Delivery-Date Coordination in an Internal Market via Risk Sharing

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Abstract

We study the problem of delivery-date coordination between the marketing and engineering divisions within an engineer-to-order firm. Marketing concerns the customer who has a preferred delivery-date for his order but is willing to compromise in return for price discounts. Engineering concerns the efficient utilization of resources and is willing to offer a higher service level if the additional cost is compensated. Operating in a project management environment, we design a Nash game between marketing and engineering where the two divisions share the responsibility for timely delivery. Marketing quotes a due date and engineering sets the capacity level based on their corresponding utilities; the utility functions are defined by due-date extension cost, flowtime distribution, tardiness penalty, and capacity expansion cost. We characterize the basic properties of the players' utilities and show that the game between two parties is supermodular, implying the existence of a Nash equilibrium. We show that equilibrium behaviors of both parties never coincide with the system optimum. We develop an incentive scheme for marketing and engineering in such a way that the system optimum can be achieved at equilibrium. We conduct sensitivity analysis on the transfer payments such that they could be tailored for alternative utilities.

Keywords: Game Theory; Supermodular Games; Due-Date Quotation; Supply Chain Coordination

1. Introduction

The literature on due-date-based planning and scheduling typically assumes that order due-dates are exogenous and given. In most engineer-to-order (ETO) environments, such as construction, customized industrial machinery, and aerospace, the delivery dates are negotiable, and due-date quotation is a marketing and sales function. However, when quoting due-dates, marketing must consider the customer's preferences together with internal constraints, such as production capacity, and other critical resource requirements at the firm. Most internal constraints are controlled by the engineering (or project management) division of the firm, which sets the pace for each project that would ultimately influence the order completion time. To properly integrate the due-date quotation and project management decisions, the marketing and engineering divisions must coordinate. However, division managers are typically rewarded based on the performance of their local units, which have distinctly different cost structures. Since the best interests of individual divisions rarely coincide with the firm's optimal (profit maximizing) policy, the firm must provide proper incentives for marketing and engineering to coordinate. Most ETO firms produce capital-intensive products with long lead time at an extremely low ordering frequency. According to a study conducted by Konijnendijk (1994), more than 60% of the surveyed ETO companies service less than 10 orders a year. In addition to pricing, significant emphasis is put on due-date quotation and subcontracting/outsourcing decisions. Another survey by Spencer and Cox (1994) supports this observation by concluding that most make-to-order firms compete on customization, flexibility, and delivery performance, as opposed to commodity producers and make-to-stock firms that have a primarily focus on pricing. In a typical ETO setting, detailed production specifications that influence the job completion time are not available at the initial price quotation phase. However, a finalized due-date quotation must be confirmed with the customer before the order becomes binding.

Our work is motivated by the operation of the industrial pump division of a large U.S. industrial machinery manufacturer. When placing the order, say, for a set of customized pumps to be used in a nuclear power plant, the customer may provide the design specifications while requesting pricing and delivery-date information. A typical order of this magnitude involves from several months to over a year to complete, with the order valued at a few million dollars. The marketing and sales division of the manufacturer communicates the order information to the engineering division, which in turn lays out the steps required to complete the project and determines the resource requirements. Engineering may determine that the existing capacity is not sufficient to handle the project, in which case they need to estimate costs for capacity expansion or outsourcing. Using this information, marketing negotiates with the customer on final pricing and the delivery date. This paper focuses on the internal coordination between marketing and engineering for due-date quotation. In the context of the ETO operation described above, due-date quotation involves two main considerations: the customer preferences and the capacity requirements. A customer may have a preferred delivery date for an order based on a certain utility, which is usually a function of time (Tang and Tang, 2002), i.e., a prolonged delivery date may degrade the customer's utility. On the other hand, capacity constraints facing the engineering division might force the marketing division to deviate from customer preferences on delivery dates; consequently, customers must be compensated for such deviations. We model this compensation via price discounts that are proportional to the deviation between the quoted and preferred delivery dates. Keeping this deviation within reasonable range oftentimes necessitates the cooperation of the engineering division. The engineering division can give more room to marketing by increasing its capacity to that required to produce the order. However, in most decentralized environments, engineering has little incentive to do so. We design a Nash game to investigate the incentives and competition between the marketing and engineering divisions where marketing determines the delivery date to be quoted for the customer, while engineering decides on capacity. Since the timely delivery of the order is the responsibility of both divisions, when the completion time exceeds the promised delivery date, they share the tardiness cost based on pre-specified terms. To generate proper incentives that align local decisions with the system optimal, we derive a payment scheme between marketing and engineering which allows for the system optimal being achieved at Nash equilibrium.

Two areas of literature are directly relevant to our study: marketing and engineering coordination and due-date quotation. Next, we provide a brief overview of the literature.

Marketing/Production Coordination

The need for coordinating marketing and production decisions has been recognized by researchers for more than two decades (c.f., Davis (1977), Shapiro (1977), Montgomery and Hausman (1986) and Karmarkar and Lele (1989)). In general, this line of research defines the need for marketing and production coordination in companies producing industrial goods and discusses the nature of the problem as "necessary cooperation but potential conflict." Areas of coordination include capacity planning and allocation, forecasting, scheduling, delivery and distribution, quality assurance, cost control, product design, and adjunct services. A broad survey of these approaches are reported by Eliashberg and Steinberg (1993).

Porteus and Whang (1991) propose a different approach to the problem by developing an incentive plan that would reward the division managers for acting in a system-optimal way. They propose a plan where product managers receive all revenues from the sales, while they pay the manufacturing manager the realized marginal value of capacity. While this "internal market" induces optimal local behavior, the firm needs to provide subsidies. Kouvelis and Lariviere (2000) present a generalization of the internal market mechanism based on linear transfer payments between functions, i.e., a market maker buys from upstream managers and resells it to downstream managers, where the buying/selling prices are set in such a way that they lead to system optimal actions. Desai (1996) compares three different contracts in a marketing-engineering channel faced with seasonal demand. He discusses Stackelberg games under fixed retailer processing rate, fixed engineering price, and a general case without variable fixing. Desai and Srinisavan (1995) consider a signaling game between a franchisee and franchisor where they investigate the deviation of the franchiser's pricing decisions from the first-best solution when the franchisee's efforts cannot be observed. Kim and Lee (1998) study optimal coordination strategies for short-term production and marketing decisions. They propose a scheme where manufacturing determines the production volume based on the marginal revenue given by marketing using the previous demand rate. Celikbas *et al.* (1999) investigates coordination mechanisms based on different penalty schemes that enable the firm to match demand forecasts with production. They consider both centralized and decentralized organizational structures and show that by setting appropriate penalty levels the decentralized system could operate similar to the centralized one. In many cases, the work in the contracting and coordination between retailers and manufacturers can be applied to marketing/production coordination with little modification. Analytical studies of such problems can be found in Cachon and Zipkin (1999) and Agrawal and Tsay (2001). In this paper, we will focus our attention on marketing and engineering coordination in the project management context of an engineer-to-order firm. In this environment, the engineering division has more control over project resources, while meeting due date at each stage of the project is just as important to engineering as it is to marketing.

