following optimization problem:

$$Max\ \Pi_S(q_S, w_S) = (w_S - c)q_S$$

$$s.t. \qquad (P(q_S) - w_S - c)q_S \geq \alpha_B$$

Similarly, the constraint is binding, i.e., $(P(q_S) - w_S - c)q_S{=}\alpha_B$, by substituting w_S we will get the optimal order quantity

$$q^*_S = \frac{1}{2}[a - b(s + c)], \text{ and then } w^*_S = \frac{1}{2}\frac{-(a - cb)^2 + s^2b^2 + 4b\alpha_B}{b(-a + cb + sb)}$$

Due to the random proper assumption stated earlier, no matter which player makes the offer, each bargaining iteration is a subgame of the same structure. Thus, the perfect equilibrium strategies of the players are the same in each subgame. Thus, comparing to equation (4), we know that the system-optimum order quantity is also optimal in SPE, i.e., $q^*_{SPE} = q^*_B = q^*_S = \frac{1}{2}[a - b(s+c)] = q^*$.

□

The *proposition* shows that in *subgame perfect equilibrium* the channel coordinated optimal quantity is also optimal for the players. Recall that the total system profit is not a function of the wholesale price, and the wholesale price merely defines an *internal* transfer between the buyer and the supplier. Thus, given the channel coordinated order quantity, the players only need to negotiate to split of the total surplus, $\pi = \frac{1}{4}\frac{[a-b(s+c)]^2}{b}$, corresponding the system optimal profit. We thus revise step 1 of our bargaining game as follows:

1$'$. With equal probability, one of the two players proposes a split of the total contract surplus π

The remainders of the game stay the same. We now analyze this *surplus-splitting bargaining game* in a time-line of offers, and find the surplus splitting in SPE. Note that this is an infinite horizon, complete information bargaining game similar to the alternating offer bilateral bargaining game proposed by Rubinstein (1982). Our goal is to show that there is a *unique* subgame perfect equilibrium strategies of the buyer and the supplier. This is important as knowing the unique SPE strategy will allow us to predict the outcome of bargaining, and the actual bargaining game will terminate in one round where the players each use their SPE strategy. In order to show this, we will use an approach similar to that taken in Shaked and Sutton (1984), Sutton (1986), and Muthoo (1999), where they show, for an alternating offer bargaining game, there is a unique SPE that satisfies two essential properties of the game: (1) *no delay*: a player's equilibrium offer will be always accepted by the other player, and (2) *stationarity*: in equilibrium, a player always make the same offer. We will make use of the same properties.

We introduce the following additional notations:

$M_B\,(M_S)$: The maximum payoff the buyer (the supplier) receives in the SPE of any subgame starting with her offer.

$m_B\,(m_S)$: The minimum payoff the buyer (the supplier) receives in the SPE of any subgame starting with her offer.

p : The probability that the negotiation will continue to the next round.

Starting by assuming that in SPE there is an infinite number of solutions that lead to payoffs ranging from m_B to M_B for the buyer, and m_S to M_S for the supplier. We aim to show that there is a unique SPE strategy for the buyer and supplier, say X_B and X_S, respectively, such that $M_B = m_B = X_B$, and $M_S = m_S = X_S$.
We first find the expression for M_B (m_B) by constructing the scenario under which the buyer's payoff would be maximized (minimized). As shown in Figure 1, we define a subgame beginning with the buyer's offer where the buyer gets the maximum SPE payoff. The nodes represent the player who makes the next offer, or the breakdown event, while the arc points to the subsequent event its associated probability. The payoffs labeled in tree represent the supplier's point of view where she receives the minimum possible share in perfect equilibrium (this corresponds to the maximum share the buyer would gain).

