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Procurement Auction with Supplier Coalitions: Validity Requirements and Mechanism Design

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Abstract: We study the formation of supplier coalitions in the context of a buyer-centric procurement market. Considered under a *second-price descending seal-bid auction*, we propose a two-stage auction mechanism that allows suppliers to form coalitions with one another. Building on the foundations of core games and bidding-rings, we explore the idea of "managed collusion" which provides a means to enhancing bidder profitability. We identify six basic requirements for a *valid coalition mechanism* including characteristics such as individual rationality, welfare compatibility, maintaining competition, and financial balancedness. We show that such mechanism could be constructed so that the buyer does not loose the advantage from supplier competition, and that a stable coalition structure could be formed. We propose a profit distribution scheme among members in the supplier coalition and show that the proposed scheme provides proper incentive such that (1) the best strategy for a coalition member is to comply with the coalition agreement, and (2) bidding the true cost is the best strategy so long as the bids are uniformly distributed and the bidder's cost is above a certain threshold. We also investigate the stable coalition structure under the proposed mechanism and show that under symmetric information there exists one unique strongly stable coalition structure.

Keywords: Supply Chain Management, Coalition Structures, e-Commerce, Procurement Auctions, Mechanism Design

1. Introduction

Business-to-business electronic commerce, the use of electronic means to conduct business transactions within or across business entities, has been touted as the most important economic development of the decade. The 2001-2002 economic downturn has significantly dampened the development of eCommerce initiatives. Nonetheless, according to a June 2002 survey by AMR research [2], leading companies are continuing to deploy on-line commerce application such as Private Trading Exchanges (PTXs), while others have implemented more basic means of on-line transactions. They surveyed companies in several industry sectors including Chemical, Industrial Equipment, High-Tech and Electronics, Oil and Gas, and Consumer Goods. These companies report an average investment of $6.57M in building their "on-line commerce system," while the expected Return on Investment (ROI) averages around 7%. The survey cited the (lack of) readiness and acceptance of supply partners as the two biggest hurdles to implement B2B eCommerce. Moreover, 52% rated *value creation* and *enablement* for supply partners as being the most critical.

From the viewpoint of supply chain management, an on-line market such as PTX includes three main parties: the buyers, the suppliers and the market maker. The role of the three parties can be simply stated as follows: the buyer tries to find the most economical ways to fill his orders, the supplier desires to improve his market share while protecting his profit margin, the market maker extracts transaction fees or delivers packaged services by matching the buyers with the suppliers. The market maker's short-term goal maybe to establish credibility in niche commerce areas, and in the long-run increase market participations, therefore higher profit potentials. Of interest to all three parties is market efficiency. We will define market efficiency more precisely in our analysis, but intuitively efficiency is built on the premise of fair competition and fair distribution of surplus among market participants. In existing eProcurement markets, the buyers tend to be represented by a few large firms. The suppliers, on the other hand, are much more numerous and the competition among them is fierce. For example, Freemarkets [1] report that in the first quarter of 2002 that a total of 125 buyers traded in their procurement auctions, with some 21,000 suppliers participating. All auctions were triggered by buyers' demand, and each auction has one buyer and many more suppliers. Freemarkets claimed a total saving of $800 million (for the buyers) during this period, over a total auction value of about $4 billion. One may infer from the above that the buyer has significant advantage over the suppliers, and the supplier's margin may erode significantly.

Many would attribute the early success of eProcurement to the apparent savings it brings to the buyers. Through electronic markets, the buyers have direct access to a large number suppliers and therefore lower prices. They would urge their existing supply partners to join the on-line market, who may do so just to secure the customer base. Once joining the market, the supplier may quickly lose previous benefits from long-term business relations, while facing fierce competition in a global scale. These suppliers are likely to suffer from undercut profit margins, and facing the dilemma of staying in or fleeing the market. Suppliers who have leverage other than that of pricing (e.g., unique technology) are likely to avoid on-line commerce altogether. As suggested by Wise and Morrison (2000) the above buyer-dominant phenomenon is among the "fatal flaws" of earlier B2B model. This observation echoes the findings from the AMR survey, as the main difficulty in implementing on-line commerce is to convince the supply partners that the new business paradigm creates value for *them* as well. Otherwise, there is little incentive for the supplier to invest in the multi-million dollar infrastructure required for long-term, stable on-line transactions. A main purpose of this paper is to explore mechanisms that would systematically improve the suppliers' margin without sacrificing competition, thus market efficiency. We believe such mechanisms are crucial to sustain the long-term viability of on-line commerce.

We submit that market efficiency and stability are the keys to its long-term success. We examine a mechanism for procurement auction where suppliers are encouraged to form coalitions with one another under market rules with the goal of increasing supplier profits but without degrading market efficiency. We will show that such mechanism could be constructed so that the buyers do not loose the advantage from supplier competition, and that a stable coalition structure could be formed.

In traditional supply chain operations, where each buyer tends to deal with a limited number of suppliers via long-term, off-line negotiations, the interaction among the suppliers is limited (and sometimes considered as illegal collusion). From a modeling viewpoint, it is sufficient to consider supply chain coordination via one-buyer one-supplier negotiation pairs, as is the case in a majority of the supply chain literature. However, the emergence of on-line commerce makes inter-supplier interactions more direct and much more critical. It is of importance to consider not only supplier-buyer interactions but also the interactions among competing suppliers who may be at the same time collaborating. It is our intention to examine how this interaction could be directed toward improved market efficiency.