Due-Date Quotation Problems and Coordination

The importance of setting reliable job due dates in make-to-order production system is well recognized in the literature for at least two decades. Most early work on due-date setting uses generalized but ad hoc decision rules. An extensive survey of earlier research regarding traditional due-date setting problems is provided by Cheng and Gupta (1989). A more recent survey is given by Keskinocak and Tayur (2003). A vast majority of this literature does not consider customer preferences when setting due dates, assuming that any due date quoted will be accepted without any friction with the customer. However a recent survey of US manufacturing practices in make-to-order companies by Wisner and Siferd (1995) reveals that in over 60% of the cases customers' specifications and preferences are the main determinant in due-date quotation. The due-date setting problems are typically studied using centralized and monolithic models where the decision regarding due dates is considered in conjunction with decisions such as capacity utilization, sequencing and scheduling, pricing etc. Some of the recent work includes Wein (1991), Duenyas and Hopp (1995), Zijm and Buitenhek (1996), Spearman and Zhang (1999) and Weng (1999). Chatterjee *et. al.* (2002) provide a broad survey of recent papers on the subject.

Lawrence (1994) finds flowtime distribution estimations as the most important factor in achieving competitive due-date quotation among manufacturing, marketing and the customers. He acknowledges that flowtime distributions allow the construction of managerially useful tradeoff curves contrasting order completion probabilities and expected tardiness costs with order lead times. Van der Maijden *et al.* (1994) underlines the importance of setting goals as a result of negotiation between departments especially under demand uncertainty. Elhafsi and Rolland (1999) propose a due-date quotation model based on the congestion level of the manufacturing shop floor and the operating cost. Easton and Moodie (1999) discuss a procedure where the manufacturer bids the price and lead time for the customer, and the customer may accept, reject or modify the terms. As a hedging strategy, the manufacturer may bid on other projects. In case more customers accept the bid, some orders will be delayed. Thus, the hedging strategy must balance potential profits with the tardiness penalty. Tang and Tang (2002) study price discounts offered to customers in a build-to-order environment for extended lead times. They show that with customers sensitive to delivery times, discounts can in fact result in higher revenues. While the authors concentrate on the interaction between buyers and sellers, we investigate the price discounts in the context of cross-functional operations in an engineer-to-order environment.

Weng (1999) studies the impact of quoted due dates and order acceptance rates on expected profit. His results apply to cases where the flowtime follows a general phase-type distribution function of the order acceptance rate. Palaka *et al.* (1998) and So and Song (1998) consider customers who are sensitive to quoted due-dates and prices. They propose nonlinear optimization models to find the "jointly" optimal due-date, capacity utilization, and price that maximize the firm's profit.

Competitive due-date quotation has been investigated to a limited extent in the literature. Lederer and Li (1997) investigates the competitive equilibrium among multiple buyers (customers) and suppliers (firms) over selecting prices, production rates, and scheduling policies. Lead time (thus due date) represents a function of production rate and scheduling policy, which specifies how arriving jobs are sequenced. In this setting, firms differ in operation costs, mean processing times and processing time variability, while customers are differentiated based on their delay costs. A competitive equilibrium is found when the Kuhn-Tucker conditions for the firm's optimization problem and the market clearing condition are simultaneously satisfied.

Customers who carry delay costs are also considered by Ha (1998). In this setting a G1/G1/1 service queue is assumed where the customers choose the service rates and linear delay costs while the firm sets a price for each customer served. It is shown that when customers choose the service rates based on their local cost structure, the resulting system service rate and arrival rate are always smaller than the optimum due to externalities. The author proposes incentive-compatible pricing consisting of a fixed admission fee and a variable fee that is proportional to the actual service time. Grout (1996) proposes an incentive-inducing contract between a buyer and a supplier aiming at the timely delivery of orders. In his setting, the buyer dominates the supplier and moves first by selecting an incentive scheme that consists of an on-time delivery bonus and a tardiness penalty. The optimal probability for on-time delivery can be ensured if the supplier responds to the incentive scheme by selecting a flowtime allowance that would minimize his own expected cost.

In the following section, we describe our model in detail and state related assumptions. In Section 3, we present the global optimal model from the firm's point of view. We then present the marketing and the engineering models and provide equilibrium analysis in Section 4. Section 5 describes the mechanism that achieves coordination between the two departments and follows with an examination of our approach assuming quadratic costs and Weibull flowtime distribution in Section 6. Last section concludes the paper.

2. Delivery-Date Quotation in an Internal Market: Model Description

We consider a cost minimization model for the delivery-date quotation process between the decision makers of marketing and engineering divisions in an ETO company. Our focus is on the cross-functional operations within the firm, and thus the customer incentives are assumed to be exogenous. By design, decisions regarding due-date quotation and capacity utilization are closely related. While the due dates quoted for customers hinge on the capacity utilized by engineering, engineering is encouraged to make capacity adjustments based on a target delivery performance shaped by customer preferences. Suppose that each customer has a preferred delivery date *cd* for his orders, and he states *cd* truthfully. We assume that the customer is always willing to accept a later delivery date, dd, which is quoted by marketing, in return for a price discount that is proportional to the difference between *dd* and *cd*. The price discount offered to the customer is referred to as *due date extension cost* and represented by $\mathcal{D}([dd - cd]^+)$, where x^+ denotes $\max(0, x)$. Clearly, such cost occurs only when *dd*>*cd*.

