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Industrial and Systems Engineering

Equipment Replacement Under Continuous And Discontinuous Technological Change

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Report No. 03T-003

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Abstract

Technological change is a major motivator for replacing equipment. Models in the literature generally assume that technology evolves continuously according to some known function or that there are breakthroughs in technology in a given (or uncertain) future time period. We model the situation where breakthroughs occur periodically, providing a new vintage of challenger, *and* each challenger within a vintage evolves according to a known function. We examine the economic life of an asset with respect to models that only assume continuous or discontinuous technological change. For the case of exponential technological change, we illustrate that the economic life tends to be shorter in our models and that significantly different solutions can occur when breakthroughs are ignored. We believe this work leads to new questions in replacement analysis which examine the combined effect of continuous and discontinuous technological change.

1 Introduction

The replacement of assets is generally motivated by deterioration of the asset currently owned (defender) or technological advances of possible replacement assets (challengers). Deterioration is generally modelled through increasing operating costs and/or decreasing salvage values. Technological change is generally quantified by improved efficiency, such as higher production rates, reduced operating and maintenance costs, and/or higher salvage values for future challengers.

There is a large body of literature in replacement analysis which considers technological change. We classify this work according to the following three categories:

1. **General Technological Change:** In this research, no general assumptions are made on the costs associated with challengers. Rather, specific cash flows are forecasted for future capital costs (purchase and salvage values) and operating costs (operating and maintenance costs).

   The most common form of analysis with these models is dynamic programming, as it can readily solve problems with any cash flow estimates. Bellman [3] introduced the first dynamic programming formulation to consider a single challenger in each period which may improve periodically. Dreyfus [8] generalized this to include used asset purchases. Sethi and Chand [20] present a forward algorithm for the problem. Oakford et. al [18] allow for general technological change modeling according to individual capital and operating costs. Christer [7] determines the optimal economic life of the current asset when technological change is expected and the cash flows are forecasted. Bylka et al. [5] consider multiple replacement alternatives in each period. Bean et. al [2] and Chand and Sethi [6] consider the infinite horizon problem.

2. **Continuous Technological Change:** In this research, the arrival time of new technology (challengers) is constant. In this case, the costs of the current technology are known, and the cost of future available technology is some function of those known costs.

   Grinyer [10] formulates an economic life model that considers a constant rate of technological change and the effects of obsolescence on the current asset. Bean et. al [1] formulate a dynamic programming problem in which the cash flow of the future challenger is related to the current asset by a multiplicative factor. Kusaka [13] considers that there is a forecast of technological advances in which the improvements are gradual and known. Later, Kusaka and Suzuki [14] formulate a control limit policy to determine the economic life of the current asset under the assumption that the replacements are (deterministic) technological improvements. Bethuyne [4] defines a function for the rate of technological change. The model determines the optimal replacement considering that technology is improving (by this function) and considers wear on the current asset, depending on utilization.

3. **Discontinuous Technological Change:** This research considers breakthroughs in technology. The arrival time of new technology may or may not be known with certainty and generally only one technologically advanced product arrives during the problem horizon. In addition, the costs associated with the new technology may or may not be known in advance.

   Goldstein et al. [9] consider the economic life of the current asset when the arrival of new technology is described by a geometric distribution with a constant hazard rate. Hopp and Nair [11] consider the optimal keep or replace decision when a future breakthrough expected at an uncertain time. Nair and Hopp [17] formulate a forecast horizon model with improved technology currently available and the possibility that a more significant improvement will be available in the future. Hopp and Nair [12] generalize this to the case with Markovian deterioration.

   Mehrez and Berman [16] formulate a finite horizon model in which there is a machine with a continuous rate of technological change and another improved machine with an unknown arrival time. Kusaka and Suzuki [15] formulate a control limit policy to determine the replacement time given that there are gradual technological advances and a single breakthrough that is forecasted to occur at a known time. Rajagopalan et al. [19] consider technological progress as a Markov process.

This paper is motivated by assets which advance through continuous *and* discontinuous technological change. These may also be referred to as constant and breakthrough technological change in the literature. We consider breakthroughs that occur far in the future by assuming they arrive cyclically.

