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Optimal Tax Depreciation and Loss Carry-Forward and Backward Options

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Abstract

The choice of depreciation method from among straight-line and accelerated methods can have a significant impact on the present value of expected tax payments. This is a problem that has been studied for decades, with most results indicating the optimality of accelerated methods. Recent research has begun to question this claim by relaxing one of the original assumptions of positive taxable income. The situation where net-operating losses may be carried-forward and backward in time to when a profit is made is the subject of this paper. This paper models this situation and establishes the conditions that allow straight-line depreciation to be preferred over accelerated methods. The results are focused around a threshold number of periods of consecutive losses, which are determined by the allowable periods to carry a loss forward. For consecutive losses beyond this threshold, straight-line will always be optimal. When the cumulative depreciation charges up to and including the window are guaranteed to be applied on or before the threshold period, then straight-line will never be optimal.

Keywords: Decision Analysis; Depreciation; Tax minimization; Cash Flow Analysis

1. Introduction

The general problem of selecting a depreciation method is important because it directly affects the amount of taxes paid in a given period. Payments made in later periods receive greater discounting, so there is an obvious advantage to postponing payments through depreciation in order to minimize the present value (assuming a constant discount rate). Since accelerated methods provide a greater deduction in earlier periods than the straight-line method, they have typically been identified as the optimal methods. Numerous authors have reached this conclusion, including Davidson [3], Davidson and Drake [4,5] and Wakeman [12]. There are

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certain conditions when this is not the case though and therefore this problem has continued to receive attention in recent years. When cash-flows are uncertain and negative taxable income is expected, the choice of depreciation method is not as apparent. Motivated by Berg et al. [2], this paper examines the case where cash flows are growing and becoming more stable with time, leaving the opportunity for negative taxable income in early periods. Given this, an optimal depreciation policy is determined when losses can be carried-forward and applied to future periods, where a profit is realized. The loss carry-backward case is also modeled.

Current tax law allows a choice between straight-line (SL) and approved accelerated depreciation methods, namely Double Declining-Balance (DDB) and Sum of the Years-Digits (SYD). Straight-line depreciation applies a constant charge each period of the asset's life. Accelerated methods are those that initially apply a deduction larger than SL and then taper off to a deduction smaller than SL by the end of the asset's life (N). For SYD, the depreciation percentage is a declining fraction where the denominator always equals the sum of the digits from 1 to N, and the numerator begins at N and decreases by 1 each year. DDB takes a fraction of the residual value each period. Under this method, the asset would never be fully depreciated, so it is typically adjusted in some way at or near the end so that the residual value is zero after year N. With this choice of depreciation has come significant study of the optimal method in terms of the maximum present value of taxes saved each period by depreciation.

Early papers on the subject [3] quickly and easily showed that the accelerated methods are preferred over straight-line. Later work [4,5,11] then studied the two accelerated methods more closely and identified the conditions that make one preferable over the other.

Eventually, research turned to a closer examination of the superiority of accelerated methods and formulated specific decision rules to select the optimal depreciation method. Work by Schoomer [12] and Wakeman [14] compared both the accelerated methods to SL and showed that there are cases when SL is preferred to DDB if there is a low rate of return and a low salvage value. Wakeman's [14] final conclusion selected DDB switching to SYD as the new optimal method. Given that the switch is made in the year which maximizes the present value of the tax savings, this method will always be preferred over any other allowable method.

This idea of switching is another topic that has received much attention. Authors such as Greene [7] and Ricks [10] studied the timing of the switch to SL and developed criteria and strategies for optimum switching. An optimum switching rule, used by Wakeman [14] was developed by Schwab and Nicol [13].

The early research in tax minimizing depreciation assumed a constant tax rate, cash flows large enough to cover the depreciation expense each period, and did not address asset disposition. More recently, research has been extended to consider these situations where these assumptions do not hold.

The effect of uncertain cash flows on the optimal depreciation method was first studied by Berg and Moore [2]. They found that when negative taxable income resulted in lost depreciation deductions, then SL depreciation could be preferred to an accelerated method. Berg et al. [1] continued this study with a particular emphasis on cash flow probability distributions that represent a company in a growth stage, where cash flows increase and their standard deviation decreases with time. They found that SL depreciation can be preferred when early periods have a higher probability of negative income, assuming that a carry-forward or carry-backward does not occur with negative income. These two papers [1,2] also examine the issue of a non-constant tax rate in the form of a progressive tax system. The result is that stable or growing future cash flows in a progressive tax system cause SL depreciation to be optimal in order to offset the chance of moving to a higher tax bracket. Another recent paper [15] focuses exclusively on a progressive tax system without the uncertain cash flows. This paper includes replacement investments and examines firms in a steady state where reinvestments are made to offset technical deterioration and keep capital stock constant. The results of this paper are that accelerated depreciation methods may not be optimal, even if taxable income is always positive.

Fleischer et al. [6] examined premature asset disposal and its effect on the optimal depreciation method. When an asset is salvaged early, the total depreciation is dependent on the method used and may result in different taxable gains. The results are difficult to characterize in a general sense because they depend on the property class, discount rate, and year of asset disposal.

We specifically analyze the case where losses can be carried forward or backward in time and their impact on the optimal depreciation method. This paper proves that an accelerated method remains optimal as long as the number of consecutive periods of loss does not exceed the carry-forward period. In other words, when the carried-forward deductions are not lost, the most accelerated method will result in the minimum present value of tax payments. When the number of consecutive periods exceed the permitted carry-forward time frame, straight-line may be optimal depending on when the losses began and if they end before the conclusion of the equipment's depreciation time frame.

This paper is structured as follows. Section 2 discusses the model used to select an optimal depreciation method in order to minimize the present value of the expected tax payments. There are three versions of the model presented, each handling different carry-forward possibilities, and a five-period example is evaluated by each version. Section 3 develops several conditions on the nature of the cash flows, which will determine the optimal depreciation method. The final section concludes with a summary of the contributions of this paper and ideas for further research in this area.

2.0 Models and Examples

This section presents models for different cases of losses being carried backwards or forwards in time. Notation used throughout the section is presented first.

2.1 Notation and Formulation

For each case, the general model will be explained and then applied to the small example. First, the basic notation and equations necessary for the model are presented. For consistency, the notation is similar to that provided by Berg et al. [1].

i = period

D = initial value of asset

N = number of periods over which the asset is depreciated

d = straight-line depreciation deduction taken each period through N

d_i = accelerated depreciation deduction in period i

C_i = cash flow in period i prior to deducting depreciation. This is a random variable with a probability distribution of $F_i(x) = P(C_i \le x)$.

$T \in (0,1]$ = constant tax rate

$\alpha \in [0,1]$ = constant single period discount factor.

The analysis compares the straight-line depreciation method (SDM) to accelerated methods (ADM), represented by DDB and SYD. The formulas for calculating the depreciation charges for each method are shown below.

Straight-line (SL):

$$d = \frac{D}{N}$$

Sum of the Year's-Digits (SYD):

$$d_i = \frac{2(N-i+1)D}{N(N+1)} \quad \text{for } i = 1,\ldots, N$$

Double Declining-Balance (DDB):

$$d_i = D\left(\frac{2}{N}\right)\left(1-\frac{2}{N}\right)^{i-1} \quad \text{for } i = 1,\ldots,N\text{-}1$$

$$d_N = D\left(1-\frac{2}{N}\right)^{N-1}$$

To compare SDM to ADM, the present values of the expected tax payments are calculated for each method and compared. Since a lower payment is desired, the smaller of the two tax payments determines the optimal method. Similar to Berg et al. [1], we use the following formula from Nahmias [9] for calculating the expected positive value:

$$E[\max(C_i - x,0)] = \int_x^\infty (C_i - x) f(x) dx$$

where $f(x) = \frac{e^{\frac{-(x-\mu)^2}{2\sigma^2}}}{\sigma\sqrt{2\pi}}$ for the normal distribution with a mean of μ and a standard deviation of σ.