2. Related Literature

We will review three areas of research that are directly related to the paper: auction mechanisms of various forms, coalition formation in non-cooperative games, and bidding-rings. Auction is a popular form of price determination in eCommerce, owing to its simplicity and low information requirements. Basic forms of auction exist in human economy for a long time, but the research literature on auction theory exploded after the seminal paper by Vickrey [1961], which proposes the concept of the *second-price sealed-bid auction*. Second-price auction awards the lowest bidder the second lowest bid price, which is proposed as a means to encouraging all sellers to bid at their true costs. Vickrey proves analytically that this scheme is *incentive compatible* since neither bidding above nor below the true cost would make the bidder better off. Sandholm [1996] point out some limitation of the Vickrey auction in computational multi-agent systems. For other well-known forms of auctions the readers are refer to surveys such as the one given by Rasmusen et al [1992]. More recently, Beam and Segev [1998] and Herschlag and Zwick [2000] provide surveys on various Internet auctions. McMillan [1994] discusses mechanism design for the spectrum auction and compared open vs. sealed-bid auction, simultaneous vs. sequential auction, and several stopping rules. Milgrom [1998] discusses the *simultaneous ascending auction* for radio frequency spectrum in some detailed. Klemperer [1999] provides an excellent survey on the economic theory of auctions.

Coalition formation has been studied intensively by the game theorists. A primary motivation for players in a game to form coalitions is to improve their surplus. There are three basic problems in coalition formation in a given game: whether a *stable coalition* exists, how to *share profit* among the participants, and who should be in which coalition (*coalition generation*). Among various theoretic developments, the *core* introduced by Gillies [1953] is the earliest and most well accepted for *coalition formation* problems. The *core* is the set of all feasible payoffs to the players that no player or group of players could improve upon by acting unilaterally. The existence of a nonempty core implies that the game will have a stable solution and the players will have incentive to form the grand coalition. For the detailed discussion on the *core*, refer to Kannai [1992], Peleg [1992], Anderson [1992] and Gabszewicz [1992]. The literature on *core game* typically considers the stability of the grand coalition in a game, and does not offer practical solution for coalition formation.

Another related line of literature in economic theory is *collusion* in auctions. A primary focus of this literature is the presence of illegal *cartels* among bidders (also known as "bidding-rings"), who try to optimize joint expected profits, thus undercut the seller's profit. A typical assumption (under single item auction) is that the cartel would select a designated winner who follows a particular bidding strategy, while requesting other members to be inactive during the auction. There are various strategies possible for the bidders to conceal the existence of the cartel by creating bidding patterns that mislead the seller. However, there are enforcement issues in the cartel as the "designated losers" may have incentive to deviate from the cartel agreement. Robinson [1985] shows that cartels are stable (i.e., incentive-compatible) if the seller uses open ascending-bid auctions, but not if he uses sealed high-bid auctions (e.g., Dutch auctions). Under the latter setting, McAfee and McMillan [1992] make the distinction between *weak* and *strong* cartels, where only the latter can exclude new entrants and make transfer payments among cartel members. They show that only the *strong cartel* could indeed achieve "efficiency" (maximize the bidders' expected profit), but in effect re-auctions the good among its members. Graham and Marshall [1987] examine collusion in second-price auctions. In their setting, the identity of the winning bidder and the price paid (which was the second highest bid) are common knowledge, while the information concerning bid valuation is private. All members share the gains obtained by the ring. They propose a second-price pre-auction knock out (PAKT) within the ring, which generates the bid to be used in the main auction. They show that it is a dominant strategy for the bidders to corporate in the ring, and that an *all-inclusive* ring is optimal. Malaith and Zembsky [1991] also examine collusion in second-price auctions. Guth and Peleg [1996] consider the second-price auction setting where PAKT precedes the main auction. They introduce the notion of *envy-freeness* (i.e., no bidder prefers another bidder's net trade to his own) in this context, and offered an *impossibility theorem,* which states that none of the transfer payment scheme satisfying envy-freeness condition is *incentive compatible*, i.e., bidders do not bid truthfully; A ring member will *overbid* his true value to induce a higher transfer payment from the ring's representative, and *underbid* if he believes that he will be chosen as the representative.

In this paper, we examine the idea of "managed collusion" as a means of coalition formation in reverse auctions, with the purpose of improving the bidders' expected profit. Building on the theoretical foundation established in auction theory, core games, and bidding rings, we intent to offer an analytical framework that tackles the theoretical underpinnings of this problem.

3. Coalition in an Auction Market

3.1 Settings of the Auction Market

We consider an electronic market where the auctioneer uses auction as a pricing mechanism to identify the *lowest* possible supply price for the item specified by the buyer. The buyer may specify a reserve price, and is entitled to reject the auction outcome if the final price exceeds the reserve price. The item for auction is a buyer- specified order, where the suppliers must respond to one order at a time via an auction mechanism. The buyer must announce *a priori* all order parameters before the auction starts. Each supplier computes pricing based on his own cost structure relative to the announced specifications. The suppliers may form alliance with one another under a coalition mechanism instigated by the auctioneer. Such coalition mechanism must satisfy a number of basic properties so that it not only improve the suppliers' margin but also improve overall efficiency of the market, thus protecting the buyer's interests. This will be the focus of the following analysis.

Several different auction mechanisms are possible in this environment as described in Section 2. We will focus our analysis on the *second-price, seal-bid simultaneous descending auction* similar to that of Milgrom [1998]. In each round of the auction, the auctioneer decreases the price by one unit, the bidders whose intended bid higher than the current price will immediately drop out. The auction continues until there is only one supplier left in the system, and this supplier is rewarded the price when the last supplier drops out, i.e., one unit below the second lowest price among all participating suppliers.

