Lead-Time Coordination Between Marketing and Operations in an Internal Market

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LEAD-TIME COORDINATION BETWEEN MARKETING AND OPERATIONS IN AN INTERNAL MARKET

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ABSTRACT

Motivated by the operational environment in high-tech industries, this paper examines the coordination between *marketing* and *operations* when time to market and capacity utilization are both main factors. In contrast to earlier joint lot-sizing literature which focuses on price-quantity coordination, our model captures the inter-relationship between order quantity, capacity level, and lead-time. Besides the setup and inventory holding costs considered in EOQ-type models, our model incorporates additional cost components for capacity consumption, WIP inventory, and lead-time (as related to the customer's safety stock cost). We first analyze the centralized system-optimum solution and a decentralized Stackelberg game solution for the model, then compare their asymptotic performance as different cost components vary. We examine three coordination schemes for *marketing* and *operations*. We show that the well-known *quantity discount* scheme does not coordinate the system in this setting. We propose a *lead-time reduction* scheme where *operations* offers a more favorable lead-time provided that *marketing* convinces the customer to place a larger order. We show that lead-time and order-size alone do not guarantee to coordination the system. It is essential to add a price-adjustment scheme in the form of a discount or extra-payment in order for the system to be perfectly coordinated.

1. INTRODUCTION

Capacity is the most significant planning element in high-tech industries such as semiconductor and optoelectronics. Effective management of production capacity have significant cost and lead-time implications, and oftentimes drives the competitiveness of the firm. In this paper, we consider a general setting where a certain manufacturing capacity is allocated to a product type as a customer order arrive. The amount of allocated capacity drives the *lead-time* for the customer order, while at the same time has an impact on system utilization and inventory. A typical industry practice is to delegate the handling of *customer orders* and the management of *capacity allocation* to different decision entities in *marketing* and *operations* divisions, respectively. This is setup to establish proper check and balance, and to maintain accountability (Karabuk and Wu, 2001). However, since *marketing* and *operations* are typically rewarded based on different performance metrics (e.g., sales revenue vs. operational efficiency), coordinating their activities for the greater benefits of the firm is a managerial challenge. Motivated by the operation of a major U.S. semiconductor firm, we consider an *internal market* mechanism between marketing and operations, where *marketing* interacts with outside customers concerning price and order quantity, while *operations* negotiates with marketing to set the capacity level (thus the *lead-time*) for the orders. The *marketing* and *operations* entities are rewarded according to their respective performance measures, while an internal transfer between the two entities is used by the firm to facilitate coordination and a higher overall efficiency.

A well-established framework for production coordination is joint lot-size determination in a tightly coupled system with lot-for-lot production over a long-term. The main decision is to jointly choose a lot-size for two or more parties with conflicting objectives. Typically, EOQ-type models based on setup and inventory costs are used to characterize different decision problems for the supplier, the buyer, and the channel coordinator. The different perspectives of the decision makers stem from the fact that the supplier's setup cost and the buyer's inventory costs, respectively, are higher than their counter part's. Consequently, the buyer prefers a smaller lot-size than that desired by the supplier, and the system-optimum. Starting with Goyal (1976), Monahan (1984), and Lee and Rosenblatt (1986), the joint economic lot-sizing literature has examined various mechanisms where the supplier induces the buyer to choose a larger lot-size than she would of her own accord. *Quantity discounts* is among the most commonly used coordination schemes proposed in this context. Goyal and Gupta (1989) reviewed the line of literature that use *quantity discounts* as a means to achieving coordination in joint lot-sizing models with deterministic demands. More recently, Abad (1994) and Weng (1995) examined the case where customer demand is a price-sensitive function: the former introduces a cooperative game allowing the production lot-size and the order size to be different, the latter shows that quantity discounts is not sufficient to achieve channel coordination when demand is price-sensitive. Corbett (1999) considers the asymmetric information case where the players do not have full information about their opponent's costs. He shows that mechanisms achieving coordination under complete information may fail to do so under asymmetric information.

In contrast to the inventory coordination and joint lot-sizing literature, we propose to examine the inter-relationship between *order quantity, capacity level*, and *lead-time*, a much overlooked subject. What is essential in our analysis is to replace the EOQ-based decision model with a lead-time based model for the decision makers involved. While the same analytical approach could be applied to general supplier-retailer coordination, the level of information sharing required in our analysis is more likely to occur in closely coupled internal systems. We consider an internal market where *marketing* and *operations* are viewed as independent, self-interested market participants rather than unconditionally cooperating parties in a monolithic system. This provides a more accurate representation of the dynamics we have observed in the semiconductor industry.

The introduction of *lead-time* and related costs such as *WIP inventory* and *safety stock* allow us to address some practical operational concerns that have not been examined in the EOQ-based coordination literature.

The work by Zipkin (1986) and Karmarkar (1993) offer much insights on lead-time estimation using basic elements of congestion in the production environment. The setup cost in classical EOQ models is typically excluded from the lead-time model since it is not part of the basic trade-off. Thus, the cost structure we considered in our analysis deviates from both the inventory coordination and the lead-time literature. Another main difference of our analysis is the inclusion of capacity levels as the operations' decision problem. With a few exceptions (c.f., Porteus 1985,1986), most existing models in supply chain coordination consider only order-size decisions. Our model considers both *order size* and *capacity allocation* decisions which allow us to bring the lead-time based lot-sizing results to the supply chain coordination setting.

Several researchers also consider lead-time in the context of coordination. However, in most cases, lead-time is modeled as fixed parameter or as a realization of a random variable (c.f., Grout and Christy (1993)), in either case it is not affected by the player's decisions. We propose a model where the actual order lead-time depends on the *order-size* set by *marketing,* and the *capacity allocation* given by *operations*. A few researchers do consider lead-time as influenced by the decision variables, but focusing on quite different aspects of the problem (c.f., Haussman 1994, Barnes-Schuster 1997, and Iyer and Bergen 1997, Moinzadech and Ingene, 1993). Most relevant to this paper is the work by Caldentey and Wein (1999) where the supplier determines her capacity levels thereby effecting the lead-time, while the buyer adjusts the base-stock levels using the (s-1,s) policy. They analyze both centralized and decentralized cases in this environment, and propose coordination mechanisms for the latter case. This differs from our work in that they assume a base-stock policy for the buyer, and hence there is no customer satisfaction considerations. We consider the case where the order-size is a marketing (buyer) decision justified by the lead-time cost (as a measure of customer satisfaction), and other system costs such as setup and inventory.