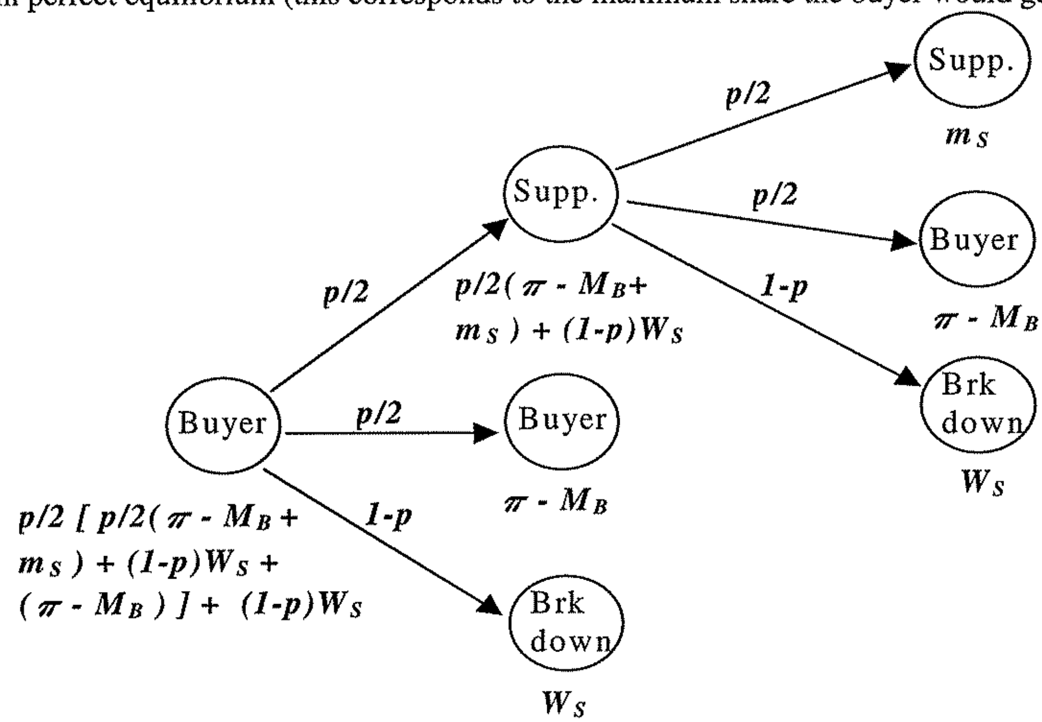

Figure 1. A tree defining the largest share the buyer could obtain in a subgame perfect equilibrium

Figure 1 shows all the possible branches for the subgame starts by the buyer. Because of the *no-delay* and *stationarity* properties, we only need to include up to two rounds of the game although the game has infinite horizon. To derive the SPE condition for this buyer-initiated subgame we evaluate the tree backward from the leaf nodes. The first leave node (following the path buyer-supp-supp) corresponds the supplier's offer. As this tree corresponds to the scenario where the supplier receives the minimum possible share in perfect equilibrium, the supplier receives m_S. Similarly, in the second leave node (buyer-supp.-buyer), the buyer makes the offer (and receives the maximum payoff M_B) and the supplier would receives $\pi - M_B$. In the case when the bargaining breaks down the supplier gets her outside option W_S. The second tier of the tree is constructed following the same logic. But now the supplier's payoff is the expected value computed from the third tier, which is equal to

$$(1-p)W_S + \frac{p}{2}(\pi - M_B + m_S)$$

When the buyer makes the offer, the supplier again receives $\pi - M_B$. In the event the bargaining breaks down, the supplier again receives W_S. Going back one offer to the root node, we see that in the perfect equilibrium the supplier would expect to gain the minimum share of

$$\frac{p}{2}\Big[(1-p)W_S + \frac{p}{2}(\pi - M_B + m_S) + \pi - M_B\Big] + (1-p)W_S$$

Thus, the maximum possible share the buyer could obtain in a SPE is as follows:

$$M_B = \pi - \left[\frac{p}{2}\Big[(1-p)W_S + \frac{p}{2}(\pi - M_B + m_S) + \pi - M_B\Big] + (1-p)W_S\right] \quad (6)$$

With slight modification, we can also derive the minimum SPE share that the buyer could get in a subgame starting with the buyer's offer. In specific, we only need to replace M_B with m_B, and m_S with M_S in the above equation. Thus, the minimum payoff the buyer receives in the subgame is as follows:

$$m_B = \pi - \left[\frac{p}{2}\Big[(1-p)W_S + \frac{p}{2}(\pi - m_B + M_S) + \pi - m_B\Big] + (1-p)W_S\right] \quad (7)$$

Since the roles of the supplier and buyer are symmetrical in the game, we can easily write the expressions for M_S and m_S by changing the indices. Given the four different sets of relationships, we end up with four linear equations with four unknowns. The following proposition summarizes the subgame perfect equilibrium descriptions.