Throughout the following models, the problem horizon is considered to be equal to the life of the asset. When expected tax payments are compared, they are all assumed to end at period *N*. It is also assumed that no value is received or paid for the disposal of the asset at the end of its life.

2.2 No Carry-Forward of Losses

We briefly summarize the notation and an example from Berg et al. [1], as it forms the basis for comparison. Without the carry-forward of losses, the taxable income in each period is determined solely from $(C_i - d_i)$. If this value is positive, then a tax of $T(C_i - d_i)$ is paid. If it is negative, the tax payment is zero and the excess depreciation deduction is lost. As shown in Berg et al. [1], the formulas for the present value of the expected tax payments without carry-forward for straight-line and accelerated methods are:

$$Tax_S(\alpha) = T\sum_{i=1}^{N}\alpha^i(\max(C_i - d, 0)) = T\sum_{i=1}^{N}\alpha^i\int_{d}^{\infty}(C_i - x)f(x)dx$$

$$Tax_A(\alpha) = T\sum_{i=1}^{N}\alpha^i(\max(C_i - d_i, 0)) = T\sum_{i=1}^{N}\alpha^i\int_{d_i}^{\infty}(C_i - x)f(x)dx$$

Example 2.1 The data for a five-period example comes from Example 3.1 in Berg et al. [1]. Straight-line depreciation is compared to accelerated methods represented by DDB and SYD.

$N = 5$

$D = 5$

$T = 0.2$

$C_1 \sim N(2,2)$ $C_2 \sim N(3,2)$ $C_3 \sim N(4,2)$ $C_4 \sim N(4,1)$ $C_5 \sim N(4,1)$

SL: $d = \frac{D}{5} = 1$

DDB: $d_1 = \frac{2}{5}D$ $d_2 = \left(\frac{2}{5}\right)\left(\frac{3}{5}\right)D$ $d_3 = \left(\frac{2}{5}\right)\left(\frac{3}{5}\right)^2 D$ $d_4 = \left(\frac{2}{5}\right)\left(\frac{3}{5}\right)^3 D$ $d_5 = \left(\frac{3}{5}\right)^4 D$

$d_1 = 2$ $d_2 = 1.2$ $d_3 = 0.72$ $d_4 = 0.432$ $d_5 = 0.648$

SYD:

$$d_1 = \frac{5}{15}D \;\; d_2 = \frac{4}{15}D \;\; d_3 = \frac{3}{15}D \;\; d_4 = \frac{2}{15}D \;\; d_5 = \frac{1}{15}D$$

$$d_1 = \frac{5}{3} \;\; d_2 = \frac{4}{3} \;\; d_3 = 1 \;\; d_4 = \frac{2}{3} \;\; d_5 = \frac{1}{3}$$

Using the above tax payment equations, expected values can easily be obtained from the data above for any value of α. The results coincide with those shown by Berg et al. [1] and are given in Table 2.1 below for α from 0.80 to 1.00.

Table 2.1 *No Carry-Forward Expected Present Value*

α	SL	SYD	DDB
0.80	1.256250041	**1.225015062**	1.235950446
0.81	1.30303287	**1.274266051**	1.285875476
0.82	1.351309985	**1.325194491**	1.33747795
0.83	1.401120709	**1.377846321**	1.390802534
0.84	1.452505102	**1.43226836**	1.445894723
0.85	1.505503962	**1.488508319**	1.502800852
0.86	1.560158839	**1.546614807**	1.561568101
0.87	1.616512039	**1.606637341**	1.622244508
0.88	1.674606628	**1.668626355**	1.684878973
0.89	1.734486447	**1.732633208**	1.749521266
0.90	**1.796196112**	1.798710195	1.816222037
0.91	**1.859781024**	1.866910554	1.885032825
0.92	**1.925287377**	1.937288472	1.956006062
0.93	**1.992762163**	2.009899102	2.029195083
0.94	**2.062253183**	2.084798562	2.104654138
0.95	**2.133809048**	2.16204395	2.182438392
0.96	**2.207479194**	2.241693352	2.262603941
0.97	**2.283313881**	2.323805849	2.345207814
0.98	**2.361364209**	2.408441528	2.430307985
0.99	**2.441682115**	2.495661487	2.517963379
1.00	**2.52432039**	2.585527851	2.608233882

Without carry-forward, negative taxable income in a given period results in a loss of the depreciation deduction. This example shows that in situations where there is a higher probability for a loss in early periods, accelerated methods are not necessarily preferred. This is because higher depreciation deductions may be wasted in early periods (and lost) rather than saving the deduction for a time when the cash flows are higher.

There is a threshold value of α around 0.895. Below this value, SYD provides a lower expected tax payment and above it SL is preferred. DDB, which is the more accelerated method, is completely dominated by SYD.

2.3 Infinite Carry-Forward Example

Here, we model the ability to carry losses forward rather than losing any depreciation in excess of the cash flow C_i. The simplest case to model allows losses to be carried-forward for an infinite number of periods. This not only lessens the complexity of tracking the carry-forward value, but it means that as long as the firm is eventually profitable, the depreciation deductions can never be lost, though their impact on the present value of the tax payments decreases as they are applied farther into the future.

Let y_i represent the total losses carried-forward from period i when the carry-forward period is infinite. It is calculated as:

$y_i = \max(d + y_{i-1} - C_i, 0)$ for all $i = 1,\ldots,N$ for the straight-line method

$y_i = \max(d_i + y_{i-1} - C_i, 0)$ for all $i = 1,\ldots,N$ for accelerated methods

$y_0 = 0$

The present value of the tax payments made using SDM and ADM can then be represented by the following expressions.

$$Tax_S(\alpha) = T\sum_{i=1}^{N}\alpha^i(\max(C_i - d - y_{i-1}, 0))$$

$$Tax_A(\alpha) = T\sum_{i=1}^{N}\alpha^i(\max(C_i - d_i - y_{i-1}, 0))$$

The model consists of building probabilistic paths of profits where each node is represented by the cash flows, minus depreciation for the period and any carry-forward deductions. The branches at each node are for the cases of positive or negative taxable income.

In the first period, we have a cash flow of C_1 and depreciation deduction of d_1. The tax payment in period 1 is $T[\max(C_1\text{-}d_1,0)]$. The taxable income is positive with a probability of $P(C_1>d_1)$, resulting in no losses carried-forward. The probability that it is negative is $P(C_1<d_1) = 1\text{-} P(C_1>d_1)$. In this case, the expected loss $E[\max(d_1 - C_1,0)]$ is carried-forward and applied to the next period. This is illustrated in Figure 2.1.