There is a *tardiness penalty* in case the completion time of the job exceeds the quoted due date *dd*. We denote this cost function by $\mathcal{T}([c - dd]^+)$, where c is the realized completion time. The job completion time follows a publicly known *flowtime distribution*, which is a continuous probabilistic density function, $f(c|\theta)$, where θ represents the capacity provided for the order. The capacity level θ is a variate of $f(c|\theta)$, and it is a decision variable for the engineering division. Suppose, without the loss of generality, that at the time of order arrival the engineering division has a nominal, fixed-cost capacity given by θ_o. Let $F(c|\theta)$ be the cdf of the flowtime distribution and we assume that $F(c|\theta)$ increases in θ. Consequently, for any $\theta > \theta_o$, $F(c|\theta) > F(c|\theta_o)$. We will refer to this as the *stochastic ordering* assumption, that is, the random numbers drawn from $F(*|\theta)$ stochastically dominate those from $F(*|\theta_0)$ in both first and second orders (Shaked and Shantikumar 1994). We represent the cost function of improving capacity from θ_o to θ by $\mathcal{Z}([\theta - \theta_o]^+)$. We assume that capacity increment is achieved by adding temporary work force, subcontracting, or outsourcing, and a certain capacity level can be directly associated with (charged to) a particular order, which are common practices in ETO environments (Hicks *et. al.* 2000). This assumption allows us to analyze due-date quotation for each order independently. Later in the paper, we will discuss how our analysis can be generalized to cases where multiple jobs share the common capacity.

As summarized above, there are three main cost components in the studied system: $\mathcal{D}$, $\mathcal{Z}$, and $\mathcal{T}$. To streamline the analysis, we adopt the following assumptions regarding the cost functions:

Assumption 1. *$\mathcal{D}([dd - cd]^+)$ and $\mathcal{Z}([\theta - \theta_o]^+)$ are continuous and differentiable functions, where $\mathcal{D}$ is twice differentiable over dd, and $\mathcal{Z}$ twice differentiable over θ.*

Assumption 2. *$\mathcal{T}([c - dd]^+)$ is continuous and differentiable over dd. In addition, $\mathcal{T}(y)$ is 0 for all $y \leq 0$, and is convex increasing in y for all $y > 0$.*

Note that we do not require convexity for functions $\mathcal{D}$ and $\mathcal{Z}$. The second assumption states that the tardiness cost function is convex in lateness, $[c - dd]^+$. We have two additional assumptions regarding the due-date extension and capacity costs, which are intuitive.

Assumption 3. *$\mathcal{D}(dd - cd) = 0$ for all $dd \leq cd$ and increases in dd otherwise.*

Assumption 4. *$\mathcal{Z}(\theta - \theta_o) = 0$ for all $\theta \leq \theta_o$ and increases in θ otherwise.*

The sequence of events for due-date quotation is as follows: (1) the customer places an order with a preferred delivery-date cd; (2) based on their local utilities, marketing announces the quoted *due-date, dd,* and engineering announces the capacity level to be committed for this order, θ; (3) if $dd > cd$, marketing must offer a discount to the customer proportional to the difference between dd and cd; (4) production occurs, the order is filled at time c and tardiness costs are charged based on the difference between c and dd. Since tardiness is not only a function of the due date quoted by marketing, but also the capacity level allocated by engineering, both divisions are responsible for the tardiness cost. Otherwise, marketing would have no incentives to deviate from the customer preferred delivery date, and engineering would have no incentive to improve the capacity level. We consider a setting where the tardiness cost is split between marketing and engineering according to a parameter γ, where $0 \leq \gamma \leq 1$, i.e., given that $c > dd$, marketing pays $\gamma\mathcal{T}(c - dd)$ while engineering pays $(1-\gamma)\mathcal{T}(c - dd)$. We assume perfect information where all parties can observe all system parameters. For the rest of the analysis let $\overline{F}(x)$ denote the tail distribution for the completion time and $E(x)$ be the expected value of x. For any function U, $U^{(x)}$ is $\partial U / \partial x$ and $U^{(x,y)} = \partial^2 U / (\partial x \partial y)$.

3. The System (Firm's) Optimization Model

At the time of due-date quotation, the firm's objective is to minimize the total transaction costs due to the due-date concession, capacity adjustments, and the expected tardiness costs due to flowtime uncertainty. The system cost function is given as follows;

$G_o = \mathcal{D}([dd - cd]^+) + \mathcal{Z}([\theta - \theta_o]^+) + E[\mathcal{T}([c - dd]^+)]$
where

$$E[\mathcal{T}([c - dd]^+)] = \int_{dd}^{\infty} \mathcal{T}(x - dd) f(x|\theta) dx \tag{1}$$

Note that γ does not influence the system's model since it defines an internal transfer between marketing and engineering. The system optimal due date quotation and capacity extension strategy must minimize G_o. In order to analyze the system optimal behavior we first offer the following observation:

LEMMA 1. *If $F(*|\theta)$ is an increasing function of θ, then the expected tardiness cost is decreasing in θ.*

Proof. See the appendix.

The foregoing result shows that by improving capacity, engineering decreases the probability for the system to pay a higher tardiness penalties. Suppose dd^o and θ^o are the due date and capacity values that minimize the system cost, G_o. Lemma 1 leads us to the following conclusions.

THEOREM 1. *$dd^o > cd$ and $\theta^o > \theta_o$.*

Proof. From Assumption 3, the due date extension cost is 0 for $dd \geq cd$ whereas from Assumption 2, tardiness cost decreases in dd. Consequently, for $dd \leq cd$, the system optimal cost strictly decreases in dd. Hence, $dd^o > cd$ must hold. Similarly for $\theta^o \leq \theta_o$, the system does not incur any capacity cost (Assumption 4), and from Lemma 1 we know that tardiness cost decreases in θ. Consequently, for $\theta^o \leq \theta_o$, the system optimal cost strictly decreases in θ. Hence, $\theta^o > \theta_o$ must be true. □

Since we do not assume convexity or unimodularity, G_o may have multiple local optima. Clearly, at any stationary point $(\widehat{dd}, \widehat{\theta})$ the following conditions must hold:

$$\mathcal{D}^{(dd)}(dd^o - cd) + \int_{dd^o}^{\infty} \mathcal{T}^{(dd)}(x - dd^o) f(x|\theta^o) dx = 0 \tag{2}$$

$$\mathcal{Z}^{(\theta)}(\theta^o - \theta_o) + \int_{dd^o}^{\infty} \mathcal{T}^{(x)}(x - dd^o)\overline{F}^{(\theta)}(x|\theta^o)dx = 0 \tag{3}$$

Theorem 1 implies that at least one of the stationary points in the system cost function must be a local minimum. Finding the optimal values may not be trivial for the general case. However, in case of unimodularity, a simple recursive search can be employed to approximate the optimal solution in a reasonable time even if the optimality conditions are not in closed form. Without the loss of generality, we assume that there is no constraint-enforcing upper bounds on due-date dd and capacity θ.