Proposition 2: The following system of equations defines the buyer's and the supplier's payoffs in the subgame perfect equilibrium:

$$
\begin{aligned}
M_B &= \pi - \left[\tfrac{p}{2}\left[(1-p)W_S + \tfrac{p}{2}(\pi - M_B + m_S) + \pi - M_B\right] + (1-p)W_S\right] \\
m_B &= \pi - \left[\frac{p}{2}\left[(1-p)W_S + \frac{p}{2}(\pi - m_B + M_S) + \pi - m_B\right] + (1-p)W_S\right] \\
M_S &= \pi - \left[\frac{p}{2}\left[(1-p)W_B + \frac{p}{2}(\pi - M_S + m_B) + \pi - M_S\right] + (1-p)W_B\right] \\
m_S &= \pi - \left[\frac{p}{2}\left[(1-p)W_B + \frac{p}{2}(\pi - m_S + M_B) + \pi - m_S\right] + (1-p)W_B\right]
\end{aligned}
$$

Solving the linear equations in Proposition 2 gives us the exact expressions for M_B, m_B, M_S, and m_S. We can now specify the subgame perfect equilibrium strategies of the buyer and supplier as follows.

Proposition 3: The unique subgame perfect equilibrium strategies of the players are given as follows:

- If the buyer is the offering party, she will ask for X_B share of the surplus and leave $\pi - X_B$ to the supplier.

- If the supplier is the offering party, she will ask for X_S share of the surplus and leave $\pi - X_S$ to the buyer, where X_B and X_S are defined as follows :

$$
\begin{aligned}
X_B &= \pi - W_S - \frac{p^2}{2(2-p)}(\pi - W_B - W_S) \\
X_S &= \pi - W_B - \frac{p^2}{2(2-p)}(\pi - W_B - W_S)
\end{aligned}
$$

Proof: If we solve the system of equations describing the SPE for the players given in *Proposition* 2, we find that

$X_B = M_B = m_B$, *and* $X_S = M_S = m_S$

In other words, the player's maximum and minimum SPE share are equal, thus X_B and X_S correspond to the unique SPE offers that the buyer and the supplier should follow, respectively. □

From the expressions for X_B (X_S), we can see that the buyer's (supplier's) equilibrium strategy is to offer the supplier (the buyer) her outside option, and a "risk premium" ($\frac{p^2}{2(2-p)}(\pi - W_B - W_S)$). Note that the term $(\pi - W_B - W_S)$ represents the mutual gain for both players. Thus, the risk premium corresponds to a split of the mutual gain considering the breakdown risk p associate with current and future bargaining iterations.

It should be clear that so long as the mutual gain is non-negative, it is always more attractive for the players to accept the current offer than taking the outside option.

Proposition 1 earlier states that in SPE, the system-optimum order quantity is also optimum for the players, and determining the wholesale price is equivalent to bargaining for the split of the contract surplus, π. We now write the expressions for the wholesale price offers. If the contract (q^*, w) is to be offered by the buyer, she would ask for a share of size X_B, thus the following equation should hold;

$$(P(q^*) - c - w_B)q^* = X_B \quad \Rightarrow \quad w_B = P(q^*) - \left(\frac{X_B}{q^*} + c\right)$$

So in SPE, it the buyer's best strategy to offer a wholesale price at market price less her handling cost c, and a marginal surplus such that she could obtain X_B share from the total surplus. Similarly, the suppliers' wholesale price offer satisfies;

$$(w - s)q^* = X_S \quad \Rightarrow \quad w_S = \frac{X_S}{q^*} + s$$

Recall that q^* is the system optimal order size.

4. Analysis of the Bargaining Game

An important implication from the SPE analysis is that the bargaining game should end in one iteration when one of the players initiates negotiations by making the SPE offer, and the other player will accept the offer. This is true because the subgame perfect equilibrium strategies of the players is to make an offer such that the opponent is indifferent between accepting the current offer or waiting for future offers. Since the general framework is an alternating offer bargaining game, one important issue remains is whether there exists a first mover advantage in the game. We attend to this matter in the following proposition.

Proposition 4: There exits a first mover advantage in the game. First mover advantage diminishes as the probability of breakdown decreases, and goes to zero if the probability of breakdown is zero.

Proof: If we take the difference between the SPE shares of the players we get;

$$X_B - (\pi - X_S) = X_S - (\pi - X_B) = \frac{(\pi - W_B - W_S)(2 - p^2 - p)}{2 - p}$$

Since $2 \geq p^2 + p$ and $(\pi - W_B - W_S) \geq 0$, the right hand side of the expression always yield a value greater than or equal to zero. It becomes zero when $p = 1$, or equivalently when the probability of breakdown is zero.□

The proposition shows that there is indeed a first-mover advantage in the game, but when the probability of breakdown diminishes, so is the first mover advantage. In any case, since the two players have equal probability to initiate the bargaining, the game remains fair. It should be intuitively clear that in an SPE offer, each player should receive an offer no less than her outside option. However, a question remains is that whether the first-mover advantage has an effect on this property. This is addressed by the following proposition.