Our research is motivated by two common assets: automobiles and processors for personal computers. Automobile manufacturers produce vehicles of a certain class each year. Generally, there are small improvements, possibly in efficiency or performance, with each successive production year. However, there are larger leaps in efficiency or performance each time the vehicle is re-designed. While small improvements occur annually, the total re-design of a vehicle may only occur every 4-6 years.

The chips which perform the operations to run a personal computer also follow this pattern of continuous and discontinuous technological change. Incremental improvements in the chips are made constantly (quarterly) while breakthroughs occur every 1-2 years. For example, each generation of Pentium chip can be viewed as a breakthrough, as each generation has provided more capability, while the release of faster chips within each class can be seen as constant technological change. One could argue that these chips have another layer of technological change with the eventual shift from 32-bit to 64-bit processors.

Previous research in replacement analysis has generally assumed either continuous or discontinuous technological change, but has rarely assumed that they occur simultaneously. We believe that this work is significant in that most assets undergo both constant improvements *and* occasional breakthroughs. In many industries, such as automobiles and computer chips, these breakthroughs are fairly predictable, while in other industries, the breakthroughs are not easily forecasted.

In this paper, we make the following two assumptions with regards to technological change:

1. Every B periods, a new vintage of challenger is released which improves upon the previous vintage of challenger according to some known function or parameter.

2. For a given vintage, each successive challenger (every period) improves upon the previous challenger according to some known function or parameter.

If one were to model technology according to productivity and assume a linear increase with time, the situation that we are considering may follow the graph in Figure 1. Here, in each period, productivity increases according to the linear function and at each breakthrough, a step function places the productivity higher and the process repeats itself. Note that any function can be used for the continuous (linear) change and discontinuous (step function) change.

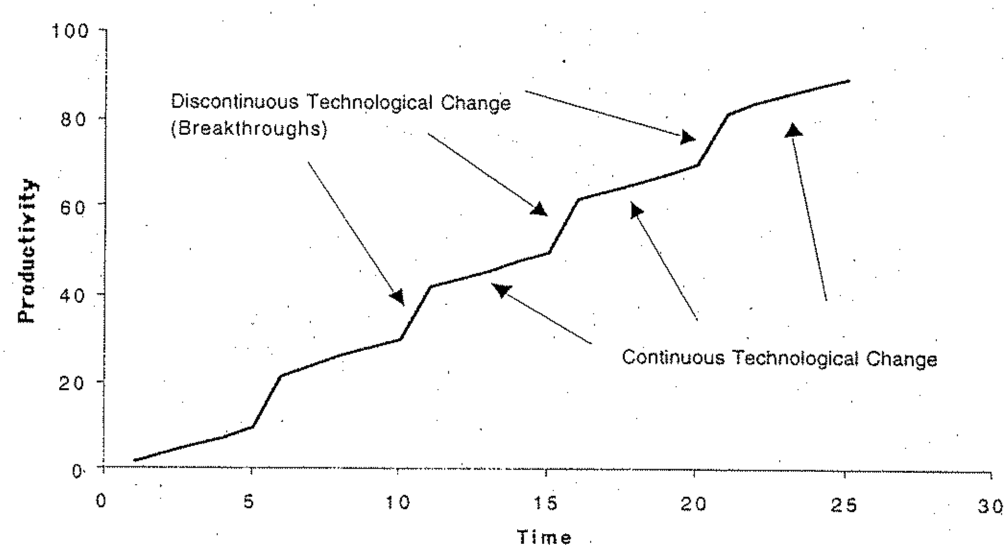

Figure 1: Cyclic continuous and discontinuous technological change.

In this research, we assume that we know the continuous and discontinuous functions of technological change and the periodic timing of the breakthroughs. Clearly, the future of this research is to fully understand the interaction of continuous and discontinuous technological change when the breakthroughs may not be periodic, their timing may be uncertain, their impact may be uncertain, or some interaction of these different assumptions.

While we examine a specific model where assets improve according to an exponential function each period and an exponential function with each vintage, we believe the results illustrate that there is much to be learned in the area. We also illustrate that ignoring breakthroughs can lead to strikingly different replacement schedules.

2 Modeling Technological Change

As noted earlier, we consider a basic model of continuous and discontinuous technological change. A new vintage (breakthrough) of technology arrives every B periods. Improvements occur with each new vintage according to known parameters. Additionally, improvements occur each period for a given vintage according to known parameters. The specific assumptions and notation follow.