4. The Due-Date Quotation Game

We now define a Nash game corresponding to the due-date quotation procedure described in Section 2. The game, Ω, consists of marketing and engineering decision makers as independent players who act simultaneously and choose their strategies. The strategy space for marketing, σ_{m}, has a lower bound, cd and has no upper bound. Hence, $dd \in \sigma_{\mathrm{m}} = [cd, M_{\mathrm{m}}]$ where M is a large arbitrary constant that will never constrain marketing in its decision. Likewise, the strategy space for engineering, σ_{e}, is bounded by θ_o, i.e., $\theta \in \sigma_{\mathrm{e}} = [\theta_o, M_{\mathrm{e}}]$. Both players have complete information about each others' cost functions, and thus all parameters in the model are common knowledge. The flowtime distribution is public (e.g., computed from historic information) and therefore identical for both players. In this model, the central management can be seen as an information intermediary that can observe the actions of both sides and make sure the information is fully shared across divisions.

Let $H_j(dd, \theta)$ denote the player j's expected cost when players adopt the joint strategy of (dd, θ). Let j be m for marketing and e for engineering. The best response mapping for player j is a set-valued function corresponding to each strategy of player k $(k \neq j)$, with a subset of σ_j and formally defined as follows for each player in this game;

$$r_{\mathrm{m}}(\theta) = \left\{ dd \in \sigma_{\mathrm{m}} \,|\, H_{\mathrm{m}}(dd, \theta) = \min_{x \in \sigma_{\mathrm{m}}} H_{\mathrm{m}}(x, \theta) \right\}$$

$$r_{\mathrm{e}}(dd) = \left\{ \theta \in \sigma_{\mathrm{e}} \,|\, H_{\mathrm{e}}(dd, \theta) = \min_{x \in \sigma_{\mathrm{e}}} H_{\mathrm{e}}(dd, x) \right\}$$

In this setting, a pure strategy Nash equilibrium is a pair of due date and capacity level, (dd^q, θ^q), such that each player chooses a best response to the other player's equilibrium decision, i.e., $dd^q \in r_{\mathrm{m}}(\theta^q)$ and $\theta^q \in r_{\mathrm{e}}(dd^q)$.

4.1. Decision Models for the Players

As described earlier, marketing is charged for the deviation between the quoted due date (*dd*) and the customer preferred due date (*cd*), and engineering pays for capacity expansion. The tardiness penalty is shared by the two divisions (specified by parameter γ). We define the expected cost function of marketing as follows:

$$H_{\mathrm{m}}(dd, \theta) = \mathcal{D}([dd - cd]^{+}) + \gamma E[\mathcal{T}([c - dd]^{+})]$$

Similarly, the expected cost function of the engineering division is as follows:

$$H_{\mathrm{e}}(dd, \theta) = \mathcal{Z}([\theta - \theta_o]^{+}) + (1 - \gamma)E[\mathcal{T}([c - dd]^{+})]$$

Based on the local cost structures we may derive the following generalization regarding the players' behaviors.

THEOREM 2. *It is a dominant strategy for marketing to quote a due date dd, such that dd > cd, and for engineering to increase its capacity* θ *beyond* θ_o.

The proof follows directly from Theorem 1. Both players pay a portion of the tardiness penalty, and the expected tardiness penalty decreases in both dd and θ. Regardless of its opponent's action, the player's total cost is decreasing for any decision variable value below its lower bound in the strategy space. Therefore, one must look for the equilibria in the dominant strategy space of each player. This is examined in the following section.

4.2. Analysis of Equilibria

We establish the existence of equilibrium based on a result regarding *supermodular* games. A function $g(x_1, x_2)$ is said to be supermodular in (x_1, x_2) if for all $(\widehat{x}_1, \widehat{x}_2) \geq (\overline{x}_1, \overline{x}_2)$ the inequality of $g(\widehat{x}_1, \widehat{x}_2) + g(\overline{x}_1, \overline{x}_2) \geq g(\widehat{x}_1, \overline{x}_2) + g(\overline{x}_1, \widehat{x}_2)$ holds. Observe that supermodularity does not require convexity, and it is a somewhat relaxed condition. In a supermodular game each player's best response mapping increases in the other player's strategy. With such monotonicity on best response functions, the equilibrium is straightforward to establish due to the conclusions of Topkis (1979): if the strategy space of a game is a complete lattice, the joint payoff function is upper semicontinuous, and each player's payoff function is supermodular, then there exists a pure strategy Nash equilibrium. A comprehensive discussion on supermodular games can be found in Sundaram (1996) and Topkis (1998). See Lippman and McCardle (1997) and Cachon (2001) for examples of game theoretical applications that utilize the theory of supermodularity. The following theorem shows that the due date quotation game between marketing and engineering satisfies the conditions for supermodular games.

THEOREM 3. *A pure-strategy Nash equilibrium exists for the due-date quotation game* Ω.

Proof. Assumptions 1 and 2 assert that the joint payoff function is upper-semicontinuous. Let's first focus on marketing cost function, H_{m}. For supermodularity we need to show that $H_{\mathrm{m}}(dd_1,\theta_1) - H_{\mathrm{m}}(dd_2,\theta_1) \geq H_{\mathrm{m}}(dd_1,\theta_2) - H_{\mathrm{m}}(dd_2,\theta_2)$ for all $dd_1 \geq dd_2$ and $\theta_1 \geq \theta_2$. Notice that the inequality holds when $H_{\mathrm{m}}(dd_1,\theta) - H_{\mathrm{m}}(dd_2,\theta)$ is nondecreasing in θ for all $dd_1 \geq dd_2$. This property implies increasing differences that are, in fact, a consequence of supermodularity. We can check whether the difference increases in θ or not by simply taking the first derivative. First observe that

$$\frac{\partial H_{\mathrm{m}}(dd_1,\theta)}{\partial\theta} = H_{\mathrm{m}}^{(\theta)}(dd) = \gamma \int_{dd}^{\infty} \left(\frac{\partial \mathcal{T}(x-dd)}{\partial x} \cdot \frac{\partial \overline{F}(x|\theta)}{\partial \theta} \right) dx$$