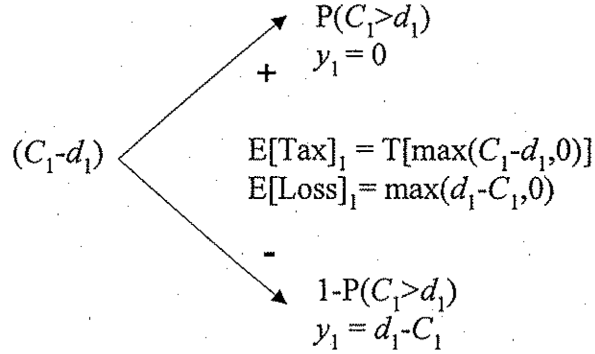


Figure 2.1 *Payments, Losses, and Probabilities in Period 1*

If the taxable income in period 1 is positive and the top branch is followed, then the same procedure is repeated in period 2, as shown in Figure 2.2. The probabilities on the branches are now conditional, since they depend on what has occurred in previous periods.

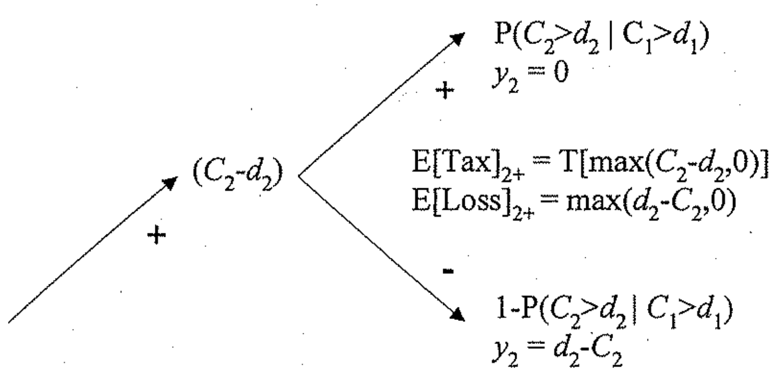


Figure 2.2 *Payments, Losses, and Probabilities in Period 2 (Upper Branch)*

Following the bottom branch in period 1, the carry-forward amount of $(d_1\text{-}C_1)$ must be deducted from C_2 along with d_2. This also affects the calculation of the probabilities of positive and negative taxable income. These new values are used to calculate $P(C_2+C_1>d_2+d_1 \mid C_1<d_1)$. The expected value of the tax payment consists of $(C_2\text{-}d_2)$ minus the expected loss carried-forward from period 1. Clearly, this value is lower than the tax payment expected when a loss did not occur in the previous period. Figure 2.3 illustrates this situation.

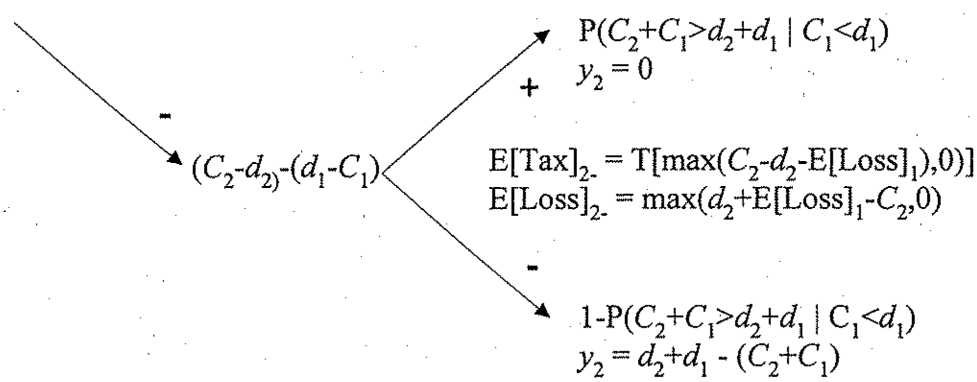


Figure 2.3 *Payments, Losses, and Probabilities in Period 2 (Lower Branch)*

In a given period i, there are a finite number of possible carry-forward scenarios. Only 0, 1, 2, … , $i-1$, or i periods of consecutive loss can be brought forward. Regardless of the path taken to get there, all the branches with 2 periods of loss carried-forward are the same in a given period. This results in many of the nodes being the same and they can be combined to keep the tree to a manageable size. The overall expected tax payments in each period are calculated using the cumulative probability of each node. For example, in period 2:

$E[Tax]_2 = T[\max(C_2\text{-}d_2,0)]*P(C_1>d_1) + T[\max(C_2\text{-}d_2\text{-}\max(d_1\text{-}C_1,0),0)]*[1\text{-}P(C_1>d_1)]$

The present value as a function of α is then computed from the expected payments in each period.

Example 2.2 The same example data is used to show some of the calculations in the infinite carry-forward method. The cash flows C_i are assumed to be normally distributed and independent, which allows the probability of traversing each branch to be easily obtained. When necessary, multiple independent, normal random variables can be combined relatively easily. (For example, combining C_1 and C_2 would result in a combined mean of $\mu_1 + \mu_2$, and a standard deviation of $\sqrt{\sigma_1^2 + \sigma_2^2}$.)

Using DDB depreciation, the expected value of the tax payment in period 1 is $E[Tax]_1 = E[\max(C_1\text{-}2,0)]*0.2 = 0.1596$. The probability that taxable income is positive and no losses are carried-forward is $P(C_1 \sim N(2,2) > 2) = 0.5$. The probability of a loss occurring is 0.5, and the expected value of the loss carried-forward to period 2 is $E[\max(2\text{-}C_1,0)] = 0.79788$. When the taxable income in period 2 is (C_2-1.2), the expected tax payment is 0.4002, and when the taxable income is (C_2-1.2-0.7979) the expected payment is 0.2794. Using the probabilities from period one of reaching each node, $E[Tax]_2 = E[Tax]_{2+}(0.5) + E[Tax]_{2-}(0.5) = 0.4002*0.5 + 0.2794*0.5 = 0.3398$. Each branch is calculated similarly. Figure 2.4 shows the tree for the first two periods.

For α from 0.8 to 1.0, the expected present values for the five-period example are shown in Table 2.2. Note that now, the most accelerated depreciation method (DDB) dominates the other two in providing the lowest present value of expected tax payments. SYD is only preferred over SL for $\alpha \leq 0.94$.