Our experiments examine the change in economic life of an asset under the assumptions of no technological change, continuous, discontinuous, and both continuous and discontinuous technological change. For comparison, we determine the best *constant* economic life for a given set of data under these four scenarios. While it clearly may not be optimal to have a constant economic life, we find this constant replacement policy in order to make comparisons amongst the models.

2.1 Model Notation

We utilize the following notation, much of it as in Grinyer [10], as we assume exponential functions of technological change:

- PW: Present Worth of utilizing the current asset for T periods.
- A: Purchase cost of the current asset.
- X: Operating cost parameter.
- Y: Operating cost parameter.
- k: Salvage value parameter.
- n: Index for replacements; i.e., the initial purchase is $n = 0$, the first replacement is $n = 1$, the second replacement is $n = 2$, etc.
- B: The interval between technological breakthroughs.
- T: The lifetime of an asset.
- r: Discount factor.
- $f(n)$: The function describing the technological change associated with the breakthrough, and the subsequent improvements before the new vintage. The function shows the change in the present worth of the replacement compared to current asset.
- β_0: Value indicating technological change.
- β_1: Value indicating the change associated with the breakthrough technology.

The problem has an infinite horizon with breakthroughs occurring at a certain interval, B. The costs associated with future replacements are a function of the present worth, PW, of the current asset. T is the lifetime for each asset and r is the discount factor. The function $f(n)$, with known parameters, describes the change in costs of the new technology. In this model, an exponential function is used to describe both technological change and discounting. The function $f(n)$ also describes the improvement with all assets of a particular vintage available before the next breakthrough. Because the lifetime of the current asset and its replacements is a constant value, T, the breakthrough may occur before the end of life of an asset. In this case, the breakthrough will not be purchased until period nT, the end of the life of the n^{th} replacement.

2.2 No Technological Change

As in Grinyer [10], we assume that the present worth of keeping the asset for keeping the asset for T periods is calculated as follows:

$$PW = A + \sum_{t=0}^{T} (X + Yt)\, e^{-rt} - Ae^{-kT} e^{-rT}$$

The parameters X and Y define the operating and maintenance costs while the salvage value is a function of the purchase price A.

Under the assumption of no technological change, the present worth of the asset for a given lifetime T does not change with time. Over an infinite horizon with continuous discounting, the total present worth is:

$$PW \left(\sum_{n=0}^{\infty} e^{-nrT} \right) \tag{2.1}$$

2.3 Continuous Technological Change

In this model, there is a continuous rate of technological change, described by the parameter β_0 in an exponential function. Again, the present worth is calculated for the current asset (at time zero), then this value is used to calculate the present worth of the future available technology. With replacements every T periods over an infinite horizon, the present worth of the costs is:

$$PW \left(\sum_{n=0}^{\infty} e^{-nrT} \left(e^{-\beta_0 nT} \right) \right) \tag{2.2}$$

2.4 Discontinuous Technological Change

This model is developed from the model with no technological change. Improvements in technology are described by the parameter β_1. Because n represents the index of any replacement and T represents the time between those replacements, nT represents a particular time period. Therefore, the most recent vintage introduced is described by $\lfloor \frac{nT}{B} \rfloor$. Again, the present worth is calculated at time zero, for the current asset, and the discounted present worth over an infinite horizon is:

$$PW \left(\sum_{n=0}^{\infty} e^{-nrT} \left(e^{-\beta_1 \lfloor \frac{nT}{B} \rfloor} \right) \right) \tag{2.3}$$

2.5 Continuous and Discontinuous Technological Change

This model combines the previous two models of discontinuous and continuous technological change. The technological change associated with a particular asset is described by the following function of n, the replacement index:

$$f(n) = e^{-\beta_0 nT} e^{-\beta_1 \lfloor \frac{nT}{B} \rfloor}$$

Discounting and summing over the infinite series of replacements results in the following present worth:

$$PW \left(\sum_{n=0}^{\infty} e^{-\beta_0 nT} e^{-\beta_1 \lfloor \frac{nT}{B} \rfloor} e^{-nrT} \right) \tag{2.4}$$

3 Experimental Results

We examine the four models of technological change defined above through a series of experiments. For a given set of parameter values, we minimize the economic life T in each of the above equations. This allows us to compare economic lives between the models. Note that only in the model without technological change can it be assumed that a constant replacement cycle is optimal. However, our method does find the best constant replacement cycle for comparing the models.