From our assumption related to stochastic ordering of flowtime distributions it is straightforward to see that the foregoing function is always negative. As pointed out above, increasing differences are guaranteed when $H_{\mathrm{m}}^{(\theta)}(dd_1) - H_{\mathrm{m}}^{(\theta)}(dd_2)$ is non-negative. To see that this condition is satisfied it is sufficient to show that $\partial H_{\mathrm{m}}^{(\theta)}/\partial dd \geq 0$ since $dd_1 \geq dd_2$.

$$\frac{\partial^2 H_{\mathrm{m}}(dd,\theta)}{\partial\theta\partial dd} = H_{\mathrm{m}}^{(dd,\theta)} = \gamma \int_{dd}^{\infty} \left(\frac{\partial^2 \mathcal{T}(x-dd)}{\partial x \partial dd} \cdot \frac{\partial \overline{F}(x|\theta)}{\partial \theta} \right) dx \tag{4}$$

We know from Lemma 1 that $\mathcal{T}^{(x)}(x-dd) = \partial\mathcal{T}/\partial y$ where $y = x - dd$. Then from chain rule

$$\frac{\partial^2 \mathcal{T}(x-dd)}{\partial x \partial dd} = \partial(\partial\mathcal{T}/\partial y)/\partial dd = \frac{\partial y}{\partial dd} \cdot \frac{\partial^2 \mathcal{T}(y)}{\partial y^2} = -\frac{\partial^2 \mathcal{T}(y)}{\partial y^2} < 0$$

The right hand side relation above is from Assumption 2. Consequently, the right hand side of (4) is non-negative. implying that $H_{\mathrm{m}}(dd_1,\theta_1) - H_{\mathrm{m}}(dd_2,\theta_1) \geq H_{\mathrm{m}}(dd_1,\theta_2) - H_{\mathrm{m}}(dd_2,\theta_2)$ for all $dd_1 \geq dd_2$ and $\theta_1 \geq \theta_2$. Therefore H_{m} is supermodular in dd and θ. With a similar approach, we can show that $H_{\mathrm{e}}^{(dd,\theta)} = (1-\gamma)/\gamma \cdot H_{\mathrm{m}}^{(dd,\theta)}$, and thus it is non-negative as well since $\gamma \leq 1$. Hence, H_{e} is also supermodular in dd and θ. As a result, we can conclude that Ω is a supermodular game and thus equilibrium exists. $\square$

Although the foregoing theorem establishes the *existence* of Nash equilibrium it does not establish its *uniqueness*. Depending on the flowtime distribution and the cost structure, there might be multiple Nash equilibria. We can make several observations about the characteristics of the equilibria. Based on the implicit function theorem, the derivatives $r_{\mathrm{m}}^{(\theta)}$ and $r_{\mathrm{e}}^{(dd)}$ can be given as follows;

$$r_{\mathrm{m}}^{(\theta)} = -\left(H_{\mathrm{m}}^{(dd,\theta)}/H_{\mathrm{m}}^{(dd,dd)}\right) = \frac{-\gamma E^{(dd,\theta)}[\mathcal{T}([c-dd]^{+})]}{\mathcal{D}^{(dd,dd)} + \gamma E^{(dd,dd)}[\mathcal{T}([c-dd]^{+})]}$$

$$r_{\mathrm{e}}^{(dd)} = -\left(H_{\mathrm{e}}^{(dd,\theta)}/H_{\mathrm{e}}^{(\theta,\theta)}\right) = \frac{-(1-\gamma)E^{(dd,\theta)}[\mathcal{T}([c-dd]^{+})]}{\mathcal{Z}^{(\theta,\theta)} + (1-\gamma)E^{(\theta,\theta)}[\mathcal{T}([c-dd]^{+})]}$$

Observe that if the due date extension cost is convex in dd then $r_{\mathrm{m}}^{(\theta)} < 0$ for all dd and θ. Similarly if the engineering cost function is convex in θ, then $r_{\mathrm{e}}^{(dd)} < 0$. Under the assumption of convexity we may conclude the following.

LEMMA 2. *Assume that both marketing and engineering cost functions are convex, if, for all dd and θ,*

i) $r_m^{(\theta)} \geq -1$ *and* $r_e^{(dd)} \geq -1$ *or;*
ii) $r_m^{(\theta)} < -1$ *and* $r_e^{(dd)} < -1$

then the pure strategy Nash equilibrium is unique.

Proof. See the appendix.

To derive general conclusions, we may utilize the properties of supermodularity.

LEMMA 3. *Let* (dd_1^q, θ_1^q) *and* (dd_2^q, θ_2^q) *are any two equilibrium points in* Ω. *If* $dd_1^q > dd_2^q$ *then* $\theta_1^q < \theta_2^q$.

Proof. See the appendix.

Lemma 3 implies that there exists an equilibrium where engineering chooses a higher level capacity (higher θ) and marketing quotes a lower due date than in any other equilibrium whereas in another equilibrium θ is lower and dd is higher than in any equilibrium. We call the former equilibrium the highest (lowest) equilibrium and the latter one as the lowest (highest) equilibrium for engineering (marketing). Next, using envelop theorem we can deduce the following conclusion:

LEMMA 4. *The highest equilibrium for a division incurs the worst financial outcome for that division while providing the best financial outcome for the opposing division among all existing equilibria.*

Proof. See the appendix.

Note that the foregoing observations imply that if the highest and the lowest equilibrium points coincide then the equilibrium must be unique. Whether these points are the same or not can be determined by applying a process known as *iterated deletion of dominated strategies*. A detailed description of the process is given in the appendix. In essence, to find the lowest (highest) equilibrium for marketing (engineering), we start with $dd = cd$ and find the best response of engineering, $\widehat{\theta}$. Given $\widehat{\theta}$, we calculate the best response of marketing. We continue in the same fashion until the best response functions converge to an equilibrium point (The proof is detailed in the appendix). We start with strategy $\theta = \theta_o$ for engineering and repeat the same procedure to find the smallest (highest) equilibrium for engineering (marketing). If both equilibrium points are identical then we can conclude that the equilibrium is unique. Although we cannot guarantee the uniqueness of Nash equilibrium under the general case of supermodularity, we could still conclude that none of the equilibria coincides with the system optimum. This is shown in the following theorem.