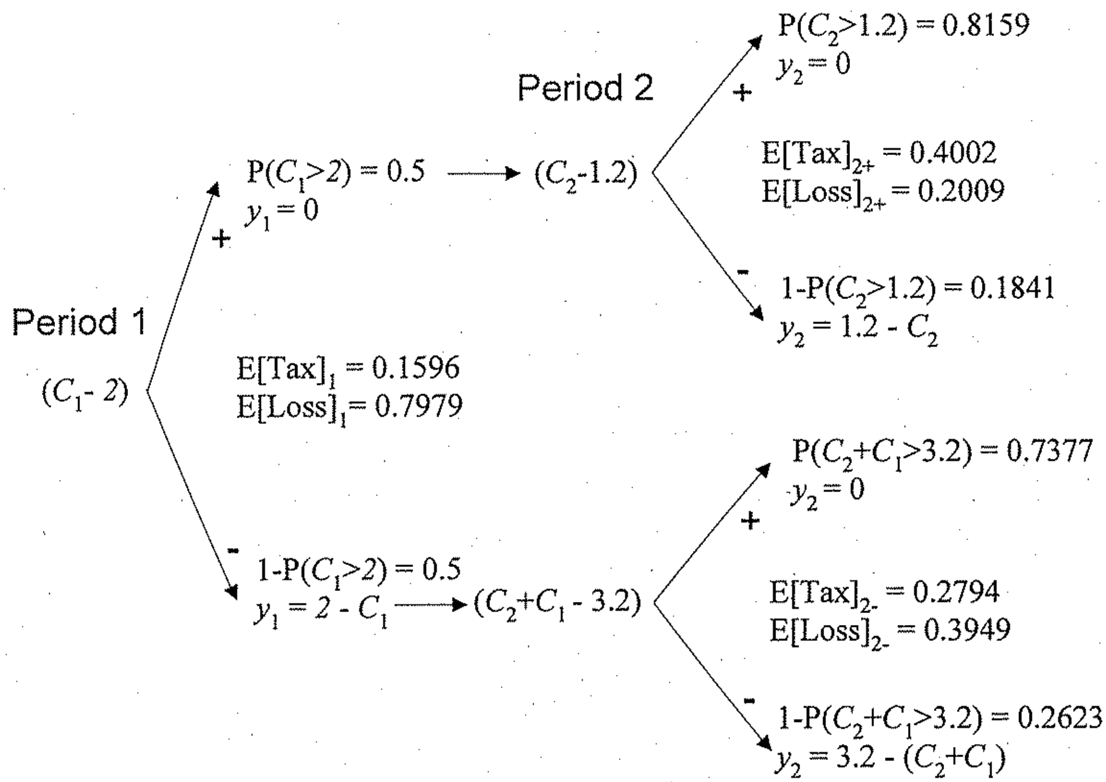


Figure 2.4 *Numerical Example of Infinite Carry-Forward Tree*

Table 2.2 Infinite Carry-Forward Expected Present Value

α	$E[Tax_{SL}]$	$E[Tax_{SYD}]$	$E[Tax_{DDB}]$
0.80	1.24045	1.191716	**1.1673**
0.81	1.286792	1.24004	**1.214509**
0.82	1.383977	1.341722	**1.313721**
0.83	1.383977	1.341722	**1.313721**
0.84	1.434899	1.395172	**1.365808**
0.85	1.487429	1.450425	**1.419606**
0.86	1.541607	1.507529	**1.475159**
0.87	1.597475	1.566534	**1.532514**
0.88	1.655078	1.62749	**1.591714**
0.89	1.714459	1.690448	**1.652806**
0.90	1.775661	1.755461	**1.715838**
0.91	1.838732	1.822581	**1.780858**
0.92	1.903716	1.891863	**1.847915**
0.93	1.970661	1.963361	**1.917058**
0.94	2.039614	2.037133	**1.988338**
0.95	2.110625	2.113234	**2.061806**
0.96	2.183741	2.191723	**2.137515**
0.97	2.259015	2.272659	**2.215518**

0.98	2.336496	2.356101	**2.295868**
0.99	2.416237	2.442111	**2.378621**
1.00	2.49829	2.530752	**2.463832**

2.4 Finite Carry-Forward of Losses

The above two examples represent extreme scenarios of either all or nothing carry-forward. SDM can dominate ADM when carry-forward is not allowed and DDB dominates when carry-forward is allowed over an infinite horizon. Therefore the length of n, defined as the number of periods that losses may be carried-forward, determines if SDM can ever be optimal.

When losses may only be carried-forward for n periods, the model becomes more complicated. Earlier losses carried-forward should be applied as soon as possible to avoid losing any unused portion of the deduction. Once the allowed carry-forward horizon has been exceeded, any remaining value of the losses must be eliminated from the total carry-forward deduction. In order to track this, the annual carry-forward values must be kept separate from one period to the next and updated to reflect any changes from applying all or part of the carried-forward loss.

To accomplish this tracking, a second index is added to the carry-forward variable such that it becomes $y_{k,i}$. The first index represents the period from which the loss is originally carried-forward. The second index represents the period in which the value of the loss $y_{k,i}$ still remains. For example, a loss occurring in period 1 is $y_{1,1} = 10$. In period 2, if there is a positive taxable income of 7, the amount remaining after period 2 is $y_{1,2} = 10\text{-}7 = 3$.

The positive taxable income in each period is defined as:

$\max(C_i - d - \sum_{j=i-n}^{i-1} y_{j,i-1}, 0)$ for the straight-line method or

$$\max(C_i - d_i - \sum_{j=i-n}^{i-1} y_{j,i-1}, 0) \text{ for accelerated methods.}$$

To calculate the initial loss carried-forward, $y_{i,i}$, the equation is the same as the infinite carry-forward case. As long as $(i - k) \leq n$, then a new current value for $y_{k,i}$ is assigned each period according to the following equations:

$$y_{i-k,i} = \begin{cases} \max(y_{i-k,i-1} - (C_i - d_i - \sum_{j=i-n}^{i-k-1} y_{j,i-1}), 0), & \text{if } (C_i > d_i + \sum_{j=i-n}^{i-k-1} y_{j,i-1}) \\ y_{i-k,i-1}, & \text{otherwise} \end{cases} \quad \text{for } k = n, n\text{-}1, \ldots, 1$$

$$y_i = 0 \text{ for all } i < 1$$

These update equations take the amount of loss remaining from period *i-k* and subtract the remaining positive taxable income in period *i* represented by the cash flow in *i*, minus the depreciation deduction in *i*, minus the losses carried from periods prior to *i-k*. Since the updates are made starting with $k = n$ and working towards $k = 1$, this guarantees that losses from earlier periods are used before those from later periods. This update procedure is only used while there is remaining profit to which the losses can be applied. As soon as $C_i < d_i + \sum_{j=i-n}^{i-k-1} y_{j,i-1}$ (remaining taxable income is ≤ 0), any losses that cannot be applied in the current period carry-over to the next, assuming they have not expired.

The present value of the tax payments made using SDM and ADM can be represented by the following expressions:

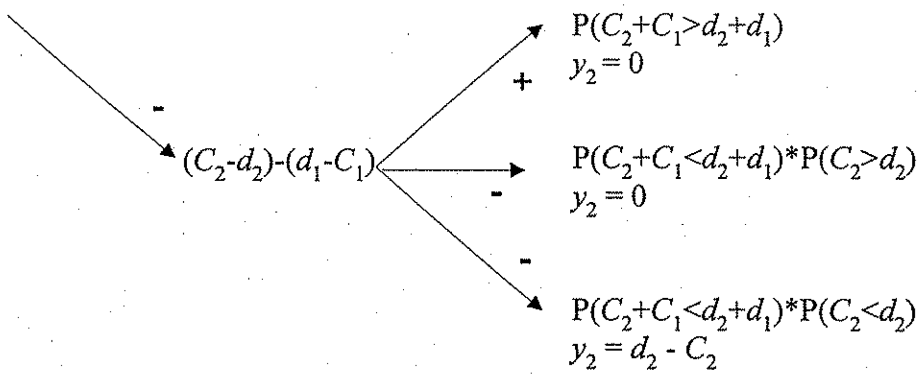
$$Tax_S(\alpha) = T\sum_{i=1}^{N} \alpha^i (\max(C_i - d - \sum_{j=i-n}^{i-1} y_{j,i-1}, 0))$$

$$Tax_A(\alpha) = T\sum_{i=1}^{N} \alpha^i (\max(C_i - d_i - \sum_{j=i-n}^{i-1} y_{j,i-1}, 0))$$

The next example allows carry-forward for one period ($n = 1$). At any period, only new losses can move to the next period. Therefore, the total loss carried-forward to period $i + 1$ is either (d_i-C_i) or 0. This effectively adds an extra branch on the tree from any node that already contains a loss carried-forward. Period 1 will not change, but the bottom node in period 2, which has a taxable income of $\max(C_2-d_2-(d_1-C_1),0)$, will have three branches instead of two. The first branch represents a cash flow C_2 large enough to cover the depreciation expense in that period and the loss carried-forward. This results in a positive tax payment, no losses carried-forward to period 3, and occurs with probability $P(C_3+C_2>d_3+d_2 \mid C_2<d_2)$. The second branch is the case where $(C_2-d_2-(d_2-C_2))<0$, but $C_2>d_2$. No taxes would be paid in this period, but the carry-forward amount would be zero. This branch occurs with a probability of $P(C_3+C_2<d_3+d_2 \mid C_2<d_2)*P(C_2>d_2)$. The final branch is the case where $C_2<d_2$, which results in no taxes paid and a loss of d_2-C_2 to be brought forward to period 3.