Proposition 5: In the SPE, both the offering party and the opponent receive a share of the total surplus that is greater than or equal to their respective outside options.

Proof : For the player who initiate the offer, we can find the difference between her SPE share and her outside options is as follows:

$$X_B - W_B = X_S - W_S = \frac{1}{2}\frac{(\pi - W_B - W_S)(4 - p^2 - 2p)}{2 - p}$$

The right hand side of the expression is always positive. Hence the initiating player will always receive an offer no less than her outside option. For the player who does not initiate the offer, the difference between her SPE share and her outside option is as follows;

$$(\pi - X_S) - W_B = (\pi - X_B) - W_S = \frac{1}{2}\frac{(\pi - W_B - W_S)p^2}{(2 - \mathrm{p})}$$

Again, the right hand side of this expression is always positive. Thus, both players receive an amount no less than their outside options in SPE, regardless of who initiates the offer. □

Another question of interest is how does the size of the player's (and her opponents) outside option affect the SPE share. Intuitively, the larger the player's outside option is , the higher she would expect the current offer to be. This relationship is further explored in the following propositions.

Proposition 6: As long as the breakdown probability p>0, the SPE share of the offering party is increasing in her outside option, and decreasesing in the opponent's outside option.

Proof : The proposition can be easily confirmed by first taking the first and second derivatives of the SPE offer of the buyer with respect to W_B, W_S:

$$\frac{\partial X_B}{\partial W_B} = \frac{p^2}{2(2-p)} \geq 0\,, \quad \frac{\partial^2 X_B}{\partial^2 W_B} = 0$$

$$\frac{\partial X_B}{\partial W_B} = \frac{-\,(4-p^2-2p)}{2(2-p)} < 0\,, \quad \frac{\partial^2 X_S}{\partial^2 W_S} = 0$$

We cam confirm the same for the supplier.

□

One interesting aspect of the game is that the offering party's SPE share maximizes when the breakdown probability approaches 1. This is described in the following proposition.

Proposition 7: The SPE share of the offering party is maximized when the probability of breakdown goes to 1, and the maximum share equals to the total surplus less the outside option of the other party.

Proof : If we take the first derivative of the SPE offer of the buyer with respect to p, we have:

$$\frac{\partial X_B}{\partial p} = \frac{1}{2}\frac{p[(p-4)(\pi - W_B - W_S)]}{(2-p)^2}$$

Here $\frac{\partial X_B}{\partial p}$ is less than or equal to zero for $0 \leq p \leq 1$, since we assume that $\pi - W_B - W_S \geq 0$. Hence, X_B is maximized at $p = 0$. Since the roles of the buyer and the supplier are symmetrical, the same reasoning follows for the supplier. □

The Proposition is intuitive in that if the offering party knows that the negotiation is likely to breakdown, in which case the opponent will only receive her outside option, the offering party would have no incentive to make an offer larger than the opponent's outside option.

5. The Negotiation Sequencing Problem

Up to this point we have been focusing on the bargaining game between a single buyer-supplier pair. We now illustrate the use of this bargaining analysis as a building block for multiple buyer-supplier environments. We are interested in the environment where the buyer faces a large number of potential sourcing suppliers. In this setting, the buyer could obviously benefit from negotiating with more than one supplier, but more importantly, the buyer could optimize the *sequence* of which she negotiates with the suppliers. We will show that the negotiation sequence is important. The negotiation sequencing problem is motivated by recent developments in electronic markets. *Freemarkets,* a leading eProcurement service provider, reports that in the first quarter of year 2000 the number of buyers participating in their procurement markets was less than 50, while the number of suppliers was more than 4,000. While current electronic markets rarely offer negotiation services, semi-automated and off-line negotiation after the buyer-supplier matching is not unusual.

As the buyer negotiates with a list of suppliers in a sequence, there are two alternative assumptions concerning previously negotiated deals: (1) all previously negotiated deals (with the suppliers) are recallable, and (2) the previous deals are not recallable. In our analysis, we will assume that the sequence of negotiations occur in a relatively short time, and all deals are recallable. This assumption applies to industries where the buyer has favorable bargaining power, and could ask for a time period during which a negotiated deal stay valid unless the buyer decides otherwise. From our earlier analysis, it should be clear that the outside options play an important role in the bargaining result. The better outside options a buyer has (from previous negotiations), the better deal she is likely to get in subsequent negotiations. After the negotiation with a particular supplier ends with a deal, this deal becomes an outside option for the buyer since all deals are recallable. Note that in our model we restrict outside option to be deals at hand, rather than potential future deals. An advantage is that when the outside options are deals at hand, they carry a more credible threat to the opponent.