For the models presented here, unless otherwise indicated, the parameter values used are as follows:

- $r = 0.1$
- $k = 0.35$
- $A = 1000$
- $X = 300$
- $Y = 20$

3.1 General Results

The following sections highlight the general results when varying the parameters, including β values, A, B, r, k, X and Y. We provide some general results for each of the models, but elaborate on the comparisons between continuous and combined continuous and discontinuous technological change. Specific results for all four models can be found in the Appendix.

3.1.1 No Technological Change

The model with no technological change depends only on the present worth of the asset purchased at time zero. As the salvage value parameter (k) increases, the economic life (T) increases. A low value of k indicates that the asset is retains most of its value over its lifetime. In addition, as the interest rate increases (r), the economic life increases.

3.1.2 Continuous Technological Change

The model with continuous technological change depends on the present worth of the current asset, the technological change parameter β_0, and time. Time can be represented as the index of the replacement times the replacement interval, nT. For the experiments on this model, values of 0.001 to 0.05 were used for β_0. For low rates of technological change (low values of β_0), the economic life is the same as in the model with no technological change. As the rate of technological change increases (β_0 increases), the economic life decreases. The results are similar (as expected) to those in Grinyer [10].

3.1.3 Discontinuous Technological Change

The model including only discontinuous technological change (a breakthrough every B periods) depends on time (nT), the present worth of the current asset, and the rate of technological change. However, as the improvements do not occur every period, a replacement machine may be of the same vintage as the previously owned asset, depending on the replacement interval and the breakthrough interval. For these experiments, values of β_1 ranged from 0.002 to 0.9. As the technological change parameter, β_1, increases, the economic life decreases. For very small β_1, the economic life is the same as in the model with no technological change, regardless of the breakthrough interval.

3.1.4 Discontinuous and Continuous Technological Change

The model including both types of technological change depends on the present worth of the current asset, time (nT), and both technological change parameters β_0 and β_1. For low rates of technological change (low β_0 and β_1), the economic life is the same as the model with no technological improvements, regardless of the breakthrough interval. In addition, the model does not often select the breakthrough period for replacement when there is only a small rate of technological improvement, and a breakthrough interval of at least five.

Figure 2 illustrates the decreasing economic life with respect to an increasing rate of technological change associated with the breakthrough technology (β_1) The results in Figure 2 assume $\beta_0 = .001$.

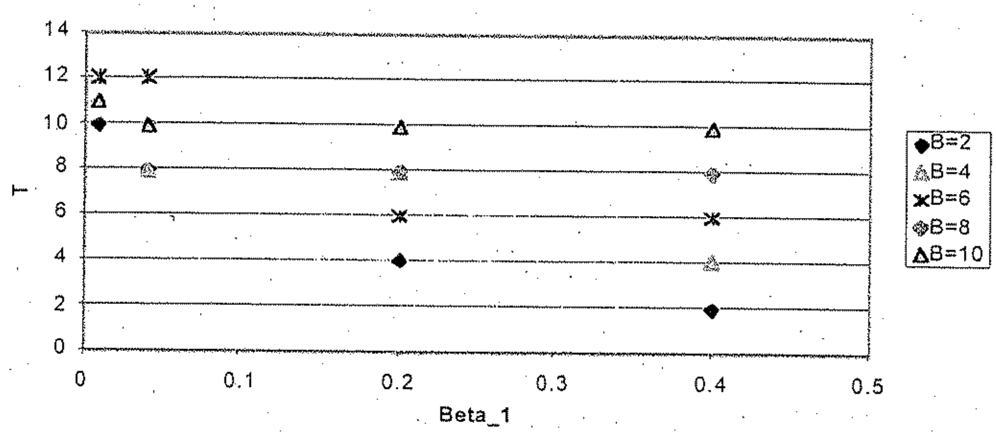

Figure 2: Change in economic life with respect to β_1 with $\beta_0 = .004$.