We now explain the basic principle that provides the surplus to sponsor a supplier coalition. Consider the cost scenario for a two-supplier example depicted in Figure 1. The buyer announces in the market an order with specification q, which could be any attribute describing the order, such as quantity or quality. For all orders to be considered in the market, q is a random variable described by density function $f(q)$. $s_1(q)$ and $s_2(q)$ represent the cost functions of supplier s_1 and s_2 in q, respectively. The buyer's reserve price is $r(q)$. Define $s_m(q)$ as the collective cost function of all *other* suppliers in the market S, i.e., $s_m(q)$ represents integrated market information.

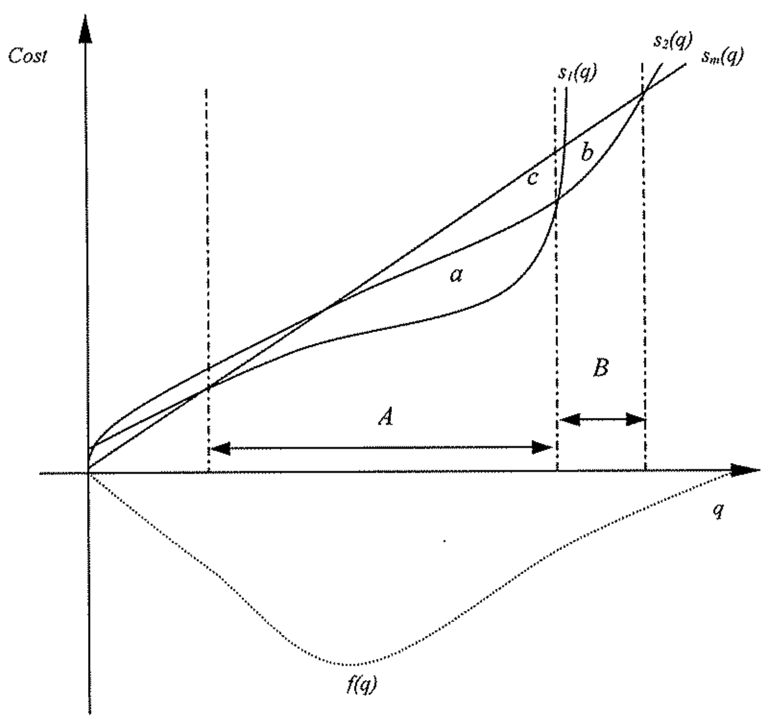

Figure 1. An Example Cost Structure for Suppliers in the Market

The *second-price descending seal-bid simultaneous auction* is used to determine which supplier(s) wins a given order in the market. As shown in the example, when the order attribute q falls into region A, supplier s_1 would win the order and awarded the bid $min\{s_2(q), s_m(q)\}$; when the order fall into region B, supplier s_2 would win the order and awarded $min\{s_1(q), s_m(q)\}$. Thus, the expected profit for s_1 is defined by area a, and the expected profit for s_2 is b. If suppliers s_1 and s_2 form a coalition, (i.e., they still bid at their true costs, but now they win the bid for the coalition) they will win the orders fall in either region A or B, *and* gain additional expected profit represented by area c, which they may share. This provides the basic incentive for suppliers to form coalition in the auction market.

3.2 Previous Results on Coalition Formation: The Core Game

Our proposed specification for supplier coalition mechanism rooted from the principles in a classical *core game*. A detailed discussion on various aspects of the *core* can be found in [Kannai, 1992]. Let $N=\{1,2,...n\}$ be the set of all players. A subset α of N is called a *coalition.* The *characteristic function* is a real-valued function $v(\cdot)$ defined on the coalition α which represents the outcome for α and $v(\phi)=0$. An outcome of the game is an n-dimensional vector $x=\{x_1, ..., x_n\}$, which represents how much each player $i \in N$ receives as payoff. Almost all existing coalition theory requires the assumption that $v(\alpha)$ is *well defined*, i.e., the value of a specific coalition is independent from the other coalitions. Gillies [1953] proposed the concept of *core game* for the *grand coalition.* A *core* is the set of all feasible outcomes that no player or group of players could improve upon by forming a subset coalition. An outcome is in the core when it satisfies the following:

$$\sum_{i \in \alpha} x_i \geq v(\alpha) \qquad \forall \alpha \subset N \qquad (1)$$

Much of the theoretical development in the *core game* sets out to answer the question: "Whether the core is empty for a the set of players N in a given game?" If the core is nonempty, the *grand coalition* has one or more stable outcome solutions. An important concept in the core game is that of a *balanced collection of coalitions.* A collection $\{\alpha_1, ..., \alpha_k\}$ of coalitions is *balanced* if there exists positive numbers (called *balancing weights*) $\lambda_1, ..., \lambda_k$ such that for *every*

$i \in N$, $\sum_{j, i \in \alpha_j} \lambda_j = 1$. Shapley [1967] and Bondareva [1963] give the following theorem for this classical problem:

Theorem *The core of the game v is non-empty iff for every balanced collection $\{\alpha_1, \ldots, \alpha_k\}$ with balancing weights $\{\lambda_1, \ldots, \lambda_k\}$, the following inequality holds:*

$$\sum_{j=1}^{k} \lambda_j v(\alpha_j) \le v(N). \tag{2}$$

Clearly, the core for a given set of players N could be empty. Shapley and Shubik [1973] suggest constraints for coalition formation such that non-empty coalitions could be formed.

The core game assumes that the coalition maximizes some measure of system's welfare, and studies whether the players in the system have proper incentives to form such coalition. However, answering the question "whether the *core* exist" may not be sufficient to model coalition formation in an auction market. Additional insights might be gained by modeling the players' choice based on the *profit distribution* scheme within the coalition. Later in Section 4, we propose such a mechanism. Note that the classical core game does not model this aspect of coalition formation. In the following, we first specify the requirements for a proper coalition mechanism in the auction market.