THEOREM 4. *System's optimal solution is never a Nash equilibrium in Ω.*

Proof. Optimal due date and capacity decisions satisfy first order optimality conditions of the integrated system given in (2) and (3). First derivative of marketing cost function with respect to dd at system optimal capacity, θ^o, is

$$H_{\mathrm{m}}^{(dd)}(dd,\theta^o) = \mathcal{D}^{(dd)}(dd-cd) + \gamma\int_{dd}^{\infty}\mathcal{T}^{(dd)}(x-dd)f(x|\theta^o)dx \tag{5}$$

Observe from (2) and Assumption 2 that for $\gamma < 1$, the foregoing function is positive at $dd = dd^o$ implying that the system optimal due date cannot be a best response to θ^o for the marketing division. On the other hand, for optimal due date dd^o, first derivative of engineering cost function with respect to θ is

$$H_e^{(\theta)}(dd^o, \theta) = \mathcal{Z}^{(\theta)}(\theta - \theta_o) + (1-\gamma)\int_{dd}^{\infty} \mathcal{T}(x - dd) f^{(\theta)}(x|\theta^o)dx \tag{6}$$

Clearly, for $\gamma > 0$, the expected cost function of the engineering division is increasing at θ^o for $dd = dd^o$. Therefore, θ^o cannot be a best response to dd^o for engineering. This completes the proof. □

A straightforward analysis of (5) will reveal that if the marketing cost function is unimodular, any equilibrium due-date decision is strictly smaller than dd^o. Likewise, under unimodularity, (6) indicates that engineering will always keep its capacity below θ^o. To encourage both divisions to improve their efforts, incentive compatible mechanisms are needed.

5. Coordinating Marketing and Engineering Decisions

In Theorem 4, we show that the marketing-engineering competition degrades system efficiency in the due-date quotation game. A coordination mechanism that gives proper incentives for the players to coordinate could lead to higher overall efficiency. Several different coordination schemes have been proposed in the literature. For instance, Porteus and Wang (1991) and Kouvelis and Lariviere (2000) propose cross-functional coordination schemes where the central management subsidizes the divisions in such a way that the induced local cost functions lead to an equilibrium that coincides with the system optimum. In this setting, the firm could make a payment to marketing that is linear in the due date extension cost and the (marketing's) tardiness penalty, while making a payment to engineering that is linear in the capacity expansion cost and (engineering's) tardiness penalty. Although this method is straightforward to implement, it deos not guarantee budget balanceness. In other words, the central management may need extra funds to subsidize the trade. Alternatively, Lee and Whang (1999) and Cachon and Zipkin (1999) propose schemes using transfer payments between the divisions. In essence, the transfer payment imposes a cost sharing scheme for the divisions based on their local utilities. Using the transfer payment, it is possible to align the divisional cost margins with those of the system's. Since the transfers are paid by one division to the other, the firm does not need to subsidize. In this paper, we propose a coordination mechanism that employs the transfer payment scheme. The transfer payments between divisions are devised based on due date extension, capacity expansion, and realized completion times. Our mechanism employs only cost parameters that are independent of the flowtime distribution; thus, the central management does not need to know the optimal solution in advance to coordinate the decisions. However, it might need to act as the information intermediary to verify and broadcast the quoted due dates and capacity arrangements across divisions.

5.1. Coordination via Transfer Payments

Suppose a transfer payment is established between the marketing and engineering divisions based on constant parameters and realized values of all cost components. Specifically, let T denote the transfer amount from marketing to engineering and define the parameters β_1, β_2 and β_3 such that

$$T = \beta_1\mathcal{D}([dd - cd]^+) + \beta_2\mathcal{T}([c - dd]^+) + \beta_3\mathcal{Z}([\theta - \theta_o]^+) \tag{7}$$

Clearly, the payment is proportional to due date extension cost, realized tardiness penalty, and capacity expansion cost. It should be noted that no sign restrictions are set for the coefficients. A negative value for a coefficient represents a payment in the reverse direction, that is, from engineering to marketing. The goal is to determine the set of contracts, (i.e., the *value ranges* for the coefficients in T) such that the Nash equilibrium solution coincides with the optimal solution.

In application, the introduced transfer payment can be designed as part of a revenue sharing scheme. Suppose the firm distributes a certain proportion of the revenue across divisions. Specifically, let $L_m - T$ and $L_e + T$ be the share of marketing and engineering divisions in revenues respectively where L_m and L_e are fixed fractions of the revenue (before the discount given to the customer as a result of due date extension). Since L_m and L_e are constants the analysis will not be affected by what proportion of the total revenue they constitute. While the final value of T is realized after job completion, L_m and L_e are known *a priori*.

With the transfer payments, expected cost functions of the players will be $T_m = H_m + E[T]$ and $T_e = H_e - E[T]$. In devising T, first, we determine the allotments in which dd^o satisfies marketing's optimality conditions for θ^o, and θ^o satisfies engineering's optimality conditions for dd^o. Afterwards, we need to identify the subset of these allotments that also satisfies the conditions for supermodularity. We write the first order conditions for the players' cost functions after the transfer payments. Then we compute the coefficient values with which the first order conditions are met at dd^o and θ^o. That is,

$$\begin{aligned}\frac{\partial T_m(dd^o, \theta^o)}{\partial dd} &= (1+\beta_1)\mathcal{D}^{(dd)}(dd^o - cd) + (\gamma + \beta_2)\int_{dd^o}^{\infty} \mathcal{T}^{(dd)}(x - dd^o) f(x|\theta^o)dx \\ &\quad + \beta_3\mathcal{Z}^{(dd)}(\theta^o - \theta_o) = 0\end{aligned} \tag{8}$$

$$\frac{\partial T_e(dd^o,\theta^o)}{\partial\theta} = -\beta_1\mathcal{D}^{(\theta)}(dd^o-cd)+(1-\gamma-\beta_2)\int_{dd^o}^{\infty}\mathcal{T}(x-dd^o)f^{(\theta)}(x|\theta^o)dx$$
$$+(1-\beta_3)\mathcal{Z}^{(\theta)}(\theta^o-\theta_o)=0 \tag{9}$$

Solving (2), (3), (7) and (8) for new cost coefficients yields the following equations:

$$(i) \qquad \beta_1=\beta_2-(1-\gamma) \tag{10}$$

$$(ii) \qquad \beta_3=\beta_2+\gamma \tag{11}$$

As a last step we need to ensure that the coordinating game, Ω^o, with the new cost functions is still supermodular and (dd^o,θ^o) incur global optimal solutions for both player cost functions. This is accomplished by imposing further restrictions on the value range of the foregoing coefficients as specified in the next theorem.