The next period begins with the same two scenarios from period 2. There is either a cash flow of (C_3-d_3) or (C_3-d_3-(d_2-C_2)) if a loss of (d_2-C_2) has been brought forward. Figure 2.5 illustrates the change in the second period from an infinite to a 1-period carry-forward example (compare to Figure 2.3).

Given an n period carry-forward limit, the tree will not change from the infinite case until period n+1. Moreover, the expected tax payment is not affected by the new branches and different probabilities until period n+2. The most prominent effect of the carry-forward limit is to increase the probability of states occurring that have lower losses carried-forward. Higher carry-forward values correspond to lower expected tax payments, so by eliminating some of the cumulative losses carry-forward, the expected tax payments become higher.



$$E[\text{Tax}]_{2-} = T[\max(C_2 - d_2 - E[\text{Loss}]_1), 0)]$$
$$E[\text{Loss}]_{2-} = \max(d_2 + E[\text{Loss}]_1 - C_2, 0)$$

Figure 2.5 *1-Period Carry-Forward Tree (Period 2)*

To illustrate this, compare the probabilities of the possible taxable income states given an infinite and a 1-period carry-forward limit. This is shown in Table 2.3 for DDB in period 3 of the same example used previously.

	State	E[Tax]	**INFINITE** CARRY-FORWARD Probability	**1-PERIOD** CARRY-FORWARD Probability
1	(C_3-d_3)	0.66446	0.77684	0.88383
2	$(C_3+C_2-(d_3+d_2)$	0.62653	0.09203	0.11617
3	$(C_3+C_2+C_1-(d_3+d_2+d_1)$	0.59036	0.13113	0.00000
		$E[\text{Tax}]_3$ =	**0.65125**	**0.66005**

Table 2.3 *Branch Probability Comparisons*

The expected value of the tax payment in period 3 is higher for 1-period carry-forward than for infinite. The same is true for all periods $i > n+1$. Specifically, the 1-period example cannot have the third state where losses are carried-forward from period 1 and 2. Therefore the probability of state 3 occurring must be divided over the other two states. The amount sent to the (C_3-d_3) cash flow state is proportionate to $P(C_2>d_2)$. Likewise, the amount sent to the $(C_3+C_2-(d_3+d_2))$ state is proportionate to $P(C_2<d_2)$.

Example 2.3 Once again, applying this procedure to the five-period example results in the expected tax payments shown in Table 2.4. Since losses may only be carried-forward one period, we have the situation where some of the losses in the early periods are lost, since the cash flows are not large enough to cover the current depreciation and the loss from the previous period. For $\alpha \geq 0.937$, SDM is optimal in this example. When compared to the results shown in Table 2.1 for the case where losses cannot be carried-forward, the threshold value for the optimality of SDM has shifted downward from 0.895 to 0.937. Allowing carry-forward for one period causes the range for which SDM is preferred over ADM to decrease. A further downward shift can be expected as n is increased until at some point ADM will completely dominate SDM.

Table 2.4 *1-Period Carry-Forward Expected Present Value*

α	$E[Tax_{SL}]$	$E[Tax_{SYD}]$	$E[Tax_{DDB}]$
0.80	1.241371	1.195128	**1.194896**
0.81	1.287749	**1.243582**	1.243755
0.82	1.335615	**1.293703**	1.294278
0.83	1.385007	**1.345535**	1.346508
0.84	1.435967	**1.399125**	1.40049
0.85	1.488536	**1.454522**	1.456273
0.86	1.542753	**1.511774**	1.513901
0.87	1.598663	**1.570929**	1.573424
0.88	1.656308	**1.632039**	1.634891
0.89	1.715731	**1.695155**	1.69835
0.90	1.776977	**1.760329**	1.763854
0.91	1.840093	**1.827615**	1.831452
0.92	1.905123	**1.897066**	1.901198
0.93	1.972114	**1.968737**	1.973145
0.94	**2.041115**	2.042685	2.047347
0.95	**2.112175**	2.118967	2.123859
0.96	**2.185342**	2.19764	2.202738
0.97	**2.260666**	2.278764	2.28404
0.98	**2.338199**	2.362399	2.367823
0.99	**2.417993**	2.448605	2.454146
1.00	**2.500101**	2.537446	2.543069

2.5 Finite Carry-Forward and Backward of Losses

When losses can be carried-backward, the model must now incorporate prior profitability into the carry-forward variable y. Either a positive value is carried-forward, representing a prior profit, or a negative value is carried-forward to represent a loss. The equations to calculate the $y_{k,i}$ value remain the same as before when a loss is being carried-forward. When profits are being brought forward (equivalent to carrying losses backward), then the equations to calculate update $y_{i-k,i}$ are:

$$y_{i-k,i} = \begin{cases} \min(y_{i-k,i-1} - (C_i - d_i - \sum_{j=i-n}^{i-k-1} y_{j,i-1}), 0), & \text{if } (C_i < d_i + \sum_{j=i-n}^{i-k-1} y_{j,i-1}) \\ y_{i-k,i-1}, & \text{otherwise} \end{cases} \quad \text{for } k = n, n\text{-}1, \ldots, 1$$

$$y_i = 0 \text{ for all } i < 1$$

When a loss occurs, a refund would be returned to the firm for the minimum of the current loss or the sum of the taxable income over the carry-backward period, each multiplied by the tax rate. If the loss exceeds the profits in the time-frame, any leftover loss would then be carried-forward. The equation to represent taxable income when a loss is carried-forward remains the same, but the equation when a profit is carried-forward is

$$\min\left(C_i - d, \sum_{j=i-n}^{i-1} y_{j,i-1}\right) \text{ for the straight-line method or}$$

$$\min\left(C_i - d_i, \sum_{j=i-n}^{i-1} y_{j,i-1}\right) \text{ for accelerated methods.}$$

This method would not only provide tax savings at an earlier point in time, but could reduce the potential for losing a portion of the loss if the carry-forward horizon does not produce a sufficient profit.

In a one period carry-forward and one period carry-backward example, the tree again remains the same in the first period. In the second period, the tree hanging from the top branch of period one carries-forward a negative quantity representing the profit (negative loss) achieved in period 1.