3.3 Required Properties of the Coalition Mechanism

In this section, we define basic requirements for a *supplier coalition mechanism* in the auction market specified above. These requirements form the criteria through which we may evaluate the design of a coalition mechanism. We first define as follows the expected profit of supplier $s_i \in S$ before he joins any coalition:

$$E[p_i(q)] = \int_0^\infty [\min\{r(q), s_j(q) \mid j \in S - \{i\}\} - s_i(q)]^+ f(q)dq \tag{3}$$

A coalition mechanism Ω defines the basic rules that allow suppliers to form efficient alliances with one another. Specifically, we define a coalition mechanism as two basic components:

(1) *coalitions generation*, where the membership of each coalition is indentified,

(2) *profit distribution* (among coalition members) once the coalitions are formed.

We argue that *profit distribution* is the key to a coalition mechanism. Once the *profit distribution* scheme is properly defined, *coalition generation* follows by considering each player's incentive via computed expectation of the player's profit. Thus, our discussion in the rest of the paper will focus on the properties of profit distribution. Following the convention of the core game, if under mechanism Ω the total profit for any given coalition is independent from all other coalitions, we say mechanism Ω is *well defined.* Denote a specific supplier collation α_j, where $\alpha_j \subseteq S$. A *coalition structure* is a set of coalitions $\{\alpha_j\}$ that are mutually exclusive and exhaustive, i.e.,$\bigcup_j \alpha_j = S$ and $\alpha_j \cap \alpha_k = \phi$. Clearly, the possible number of coalition among n suppliers is 2^n-1. Under a specific coalition mechanism Ω, we denote supplier s_i's profit as $p_{\Omega,i}(\alpha,q)$. We now list requirements for a *valid* coalition mechanism:

1. ***Individual Rationality****:* A fundamental requirement for a coalition membership is that all members in the coalition have an expected profit higher than his own expected profit by acting alone. In other words, the coalition mechanism should satisfy individual rationality, which is defined as follows:

$$E[p_{\Omega,i}(\alpha,q)] \geq E[p_i(q)] \quad \forall i \in \alpha\,, \tag{4}$$

where $E[p_{\Omega,i}(\alpha,q)]$ is the expected profit of supplier s_i after joining coalition α under mechanism Ω.

2. ***Private and Distributed Information****:* The coalition mechanism should *not* require the members to reveal their cost structure or other private information. Although information could be communicated rapidly and conveniently in the electronic market, information essential for a firm's competitiveness must be kept private. As in normal game theoretic assumptions, even if an agent is asked to provide pricing information, there is no guarantee that the information provided is truthful.

3. ***Observability and Controllability***:* Since each player maintains his private information in a distributed fashion, only information observable by all could be utilized by the coalition mechanism, i.e., the mechanism must be implemented solely on *observable* information. Further, all actions to be taken by the mechanism should be *controllable* by the market, e.g., a meaningful punishment could be imposed on the members who violate the market rules.

*4. **Social Welfare Compatibility:*** From the viewpoint of the buyers, an efficient market is one where every order is processed with the lowest achievable cost in regard to the order. A *cartel* in the tradition sense doesn't satisfy this condition as it might intentionally assign some orders to members with higher cost for the interest of increasing profit. Such coalition is unlikely to survive in a competitive market.

*5. **Maintaining Competition:*** Most countries prevent monopoly by anti-thrust regulations, as monopoly diminishes the incentive for true competition. Thus, the coalition forming mechanism must maintain competition among its members, i.e., regardless of which coalition they belong; every player in the market should have the incentive to reduce cost in order to compete with other players.

*6. **Financially Blancedness:*** The mechanism should guarantee that the formed coalition is financially balanced, i.e., the market maker doesn't need to provide subsidy to any coalition under the mechanism.

$$\sum_{i\in\alpha} p_{\Omega,i}(\alpha,q) = v_{\Omega}(\alpha,q) \quad \textit{for any } \alpha \textit{ and } q \tag{5}$$

4. An Auction Mechanism with Supplier Coalitions

In the following, we describe an auction mechanism that facilitates supplier coalitions. We propose a *profit distribution* scheme, which we will show providing the proper incentives for suppliers to form *valid* coalitions, i.e., a coalition mechanism defined based on this profit distribution scheme is *well defined* and satisfies the six basic requirements specified above.

We first define the meaning of *profit distribution* for supplier coalitions.

Definition 1. *Profit distribution within a supplier coalition refers to the splitting of the portion of the overall profit exceeding what any individual suppliers could achieve by acting alone.*

We now describe an auction mechanism in conjunction with a profit distribution scheme given a *coalition structure* $\{\alpha_i\}$. The mechanism consists of three main phases that includes two sub-auctions both supervised by the market maker.

Auction Mechanism with Supplier Coalitions (AMSC)

Phase 1. All suppliers compute their bids based on their own cost structure for a posted buyer order. A supplier coalition submits the bid that is the lowest among its members, and will be viewed as a single supplier during the auction. The *second-price, seal-bid simultaneous descending auction* takes place, i.e., in each round of the auction; the auctioneer decreases the price by one unit if the number of bidders is more than one. The auction stops when there is only one supplier left in the system. This is the lowest-cost supplier who will be reward the second lowest bid price (outside his coalition). Note that suppliers belonging to the same coalition do not compete against each other in the view of the auctioneer.