2. THE MODEL

We consider the setting where *marketing* negotiates orders with customers in the market, and *operations* controls the manufacturing capacity necessary to producing the item. We consider a single product where the market demand is stochastic but stationary with rate D. Items are produced in a lot-for-lot basis where the lot-size (Q) is set identical to the order-size placed by *marketing*. We characterize the congestion in the manufacturing process as a M/M/1 queue with FIFO discipline. The arrival rate to the manufacturing facility is defined as D/Q. Actual production capacity to be allocated for the item is determined by *operations,* and we define capacity in terms of lot processing-rate, u. Following the analysis in (Zipkin, 1986) we assume that u is independent of lot-size. Given these assumptions, the expected manufacturing lead-time (L) for an order with size Q is given by:

$$L = \frac{1}{u-\frac{D}{Q}} \tag{1}$$

We consider two major cost components for *operations:* holding cost for work-in-process (WIP) inventory, and the capacity consumption cost. The WIP holding cost (C_{wip}) is defined as follows:

$$C_{wip} = h_{wip} DL \tag{2}$$

where h_{wip} is the unit holding cost. The capacity consumption cost (C_{cap}) is defined with a unit cost m as follows.

$$C_{cap} = mu \tag{3}$$

Thus, we define the *operations'* profit function as follows:

$$\pi_o = -h_{wip} D \frac{1}{u-\frac{D}{Q}} - mu \tag{4}$$

We assume that *marketing*'s handling cost is similar to that of a typical retailer/wholesaler. Thus, we consider the setup and finished goods inventory costs as in a EOQ-type model. Further, to characterize customer satisfaction we consider the lead-time cost C_L given per unit cost k_L as follows :

$$C_L = k_L L^b \quad where\ 0.5 \leq b \leq 1 \tag{5}$$

To streamline the analysis, we will set $b = 1$ throughout the paper. Further, to draw contrast with the *operations'* first cost component we define a new parameter $h_L = \frac{k_L}{D}$, such that $C_L = h_L DL$. Since D is a parameter, this would have no effect on the analysis. Based on these considerations, the *marketing's* profit function is as follows:

$$\pi_m = -h_L D \frac{1}{u - \frac{D}{Q}} - \frac{KD}{Q} - \frac{hQ}{2} \tag{6}$$

Based on their corresponding profit functions, *marketing* determines the order size (Q), while *operations* sets the manufacturing capacity (u) for the item.

2.1 The Centralized Solution

We now define the centralized solution from the firm's perspective so as to establish a reference point for decentralized coordination between *marketing* and *operations*. Following the convention in the supply chain coordination literature, we construct the centralized solution assuming a monolithic system optimizer would determine the manufacturing capacity (u) and order size (Q) so as to maximize the firm's profit (i.e., to achieve marketing-operations coordination). The steady-state expected profit for the system optimizer is defined as follows:

$$\begin{aligned} \pi &= \pi_o + \pi_m \\ &= -h_{wip} D \frac{1}{u - \frac{D}{Q}} - mu - h_L D \frac{1}{u - \frac{D}{Q}} - \frac{KD}{Q} - \frac{hQ}{2} \\ &= -(h_{wip} + h_L) D \frac{1}{u - \frac{D}{Q}} - mu - \frac{KD}{Q} - \frac{hQ}{2} \end{aligned} \tag{7}$$

In the following, we characterize the solution of this centralized optimization problem.

Proposition 1: *There is a unique solution to the centralized profit-maximizing problem and is given by:*

$$u^* = \sqrt{\frac{Dh}{2(K+m)}} + \sqrt{\frac{(h_{wip}+h_L)D}{m}} \qquad (8)$$

$$Q^* = \sqrt{\frac{2(K+m)D}{h}} \qquad (9)$$

The corresponding profits are as follows:

Operation's profit:

$$\pi_o^* = -\frac{2h_{wip}+h_L}{\sqrt{h_{wip}+h_L}}\sqrt{mD} - \frac{m}{\sqrt{2(K+m)}}\sqrt{Dh} \qquad (10)$$

Marketing's profit:

$$\pi_m^* = -\frac{h_L}{\sqrt{h_{wip}+h_L}}\sqrt{mD} - \frac{(2K+m)}{\sqrt{2(K+m)}}\sqrt{Dh} \qquad (11)$$

Firm's total profit :

$$\begin{aligned}\pi^* &= -\frac{2h_L+2h_{wip}}{\sqrt{h_{wip}+h_L}}\sqrt{mD} - \frac{(2K+2m)}{\sqrt{2K+2m}}\sqrt{Dh} \\ &= -2\sqrt{D}\left(\sqrt{(h_L+h_{wip})m} - \sqrt{2(K+m)h}\right)\end{aligned} \qquad (12)$$

Proof: *π is a concave function with two variables. Using the first-order condition w.r.t to capacity u, i.e.,* $\frac{\partial \pi}{\partial u} = 0$, *we have the following solution:*

$$u^*(Q) = \frac{D}{Q} + \sqrt{\frac{(h_{wip}+h_L)D}{m}} \qquad (13)$$

All expressions in the proposition are direct consequences of (13). □

Interestingly, the optimal order-size Q^* is similar to the solution one would derive from an EOQ-type decision model by substituting the capacity cost parameter m with the setup cost in typical EOQ models. However, there is a significant difference between this model and EOQ-type models in that Q^* is optimal only when the manufacturing capacity is set to the optimal level u^*, i.e., both decisions must be made at the same time to achieve optimality. As we will show, this distinction yields quite different results and conclusions in the decentralized setting. Furthermore, the model assumes linear capacity adjustment cost as well as a batch-size independent processing rate. The similarity to EOQ model diminishes when capacity can not be adjusted, adjustment cost is non-linear, or the processing rate is batch-size dependent.

Consider the intuition behind equation (13): if the order-size is Q, we know that feasibility can be maintained at a capacity level of $\left(\frac{D}{Q}\right)$, but an additional amount of capacity $\sqrt{\frac{(h_{wip}+h_L)D}{m}}$ must be added to achieve optimality. Note that this latter term is proportional to sum of the WIP holding (h_{wip}) cost and the lead-time cost (h_L), while inversely proportional to the capacity cost m. This is also intuitive, as the lead-time related costs h_{wip} and h_L increase, *operations* should add more capacity, while the unit capacity cost m obviously has an adverse effect.

2.2 The Decentralized Stackelberg Solution

To examine the coordination between *marketing* and *operations*, we now consider the case where the two decision entities are decentralized and making their decisions in a sequential fashion as in the following Stackelberg game:

1. Based on direct interactions with the customer, *marketing* determines the order size (Q) for the product.
2. Given the order quantity Q, *operations* determines the capacity level (u) to be allocated to the product.
3. The production takes place and the order is fulfilled.

We assume that the players (*marketing* and *operations*) have complete information of the opponent's costs and profit function. In the following, we characterize the decentralized solution from the Stackelberg game.

Proposition 2: *The equilibrium solution for the Stackelberg game is given by*

$$u^0 = \sqrt{\frac{Dh}{2K}} + \sqrt{\frac{h_{wip}D}{m}} \tag{14}$$

$$Q^0 = \sqrt{\frac{2KD}{h}} \tag{15}$$

The corresponding profits are as follows:

Operations' profit:

$$\pi_o^0 = -\frac{2h_{wip}}{\sqrt{h_{wip}}}\sqrt{mD} - \frac{m}{\sqrt{2K}}\sqrt{Dh}$$

Marketing's profit:

$$\pi_m^0 = -\frac{h_L}{\sqrt{h_{wip}}}\sqrt{mD} - \frac{2K}{\sqrt{2K}}\sqrt{Dh} \tag{16}$$

Firm's total profit:

$$\pi^0 = -\frac{h_L+2h_{wip}}{\sqrt{h_{wip}}}\sqrt{mD} - \frac{2K+m}{\sqrt{2K}}\sqrt{Dh} \tag{17}$$