THEOREM 5. *Assuming equations (10) and (11) hold, there exists a pure strategy Nash equilibrium corresponding to the system optimal solution with the new cost settings if and only if* $-\gamma<\beta_2<1-\gamma$

Proof. In order to see that the coordinating game is supermodular it is sufficient to show that both $\partial^2T_m/(\partial dd\partial\theta)$ and $\partial^2T_e/(\partial dd\partial\theta)$ are positive. Observe that

$$\frac{\partial^2T_m}{\partial dd\partial\theta}=\frac{(\gamma+\beta_2)}{(1-\gamma-\beta_2)}\frac{\partial^2T_e}{\partial dd\partial\theta}=E^{(dd,\theta)}[\mathcal{T}([c-dd]^+)]$$

is positive from Theorem 3 if $-\gamma<\beta_2<1-\gamma$. Hence, the game is supermodular implying that there exists a pure strategy Nash equilibrium. Observe from (8)-(11) that

$$(\gamma+\beta_2)\left(\mathcal{D}^{(dd)}(dd^o-cd)+\int_{dd^o}^{\infty}\mathcal{T}^{(dd)}(x-dd^o)f(x|\theta^o)dx\right)=0$$

$$(1-\gamma-\beta_2)\left(\int_{dd^o}^{\infty}\mathcal{T}(x-dd^o)f^{(\theta)}(x|\theta^o)dx+\mathcal{Z}^{(\theta)}(\theta^o-\theta_o)\right)=0$$

indicating that the first order optimality conditions for marketing and engineering divisions, as well as the second order optimality conditions, are satisfied at (dd^o,θ^o). Consequently, since the pair (dd^o, θ^o) is optimal for G_o, they are optimal for both divisions. At this point, neither parties have the incentive to deviate, and thus it is an equilibrium. $\square$

Although the system optimal solution is now guaranteed to be an equilibrium point under the new setting, there might be other equilibria. Since supermodularity is preserved, the results of Lemmas 2 and 3 can be applied to Ω^o. In the case of multiple equilibria, there are maximum and minimum equilibrium points as in Ω. These equilibrium points can be computed using the iterative deletion of dominant strategies introduced in the previous section. However, under the new scheme, the maximum and minimum equilibrium points may not correspond to the players' financially best (or worst) outcomes; this is due to the envelop theorem, which does not lead us to the same conclusions as in Ω, i.e., Lemma 4 is not valid for the coordination contract. In fact, the best outcome for each division is given by the system optimal solution, which is not necessarily the highest or lowest equilibrium for any particular division. In essence, with the transfer payments, the two parties share all the costs incurred by their decisions, and they are responsible for a constant portion of the total system costs as specified by β_2. Therefore, no other equilibrium strategy can incur less costs for both parties, implying that (dd^o, θ^o) Pareto dominates the other equilibrium strategies. Although players do not necessarily select the Pareto dominant equilibrium (Huyck *et. al.* 1991), it has been empirically observed that they tend to do so if they are aware of such option before playing (Cachon and Camere 1996).

The main implication of the coordination contracts, as pointed out above, is that both parties share the overall cost. Since β_2 cannot be equal to $-\gamma$ or $1-\gamma$ (hence, $\beta_1 \neq 0$ or 1 and $\beta_4 \neq -1$ or 0) no cost entry is charged to only one division. Marketing shares the cost of capacity expansion with engineering whereas the latter becomes responsible for a certain proportion of the due date extension cost. Notice that the equations in (10) and (11) imply that $\beta_3 - \beta_1 = 1$. Subsequently it is straightforward to see that the proportions of the due date extension cost, tardiness penalty and the capacity expansion cost that are allocated to a division will be equal. Moreover, if $\beta_2 = 1/2 - \gamma$, cost for each entry is equally shared by the departments. Moreover, if $L_1 = L_2$ then the revenue is also shared evenly and as a result departments make the same profit at equilibrium. As β_2 increases, marketing's share in cost increases whereas the opposite occurs for increasing values of β_2.

As pointed out by Theorem 5, the coordination contracts stipulate direct payments between divisions and result in supermodular games with an equilibrium corresponding to the system optimal solution. An alternative interpretation and implementation of the payments can lead to the same (optimal) solution but a slightly different game. Suppose that the central management gives the engineering division the freedom of choosing its own due date. Thus, engineering becomes responsible for the due date, say dd_e, rather than what has been quoted by marketing, dd_{m} to the customer. However, engineering must pay a due date extension cost to marketing that is equal to the amount $\widehat{\beta}_1 D([dd_{\mathrm{e}} - cd]^+)$ where $\widehat{\beta}_1 = -\beta_1$. Additionally, even though engineering pays for tardiness with respect to dd_{e}, its share increases to $1-\gamma+\widehat{\beta}_2$ ($\widehat{\beta}_2 = -\beta_2$) which is directly paid to marketing. On the other hand, marketing buys capacity (expansion) from engineering by paying $\beta_3 \mathcal{Z}([\theta - \theta_o]^+)$. While the price is set by the central management, the amount to be purchased (expanded) is determined by engineering. All decisions are made simultaneously. At the end, the expected cost functions for marketing and engineering is as follows:

$$\widehat{H}_{\mathrm{m}} = \mathcal{D}([dd_{\mathrm{m}} - cd]^+) + \beta_1 D([dd_{\mathrm{e}} - cd]^+) + \int_{dd_{\mathrm{m}}}^{\infty} \mathcal{T}(x - dd_{\mathrm{m}}) f(x|\theta) dx$$
$$- (1-\gamma-\beta_2) \int_{dd_{\mathrm{e}}}^{\infty} \mathcal{T}(x - dd_{\mathrm{e}}) f(x|\theta) dx + \beta_3 \mathcal{Z}([\theta - \theta_o]^+)$$

$$\widehat{H}_{\mathrm{e}} = -\beta_1 D([dd_{\mathrm{e}} - cd]^+) + (1-\gamma-\beta_2) \int_{dd_{\mathrm{e}}}^{\infty} \mathcal{T}(x - dd_{\mathrm{e}}) f(x|\theta) dx + (1-\beta_3) \mathcal{Z}([\theta - \theta_o]^+)$$

First observe that engineering's due date decision is independent of dd_{m}. Moreover, assuming that equations (10-11) hold, it is straightforward to see that $\widehat{H}_{\mathrm{e}} = (1-\gamma-\beta_2)G_o$ (from (1)) implying that $dd_{\mathrm{e}} = dd^o$ and $\theta = \theta^o$ is a dominant strategy for engineering. Consequently, since dd_{m} becomes independent of dd_{e}, $dd_{\mathrm{m}} = dd^o$ is the best response of marketing to capacity level θ^o. Hence the equilibrium coincides with system optimality. Notice that if the optimal solution to G_o is unique then the equilibrium is also unique.