***Phase* 2.** Suppose supplier s_i wins the order with price p_1 and $s_i \in \alpha_j$, we say that coalition α_j wins this order with price p_1 (since supplier s_i's bid is not compared against other member of his coalition, the second lowest bid price p_1 is offered by a supplier outside coalition α_j). The market maker starts a *second-price sealed bid auction* within coalition α_j for this order and the supplier with the lowest bid wins the order with the second lowest price p_2. If there is a tie for the lowest bid within the coalition, then the players with the lowest bid has equal probability to process the order. Furthermore, allocation of profits within the coalition is based on the lowest among the bids that are strictly greater than the lowest bid, not on the second number in a sorted list of all bids.

***Phase* 3.** The coalition shares the *addition* profit $(p_1 - p_2)$ evenly among the members, i.e., each receives $(p_1 - p_2)/|\alpha_j|$.

Again, consider the example in Figure 1. Suppose an order described by q falls into region A. The bid would be won by the s_1-s_2 coalition (due to supplier s_1) with reward $p_1 = s_m(q)$ (since s_2 does not compete against s_1) given to the coalition. In the *Phase 2* auction within the coalition, supplier s_1 win the bid with reward $p_2 = s_2(q)$. The two suppliers split the additional profit $(p_1 - p_2)$ evenly. As a result, supplier s_1 receives profit $s_2(q) - s_1(q) + (s_m(q) - s_2(q))/2$, supplier s_2 receives $(s_m(q) - s_2(q))/2$.

The reason that the Phase 1 and Phase 2 auctions are both needed in the mechanism is due to the truth revealing property of the *second-price seal-bid auction.* An implicit but important property here is that the second-price auction will always induce the suppliers *participating in the auction* to reveal their true costs. Suppose we eliminate *Phase 2* auction but simply record in *Phase 1* the lowest bid and the second lowest bid in each coalition, say p_{min} and $p_{mis,}$ respectively. At the end of the auction, the supplier with bid p_{min} would win a bid at p_1 for the coalition. Since the second lowest bid in the coalition is already recorded, ($p_{mis} = p_2$), we would have no problem computing the additional profit ($p_1 - p_2$) brought by the coalition. This makes the Phase 2 auction seemly unnecessary. Nevertheless, without the Phase 2 auction supplier s_{mis} who has a true cost of p_{mis} would not actually participate in the auction since the supplier with p_{min} would represent the coalition. Consequently, there is no proper incentive that would ensure supplier s_{mis} to reveal his true cost. For example, in the case of equal profit sharing described in Phase 3, supplier s_{mis} actually has the incentive to lower his bid below his true cost in order to increase his share of the coalition profit $(p_1 - p_2)/|\alpha_j|$.

4.1 Incentive Compatibility

Recall that Guth and Peleg's [1996] *impossibility theorem* states that none of the transfer payment scheme (in our case, $(p_1 - p_2)/|\alpha_j|$) satisfying envy-freeness condition is *incentive compatible*. Applying their results in our situation, it would imply the following: if a coalition member believes he is the second lowest cost supplier in the coalition, he will *underbid* his true value (to lower p_2) to induce a higher transfer payment from the coalition, but he must be careful not to lower his bid below the lowest cost in the coalition otherwise he will suffer the "winner's curse." In the following, we will show that our proposed profit distribution mechanism provides proper incentive such that (1) the best strategy for the suppliers in the same coalition is not to complete with one another, and (2) bidding the true cost is the best strategy so long as the bids are uniform distributed and the bidder's cost is above a certain threshold. Given the auction mechanism described above, and a coalition structure $\{\alpha_i\}$, we state the following proposition:

Proposition 1. *(Compliance) In a second-price, seal-bid simultaneous descending auction participated by all suppliers, if profit distribution (Definition 1) is used among coalition* *members, the best strategy for a supplier s_i in coalition α_j is not to compete with other members in the same coalition α_j during the auction.*

Proof: Suppose the first and second lowest cost suppliers are s_{min} and s_{mis}, respectively. Coalition α_j may only win the bid if the lowest cost supplier s_{min} is in the collation. When coalition α_j wins the order characterized by q, the reward p_1 will be

$$p_1(\alpha_j,q) = \min\{r(q), s_i(q) | \forall i \notin \alpha_j\} \qquad (6)$$

If $p_1\ (\alpha_j\ ,q)\ <\ s_i(q)\ \ \forall i \in \alpha_j$, coalition α_j will not win this order in the first place. If coalition α_j does win the order, it is sufficient to consider two cases:(1) the second lowest cost supplier s_{mis} (where $s_{mis}(q) < p_1$) is also in α_j, and (2) otherwise

When (1) is the case, it is the best interest for the α_j suppliers other than s_{min} (including s_{mis}) not to participate in the auction since the profit to be *distributed* in the coalition is the difference between the second lowest cost outside the coalition, $p_1\ (\alpha_j\ ,q)$ and the second lowest cost inside the coalition $s_{mis}(q)$, *i.e.*, $p_1\ (\alpha_j\ ,q)\ -\ s_{mis}(q)$. If any supplier s_k in coalition α_j with cost $s_k(q)<p_1\ (\alpha_j\ ,q)$ competes in the auction, $s_k(q)$ would replace $p_1\ (\alpha_j\ ,q)$and his profit would decreases. On the other hand, for a supplier s_l in coalition α_j with cost $s_l(q)\geq p_1(\alpha_j\ ,q)$, competing in the auction will not increase his profits.

When (2) is the case, it is again the best interest for the α_j suppliers other than s_{min} not to participate in the auction since all profits $p_1\ (\alpha_j\ ,q)-\ s_{min}(q)$ go to supplier s_{min} in any case.