For this model, β_0 ranged from 0.001 to 0.05, and β_1 was set to $2\beta_0$, $10\beta_0$, $50\beta_0$, and $100\beta_0$. For low β_0 and $\beta_1 = 2\beta_0$, the economic life is at least as long as the breakthrough interval. For any β_0 and $\beta_1 \geq 50\beta_0$, economic life is equal to the breakthrough interval. For high β_0, the model chooses the breakthrough time for replacement in almost all cases. For high β_0 and low β_1, the model waits for the second breakthrough, if $B = 2$ or $B = 3$ (the economic life is twice the length of the breakthrough interval) while for $B = 9$ or $B = 10$, the model will not wait for the breakthrough to arrive (the economic life is one half or one third of the length of the breakthrough interval).

3.2 Experimental Conclusions

In general, technological change leads to a decrease in economic life under the assumption of a fixed replacement interval over an infinite horizon with technology evolving according to an exponential function. For the model with only continuous technological change, the economic life decreases as the rate of technological improvement increases. The model with only discontinuous technological change results in a decrease in economic life as compared to the model with no technological change. In addition, as the rate of technological change increases, the economic life decreases to the length of the breakthrough interval.

The model with both discontinuous and continuous technological change leads to a decrease in economic life when compared to any of the preceding models. Table 1 displays the replacement cycles for the discontinuous and combined discontinuous and continuous technological change models for three cases of β_0 and various β_1 values and breakthrough intervals between 2 and 10 periods. The results show that the economic life of the combined model is always lower than the other three models. Note that the replacement interval

Table 1: Replacement schedules under (1) discontinuous and (2) both continuous and discontinuous technological change. Differences are in bold. Replacement interval was 12 under no technological change and 12, 11 and 7, respectively, for three cases of β_0 for continuous technological change.

		Breakthrough Interval																	
		2		3		4		5		6		7		8		9		10	
β_0	β_1	(1)	(2)	(1)	(2)	(1)	(2)	(1)	(2)	(1)	(2)	(1)	(2)	(1)	(2)	(1)	(2)	(1)	(2)
.001	.002	12	12	12	12	12	12	12	12	12	12	12	12	12	12	12	12	12	12
.001	.010	10	10	12	12	12	12	11	11	12	12	11	11	12	12	12	**11**	12	**11**
.001	.050	8	8	9	9	8	8	10	10	12	12	11	11	10	10	9	9	11	11
.001	.100	6	6	9	**6**	8	8	10	10	12	12	7	7	8	8	9	9	10	10
.008	.010	10	10	12	**9**	12	**10**	12	**10**	12	**10**	12	**10**	12	**10**	12	**10**	12	**10**
.008	.080	6	6	9	**6**	8	8	10	10	12	**9**	7	7	8	8	9	9	10	10
.008	.100	6	6	9	**6**	8	8	10	10	12	**6**	7	7	8	8	9	9	10	10
.008	.500	2	2	3	3	4	4	5	5	6	6	7	7	8	8	9	9	10	10
.050	.080	6	**4**	9	**6**	8	**4**	10	**5**	12	**6**	7	7	8	8	9	**6**	10	**5**
.050	.100	6	**4**	9	**3**	8	**4**	10	**5**	12	**6**	7	7	8	8	9	**5**	10	**5**
.050	.500	2	2	3	3	4	4	5	5	6	6	7	7	8	8	9	9	10	**5**
.050	.900	2	2	3	3	4	4	5	5	6	6	7	7	8	8	9	9	10	**5**

is 12 periods with no technological change and 12, 11 and 7 periods, respectively, for the three cases of β_0 for continuous technological change.

In general, for low rates of technological change, the economic life is the same as in the model with no technological change (this is consistent with the model with only one type of technological change). As the rate of change increases, the economic life decreases to the length of the breakthrough interval (in almost all cases). When comparing the model with only discontinuous technological change to the model with both discontinuous and continuous technological change, the economic life determined by the discontinuous model is greater than or equal to the economic life in the model with both types of technological change. For both of the models that consider breakthrough technology, the optimal decision is often to wait for the breakthrough to arrive to purchase a replacement. However, for small breakthrough intervals, the optimal decision is often to wait for the second or even third breakthrough to arrive before purchasing the replacement.

In Grinyer's [10] paper, he concluded that technological change may, under some circumstances, lead to an increase in the economic life of successors and defenders. These circumstances include a low interest rate and a high Y (operating cost parameter) value, or a low interest rate and a low salvage value parameter. In addition, he stated that the economic lives of successors are sensitive to errors in estimates of the interest rate, the rate of decrease of salvage value, and the operating cost gradient. However, as the rate of technological change increases, the sensitivity to these parameters decreases.