Proof: *Based on the complete information assumption, marketing is able to compute operations' best response for a given order size, Q, i.e., by maximizing the operations' profit function,* π_o *with respect to* u *for a given* Q. *Thus, we solve* $(\frac{\partial \pi_o}{\partial u}) = 0$ *and the corresponding best response function of the operations,* $u_{best}(Q)$ *is given by:*

$$u_{best}(Q) = \frac{D}{Q} + \sqrt{\frac{hwipD}{m}} \tag{18}$$

Based on the best response function, the marketing maximizes her profit by maximizing

$$\pi_m = -h_L D \frac{1}{\frac{D}{Q}+\sqrt{\frac{hwipD}{m}}-\frac{D}{Q}} - \frac{KD}{Q} - \frac{hQ}{2} \tag{19}$$

Solving this maximization problem yields the order size:

$$Q^0 = \sqrt{\frac{2KD}{h}} \tag{20}$$

When marketing orders Q^0, *operations will adjust the capacity to* u^0 *which is given by:*

$$u^0 = u_{best}(Q^0) = \sqrt{\frac{Dh}{2K}} + \sqrt{\frac{h_{wip}D}{m}} \tag{21}$$

All other expressions are the direct consequences of substituting u^0 *and* Q^0 *in the corresponding functions.*□

Note that Q^0 is always less than or equal to Q^* which confirms that the decentralized (non-coordinated) solution always yields a smaller order-size than what is optimal. However, such simple relationship does not exist between the capacity levels u^0 and u^*, which is based on parameters D, h, K, m, h_L, and h_{wip}. One interesting observation is that as the lead-time cost h_L increases, the optimal capacity level u^* also increases while u^0 stays the same, and at some point u^* exceeds u^0 (that is, if originally $u^0 \geq u^*$). In fact, h_L does not affect the decentralized solution at all; turn out this insensitivity to the lead-time cost is one reason that the decentralized solution is inefficiency. We will analyze this effect in greater detail in the following section.

2.3 Comparing the Centralized and the Decentralized Solutions

As shown in the previous section, when *marketing* and *operations* make their local decisions in sequence as the Stackelberg leader and follower, a set of decentralized solutions results. We use the Stackelberg game results to characterize the situation when *marketing* and *operations* do not coordinate, and we are interested in the efficiency gap caused by this lack of coordination. In Table 1, we first summarize the centralized (system optimal) and the decentralized (Stackelberg) solutions.

Table 1. Summary of Results from the Centralized and Decentralized Solutions

	Centralized Solution	Decentralized Solution
Order size	$Q^* = \sqrt{\frac{2(K+m)D}{h}}$	$Q^0 = \sqrt{\frac{2KD}{h}}$
Capacity	$u^* = \sqrt{\frac{Dh}{2(K+m)}} + \sqrt{\frac{(h_{wip}+h_L)D}{m}}$	$u^0 = \sqrt{\frac{Dh}{2K}} + \sqrt{\frac{h_{wip}D}{m}}$
Lead-time	$L^* = \sqrt{\frac{m}{(h_{wip}+h_L)D}}$	$L^0 = \sqrt{\frac{m}{h_{wip}D}}$
Operations' profit	$\pi_o^* = -\frac{2h_{wip}+h_L}{\sqrt{h_{wip}+h_L}}\sqrt{mD} - \frac{m}{\sqrt{2(K+m)}}\sqrt{Dh}$	$\pi_o^0 = -\frac{2h_{wip}}{\sqrt{h_{wip}}}\sqrt{mD} - \frac{m}{\sqrt{2K}}\sqrt{Dh}$
Marketing's profit	$\pi_m^* = -\frac{h_L}{\sqrt{h_{wip}+h_L}}\sqrt{mD} - \frac{2K+m}{\sqrt{2(K+m)}}\sqrt{Dh}$	$\pi_m^0 = -\frac{h_L}{\sqrt{h_{wip}}}\sqrt{mD} - \frac{2K}{\sqrt{2K}}\sqrt{Dh}$
Firm's profit	$\pi^* = -\frac{2h_{wip}+2h_L}{\sqrt{h_{wip}+h_L}}\sqrt{mD} - \frac{2K+2m}{\sqrt{2(K+m)}}\sqrt{Dh}$	$\pi^0 = -\frac{2h_{wip}+h_L}{\sqrt{h_{wip}}}\sqrt{mD} - \frac{2K+m}{\sqrt{2K}}\sqrt{Dh}$

The following proposition further characterizes the firm's total profit under the centralized and decentralized settings.

Proposition 3: *The decentralized solution is inefficient in that* $\pi^0 \leq \pi^*$, *the equality* $\pi^0 = \pi^*$ *holds only when the lead-time and capacity costs,* h_L *and m, are both 0.*

Proof: *First we show that* $\frac{2h_{wip}+2h_L}{\sqrt{h_{wip}+h_L}}\sqrt{mD} \leq \frac{2h_{wip}+h_{ss}}{\sqrt{h_{wip}}}\sqrt{mD}$. *It is sufficient to compare* $\frac{2h_{wip}+2h_L}{\sqrt{h_{wip}+h_L}}$ *with* $\frac{2h_{wip}+h_L}{\sqrt{h_{wip}}}$ *since m, D are positive terms. Therefore, we need to examine the following:*

$$\frac{2h_{wip}+2h_L}{\sqrt{h_{wip}+h_L}} \leq ?\ \frac{2h_{wip}+h_L}{\sqrt{h_{wip}}} \tag{22}$$

or equivalently,

$$\left(\frac{2h_{wip}+2h_L}{\sqrt{h_{wip}+h_L}}\right)^2 \leq ?\left(\frac{2h_{wip}+h_L}{\sqrt{h_{wip}}}\right)^2 \tag{23}$$

or

$4h_{wip} + 4h_L \leq ?\, 4h_{wip} + 4h_L + \frac{h_L^2}{hwip}$ (24)

Since h_L, h_{wip} *are non-negative, (24) holds. Note that the equality only holds when* $h_L = 0$. *Similarly, it can be shown that* $\frac{2K+2m}{\sqrt{2(K+m)}}\sqrt{Dh} \leq \frac{2K+m}{\sqrt{2K}}\sqrt{Dh}$ (25)

together with (24), *we have* $\pi^o \leq \pi^*$. $\square$

To further characterize the degree of inefficiency in the decentralized solution, we define three efficiency-loss ratios (ELR) as follows:

$ELR = \frac{\pi^0}{\pi^*}$, $ELR_o = \frac{\pi_o^0}{\pi_o^*}$; $ELR_m = \frac{\pi_m^0}{\pi_m^*}$

These ratios turn out to be rather complicated functions, however, it is possible to analyze the asymptotic behavior of the functions with respect to the change in each parameter. We summarize these results in Table 2.