The above proposition answers a fundamental question: "If all suppliers in a coalition participate directly in the auction, do they have the incentive to act collaboratively as a member of the coalition?" The answer is yes. Thus, the rule of participation in the above auction mechanism is indeed incentive compatible. In the following, we show that giving the true cost during the auction is the best strategy for the suppliers.

Proposition 2. *(Incentive Compatibility) Given the profit distribution scheme defined by AMSC, the best strategy for a supplier s_i in coalition α is to reveal his true cost in both Phase 1 and Phase 2 auctions, so long as the bids are uniformly distributed and his cost is greater than $|\alpha|/2$.*

Proof. To prove that players will tell the truth during the second stage auction.

Suppose coalition α wins the order in the first stage auction and this auction has n members. Thus, a supplier $i \in \alpha$ would fall into one of the following categories (1) the lowest cost in α, (2) the second lowest cost in α, or (3) otherwise. Thus, the *expected profit* for supplier i with the placed price $s_i'(q)$ is as follows:

$$(\min\{s_j(q)|j \in s-\{i\}\} - s_i(q) + \frac{p_1 - \min\{s_j(q)|j \in s-\{i\}\}}{|\alpha|}) \cdot \Pr[\min\{s_j(q)|j \in s-\{i\}\} > s'_i(q)]$$
$$+(\frac{p_1 - s'_i(q)}{|\alpha|}) \cdot \Pr[\min\{s_j(q)|j \in s-\{i\}\} \leq s'_i(q) \text{ and } mis\{s_j(q)|j \in s-\{i\}\} > s'_i(q)]$$
$$+\frac{p_1 - mis\{s_j(q)|j \in s-\{i\}\}}{|\alpha|} \cdot \Pr[mis\{s_j(q)|j \in s-\{i\}\} \leq s'_i(q)]$$

We assume that no supplier knows which category he actually belongs to. However, it should be clear that if i happen to fall into categories (1) and (3), there is no incentive for him to lie (i.e., to report a $s_i'(q)$ different from the true cost $s_i(q)$) as this will not increase his profit in any way. The only supplier who may influence the *coalition profit* (as defined by the two-stage auction mechanism) is supplier in category (2). We will analyze this (second-lowest price) supplier's incentive as follows.

First of all, there is no incentive for this supplier to report a higher cost ($s_i'(q) > s_i(q)$) since this will decrease his profit. However, there may be incentive for him to report a lower cost to increase overall *coalition profit* (therefore his own share), but he needs to take the risk of the "winner's curse," in which case he will have to process the order with price lower than his true cost. To examine this case further, we introduce the following setting: As defined in the *second-price, seal-bid simultaneous descending auction,* stated bids are discrete and in a range, say, $1..m$. The coalition α has $|\alpha|=n$ players. For a category (2) supplier s_i with cost $l+1$ (between 1 and m), the expected gain to lower his announced cost with one unit is as follows:

$$E_i = \frac{l-1}{l} \cdot \frac{1}{n}(r_1) - \frac{1}{l}[(\frac{1}{2} - \frac{1}{n}) \cdot r_1 + \sum_{j=2}^{n-1} r_j (\frac{1}{j+1} - \frac{1}{n})]$$

$$Here\ r_k = \frac{\binom{k}{n-1}(m-l)^{n-k-1}}{\sum_{j=1}^{n-1}\binom{j}{n-1}(m-l)^{n-j-1}}$$

Where r_k represents the probability that k members in the coalition has the lowest cost when supplier i with cost $l+1$ has the second lowest price. Recall that if there is a tie for the lowest bid within the coalition, then the players with the lowest bid has equal probability to process the order, thus the expected production cost is shared evenly. Since supplier s_i has the second lowest price in the coalition, the lowest price in the coalition may be any discrete number between *1 and l*, with equal probability (independent to the number of suppliers in the coalition). When k=1 and $min(s_j(q)\,|\,j \in \alpha) < l$, supplier i can get $1/n$ additional units of profit based on the profit sharing scheme. When $min(s_j(q)\,|\,j \in \alpha) = l$, supplier i would have $1/(k+1)$ to process the order and lose one unit. By simplifying the equation, we have

$$E_i = \frac{2l-n}{2nl} r_1 - \frac{1}{nl}\sum_{j=2}^{n-1} r_j \left(\frac{n-j-1}{j+1}\right)$$

$$When\ l \le \frac{n}{2},\ E_i\ is\ negative\ given\ that\ r_1 \ge 0$$

Thus, the supplier whose cost $s_i(q)=l+1>n/2$ satisfy the above condition will always tell the truth.

We show that for a second lowest price player with cost above a certain threshold value, truth telling is an optimal response when all other bidders are telling the truth.

If we only consider the cases where $k=1$ and 2 (r_1 and r_2) we have

$$E_i \le \frac{3(2l-n)(m-l)-(n-3)(n-2)}{3nl} r_2$$

In theory, the coalition may control the amount of decrement in each iteration of the auction to influence the value of l, thus force the players to tell the truth. If the resulting price from the first auction is P_1 and possible lowest price is P_{min}, the coalition could set decrement as $2(P_1-P_{min})/n$ to guarantee truth revealing. In practice, when n is large enough, the second lowest price supplier may not reach the threshold value and would have the incentive to lie.

For continuous price auction, Guth and Peleg [1996] show that the dominating strategy is for the player to bid slightly below his true cost. A player can always find a small enough amount and get more profit, but this may not be relevant from the mechanism design point of view. The coalition can always adjust the unit decrement such that it is larger than this small amount thus force the truth telling behavior. The above results verify two important properties for the auction mechanism with supplier coalitions. In the following, we will evaluate this mechanism against basic requirements for coalition formation.