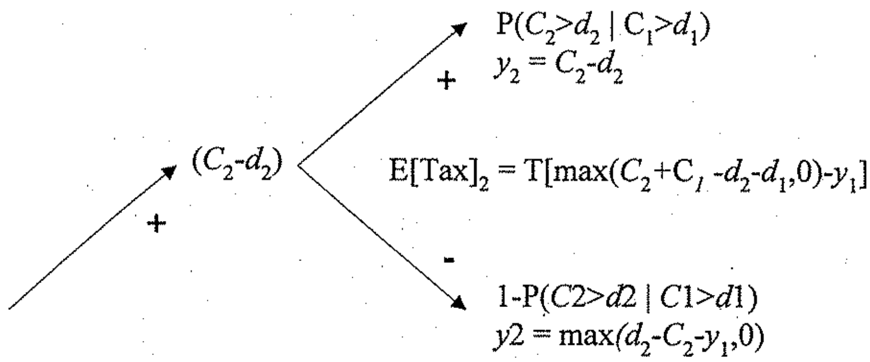
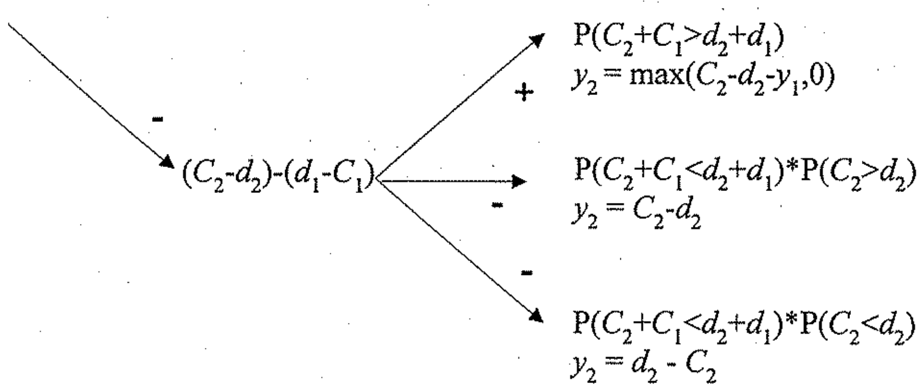


Figure 2.6 *1-Period Carry-Backward/Forward Tree (Period 2 Upper Branch)*

The lower portion of the second period tree carries forward a positive value as before, and the differences in the carry-backward tree are the expected tax payments and the possibility of carrying-forward a profit to which a loss in the next period may be applied.



$$E[\text{Tax}]_2 = T[\max(C_2-d_2-y_1,0)]$$

Figure 2.7 *1-Period Carry-Backward/Forward Tree (Period 2 Lower Branch)*

Example 2.4 Using the same five period example, the results of a one period carry-backward and one period carry-forward are shown in Table 2.5.

Table 2.5 *1-Period Carry-Backward/Forward Expected Present Value*

α	$E[Tax_{SL}]$	$E[Tax_{SYD}]$	$E[Tax_{DDB}]$
0.80	1.19944265	**1.1543595**	1.16170642
0.81	1.24457926	**1.20166346**	1.20972897
0.82	1.29118028	**1.2506133**	1.2594033
0.83	1.33928473	**1.30125473**	1.31077398
0.84	1.38893237	**1.35363434**	1.36388644
0.85	1.4401637	**1.4077996**	1.4187869
0.86	1.49301996	**1.46379889**	1.47552248
0.87	1.54754315	**1.52168148**	1.5341411
0.88	1.60377602	**1.58149756**	1.59469159
0.89	1.66176211	**1.64329825**	1.65722361
0.90	1.72154572	**1.70713561**	1.72178772
0.91	1.78317192	**1.77306261**	1.78843536
0.92	1.8466866	**1.8411332**	1.85721887
0.93	1.91213642	**1.91140228**	1.92819149
0.94	**1.97956887**	1.98392572	2.00140736
0.95	**2.04903222**	2.05876036	2.07692155
0.96	**2.12057558**	2.13596403	2.15479004
0.97	**2.19424888**	2.21559555	2.23506977
0.98	**2.27010289**	2.29771476	2.31781861
0.99	**2.3481892**	2.38238248	2.40309537
1.00	**2.42856027**	2.46966058	2.49095982

This example does not produce a drastic change in the expected tax payments due to the nature of the cash flows in the problem and the fact that only a one-period carry-backward was allowed.

3.0 Optimal Depreciation Conditions

Now that several different carry-forward scenarios have been modeled and the results for a 5-period example have been shown, some intuition may exist concerning the conditions determining the optimal depreciation method. This section will establish these conditions and discuss their realistic implications.

3.1 General Accelerated Depreciation Characteristics

For the accelerated methods, the depreciation deductions d_i are such that the values are non-increasing and must sum to the initial value of the asset. Therefore, they begin larger and end smaller than annual SL deductions. This requires the existence of some threshold period $\bar{k} \in \{1,...,N\}$ that identifies the last period where the accelerated deduction is larger than the straight-line deduction [1].

From this, there comes an important characteristic of accelerated depreciation methods that is fundamental to this analysis. If one considers the total depreciation from period 1 to any period $\leq N$, the cumulative sum is always greater for ADM than SDM. This is formally stated in the following definition.

Definition 4.1. An accelerated depreciation method is one where

$$\sum_{i=1}^{N} d_i = D,$$

$$d_i \geq d_{i+1} \;\; \forall \; i = 1,..,N-1,$$

there exists some $\bar{k} \in \{1,...,N\}$ such that

$$d_i \geq \frac{D}{N} \text{ for all } i \leq \bar{k}$$

$$d_i < \frac{D}{N} \text{ for all } i > \bar{k}$$

and

$$\sum_{i=1}^{j} d_i \geq j\left(\frac{D}{N}\right) \;\; \forall \; j \in \{1,...,N\}$$

Even without stating this final property about the cumulative depreciation, Berg et al. [1] prove that ADM provides a lower present value of tax payments when $P(C_i > d_i) = 1$ for all $i \leq \bar{k}$.

As an example, the depreciation percentages for the five-period example are shown in Table 3.1. The cumulative depreciation percentages verify that both DDB and SYD exceed SL at any time in terms of the total depreciation charged since period 1.

Table 3.1 *Cumulative Depreciation Charges for 5-Period Example*

	Depreciation Charge %			Cumulative Depreciation %		
Period	SL	SYD	DDB	SL	SYD	DDB
1	0.2	0.3333	0.4000	0.2	0.3333	0.4000
2	0.2	0.2667	0.2400	0.4	0.6000	0.6400
3	0.2	0.2000	0.1440	0.6	0.8000	0.7840
4	0.2	0.1333	0.0864	0.8	0.9333	0.8704
5	0.2	0.0667	0.1296	1.0	1.0000	1.0000

3.2 Effect of Carry-Forward on Cumulative Depreciation

When losses can be carried forward, the only way SDM is preferred to ADM is when the losses cannot be applied within the carry-forward period and are ultimately lost. For this to occur, the number of consecutive periods of losses must exceed the carry-forward length n. By examining the cumulative depreciation charge percentages in terms of a sliding window of length $n + 1$, some conclusions can be drawn about the optimal depreciation method.

At a maximum, n periods of depreciation may be brought forward to period i. If C_i exceeds d_i plus all the losses carried-forward, the total depreciation realized in period i will be $\sum_{j=i-n}^{i}(C_j - d_j)$, assuming that $C_j < d_j$ for all periods $j \le i$. Otherwise, SDM cannot yield a lower present value since ADM would apply a deduction at least as large as that of SDM given the opportunity of profit. In the case of small profits, the present value of the tax payment using SDM would at best be equal to that of ADM.

If the current period is i, the sliding window analysis requires that a loss has occurred from period 1 to $i - 1$, and

$$C_i \geq d_i + \sum_{j=i-n}^{i-1}(d_j - C_j).$$

This allows the last n periods of depreciation to be used in its entirety, but anything prior to $i - n$ has been lost. In terms of cumulative depreciation, this may allow SDM to realize a greater deduction in period i since the extra depreciation provided by ADM prior to period $i - n$ has been lost. Whether or not this is the case depends on the length of n and the period in which a profit is attained and whether the carry-forward deductions can be used.