In almost all of our experiments, technological change led to a decrease in economic life. In addition, similar to Grinyer's [10] results, the economic life is sensitive to changes in the interest rate, the salvage value parameter and the operating cost. As technological change increases, the model tends to select a breakthrough period for replacement (for small breakthrough intervals it may wait for the second breakthrough to occur). Because of the sensitivity of the model to the breakthrough interval, it is difficult to determine, even for a specific breakthrough interval, how sensitive economic life is to varying interest rates, salvage values, and operating costs.

There are some notable exceptions in which technological change or the model with only a continuous rate of technological change results in a shorter economic life than the model with discontinuous change only or the model with both types of change. For some breakthrough intervals, B, higher values of Y (operating cost parameter) lead to longer lifetimes when there is breakthrough technological change than when there is not. Also, for low purchase prices and high breakthrough intervals, the models with discontinuous improvement will generally wait for the breakthrough to arrive, rather than purchase one or two periods before its arrival (in the experiments, the model never waited more than two periods for the breakthrough to arrive). For varying interest rates (r), when comparing the model with only continuous technological change to the model with both types of change, the model will occasionally wait one more period for the breakthrough to arrive. There also exist a few exceptions for varying values of k, the salvage value parameter in which the economic life with continuous or no technological change is shorter than the economic life with only discontinuous change or with both types of change. However, when varying any of these parameters, the increase in economic life with technological change is not the norm. In general, for the parameters r, k, A, Y, and X, the economic life decreases with technological change.

4 Conclusions and Directions for Future Research

There has been a significant amount of research into different types of technological change, both continuous and discontinuous, and its effect on equipment replacement schedules. However, analysis including both types of technological change in a single model is rare. Based on Grinyer's [10] negative exponential model for continuous technological change, four models are presented with various types of technological change: no technological change, continuous, discontinuous, and both continuous and discontinuous technological change. The decisions made using each type of model are then compared for varying values of all of the parameters. As shown in previous research, ignoring technological change may result in keeping an asset beyond its economic life. A few exceptions to this rule exist, but they generally involve waiting only one more period for the breakthrough to arrive. However, including only continuous technological change may lead to a longer than optimal economic life as well. In any industry in which there are frequent breakthroughs in technology, it is important to consider their impact on the economic life of the current asset. In fact, as the rate of technological improvement increases, the difference in economic life becomes even more significant. Thus, for industries in which there is a significant improvement in technology every few periods, it is very important include in the analysis the breakthrough, as well as the additional developments that occur leading up to and following the breakthrough.

As noted in the introduction, there are many paths for future research which examine the relationship between continuous and discontinuous technological change. In this paper, the modeling of discontinuous technological change can be viewed as continuous technological change over an interval greater than one period. This could be relaxed such that the interval is not constant (possibly uncertain) and the leap in technology is sporadic. Additionally, we have only looked at the case of an exponential function. Other functions may lead to different results and the continuous and discontinuous cases may have different functions. Our model also assumes that technological advances are reflected as a function of the initial asset's present worth (as a function of T). More flexible modeling would allow variation in the relationship of purchase cost, operating costs, and salvage values. Finally, we assume a constant replacement schedule which clearly may not be optimal. It would be interesting to examine how an optimal replacement schedule (which is not fixed) is influenced by continuous and discontinuous technological change.

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5 Appendix

Tables 2 through 6 illustrate the different replacement schedules for cases of (1) no, (2) continuous, (3) discontinuous, and (4) both continuous and discontinuous, technological change. The tables illustrate changes in Y, X, A, r and k, respectively. For each table, it is assumed that $\beta_0 = 0.006$ and $\beta_1 = 0.1$. The first column in each table defines the parameters while the remaining columns define the replacement age. The column label (in parentheses) defines the technological change assumption. For models (3) and (4), breakthrough intervals of 2 through 10 were examined.

Table 2: Replacement schedules under (1) no, (2) continuous, (3) discontinuous, and (4) both continuous and discontinuous, technological change with varying cost parameter Y and $\beta_0 = .006$ and $\beta_1 = .1$.