4.2 Properties of AMSC

As pointed out earlier, a coalition mechanism includes *coalitions generation*, and *profit distribution*. While we do not address the issue of *coalition generation* in this paper, the profit distribution scheme described by *AMSC* is sufficient to evaluate the basic properties of the coalition mechanism. In the following, we will verify all the mentioned requirements for *AMSC*.

Proposition 3. *AMSC mechanism is a well-defined mechanism.*

$$E[v(\alpha,q)] = \int_0^\infty \left[\min\{r(q), s_j(q) \mid j \notin \alpha\} - mis\{s_i(q) \mid i \in \alpha\}\right]^+ f(q)dq \qquad (8)$$

Proof. Under mechanism *AMSC*, the expected profit for a coalition α is as follows:

Obviously, $v(\alpha,q)$ is independent from other supplier coalition structures, however, it relies on aggregated market information on second lowest price defined by $\min\{r(q), s_j(q) \mid j \notin \alpha\}$.□

This feature is critical to simplify the *coalition generation* problem.

Proposition 4. *Any coalition under the AMSC mechanism satisfies the Individual Rationality requirement*

Proof. For a supplier s_i in coalition α, he can always get more profit than acting alone in the market. Because for any q, the profit for one specific member i in α is as follows:

$$p_i(\alpha,q) = p_i(q) + \left[\min\{r(q), s_j(q) \mid j \notin \alpha\} - mis\{s_i(q) \mid i \in \alpha\}\right]^+ / \mid \alpha \mid \qquad (9)$$

Where $p_i(q)$ is s_i's profit by acting alone. Define the *additional profit* for a given coalition α as

$$d(\alpha,q) = \left[\min\{r(q), s_j(q) \mid j \notin \alpha\} - mis\{s_i(q) \mid i \in \alpha\}\right]^+ \qquad (10)$$

Clearly, by acting alone, $d_i(\{i\},q)=0$.

The remaining requirements can be stated in the following propositions and can be verified in words.

Proposition 5. *A coalition operates under AMSC require only distributed information that are observable and controllable.*

In *AMSC*, there is no private information required from the suppliers. All information required for the mechanism can be obtained from the auctions and therefore observable by the market maker. As following the coalition rule is incentive compatible (Propositions 1 and 2), while *AMSC* satisfies individual rationality (Proposition 4), the behavior of the suppliers is consistent with the mechanism's intent, because such behavior can lead to the maximal benefit for the suppliers. Thus, the coalition structure is also controllable, i.e., exclusion from a coalition is a meaningful punishment, as the player will suffer from less profit.

Proposition 6. *Any coalition structure operates under AMSC satisfies Society Welfare*

AMSC does not avoid competition among members of the same coalition. Every supplier has the incentive to decrease cost. Only when a supplier has the lowest cost in the market and when this price is less than the reserved price of the buyer, can his coalition win the order in *phase1* of *AMSC*. In the *phase2* auction, only the supplier with the lowest cost for this specific order can win the order. Thus, the supplier with the lowest cost will process any given order, therefore *AMSC* satisfies the society welfare requirement.

Proposition 7. *Any coalition operates under AMSC is financially-balanced.*

It is easy to verify that coalition operates under *AMSC* is financially balanced. This is because the coalition only distributes *additional* profit from the current transaction to its members, thus, equation (5) is always satisfied. The coalition is balanced not only in a long-run, but also for each order. A potential benefit from *AMSC* is that the market maker such as an eCommerce company may potentially profit from the coalition's additional profit. All features of *AMSC* mechanism remain, if the eCommerce companies charge, say, one fixed percent of $d(\alpha, q)$.

5. Stable Coalition Structure

We verify in the previous section the *profit distribution* scheme for *AMSC* satisfy all requirements for a valid coalition mechanism. A related issue is that of *coalition generation*, the formation of the coalition. While we do not address the mechanism involved in *coalition generation*, we will establish that *AMSC* sustains a stable coalition structure. First, consider the following proposition.

Proposition 8. *Under AMSC, the additional profit for the coalition satisfies the additivity property.*

Proof. Given (10), we may write the additional profit for any member in the coalition α as

$$d_i(\alpha, q) = d(\alpha, q) / |\alpha| \qquad (11)$$

For any two coalitions α and β with $\alpha \cap \beta = \phi$, we have

$$d(\alpha \cup \beta, q) = \left[\min\{r(q), s_j(q) \mid j \notin \alpha \cup \beta\} - mis\{s_i(q) \mid i \in \alpha \cup \beta\}\right]^+ \quad \textit{and, furthermore}$$
$$\min\{r(q), s_j(q) \mid j \notin \alpha \cup \beta\} \geq \max\{\min\{r(q), s_j(q) \mid j \notin \alpha\}, \min\{r(q), s_j(q) \mid j \notin \beta\}\} \quad \textit{and}$$
$$mis\{s_i(q) \mid i \in \alpha \cup \beta\} \leq \min\{mis\{s_i(q) \mid i \in \alpha\}, mis\{s_i(q) \mid i \in \beta\}\}$$

thus, the following additivity property holds:

$$d(\alpha \cup \beta, q) \geq d(\alpha, q) + d(\beta, q) \quad \textit{for any coalition } \alpha, \beta \textit{ where } \alpha \cap \beta = \phi \qquad (12)$$

The *additivity property* seems to imply that all coalitions have the incentive to merge with other coalitions and eventually lead to a grand collation, i.e., all suppliers in N belong to one coalition. We will discuss show in the following that the grand coalition is usually not the result under *AMSC*. We first need to establish a few results from the *collation formation* literature.