Table 2: Asymptotic behavior of ELR, ELR_o, and ELR_m

	ELR	ELR_o	ELR_m
$h_{wip} \to \infty$	1	1	1
$h_{wip} \to 0$	∞	$\frac{\frac{m}{\sqrt{2K}}\sqrt{h}}{\sqrt{h_L m} + \frac{m}{\sqrt{2(K+m)}}\sqrt{h}}$	∞
$h_L \to \infty$	∞	∞	$\sqrt{\frac{h_{wip}+h_L}{h_{wip}}}$
$h_L \to 0$	$\frac{2\sqrt{h_{wip}m} + \frac{2K+m}{\sqrt{2K}}\sqrt{h}}{2\sqrt{h_{wip}m} + \sqrt{2(K+m)h}}$	$\frac{2\sqrt{h_{wip}m} + \frac{m}{\sqrt{2K}}\sqrt{Dh}}{2\sqrt{h_{wip}m} + \frac{m}{\sqrt{2(K+m)}}\sqrt{h}}$	$\frac{\frac{2K}{\sqrt{2K}}}{\frac{2K+m}{\sqrt{2K+2m}}}$
$K \to \infty$	1	$\frac{\frac{2h_{wip}}{\sqrt{h_{wip}}}}{\frac{2h_{wip}+h_L}{\sqrt{h_{wip}+h_L}}}$	1
$K \to 0$	∞	1	∞
$m \to \infty$	∞	∞	$\sqrt{\frac{h_{wip}+h_L}{h_{wip}}}$
$m \to 0$	1	$\frac{\frac{2h_{wip}}{\sqrt{h_{wip}}}}{\frac{2h_{wip}+h_L}{\sqrt{h_{wip}+h_L}}}$	1
$h \to \infty$	$\frac{2K+m}{2\sqrt{K(K+m)}}$	$\frac{\sqrt{2K+m}}{\sqrt{m}}$	$\frac{\frac{2K}{\sqrt{2K}}}{\frac{2K+m}{\sqrt{2K+m}}}$
$h \to 0$	$\frac{2h_{wip}+h_L}{2\sqrt{h_{wip}(h_{wip}+h_L)}}$	$\frac{\frac{2h_{wip}}{\sqrt{h_{wip}}}}{\frac{2h_{wip}+h_L}{\sqrt{h_{wip}+h_L}}}$	$\frac{\sqrt{h_{wip}+h_L}}{\sqrt{h_{wip}}}$

Based on the asymptotic results, we can discuss more thoroughly the efficiency loss in decentralized coordination and analyze the effects of different model parameters. Note that the worst case for the firm's efficiency-loss corresponds to the increase of lead-time cost h_L and unit capacity cost m, or the decrease of WIP holding cost h_{wip} or setup cost K. Recall that in the centralized solution capacity is adjusted based on *both* the marketing's lead-time cost h_L and operations' WIP costs h_{wip}, whereas in the decentralized case, the capacity depends solely on h_{wip} (see Table 1). In the decentralized case, when the marketing's lead-time cost h_L increases, the capacity level will not be properly adjusted (i.e., *operations* is insensitive to the customer's needs). Similarly, as operations' capacity consumption cost *m* increase, marketing does not adjust the order size as in the centralized solution (i.e., *marketing* is insensitive to the manufacturing's burden). A similar observation could be made for the WIP inventory cost h_{wip} and setup cost K. Thus, the difference between the centralized solution and the decentralized solution increases essentially due to the different scope of cost consideration incorporated in the decisions.

Another observation is that the effects of parameter changes can be quite different for the marketing's and the operations' perspectives. For example as $h_L \to \infty$ or $m \to \infty$, $ELR_o \to \infty$, but, $ELR_m \to \sqrt{\frac{h_{wip}+h_L}{h_{wip}}}$.

The following proposition states the basic relationship between the centralized and decentralized solutions.

Proposition 4: *There are three cases that describe the relationship between centralized and decentralized profit functions:*

a) $\pi_o^0 \le \pi_o^*, \pi_m^0 \le \pi_m^*$ *b)* $\pi_o^0 < \pi_o^*, \pi_m^0 \ge \pi_m^*$ *c)* $\pi_o^0 \ge \pi_o^*, \pi_m^0 < \pi_m^*$

Proof. *If* $\pi_o^0 > \pi_o^*$ *and* $\pi_m^0 \ge \pi_m^*$, *or* $\pi_o^0 \ge \pi_o^*$ *and* $\pi_m^0 > \pi_m^*$ *then* $\pi_o^0 + \pi_m^0 > \pi_o^* + \pi_m^*$ *which is not possible. Cases a), b) and c) cover all remaining cases.* □

Case $a)$ in the proposition represents the case where

$$\frac{2h_{wip}+h_L}{\sqrt{h_{wip}+h_L}} - 2\sqrt{h_{wip}} \le \frac{\sqrt{mh}}{\sqrt{2K}} - \frac{\sqrt{mh}}{\sqrt{2(K+m)}}, \frac{2K+m}{\sqrt{2(K+m)}}\sqrt{h} - \sqrt{2Kh} \le \frac{h_L\sqrt{m}}{\sqrt{h_{wip}}} - \frac{h_L\sqrt{m}}{\sqrt{h_{wip}+h_L}} \tag{28}$$

Case $b)$ represents the case :

$$\frac{2h_{wip}+h_L}{\sqrt{h_{wip}+h_L}} - 2\sqrt{h_{wip}} \le \frac{\sqrt{mh}}{\sqrt{2K}} - \frac{\sqrt{mh}}{\sqrt{2(K+m)}}, \frac{2K+m}{\sqrt{2(K+m)}}\sqrt{h} - \sqrt{2Kh} \ge \frac{h_L\sqrt{m}}{\sqrt{h_{wip}}} - \frac{h_L\sqrt{m}}{\sqrt{h_{wip}+h_L}} \tag{29}$$

Case c) represents the case :

$$\frac{2h_{wip}+h_L}{\sqrt{h_{wip}+h_L}} - 2\sqrt{h_{wip}} \geq \frac{\sqrt{mh}}{\sqrt{2K}} - \frac{\sqrt{mh}}{\sqrt{2(K+m)}}, \frac{2K+m}{\sqrt{2(K+m)}}\sqrt{h} - \sqrt{2Kh} \leq \frac{h_L\sqrt{m}}{\sqrt{h_{wip}}} - \frac{h_L\sqrt{m}}{\sqrt{h_{wip}+h_L}} \tag{30}$$

Note that, in the above expressions we have simplified and rearranged the inequalities such that both sides of the inequalities are always positive, while the left-hand-side of the inequalities have the parameters, h_{wip}, h_L, m and right-hand-side of the inequalities have the parameters $K, h,$ and m. We will illustrate the insights provided by these cases using the following numerical example:

Suppose p=4, w=3, c=2, d=100, h_{wip}*=1, K=3, m=3, h=2*

for $h_L = 1$, $\pi_o^* = 51.01$; $\pi_o^0 = 48.03$; $\pi_m^* = 51.01$; $\pi_m^0 = 48.03$ *(Case a)*

for $h_L = 0.25$, $\pi_o^* = 52.01$; $\pi_o^0 = 48.03$; $\pi_m^* = 59.38$; $\pi_m^0 = 61.03$ *(Case b)*

for $h_L = 4$, $\pi_o^* = 44.45$; $\pi_o^0 = 48.03$; $\pi_m^* = 37.27$; $\pi_m^0 = 13.39$ *(Case c)*

Start with Case (a), which holds with the parameter values in the example. As h_L decreases from 1 to .25, the term $(\frac{2h_{wip}+h_L}{\sqrt{h_{wip}+h_L}} - 2\sqrt{h_{wip}})$ also decreases and the first inequality stays the same. However, the R.H.S. $(\frac{h_L\sqrt{m}}{\sqrt{h_{wip}}} - \frac{h_L\sqrt{m}}{\sqrt{h_{wip}+h_L}})$ in the second inequality reaches a point where it is larger than the L.H.S. $(\frac{2K+m}{\sqrt{2(K+m)}}\sqrt{h} - \sqrt{2Kh})$, which means it moves from Case (a) to Case (b). As h_L increases the reverse happens: the first inequality changes direction, whereas, the second inequality stays the same, i.e., it changes from Case (a) to (c). Similar analysis can be done for other parameters.