The cumulative depreciation charges shown in Table 3.1 represent the case where infinite carry-forward is allowed. The depreciation deductions are never lost, so if a profit is ever realized, ADM will apply a deduction greater than or equal to that of SDM. Tables 3.2 – 3.7 show the cumulative sliding window depreciation for $n = 0,\ldots,5$ and $N = 5$.

Table 3.2 *No Carry-Forward Cumulative Depreciation*

$n = 0$	Depreciation Charge %			Consec.
Period	**SL**	**SYD**	**DDB**	**Loss**
1	0.2	0.333333	0.4	0
2	0.2	0.266667	0.24	1
3	0.2	0.2	0.144	2
4	0.2	0.133333	0.0864	3
5	0.2	0.066667	0.1296	4

Table 3.3 *1-Period Carry-Forward Cumulative Depreciation*

$n = 1$	Depreciation Charge %			Consec.
periods	**SL**	**SYD**	**DDB**	**Loss**
1	0.2	0.333333	0.4	0
1+2	0.4	0.6	0.64	1
2+3	0.4	0.466667	0.384	2
3+4	0.4	0.333333	0.2304	3
4+5	0.4	0.2	0.216	4

Table 3.4 *2-Period Carry-Forward Cumulative Depreciation*

$n = 2$	Depreciation Charge %			Consec.
periods	**SL**	**SYD**	**DDB**	**Loss**
1	0.2	0.333333	0.4	0
1+2	0.4	0.6	0.64	1

1+2+3	0.6	0.8	0.784	2
2+3+4	0.6	0.6	0.4704	3
3+4+5	0.6	0.4	0.36	4

Table 3.5 *3-Period Carry-Forward Cumulative Depreciation*

$n = 3$	Depreciation Charge %			Consec.
periods	**SL**	**SYD**	**DDB**	**Loss**
1	0.2	0.333333	0.4	0
1+2	0.4	0.6	0.64	1
1+2+3	0.6	0.8	0.784	2
1+2+3+4	0.8	0.933333	0.8704	3
2+3+4+5	0.8	0.666667	0.6	4

Table 3.6 *4-Period Carry-Forward Cumulative Depreciation*

$n = 4$	Depreciation Charge %			Consec.
periods	**SL**	**SYD**	**DDB**	**Loss**
1	0.2	0.333333	0.4	0
1+2	0.4	0.6	0.64	1
1+2+3	0.6	0.8	0.784	2
1+2+3+4	0.8	0.933333	0.8704	3
1+2+3+4+5	1	1	1	4

Table 3.7 *5-Period Carry-Forward Cumulative Depreciation*

$n = 5$	Depreciation Charge %			Consec.
periods	**SL**	**SYD**	**DDB**	**Loss**
1	0.2	0.333333	0.4	0
1+2	0.4	0.6	0.64	1
1+2+3	0.6	0.8	0.784	2
1+2+3+4	0.8	0.933333	0.8704	3
1+2+3+4+5	1	1	1	4

The period in which the cumulative depreciation charge reaches a maximum value indicates the last period where ADM has an advantage over SDM and will remain the optimal method. This always occurs at period $n+1$, which means that none of the depreciation charges up to and including $n+1$ are lost. By definition 4.1, the first period's depreciation is the largest. In the sliding window sum, when d_1 is lost the change in the sum is $(d_{n+2} - d_1) \leq 0$, assuming losses have occurred every period. This supports the result that if initial periods of depreciation for ADM are lost, SDM can possibly dominate.

Period n+1 can then be identified as the last period where ADM provides a greater cumulative depreciation charge than SDM over the past n consecutive periods of loss. Using this threshold value, the cash flows can be divided into three categories, each resulting in a different optimal depreciation decision. The three major categories are:

(1a) $\sum_{i=1}^{\bar{k}} y_{i,i+n} = 0$

(1b) $P(C_i < d_i) = 1 \;\forall\; i > n+1$

(2) $\sum_{i=1}^{\bar{k}} y_{i,i+n} = \sum_{i=1}^{\bar{k}} d_i$

(3) $P(C_i < d_i) \in (0,1) \;\forall\; i \leq n+1$

For each of the situations mentioned above, an example will be discussed to illustrate the results before a formal proof is provided. The same five-period example will be used once again with $n = 1$. This provides a threshold value of n+1=2.

The first case has two components, each resulting in the same optimal depreciation decision for all values of α. In (1a), the situation is represented when none of the depreciation is lost over a window of periods beginning in 1 and ending in $\bar{k}$. This also provides a unifying concept between this work and Lemma 3.1 in Berg et al. [1], which requires profits prior to period $\bar{k}$. The difference is that now losses may be carried forward, as long as they are used within n periods. For $n = 0$, (1a) corresponds exactly to Berg's et al.[1] statement in Lemma 3.1. For $n > 0$, some leniency is obtained in the actual timing of profits, as long as the depreciation charges are not lost prior to $\bar{k}$.

The results in this example are provided for the scenario in which $C_i \leq 0$ for $i < \bar{k}$, and $C_{\bar{k}}$ must be greater than the entire loss carried-forward. The present values of expected tax payments are evaluated in terms of the difference between ADM and SDM for this example due to the complexity of obtaining the ADM and SDM values directly. The following equation represents the difference in periods 1 through $\bar{k}$ for this situation.

$$\Delta(\alpha) = \alpha^{\bar{k}} \left(\sum_{i=1}^{\bar{k}} (d_i - d) \right)$$

Since d_i and d represent savings, $\Delta(\alpha)<0$ indicates a lower tax payment for ADM. From periods $\bar{k}+1$ to N, the expected present value can be calculated by the method in Section 2.4, using a starting cash flow of $(C_i - d_i)$ and $(C_i - d)$ for $i = \bar{k}+1$ with a probability of 1 for ADM and SDM respectively. Table 3.8 shows the differences in the present value of expected tax payments. As this is a worst-case scenario, having higher cash-flows in early periods will only increase the savings from using ADM. The differences are negative for all values of α, therefore ADM dominates SDM in case (1a). A formal statement and proof of this follows.

Table 3.8 *Case (1a) Expected Tax Payment Differences*

α	$E[Tax_{SYD}-Tax_{SL}]$	$E[Tax_{DDB}-Tax_{SL}]$
0.80	-0.031448181	-0.056902359
0.81	-0.031148064	-0.055887555
0.82	-0.030751046	-0.05473068
0.83	-0.030251715	-0.053426389
0.84	-0.029644509	-0.051969236
0.85	-0.028923717	-0.05035368
0.86	-0.028083481	-0.048574082
0.87	-0.027117786	-0.046624702
0.88	-0.026020469	-0.044499701
0.89	-0.024785207	-0.04219314
0.90	-0.023405524	-0.039698978

0.91	-0.021874783	-0.037011071
0.92	-0.02018619	-0.034123172
0.93	-0.018332785	-0.03102893
0.94	-0.01630745	-0.027721889
0.95	-0.014102899	-0.024195487
0.96	-0.011711681	-0.020443055
0.97	-0.009126177	-0.016457817
0.98	-0.006338596	-0.012232889
0.99	-0.003340981	-0.007761276
1.00	-0.000125198	-0.003035876

Proposition 3.1. If $\sum_{i=1}^{\bar{k}} y_{i,i+n} = 0$, ADM dominates SDM. (i.e. E[Tax$_A$(α)] ≤ E[Tax$_S$(α)] for all $\alpha \in [0,1]$.)