Under the above setting, it is conceivable that a certain sequence of negotiations is better than others for the buyer. Suppose the supplier base has n players, there will be $\sum_{j=1}^{n} P(n,j)$ possible negotiation sequences in total, where $P(n,j)$ is the j-permutations of a set of size n, given by

$$P(n, j) = \frac{n!}{(n-j)!}$$

For a reasonable size n, it will not be possible to enumerate all possible negotiation sequences.

5.1 The Negotiation Sequencing Model and The Solution Methodology

Let S_i denote supplier i and E_i the expected gain of the buyer from negotiating with supplier i. Using the results from the bargaining game, we can write E_i as follows:

$$\begin{aligned} E_i &= \frac{1}{2}\Big[X_B + (\pi - X_{S_i})\Big] \\ &= \frac{1}{2}\Big[\pi + W_B - W_{S_i}\Big] \end{aligned} \tag{8}$$

As one can see, the expected gain of the buyer does not depend on the breakdown probability of the negotiation. This is due to the fact that both parties have the same probability of becoming the offering party in the game, and the effects of the breakdown probability cancel out as we take the expected value.

We now introduce the notations for the *negotiation sequencing problem*:

n : Number of suppliers in the supplier base.

x_{ij}^k : 1 if the buyer negotiates with supplier i in the jth position of a negotiation sequence of length k, 0 otherwise.

W_b^0 : The outside options that the buyer has before starting the negotiation.

d_{ij}^k : The contribution to total gain when negotiating with supplier i in the jth position of a negotiation sequence of length k,

Δ_i : The difference between supplier i's total surplus and outside option i.e., $\Delta_i = \pi_i - W_{S_i}$

We define the negotiation sequencing problem as follows:

$$Max\,\{\,V_k,\, W_b^0 \;\mid k = 1, 2, .., n\,\}$$

where

$$V_k = Max\Big[\,\sum_{i=1}^{k}\sum_{j=1}^{k} d_{ij}^k\, x_{ij}^k + \tfrac{W_b^0}{2^k} \mid \sum_{i=1}^{k} x_{ij}^k = 1,\; \sum_{j=1}^{k} x_{ij}^k = 1,\; x_{ij}^k \in \{0, 1\}\Big]$$

The second part of the objective is to take into account the contribution of the initial outside option which will be discussed later. We can see that the model for V_k is an assignment problem if we can compute the weight d_{ij}^k's *a priori*. This is addressed in the following proposition.

Proposition 8: From the buyer's perspective, negotiating with supplier i at the jth place would contribute to the buyer's overall gain by an amount d_{ij}^k as given by the following: $d_{ij}^k = \frac{\Delta_i}{2^{k-j+1}}$ (9)

Proof: Let W_j' be the outside option of the buyer after negotiating with jth supplier which can be expressed as;

$$W_j' = \frac{\Delta_{(j)} + W_{j-1}'}{2}$$

and let (j) represent the index of the supplier negotiated at jth place. The total gain of the buyer after the kth negotiation is;

$$\begin{aligned} Total\,gain &= \frac{\Delta_{(k)} + W_{k-1}'}{2} = \frac{\Delta_{(k)}}{2} + \frac{\Delta_{(k-1)}}{2^2} + \frac{W_{k-1}'}{2^2} \\ &= \frac{\Delta_{(k)}}{2} + \frac{\Delta_{(k-1)}}{2^2} + \frac{\Delta_{(k-2)}}{2^3} + \frac{W_{k-2}'}{2^3} \\ &= \sum_{j=1}^{k} \frac{\Delta_{(j)}}{2^{k-j+1}} + \frac{W_b^0}{2^k} \end{aligned}$$

Hence, the marginal contribution of negotiating with supplier i at the jth place is $d_{ij}^k = \frac{\Delta_i}{2^{k-j+1}}$.□

Given this proposition, we can define the *negotiation sequencing problem* using the pre-computed d_{ij}^k values, and solve the optimization problem for V_k , $k = 1..n$. Since each V_k is an assignment problem, we may represent the k-supplier problem as a minimum-cost network flow problem depicted in Figure 2.