Definition 2. *The set of coalition* $\{\alpha_i\}$ *is a coalition structure if* $\cup_{i \in c} \alpha_i = s$ *and* $\alpha_i \cap \alpha_j = \varnothing$ *for* $i \neq j$, Coalition structure is defined based on the idea of set partitioning. For example, if we have three players *{1,2,3}*, the possible coalition will be *{1},{2},{3},{1,2},{1,3}, {2,3}* and *{1,2,3}* and the possible coalition structures will be *({1},{2},{3}), ({1},{2,3}), ({1,2},{3}), ({1,3},{2})* and *({1,2,3})*.

Proposition 9. [Sandolhm 1999] *The number of coalition structure is* $O(n^n)$ *and* $\omega(n^{n/2})$

Clearly, to find the optimal coalition structure based on some performance measure is *NP-hard.* Sandolhm [1999] develops several *coalition generation* heuristics using both the centralized and distributed viewpoints. If we use x_k to represent the current expected additional profit for supplier *k.*

Definition 3. A *coalition structure c is stable in an n-supplier game if it satisfies the following condition.*

$$x_k \geq \max\{E[d(\alpha_i + \{k\},q)]/|\alpha|+1 \;\Big|\; E[d(\alpha_i + \{k\},q)]/|\alpha|+1 \geq E[d(\alpha_i,q)]/|\alpha|, \forall \alpha_i \in c \text{ and } k \notin \alpha_i\} \quad \text{for any } k \in N \tag{13}$$

This definition of *stable coalition* is similar to the notion of *Nash Equilibrium.* In a stable coalition, no one can get more profit by just one movement. In other words, no supplier could gain more profit by joining another coalition that is willing to accept him. Note that this definition is different from the *core* of the game. Given the cost structures of all suppliers, there will be more than one stable coalition structure, so we introduce an additional concept about *stability.*

Definition 4 *A coalition structure c is strongly stable in an n-supplier game if it satisfies the following condition.*

$$\forall \alpha \notin c, \exists k \in \alpha : E[d(\alpha,q),]/|\alpha| \leq x_k \tag{14}$$

If a market reaches the *strongly stable* coalition structure, no supplier could gain more profits with any number of movements. Obviously, a *strongly stable solution* must be a *stable solution.*

This concept is similar to the *core* discussed earlier. However, the *core game* only considers the stability of the *grand* coalition, but here the strongly stable coalition structure may be any coalition structure.

If we allow the suppliers to freely make coalitions and all cost structure information is public, we can state the following.

Proposition 10. *Under AMSC there is one unique strongly stable coalition structure.*

Proof.

We may construct (in theory) the unique strongly stable coalition structure as follows:

(Assuming all cost structure of the suppliers, order distribution and the reserved price are known).

Step 0.. Set $c=\varnothing$ and $i=1$. Set M as the set of all possible coalition, i.e., M is the set of all subset of N.

Step 1. Find coalition α in M with the highest value of $E[d(\alpha,q)]/\mid\alpha\mid$ and call this coalition α_i

Step 2. Set $c \leftarrow c \cup \alpha_i$ and $i \leftarrow i+1$

Step 3. Delete from M all coalitions that share the same member of α_i in M

Step 4. If $M \neq \varnothing$, go to step 1. Otherwise, stop.

The coalition structure c resulting from the above procedure is the unique *strongly stable coalition* structure in this game. This is true because the coalitions in c would have no incentive to accept any additional suppliers: members of the coalition with a lower index would not have the incentive to join the coalitions with higher index. On the other hand, the coalition with a lower index has no incentive to accept suppliers from the coalition with a higher index. For any other coalition structure $c' \neq c$, suppliers in the coalition $\alpha'_i \in c'$ with the lowest coalition index i (which would not have the lowest index in c, by definition) would have the incentive to form a different coalition identical to $\alpha_i \in c$. Thus, the coalition structure c' cannot be a strongly stable coalition structure.

Since the grant coalition does not necessarily have the highest value of $E[d(\alpha,q)]/|\alpha|$, the strongly stable coalition structure under *AMSC* may not be the grand coalition. This result is important from the viewpoint of market design. A converse result would imply that regardless of the *coalition generation* mechanism, the market would gravitate toward one coalition, i.e., no collation at all. In reality, the cost structure of each supplier is private information not known to the system. The system is unlikely to reach the strongly stable coalition structure immediately. In this case, we shall assume that the suppliers are not allowed to switch from one coalition to another during the auction in order to avoid operational complexity.

6. Conclusions

In this paper, we consider the supplier's coalition problem in buyer-centric procurment auctions. We specify the requirements for a valid coalition mechanism as individual rationality, private and distributed information, observability and controllability, society welfare compatibility, maintaining competition, and financially balancedness. We propose an auction mechanism with supplier coalitions *(AMSC)* and we specify a profit distribution scheme under this auction mechanism. We verify that *AMSC* satisfies all requirements as a valid coalition mechanism. Importantly, we show that the proposed *profit distribution* mechanism provides proper incentive such that (1) the best strategy for the suppliers in the same coalition is not to complete with one another (*compliance*), and (2) bidding the true cost is the best strategy so long as the bids are uniform distributed and discrete, and the bidder's cost is above a certain threshold (*incentive compatibility)*. We discuss the possibility for the auctioneer to decrease the auction pricing non-uniformly so as to discourage untruthful bidding. For theoretical interest, we show that under symmetric information, a unique strongly stable coalition structure exists under *AMSC*. However, we do not address the issue of *coalition generation* neither do we discuss the issue of unique strongly stable coalition structure under *asymmetric* information. For future study, we recommend further study of coalition generation mechanisms, and implementable rules that would force the suppliers to reach a stable coalition structure quickly.

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