Proof. The proof of this follows that of Lemma 3.1 in Berg et al. [1]. $\sum_{i=1}^{\bar{k}} y_{i,i+n} = 0$ requires that the entire depreciation charges from period 1 to $\bar{k}$ are utilized. This is equivalent to Lemma 3.1, with the additional flexibility that the depreciation charges do not need to be used immediately since they may be carried forward *n* periods. As long as d_i has been entirely used by period $i+n$ for $i \leq \bar{k}$, ADM will dominate.

The next case, (1b), requires that a loss occur every period beyond n+1. This can be easily modeled by setting the tax payments from n+2 to N equal to zero. The results for this example are shown once again in terms of the present value of the expected tax payment in Table 3.9.

Table 3.9 *Case (1b) Expected Tax Payments*

α	E[Tax$_{SL}$]	E[Tax$_{SYD}$]	E[Tax$_{DDB}$]
0.80	0.487878	0.371740	**0.345128**
0.81	0.497325	0.379116	**0.352195**
0.82	0.506855	0.386560	**0.359329**
0.83	0.516467	0.394071	**0.366531**

0.84	0.526162	0.401649	**0.373802**
0.85	0.535940	0.409294	**0.381140**
0.86	0.545800	0.417007	**0.388546**
0.87	0.555744	0.424788	**0.396020**
0.88	0.565770	0.432635	**0.403562**
0.89	0.575878	0.440550	**0.411173**
0.90	0.586069	0.448533	**0.418851**
0.91	0.596343	0.456583	**0.426597**
0.92	0.606700	0.464700	**0.434411**
0.93	0.617139	0.472885	**0.442293**
0.94	0.627661	0.481137	**0.450242**
0.95	0.638266	0.489456	**0.458260**
0.96	0.648953	0.497843	**0.466346**
0.97	0.659723	0.506297	**0.474500**
0.98	0.670576	0.514819	**0.482721**
0.99	0.681511	0.523408	**0.491011**
1.00	0.692529	0.532064	**0.499369**

The expected tax payments are lower for ADM for all values of α, therefore ADM also dominates SDM in case (1b).

Proposition 3.2. When $P(C_i < d_i) = 1 \ \forall \ i > n+1$, ADM dominates SDM.

Proof. Given that $P(C_i < d_i) = 1 \ \forall \ i > n+1$, the tax payments in $i > n+1$ are zero and the expected value only needs to be evaluated from 1 to $n+1$. By definition 4.1, $\sum_{i=1}^{n+1} d_i \geq (n+1)d$. For any C_i, $i \leq n+1$, ADM provides a depreciation deduction greater than or equal to SDM, which occurs either at the same time or earlier. Given that the time value of money discounts later periods, ADM results in a greater savings, and thus a lower tax payment.

The next case is the opposite of (1b), where a loss occurs such that all depreciation up to and including $\bar{k}$ is lost. This is modeled by using a slightly relaxed case where a tax payment of zero occurs for periods 1 through n+1, and the method from Section 2.4 is used to compute the remainder. At period n+2, a loss carried-forward of $\sum_{i=2}^{n+1}(C_i - d_i)$ occurs with a probability of 1. The results indicate that SDM is always optimal, as shown in Table 3.10.

Proposition 3.3. If $\sum_{i=1}^{\bar{k}} y_{i,i+n} = \sum_{i=1}^{\bar{k}} d_i$, SDM dominates ADM.

Proof. Given that none of the depreciation in periods 1 through $\bar{k}$ can be used, the only attainable depreciation is for periods $i > \bar{k}$. Since $d_i \leq d$ for $i > \bar{k}$, $\sum_{i=\bar{k}}^{N} d_i \leq (N-\bar{k})d$, SDM provides a depreciation deduction greater than or equal to ADM, which results in a lower tax payment.

The final case corresponds exactly to the situation modeled in Section 2.4. There is some probability between 0 and 1 associated with a loss occurring in each period. The optimal depreciation choice becomes a function of the discount rate in this situation.

Table 3.10 *Case (2) Expected Tax Payments*

α	E[Tax$_{SL}$]	E[Tax$_{SYD}$]	E[Tax$_{DDB}$]
0.80	**0.739787**	0.805088	0.832756
0.81	**0.7762**	0.845475	0.873905
0.82	**0.814006**	0.887444	0.916633
0.83	**0.853244**	0.931039	0.960984
0.84	**0.893954**	0.976308	1.007004
0.85	**0.936175**	1.023299	1.054739
0.86	**0.97995**	1.072058	1.104235
0.87	**1.025321**	1.122636	1.155541
0.88	**1.07233**	1.175083	1.208706
0.89	**1.121022**	1.22945	1.263778
0.90	**1.17144**	1.285789	1.320808
0.91	**1.22363**	1.344152	1.379847
0.92	**1.277638**	1.404595	1.440949

0.93	**1.333511**	1.467172	1.504165
0.94	**1.391296**	1.531938	1.56955
0.95	**1.451041**	1.59895	1.637159
0.96	**1.512797**	1.668267	1.707047
0.97	**1.576613**	1.739947	1.779272
0.98	**1.642539**	1.81405	1.853891
0.99	**1.710628**	1.890637	1.930963
1.00	**1.780933**	1.96977	2.010547

A natural question is "What are the consequences of this analysis?" We can only speak to current United States tax law which allow for a 20-year carry-forward period to apply net-operating losses before they are lost [8]. In light of this large value of n, the possibility of ADM optimality is greatly increased. In most cases, this 21 year window will extend beyond the life of the asset. By proposition 3.1, as long as the cumulative depreciation charges from periods 1 to min(N,21) are used on or before period 21, then ADM dominates SDM. Thus, it would appear in reality (U.S. law) that ADM would always dominate.

4.0 Conclusions and Directions for Future Research

The ability to carry-forward losses works in favor of ADM, resulting in a lower present value of the expected tax payment. When losses can be carried-forward for an infinite number of periods, the results are equivalent to the traditional analysis where losses do not occur. In this case, ADM is always optimal. When losses may only be carried-forward for a finite number of periods, the results become significantly more difficult to characterize.

To help make some general statements about the optimal depreciation decision, a threshold period of either $\bar{k}$ or $n+1$ was identified, depending on the nature of the statement. This is the last period in which a consecutive loss can occur starting from period 1 and still allow ADM to dominate SDM regardless of the discount rate. Depending on the nature of the cash flows before and after this threshold period, the optimal depreciation method can be identified. When the cumulative depreciation is guaranteed to be used by the threshold period, then regardless of what occurs after, ADM will always dominate SDM. Similarly, if the cash flows are guaranteed to be negative after the threshold period, ADM will again always provide lower expected tax payments. When losses occur with probability 1 up to and including the threshold period, then ADM has lost its advantage of greater initial depreciation and SDM will always be preferred regardless of what happens after the threshold period. Finally, in the case where losses and profits occur with a probability between 0 and 1, the optimal depreciation method depends on the discount rate given the carry-forward limit and distributions of the cash flows.

There are a few major assumptions in this paper, which may be explored in future research. The first is the problem horizon, which stops at period N when calculating the present value of the expected tax payments. When carry-forward is allowed, the possibility of having residual losses after period N to apply to future periods would affect the present value. Although the effect would be small, it would be more realistic to still consider this in the calculations.

Another area for future research concerns asset disposal or replacement, particularly when some value is received. The gain or loss from the sale of the asset will be affected by the amount of depreciation. If the asset is kept to the end of it's life, the book value will be equivalent for ADM and SDM, but if it is retired or replaced early, the book values, and therefore the gain/loss, will differ.

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