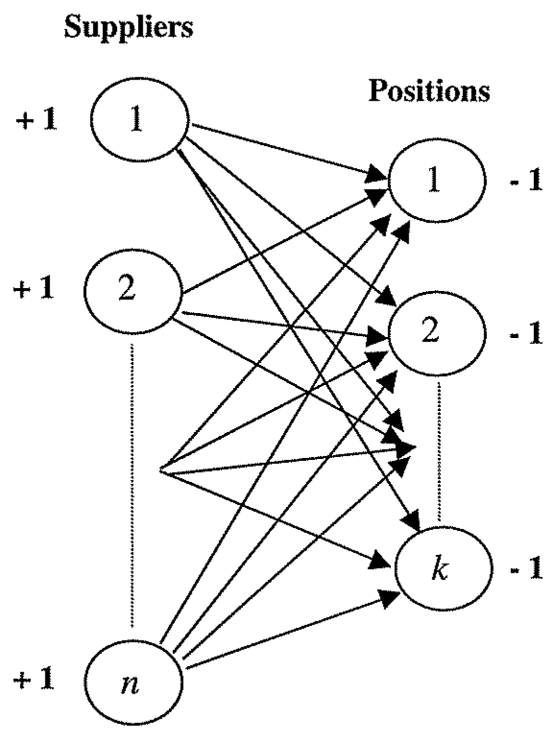

Figure 2 : The k-supplier Negotiation Sequencing Problem as a Min-Cost Flow Network

For each arc going from node i to node j we assign a weight $-d_{ij}^{k}$. The +1 and -1 values represent the surplus and demand in the network, respectively. The solution to this minimum cost network flow problem finds the assignment for each of the n suppliers to the k positions in the negotiation sequence that would maximize the total gain for the buyer. Due to the special structure of the weights d_{ij}^{k} (marginal contribution to the buyer's total gain), the optimal negotiation sequence has the following properties.

Proposition 9: In the optimal negotiation sequence, the following condition is always satisfied: $\Delta_{(j)} \leq \Delta_{(j+1)}$

Proof : Consider the contributions of negotiating with supplier i at jth place and supplier l at $j+1th$ place in an optimal sequence. Since the sequence is optimal, it must hold that ;

$$\frac{\Delta_i}{2^{k-j+1}}+\frac{\Delta_l}{2^{k-(j+1)+1}} \geq \frac{\Delta_l}{2^{k-j+1}}+\frac{\Delta_i}{2^{k-(j+1)+1}}$$

$$\Rightarrow \quad \frac{\Delta_i+2\Delta_l}{2^{k-j+1}} \geq \frac{2\Delta_i+\Delta_l}{2^{k-j+1}} \quad \Rightarrow \Delta_i \leq \Delta_l \text{ or } \Delta_{(j)} \leq \Delta_{(j+1)} \text{ in general.}$$

□

Thus, in the optimal sequence, the buyer would defer negotiating with the supplier who has a larger margin between potential surplus (π_i) and outside options (W_{S_i}) (i.e., the supplier who is potentially more fruitful for the buyer). This makes intuitive sense, since it is the buyer's best strategic to strengthen her outside options by progressively negotiating more fruitful deals.

Another consideration is that it may not be beneficial for the buyer to negotiate with all suppliers in the market. Thus, an additional question is which subset of suppliers should the buyer negotiate with. In the following proposition, we specify the criteria for the buyer to decide whether to continue negotiating with additional suppliers.

Proposition 10: In the optimal sequence of length k, the following relation should always holds true

$$\Delta_{(k)} \geq \sum_{j=1}^{k-1} \frac{\Delta_{(j)}}{2^{k-1-j+1}} + \frac{W_b^0}{2^{k-1}}$$

Proof : If we consider negotiating with k suppliers in the optimal sequence versus negotiating with the first $k-1$ suppliers in the optimal sequence, we can write the following relationship:

$$\begin{aligned}
&\frac{\Delta_{(k)}}{2} + \sum_{j=1}^{k-1} \frac{\Delta_{(j)}}{2^{k-j+1}} + \frac{W_b^0}{2^k} \geq \sum_{j=1}^{k-1} \frac{\Delta_{(j)}}{2^{k-1-j+1}} + \frac{W_b^0}{2^{k-1}} \\
\Rightarrow\ &\frac{\Delta_{(k)}}{2} + \frac{1}{2}\left[\sum_{j=1}^{k-1} \frac{\Delta_{(j)}}{2^{k-1-j+1}} + \frac{W_b^0}{2^{k-1}}\right] \geq \sum_{j=1}^{k-1} \frac{\Delta_{(j)}}{2^{k-1-j+1}} + \frac{W_b^0}{2^{k-1}} \\
\Rightarrow\ &\frac{\Delta_{(k)}}{2} \geq \frac{1}{2}\left[\sum_{j=1}^{k-1} \frac{\Delta_{(j)}}{2^{k-1-j+1}} + \frac{W_b^0}{2^{k-1}}\right] \\
\Rightarrow\ &\Delta_{(k)} \geq \sum_{j=1}^{k-1} \frac{\Delta_{(j)}}{2^{k-1-j+1}} + \frac{W_b^0}{2^{k-1}}
\end{aligned}$$