In Case (a), both *marketing* and *operations* have incentives to implement the centralized solution. In Case (b), *marketing* would suffer under the centralized solution while *operations* benefits. Thus, some internal transfer *d* from operations to marketing would be necessary to compensate for *marketing's* loss. Similarly, in Case (c) the internal transfer should be made from *marketing* to *operations* in order to implement the system optimal solution. The following proposition states that in Case (b) (Case (c)) *operations* *(marketing)* would make enough *extra* profits in a system optimal solution to compensate for the loss of *marketing* (operations).

Proposition 5: $\pi_o^* - \pi_o^0 \geq \pi_m^0 - \pi_m^*$ *for Case (b) and* $\pi_m^* - \pi_m^0 \geq \pi_o^0 - \pi_o^*$ *for Case (c).*

Proof: *Since* $\pi_o^* + \pi_m^* \geq \pi_o^0 + \pi_m^0$ *both inequalities hold automatically.* □

With the basic understanding of why the decentralized solution is inefficient, in the following section, we examine three coordination schemes that provide incentives for *marketing* and *operations* to coordinate, i.e., to implement the system-optimal solution.

3. COORDINATION SCHEMES BETWEEN MARKETING AND OPERATIONS

In this section, we propose three different coordination schemes between marketing and operations based on (1) *quantity, price discount* offers (Q, d), (2) *quantity, lead-time reduction* offers (Q, L), and (3) *quantity, lead-time, and price adjustment* offers (Q, L, d). We will use the decentralized solution to define the *individual rationality constraint* for each player, i.e., the coordination scheme should yield the level of profit that are no worse than the players' corresponding decentralized solutions.

3.1 Coordination Scheme 1: Quantity Discount

This coordination scheme is motivated by work in the supply chain contracting literature where quantity-discount schemes are popular for buyer-supplier coordination (Lariviere,1999). It has been shown that it's the supplier's best strategy to offer a (Q^*, d^*) pair, where Q^* is the system-optimum order-size for the supply chain (which is always larger than the buyer's own optimal order-size), and d^* is the corresponding discount offer that compensates the buyer for ordering a larger amount. In the following, we first summarize the previous results where EOQ-based decision models are assumed for the players.

The buyer's (typically a retailer) cost function is given as:

$\frac{K_b D}{Q} + \frac{h_b Q}{2}$, which corresponds to the setup and holding costs, in order.

The Supplier's cost function is similar but with different cost coefficients:

$$\frac{K_s D}{Q} + \frac{h_s Q}{2}$$

Suppose Q_b is the buyer's optimal order quantity considering only her own cost function, and Q_s is the supplier's optimal lot-size, and Q^* the joint optimal quantity for the supply chain, then

$$Q_b = \sqrt{\frac{2K_b \mathrm{D}}{h_b}}, \qquad Q_s = \sqrt{\frac{2K_s dD}{h_s}} \qquad Q^* = \sqrt{\frac{2(K_s+K_b)D}{(h_s+h_b)}}$$

Since it is generally assumed that setup cost is higher for the supplier while the inventory holding cost is higher for the buyer, the relationship $Q_b \leq Q^* \leq Q_s$ holds. The buyer will order Q_b unless additional incentives are offered to increase her order size. Thus, to achieve coordination, the supplier may offer a quantity discount dD to the buyer to motivate a larger order size, subject to the constraint that the buyer's original profit is protected. This problem can be stated as follows:

$$Min_{(Q,d)} \qquad \frac{K_s D}{Q} + \frac{h_s Q}{2} - dD \tag{31}$$

$$\text{s.t.}: dD + \frac{K_b D}{Q} + \frac{h_b Q}{2} = \frac{K_b D}{Q_b} + \frac{h_b Q_b}{2} \tag{32}$$

This is the same as the optimization problem below:

$$Min_{(Q)} \frac{K_s D}{Q} + \frac{h_s Q}{2} + \frac{K_b D}{Q} + \frac{h_b Q}{2} - \frac{K_b D}{Q_b} - \frac{h_b Q_b}{2} \tag{33}$$

Let Q^o is the solution of this problem. Last two terms in the function are constants therefore, $Q^o = Q^*$ and the joint optimal (centralized) solution is obtained. The corresponding d^* can be easily calculated by (4).

In our internal market model, the notion of quantity discount can be interpreted as an internal transfer offered by *operations* to stimulate larger order sizes from *marketing* (thus the outside customers). However, our model (as defined in Section 2) has a significant difference from the EOQ-type models in that *operations* (the supplier) must make capacity allocation decisions, and both *marketing* and *operations'* profit functions have a lead-time component. We summarize the sequence of events for this coordination scheme as follows:

1. *Operations* announces a quantity discount scheme *(Q,d)* where d is a discount payment to *marketing* if an order of quantity Q is placed

2. *Marketing* accepts the (Q, d) offer from *operations* if it satisfies her *individual rationality constraint,* i.e., the profit generated π^1_m is the same or better than her decentralized solution $\pi^1_m \geq \pi^o_m$

In the following proposition, we show that the quantity-discount scheme common in supply chain coordination is not sufficient to coordinate *marketing* and *operations* in our internal market setting.

Proposition 6: *Quantity discount (Q ,d) alone is not sufficient to coordinate marketing and operations (i.e., to yield the centralized solution).*

Proof: *Under the quantity discount scheme, marketing and operations communicate in terms of Q and d, and there is no explicit exchange concerning capacity adjustment and lead-time. As a result, marketing must make assumptions on operations' capacity allocation using her best response function as follows:*

$$u_{best}(Q) = \frac{D}{Q} + \sqrt{\frac{h_{wip}D}{m}} \tag{34}$$

Thus, marketing's profit function with price discount, d is as follows:

$$\pi^1_m = dD - h_L D \frac{1}{u_{best}(Q) - \frac{D}{Q}} - \frac{KD}{Q} - \frac{hQ}{2} \tag{35}$$

$$= dD - \frac{h_L}{\sqrt{h_{wip}}}\sqrt{mD} + \frac{hQ}{2} + \frac{Kd}{Q} \tag{36}$$

Marketing would only accept the (Q, d) offer from operations that satisfies the following individual rationality constraint:

$$\pi^1_m = dD - h_L D \frac{1}{u - \frac{D}{Q}} - \frac{KD}{Q} - \frac{hQ}{2} \geq \pi^o_m \tag{37}$$

$$\Rightarrow \quad \frac{h_L}{\sqrt{h_{wip}}}\sqrt{mD} + \frac{hQ}{2} + \frac{KD}{Q} - dD \leq \frac{h_L}{\sqrt{h_{wip}}}\sqrt{mD} + \frac{2K}{\sqrt{2K}}\sqrt{Dh} \tag{38}$$

$$\Rightarrow \quad \frac{hQ}{2} + \frac{KD}{Q} - dD \leq \sqrt{2KDh} \tag{39}$$

Knowing the marketing's individual rationality constraint, operations must maximize her profit function subject to (39) *and her own individual rationality constraint as follows:*

$$-\,dD - h_{wip}D\frac{1}{u_{best}(Q)-\frac{D}{Q}} - mu_{best}(Q) \geq \pi_o^{\circ} \tag{40}$$

which can be rewritten as;

$$\frac{2h_{wip}}{\sqrt{h_{wip}}}\sqrt{mD} + \frac{mD}{Q} + dD \leq \frac{2h_{wip}}{\sqrt{h_{wip}}}\sqrt{mD} + \frac{m}{\sqrt{2K}}\sqrt{Dh} \tag{41}$$

$$\Rightarrow \quad \frac{mD}{Q} + dD \leq \frac{m}{\sqrt{2K}}\sqrt{Dh} \tag{42}$$