			Breakthrough Interval																	
			2		3		4		5		6		7		8		9		10	
Y	(1)	(2)	(3)	(4)	(3)	(4)	(3)	(4)	(3)	(4)	(3)	(4)	(3)	(4)	(3)	(4)	(3)	(4)	(3)	(4)
10	19	15	8	8	9	9	12	12	10	10	12	12	14	14	16	16	18	12	20	11
20	12	10	6	6	9	6	8	8	10	10	12	6	7	7	8	8	9	9	10	10
50	6	5	4	2	3	3	4	4	5	5	6	6	7	7	8	4	5	5	5	5
75	4	3	2	2	3	3	4	4	5	5	3	3	4	3	4	4	3	3	5	3

Table 3: Replacement schedules under (1) no, (2) continuous, (3) discontinuous, and (4) both continuous and discontinuous, technological change with varying cost parameter X and $\beta_0 = .006$ and $\beta_1 = .1$.

			Breakthrough Interval																	
			2		3		4		5		6		7		8		9		10	
X	(1)	(2)	(3)	(4)	(3)	(4)	(3)	(4)	(3)	(4)	(3)	(4)	(3)	(4)	(3)	(4)	(3)	(4)	(3)	(4)
150	12	11	8	6	9	9	8	8	10	10	12	12	11	7	8	8	9	9	10	10
225	12	10	6	6	9	6	8	8	10	10	12	12	7	7	8	8	9	9	10	10
300	12	10	6	6	9	6	8	8	10	10	12	10	7	7	8	8	9	9	10	10
450	12	10	6	4	6	6	8	8	10	10	12	10	7	7	8	8	9	9	10	10

Table 4: Replacement schedules under (1) no, (2) continuous, (3) discontinuous, and (4) both continuous and discontinuous, technological change with varying cost parameter A and $\beta_0 = .006$ and $\beta_1 = .1$.

			Breakthrough Interval																	
			2		3		4		5		6		7		8		9		10	
A	(1)	(2)	(3)	(4)	(3)	(4)	(3)	(4)	(3)	(4)	(3)	(4)	(3)	(4)	(3)	(4)	(3)	(4)	(3)	(4)
700	9	8	4	4	6	6	8	4	5	5	6	6	7	7	8	8	9	9	10	10
1000	12	10	6	6	9	6	8	8	10	10	12	6	7	7	8	8	9	9	10	10
1250	14	12	8	8	9	9	8	8	10	10	12	10	14	14	12	8	9	9	10	10

Table 5: Replacement schedules under (1) no, (2) continuous, (3) discontinuous, and (4) both continuous and discontinuous, technological change with varying interest rate r and $\beta_0 = .006$ and $\beta_1 = .1$.

			Breakthrough Interval																	
			2		3		4		5		6		7		8		9		10	
r	(1)	(2)	(3)	(4)	(3)	(4)	(3)	(4)	(3)	(4)	(3)	(4)	(3)	(4)	(3)	(4)	(3)	(4)	(3)	(4)
.05	10	9	6	6	6	6	8	8	10	5	6	6	7	7	8	8	9	9	10	10
.10	12	11	6	6	9	6	8	8	10	10	12	6	7	7	8	8	9	9	10	10
.15	14	12	6	6	9	9	8	8	10	10	12	10	14	9	10	9	10	9	10	10
.20	16	14	8	8	9	9	12	8	10	10	12	10	14	14	16	12	12	11	12	11

Acknowledgements

This research was supported in part by the National Science Foundation grant No. DMI-9713690.

Table 6: Replacement schedules under (1) no, (2) continuous, (3) discontinuous, and (4) both continuous and discontinuous, technological change with varying salvage value parameter k and $\beta_0 = .006$ and $\beta_1 = .1$.

			Breakthrough Interval																	
			2		3		4		5		6		7		8		9		10	
k	(1)	(2)	(3)	(4)	(3)	(4)	(3)	(4)	(3)	(4)	(3)	(4)	(3)	(4)	(3)	(4)	(3)	(4)	(3)	(4)
.10	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
.15	7	6	6	2	3	3	4	4	5	5	6	6	7	7	4	8	5	9	5	5
.25	11	10	10	6	6	6	8	8	10	10	6	6	7	7	8	8	9	9	10	10
.35	12	11	12	8	9	6	8	8	10	10	12	12	8	7	8	8	9	9	10	10
.70	12	11	12	8	9	9	8	8	10	10	12	12	11	14	10	8	9	9	10	10