□

In other words, as soon as the condition stated in the proposition is violated, the buyer should stop negotiating with additional suppliers. Intuitively, the expected gain of negotiating with an additional supplier should be at least as good as the outside options at hand. Furthermore, the gain of the current deals is discounted in the sense that it only indirectly affects the outcome of the next negotiation if the bargaining continues. Thus, the expected gain of adding another supplier into the negotiation should be sufficient to counter the discounting effects on the current deals, otherwise there is no benefit for the buyer to continue further negotiations. In the following section, we provide a numerical example for the negotiation sequencing problem.

5.2 An illustrative numerical example

Suppose there are 15 potential suppliers. We set the following parameter values for the buyer and market demand: c = 50, a = 1000, b = 2 . We generate the suppliers' unit costs (s_i) from a uniform distribution between 1 and 100. Given the unit costs of the suppliers, we then find the system optimal solution and the corresponding surplus, π_i, for each supplier i using equation (4). Further, we assume that the outside option of each supplier is a random portion of the associated surplus π_i, distributed uniformly from 1% to 100%. The problem data produced in this fashion is given in Table 1 .

Table 1 : Example Problem data

Supplier	s_i	π_i	W_i	Δ_i
1	10	940900.0	498677.0	442223.0
2	69	884540.3	778395.4	106144.8
3	88	866761.0	615400.3	251360.7
4	26	925444.0	629301.9	296142.1
5	20	931225.0	679794.3	251430.8
6	70	883600.0	229736.0	653864.0
7	56	896809.0	538085.4	358723.6
8	39	912980.3	766903.4	146076.8
9	8	942841.0	782558.0	160283.0
10	47	905352.3	18107.0	887245.2
11	1	949650.3	911664.2	37986.0
12	82	872356.0	235536.1	636819.9
13	32	919681.0	634579.9	285101.1
14	62	891136.0	828756.5	62379.5
15	51	901550.3	820410.7	81139.5

The number of potential solutions for this small example is

$$\sum_{j=1}^{15} \frac{15!}{(15-j)!} = 3.55463\text{x}10^{12}$$

We set the outside option of the supplier to 10,000, and we construct and solve the k-supplier problem as we described in the previous section. The solution for the k-supplier problem for $k = 1..15$ is given in Table 2. From the table we can see that after negotiating with 5 suppliers, continuing with more negotiations would degrades the total gain for the buyer. Thus the buyer should only negotiate with 5 out of 15 suppliers. The sequence of suppliers that corresponds to the solution of the 5-supplier is {4, 1, 12, 6, 10}.

Table 2 : Solutions of k-supplier problems when k=1,..,15

k	Solution Value
1	481123
2	644589
3	714816
4	733080
5	735303
6	735070
7	734105
8	733329
9	732617
10	732174
11	731896
12	731732
13	731639
14	731587
15	731544

6. Issues Related to the Transaction Costs

One aspect of the negotiation process we have ignored in the bargaining model is the transaction cost, the fixed cost that both parties incur when they go through a turn of offers. This cost could be associated with the time elapses during the negotiations (the cost of delaying decisions) or the efforts required to continue the negotiation. Denote C_B and C_S the fixed unit transaction cost for the buyer and the supplier, respectively. As we described in Section 3, without the transaction costs, the smallest share that the supplier would agree to accept in SPE is given by;

$$\tfrac{p}{2}\Big[(1-p)W_S + \tfrac{p}{2}(\pi - M_B + m_S) + \pi - M_B\Big] + (1-p)W_S$$

Now, recall that this is the expected gain we derive from two rounds of offers, we need to subtract the supplier's transaction costs over two rounds of offers:

$$\frac{p}{2}\Big[(1-p)W_S + \frac{p}{2}(\pi - M_B + m_S) + \pi - M_B\Big] + (1-p)W_S - 2C_S$$