Therefore, the operations' decision problem is as follows :

$$Max_{(Q,d)} \; -\frac{2h_{wip}}{\sqrt{h_{wip}}}\sqrt{mD} - \frac{mD}{Q} - dD \tag{43}$$

$ST\,(39)$ *and* (42)

(39) *is binding in the optimal solution and therefore the operations' problem can be rewritten as follows:*

$$Max_{(Q)} \; -\frac{2h_{wip}}{\sqrt{h_{wip}}}\sqrt{mD} - \frac{mD}{Q} - \frac{hQ}{2} - \frac{KD}{Q} + \sqrt{2KDh}$$

$ST\,(42)$

Optimal solution to this problem is $Q^1 = Q^* = \frac{\sqrt{2(K+m)D}}{h}$

Operations will adjust the capacity as $u^1 = u_{best}(Q^*) = \sqrt{\frac{Dh}{2(K+m)}} + \sqrt{\frac{h_{wip}D}{m}}$ (44)

As $u^1 \neq u^*$ *the* (Q,d) *scheme fails to yield the centralized solution.* □

Note that although *Scheme 1* changes the order size from Q^o to Q^*, it fails to change the capacity level u^o. The total profit under the scheme is as follows:

$$\pi^1 = -\frac{2h_{wip}+h_L}{\sqrt{h_{wip}}}\sqrt{mD} - \frac{2K+2m}{\sqrt{2K+2\mathrm{m}}}\sqrt{Dh} \tag{45}$$

We can easily calculate the efficiency gains (EG^1) and the difference with the system optimal solution (F^1) as follows:

$$EG^1 = \pi^1 - \pi^o$$

$$= \frac{2K+m}{\sqrt{2K}}\sqrt{Dh} - \frac{2K+2m}{\sqrt{2K+2m}}\sqrt{Dh} \tag{46}$$

$$F^1 = \pi^* - \pi^1$$

$$= \frac{2h_{wip}+h_L}{\sqrt{h_{wip}}}\sqrt{mD} - \frac{2h_{wip}+2h_L}{\sqrt{h_{wip}+h_L}}\sqrt{mD} \tag{47}$$

Note that, the magnitude of F^1 depends on parameters $h_{wip,}h_L, m, and\,D$. D and m are only scaling parameters and they will be eliminated if we express the difference as a ratio.

Thus, the critical parameters are the holding costs h_{wip} and h_L. As h_L increases or h_{wip} decreases, the quantity discount scheme will start to generate poor results. This is due to the fact that *Scheme 1* focuses on order-size adjustment but fails to generate proper capacity adjustment. This results is interesting as it suggests that when lead-time and capacity are considered in the cost structure, the quantity discount scheme is not sufficient to coordination the system.

3.2 Coordination Scheme 2: Lead-Time Reduction

As discussed earlier, marketing will not order more than her EOQ from manufacturing unless some incentives are provided (e.g., quantity discount). In this section, we introduce another coordination scheme commonly seen in the industry, where the *operations* offers a reduced lead-time to *marketing* in exchange for a larger order size. Here, the sequence of events is as follows:

1. *Operations* announces a lead-time reduction scheme *(Q,L)* where *L* is the lead-time offered to *marketing* if an order of quantity *Q* is placed
2. *Marketing* accepts the (Q, L) offer from *operations* if it satisfies her *individual rationality constraint,* i.e., the profit generated π_m^2 is the same or better than her decentralized solution $\pi_m^2 \geq \pi_m^o$

Similar to the previous case, we are interested to know if a particular *(Q,L)* combination agreed upon by *marketing* and *operations* would coordinate the system. Note that lead-time is not under complete control of *operations* (lead-time is a function of *both* capacity *u* and order quantity *Q*). Nonetheless, *operations* may propose an offer *(Q,L),* in order to get the desired lead-time *L, marketing* must agree to order the specified quantity Q . For a given Q, *operations* can then adjust the capacity level, *u* to yield the desired *L.*

The *(Q,L)* scheme is more relevant in internal market coordination then the commonly seen quantity-price schemes, since lead-time, not price, is often the center of negotiation in this setting. Thus, lead-time reduction provides a focus for marketing-operation coordination. Moreover, lead-time reduction yields other qualitative benefits for the firm such as customer satisfaction and market responsiveness, which is compatible with the reward structure for marketing. Further, even for markets where price is the focus of negotiation, lead-time coordination could yield better overall solutions in a certain parameter range, as we will show in this section.

As before, we impose a *individual rationality constraint* assuming that marketing and operations would only accept offers no worse then their respective decentralized solutions. The *individual rationality constraint* for marketing is as follows:

$$-h_L D\frac{1}{u-\frac{D}{Q}} - \frac{KD}{Q} - \frac{hQ}{2} \geq \pi_m^o \qquad (48)$$

Therefore, the *operations* must determine the best (Q, u) (thus the (Q, L)) pair that would maximize her profit:

$$Max_{(Q,u)} \quad -h_{wip} D\frac{1}{u-\frac{D}{Q}} - mu$$

$$ST\,(48)$$

$$-h_{wip} D\frac{1}{u-\frac{D}{Q}} - mu \geq \pi_o^o \qquad (49)$$

Proposition 7: *There is always a feasible solution to the operation's decision problem defined above.*

The proposition is easy to prove as the non-coordinated decentralized solution (Q^o, u^o) is a feasible solution for the problem. Thus, the proposition implies that *Scheme 2* will always yield a solution that is equal to or better than the player's decentralized solution. It follows that the *individual rationality constraint* (49) for *operations* will always be satisfied in the optimal solution to the problem therefore (49) is redundant.

Since (48) is a binding constraint, we can write u in terms of Q :

$$u = \frac{D}{Q} - \frac{h_L D}{\pi_m^o + \frac{hQ}{2} + \frac{KD}{Q}} \qquad (50)$$

The problem can be thus restated as follows:

$$Min_Q \quad f(Q) = \frac{mD}{Q} - \frac{h_{wip}}{h_L}\left[\pi_m^o + \frac{KD}{Q} + \frac{hQ}{2}\right] - \frac{m h_L D}{\pi_m^o + \frac{hQ}{2} + \frac{KD}{Q}} \qquad (51)$$

Unfortunately, there is no closed-form solution for the optimization problem defined by (51). As such, it is not possible to determine *a priori* the efficiency gain of the coordination scheme. Nonetheless, we are able to provide some characterization of the optimal solution results from the coordination scheme. We will detail our analysis in the rest of the section, beginning with the relationship between the coordinated and the centralized solutions.

Proposition 8: *If the marketing's profit function is the same under the centralized and the decentralized settings,* $(i.e., \pi_m^o = \pi_m^*)$ *then coordination scheme 2 yields the centralized solution.*

The proposition is trivial to prove since the centralized solution satisfies the marketing's individual rationality constraint ($\pi_m^o = \pi_m^*$), and the solution (Q^*, L^*) is the best offer for the operations since it minimizes $\pi_m^* + \pi_o$. While the result from this proposition is not particularly useful as is, we can use this results to analyze the effects of marketing's holding cost, h. We know that if $\pi_m^o = \pi_m^*$, the following relationship would hold:

$$\sqrt{m}\left(\frac{h_L}{\sqrt{h_{wip}}} - \frac{h_L}{\sqrt{h_{wip}+h_L}}\right) = \left(\frac{2K+m}{\sqrt{2K+2m}} - \sqrt{2K}\right)\sqrt{h} \qquad (52)$$

we may express $$\hat{h} = \left(\frac{\sqrt{m}\left(\frac{h_L}{\sqrt{h_{wip}}} - \frac{h_L}{\sqrt{h_{wip}+h_L}}\right)}{\left(\frac{2K+m}{\sqrt{2K+2m}} - \sqrt{2K}\right)}\right)^2 \qquad (53)$$

Clearly, equation (52) holds if $h = \hat{h}$. Also observe that if $h > \hat{h}$, $\pi_m^o > \pi_m^*$, if $h < \hat{h}$, $\pi_m^o < \pi_m^*$. Further, when $h > \hat{h}$ ($h < \hat{h}$), as h increases (decreases), the value $|\pi_m^o - \pi_m^*|$ *increases* monotonically. Thus, the difference $|h - \hat{h}|$ provides an useful indicator for the quality of the coordination *Scheme 2.* In contrast to this result, the quality of coordination *Scheme 1* is neither affected by the holding cost, h nor the setup cost, K. We have the following observation that will lead us to some closed-form bounds for efficiency gain w.r.t. the optimal solution.

Proposition 9: *Comparing to the non-coordinated decentralized scheme (the Stackelberg game) coordination scheme 2 always yield the same or smaller on i.e.,* $L^o \geq L^2$.

Proof *: Lead-time L is equal to* $\frac{1}{u-\frac{D}{Q}}$*, since in Scheme 2,* $u{=}\frac{D}{Q}-\frac{h_L D}{\pi_m^o+\frac{hQ}{2}+\frac{KD}{Q}}$, and $L{=}\frac{-(\pi_m^o+\frac{hQ}{2}+\frac{KD}{Q})}{h_L D}$. *The latter term is maximized when* $Q=Q^o$ *and* $L^o=\sqrt{\frac{m}{h_{wip}D}}$. *Therefore, for any* Q^2 *that minimizes f(Q), we have* $L^o \geq L^2$.□

Proposition 10: *The optimal solution to function f(Q) in* (51), Q^2*, always satisfies the following inequality:* $\underline{Q} < Q^2 < \overline{Q}$ *provided that* $(\pi_m^o)^2-2KDh>0$*, where* $\underline{Q}=\frac{-\pi_m^o-\sqrt{(\pi_m^o)^2-2KDh}}{h}$, $\overline{Q}=\frac{-\pi_m^o+\sqrt{(\pi_m^o)^2-2KDh}}{h}$ *are the roots of the equation* $\pi_m^o+\frac{KD}{Q}+\frac{hQ}{2}=0$

Proof: *Since* $L=\frac{1}{u-\frac{D}{Q}}$*, in Scheme 2,* $L{=}\frac{-(\pi_m^o+\frac{hQ}{2}+\frac{KD}{Q})}{h_L D}$. *Since on is non-negative* ($L \geq 0$)*, we have* $-(\pi_m^o+\frac{hQ}{2}+\frac{KD}{Q}) \geq 0$*. Finding the roots of the equation* $(\pi_m^o+\frac{hQ}{2}+\frac{KD}{Q})=0$*, we have*

$$\frac{-\pi_m^o-\sqrt{(\pi_m^o)^2-2KDh}}{h} \leq Q \leq \frac{-\pi_m^o+\sqrt{(\pi_m^o)^2-2KDh}}{h}.$$

Since $f(Q)$ *is convex and continuous between the points* $\underline{Q}=\frac{-\pi_m^o-\sqrt{(\pi_m^o)^2-2KDh}}{h}$ *and* $\overline{Q}=\frac{-\pi_m^o+\sqrt{(\pi_m^o)^2-2KDh}}{h}$*, the optimal solution can not be at the boundary points unless* $(\pi_m^o)^2-2KDh=0$*. This proves that* $\underline{Q} < Q^2 < \overline{Q}$.□

Note that the condition $(\pi_m^o)^2-2KDh=0$ is not meaningful, since in that case the on is 0 which can be obtained only with infinite capacity.

Given the above characterization, we will examine a crude but potentially useful approximation for Q^2*:* define $\widehat{Q}=\frac{\underline{Q}+\overline{Q}}{2}=\frac{-\pi_m^o}{h}=\frac{h_{ss}}{h\sqrt{h_{wip}}}\sqrt{mD}+\sqrt{\frac{2KD}{h}}$. This is the mid-point between $\underline{Q}$, $\overline{Q}$ and the approximation would work quite well if the function $f(Q)$, for instance, is bathtub shaped and near-symmetrical in the interval $[\underline{Q},\overline{Q}]$. While it would be difficult to quantify the accuracy of this simple approximation in closed-form, we did conduct numerical studies and observed that $\widehat{Q}$ and Q^2, and their corresponding values $f(Q^2)$ and $f(\widehat{Q})$ are very close for most of the cases we have tested. More importantly, $\widehat{Q}$ is useful to establish the efficiency gain of *coordination Scheme 2* when compared with the decentralized solution. Similar to *Scheme 1*, we define the efficiency gain as follows: $EG^2=\pi^2-\pi^o=-f(Q^2)-\pi_o^o$ since we know that π_m^o is the same for the decentralized and Scheme 2 solutions. In the following, we summarize the efficiency results.

Proposition 11: *Comparing to the non-coordinated decentralized solution, the efficiency gains for coordination Scheme 2 (EG^2) is no less than*

$$\left(-\pi_o^o + \frac{h_{wip}\pi_m^{o^2} - 2KDhh_{wip} + 2mDh_L}{2h_L\pi_m^o} + \frac{2\pi h_L mD}{\pi_m^{o^2} - 2KDh}\right) \tag{54}$$

moreover, the difference from the system-optimum solution (F^2) is no more than

$$\left(\pi^* - \pi_m^o - \frac{h_{wip}\pi_m^{o^2} - 2KDhh_{wip} + 2mDh_L}{2h_L\pi_m^o} - \frac{2\pi h_L md}{\pi_m^{o^2} - 2KDh}{}^o\right) \tag{55}$$

Proof : *Note that $f(Q^2)$ is the operations' cost under scheme 2. Since $f(Q^2) \leq f(\widehat{Q})$, $EG^2 \geq -\pi_o^o - f(\widehat{Q})$, it follows that $EG^2 \geq -\pi_o^o + \frac{h_{wip}\pi_m^{o^2} - 2KDhh_{wip} + 2mDh_L}{2h_L\pi_m^o} + \frac{2\pi h_L mD}{\pi_m^{o^2} - 2KDh}$*