This is the minimum share that the supplier would accept in SPE, taking into consideration the transaction costs. Using the same reasoning as in Section 3, we can write the expression for the maximum buyer's share in SPE as follows;

$$M_B = \pi - \left[\frac{p}{2}\left[(1-p)W_S + \frac{p}{2}(\pi - M_B + m_S) + \pi - M_B\right] + (1-p)W_S - 2C_S\right]$$

Going through the same derivation for m_B, M_S, and m_S, we can rewrite the system of linear equations that define the SPE solution;

$$\begin{aligned}
M_B &= \pi - \left[\frac{p}{2}\left[(1-p)W_S + \frac{p}{2}(\pi - M_B + m_S) + \pi - M_B\right] + (1-p)W_S - 2C_S\right] \\
m_B &= \pi - \left[\frac{p}{2}\left[(1-p)W_S + \frac{p}{2}(\pi - m_B + M_S) + \pi - m_B\right] + (1-p)W_S - 2C_S\right] \\
M_S &= \pi - \left[\frac{p}{2}\left[(1-p)W_B + \frac{p}{2}(\pi - M_S + m_B) + \pi - M_S\right] + (1-p)W_B - 2C_B\right] \\
m_S &= \pi - \left[\frac{p}{2}\left[(1-p)W_B + \frac{p}{2}(\pi - m_S + M_B) + \pi - m_S\right] + (1-p)W_B - 2C_B\right]
\end{aligned}$$

Solving this system of equations yields the following results:

$$\begin{aligned}
M_B &= m_B = X_B \\
&= \pi - W_S + \\
&\qquad \frac{(p^4 + p^3 - 2p^2)(\pi - W_B - W_S) - (4p^2 + 8p - 16)C_S - 4p^2 C_B}{(2p^3 - 2p^2 - 8p + 8)} \\
M_S &= m_S = X_S \\
&= \pi - W_B + \\
&\qquad \frac{(p^4 + p^3 - 2p^2)(\pi - W_B - W_S) - (4p^2 + 8p - 16)C_B - 4p^2 C_S}{(2p^3 - 2p^2 - 8p + 8)}
\end{aligned}$$

Again, there is a unique SPE strategy for the buyer and the supplier, since $M_B = m_B$ and $M_S = m_S$. The SPE strategy of the players are the same as those described in Proposition 3 except that the definitions for X_B and X_S are as given in the above equations. In the following, we summarize the effects of the transaction costs on the players' SPE strategy.

Proposition 11: For any given value of p, the offering party's SPE share increases linearly in her opponent's unit transaction cost, and decreases linearly in her own unit transaction cost.

Proof: If we take the first derivative of X_B, with respect to C_S and C_B, we get;

$$\frac{\partial X_B}{\partial C_S} = \frac{1}{2}\,\frac{-4p^2 - 8p + 16}{p^3 - p^2 - 4p + 4}, \qquad \frac{\partial X_B}{\partial C_B} = -2\frac{p^2}{p^3 - p^2 - 4p + 4}$$

By inspection, we can see that $\frac{\partial X_B}{\partial C_S}$ will always be greater than zero and $\frac{\partial X_B}{\partial C_B}$ will always be less than zero for $0 \leq p \leq 1$.

6. Conclusions

In this paper, we propose a bargaining theoretic approach to supply chain contracting and coordination. We propose a repeated game with outside options and the breakdown probability over an infinite horizon. The game is symmetrical in that the buyer or the supplier could initiate a subgame with equal probability. We show that in the subgame perfect equilibrium, the channel-coordinated (system-optimum) order quantity is optimal for the buyer and the supplier. This reduces the bargaining game to the negotiation of the wholesale price, which in effect determines the players' split of the total surplus (generated from the coordinating contract). We show that in subgame perfect equilibrium there is a unique strategy for the players which in essence makes an offer to the opponent such that she is indifferent between accepting the current offer or waiting for future offers. This result has important implications in that the bargaining game will end in one iteration when one of the two players initiates the negotiation with the perfect equilibrium offer. We further show that there is a first-mover advantage in this game, but the advantage diminishes as the probability of breakdown approaches zero.

Using the results from the basic bargaining game, we examine the situation when a buyer is to negotiate with a set of sourcing suppliers in an open market. We formulate this *negotiation sequencing problem* as a minimum cost network flow problem, which can be solved efficiently. We show that the SPE results derived from the one-supplier, one-buyer game can be used to compute the marginal gain for the buyer which defines the arc-weight in the network flow problem. More importantly, not only can the model identify the optimal negotiation sequence, but it also identifies the subset of suppliers who the buyer should negotiate with. We conclude the analysis with a numerical example and further analysis on the effects of the transactions cost.

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