Likewise $F^2 \leq \pi^* - \pi_m^o + f(\widehat{Q})$,

therefore $F^2 \leq \left(\pi^* - \pi_m^o - \frac{h_{wip}\pi_m^{o^2} - 2KDhh_{wip} + 2mDh_L}{2h_L\pi_m^o} - \frac{2\pi h_L md}{\pi_m^{o^2} - 2KDh}{}^o\right)$.□

3.3 Coordination Scheme 3: Lead-time Reduction and Pricing Discount

The third coordination scheme is a combination of *Schemes 1* and *2* in that the *operations* could offer both lead-time reduction *and* price adjustment in exchange for a larger order-size from *marketing*. However, the price adjustment could go in either directions. In the case of a price discount, *marketing* could transfer the saving to the customer in the form of quantity discount. In the case of an extra payment, *marketing* will need to transfer the cost to the customer. However, in both cases, *marketing* will be able to offer a more favorable delivery date. This is an important feature in some industries. The sequence of events for this coordination scheme is as follows:

1. *Operations* announces a scheme *(Q,L,d)* where *L* is the ' offered to *marketing* if an order of quantity *Q* is placed. In addition, a discount (an extra payment) *d* is offered to (is collected from) *marketing*.

2. *Marketing* accepts the *(Q, L,d)* offer from *operations* if it satisfies her *individual rationality constraint,* i.e., the profit generated π_m^3 is the same or better than her decentralized solution $\pi_m^3 \geq \pi_m^o$

We will show that this somewhat more complex coordination scheme achieves marketing-operation coordination, i.e., yields the system-optimum centralized solution. In order to achieve coordination the price adjustment d offered by *operations* to *marketing* (step 2) could be negative $(d < 0)$. In this case *marketing* pays a premium for on reduction in turns of an extra payment *and* a larger order quantity. Note that d is non-negative under *Scheme 1*. For $d > 0$, *Scheme 3* correspond to an offer from *operations* as follows: "*if you (marketing) order* Q^3, *we (operations) will give you a discount* d *on the price, and will reduce the on to* L^3!" On the other hand, for $d < 0$, the offer becomes "*if you order* Q^3 *and increase the price by* d, *I will reduce on to* L^3." The following proposition states the optimality of coordination scheme 3:

Proposition 12: *Coordination Scheme 3 achieves marketing-operations coordination.*

Proof: *The operations' problem (what to offer) with the individual rationality constraints can be formulated as follows:*

$$Max_{(Q,u,d)} - dD - h_{wip}D\frac{1}{u-\frac{D}{Q}} - mu$$

$ST:$

$$dD - h_L D\frac{1}{u-\frac{D}{Q}} - \frac{KD}{Q} - \frac{hQ}{2} \geq \pi_m^o \tag{56}$$

$$- dD - h_{wip}D\frac{1}{u-\frac{D}{Q}} - mu \geq \pi_o^o \tag{57}$$

We first relax constraint (57) *and solve the above problem. In this case, it can be shown that* (56) *is a binding constraint. From the equation, d can be expressed in terms of u, and Q. If we substitute this d in the objective function, the function can be rewritten as follows:*

$$Max_{(Q,u)} \; - (h_{wip} + h_L)D\frac{1}{u-\frac{D}{Q}} - mu - \frac{KD}{Q} - \frac{hQ}{2}$$

It is easy to verify that the solution (Q^3, u^3) *to this problem (1) matches the system-optimum solution* (Q^*, u^*), *and (2) the individual rationality constraint for operations* (57) *is already satisfied for* (Q^*, u^*). □

Essentially, *operations* implements the system-optimum order quantity and lead-time pair (Q^*, L^*), while using the side payment d to either compensate for the *marketing*'s loss, or sharing the profit with *marketing*. In the latter case (when d is negative) may arise when marketing's lead-time cost, h_L is high, such that lead-time reduction (as compared to the decentralized case (Q^o, L^o)) would improve marketing's profits significantly.

4. CONCLUSION

In this paper, we model and analyze a lead-time based coordination scheme between the *marketing* and *operations* entities of a firm. Motivated by the joint lot-sizing literature and the work in supply chain contracting, we characterize the coordination problem by first defining the centralized system-optimum solution, the decentralized Stackelberg solution, and their asymptotic performance ratio as the cost parameters change. Unique to our model is the explicit consideration of *lead-time* and its relationship to manufacturing capacity. Specifically, *operations*' decision to allocate manufacturing capacity has a direct impact to the lead-time performance that *marketing* relies on for customer satisfaction. The essence of marketing-operations coordination is that *operations* would offer a more favorable lead-time provided that *marketing* convinces the customer to place a larger order. Unfortunately, we found that lead-time and order-size alone are not sufficient to coordinate the system. A price adjustment in the form of a price-discount or extra-payment will be necessary for the system to be perfectly coordinated (i.e., for the *marketing* and the *operations* entities to voluntarily implement the system-optimum solution).

Our model includes a few important cost components: in addition to the setup and inventory holding costs common in EOQ-based models, we also consider the unit capacity cost, WIP holding cost, and lead-time cost (which is typically related to the customer's safety stock cost). These cost components play important roles in defining the incentive for, thus the behavior of, the decision makers involved. For example, the relative significance of the lead-time cost to marketing's profit will help determining if additional incentive is needed for her to order the system-optimal order size. We summarize the sensitivity of these cost components in Table 2.

We propose three coordination schemes for marketing and operations. We show that the *Quantity Discount* Scheme (*Scheme 1*) popular in the joint lot-sizing literature does not coordinate the system since the *operations* must also make capacity allocation decisions in our setting. We then propose a straightforward *ard reduction* scheme (*Scheme 2*) and analyzed and compared the efficiency gains. Although there is no close-form expression that would allow us to verify that the scheme achieve perfect coordination, we are able specify its basic characteristics. The scheme shows good potential for use in practical settings due to its simplicity, and the fact that ics. (rather than price-quantity) negotiation is more relevant for marketing-operations coordination. Finally, we propose a combined *ics.* and *price-adjustment* scheme (*Scheme 3*) which we show to perfectly coordinate the system.

Note that all our analysis are based on single-point offers. The analysis can be easily extended to cases allowing a menu of offers (e.g., multiple ics. quantity combinations), or an offering function as introduced in the literature. So long as there is a single or homogenous *marketing* entity in the system, with full information, the analysis does not change significantly. Another possible extension is on the ics. function. In our analysis, we use a ics. function where the processing rate is independent of the order size. A generalized ics. expressions are also possible (c.f., Karmarkar, 1993) but the analysis will be considerably more complex and closed-form expressions are unlikely to be available for the design of perfect coordination.

Another interesting direction for extension is to model the bargaining power of the two decision parties. In our analysis we assume that *operations* makes offers, while *marketing* accepts any offer that yields a solution no worse than her decentralized solution. This can be generalize to a bargaining situation (c.f., Ertogral and Wu, 2001) where *marketing* and *operations* initiate a bargaining process to split the surplus generated from the system-optimal solution.

Finally, asymmetric information cases can be also examined (Corbett, 1999) where the players do not have full information about each other's cost data. In this case, the offer type should be carefully modeled and analyzed. For example, instead of proposing (Q,L) offers in *Scheme 2*, operations may announce a ics. function $L(Q)$. In this case, operations needs to use $L(Q)$ as an inducing mechanism that provide marketing the right incentive to place the optimal order size.

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