5.2. Additional Implementation Issues

Since our study focuses on ETO firms, capacity expansion is typically considered in the context of a particular project in the forms of outsourcing and/or temporary increase in capacity (overtime, subcontracts, temporary hiring etc.). In this context, each project and each customer is handled separately and independently. In a more generalized case, the capacity adjustment for a particular project may have an impact to capacity allocations for other existing projects, e.g., an increased expected tardiness penalty for existing projects. Assume the case where once set, the promised delivery date (or the associated due date expansion cost) cannot be renegotiated. Then, engineering capacity expansion cost function, $\mathcal{Z}$, has a component that represents the cost incurred due to shift in capacity allocation. Observe that even in this situation, as long as $\mathcal{Z}$ is continuous in θ, the above cost sharing mechanism could achieve the system optimal solution in equilibrium. However, since the incurred cost is now depending on the tardiness penalty for other projects, marketing might end up sharing this cost in the uncoordinated setting as well. Suppose marketing's share is determined by parameter $\overline{\gamma}$ where $0 \leq \overline{\gamma} \leq 1$ such that

$$H_{\mathrm{m}}(dd, \theta) = \mathcal{D}([dd - cd]^{+}) + \gamma E[\mathcal{T}([c - dd]^{+})] + \overline{\gamma}\mathcal{Z}([\theta - \theta_{o}]^{+})$$

$$H_{\mathrm{e}}(dd, \theta) = (1 - \overline{\gamma})\mathcal{Z}([\theta - \theta_{o}]^{+}) + (1 - \gamma)E[\mathcal{T}([c - dd]^{+})]$$

Then using the same approach presented earlier and from Theorem 5, one can easily see that the transfer payment where $\beta_1 = \beta_2 - (1 - \gamma)$ and $\beta_3 = \beta_2 + \gamma - \overline{\gamma}$ will coordinate the system if and only if $-\overline{\gamma} < \beta_3 < 1 - \overline{\gamma}$. Clearly, the continuity for $\mathcal{Z}$ is a strong assumption since the decisions regarding reallocation of capacity involve choosing how much capacity to shift from what project to the project under consideration; this brings combinatorics into the picture. Nonetheless, the cost sharing contract with transfer payments can still work. Consider the case where the system cost function is continuous in dd but not in θ, but there exists a unique optimal solution to the global problem. Note that with the transfer payments the cost functions of both divisions are components of the overall system cost, and their margins are aligned with the firm's. Thus, for the optimal capacity θ^{o} the marketing's best response after the transfer will be dd^{o} for which the best response of the engineering will be to choose θ^{o}, since there is a unique optimal solution for the system. Neither party would have incentives to deviate, implying an equilibrium point coincides with system optimality. Clearly, the analysis of Ω will become more complex.

Our proposed due-date coordination scheme can also be useful on a more aggregate level for production systems with steady state characteristics. Consider a job shop that serves customers with similar preferences and job requirements. The divisions need to decide on long-term capacity and lead- time policies which are uniform across customers. The sojourn time which consists of the waiting and service times in such environments can be approximated by an M/M/1 queuing system (Karmarkar, 1993 and So, 2000) with arrival rate of λ and service rate of μ. See also Palaka *et al.* (1998) and So and Song (1998) for similar approximations. In this case, the capacity expansion can be modeled simply by an increase in the service rate, *i.e.*, $\mu = \theta$. Note that the cdf of the sojourn time in the system is

$$F(x|\theta) = 1 - e^{-(\theta - \lambda)x} \tag{12}$$

and the first derivative with respect to θ is

$$F^{(\theta)}(x|\theta) = xe^{-(\theta - \lambda)x} > 0 \tag{13}$$

implying that $F(x|\theta)$ increases in θ. Clearly, this satisfies our assumption regarding the stochastic ordering. Consequently, if the assumptions 1-4 hold for costs related to lead-time extension, tardiness and service rate increase, our results for both Ω and $\overline{\Omega}$ are still valid.

6. A Case Study: Quadratic Cost Functions with Weibull(α,λ) Distribution for Flowtimes

To illustrate our approach and gain further insights, we investigate a case study where each cost component is modeled by a quadratic function. In this model, the due date extension cost, tardiness penalty and capacity expansion cost are defined as $g(dd-cd)^2$, $t(c-dd)^2$ and $v(\theta-\theta_o)^2$ respectively where g, t and v are constant coefficients. The quadratic cost function for due date extension and tardiness can be justified by the fact that in may situations the dissatisfaction of a customer due to late deliveries mounts with an increasing slope. The quadratic capacity cost function represents the increased margins in cost of capacity expansion.

We consider general Weibull distribution with a shape parameter α ($\alpha \geq 1$) and a scale parameter λ to model the flowtime distribution F. In practice, capturing the flowtime distribution may be difficult especially in complex production and/or service environments. Forecasting method may be employed and it can be as straightforward as calculating the mean and some higher moments based on the estimations provided by seasoned production managers, or schedulers. In such cases, the flowtimes can be fit to well-known distributions such as Normal, Lognormal, Erlang, Weibull, etc. We believe that employing Weibull function can provide insights for broader cases. First, it includes the Exponential and the Rayleigh distributions as special cases. Second, for the shape parameter in the neighborhood of 3.6, it is similar in shape to a Normal distribution, and with the shape parameter greater than that with some skewness value ranges it closely resembles Pearson Type VI and lognormal distributions (Johnson *et. al.* 1994). We model the capacity decision, θ as the inverse of the scale parameter so that

$$F(x|\theta) = 1 - e^{(-x\theta)^{\alpha}} \tag{14}$$

Clearly, the cdf is increasing in θ in this representation. Thus, higher values of θ imply higher capacity and our stochastic ordering assumption holds. Since all cost components are continuous functions of dd and θ, Assumptions 1-4 hold as well. Consequently we can conclude that equilibrium exists for both Ω and $\overline{\Omega}$.

LEMMA 5. *Assuming quadratic cost functions, Weibull flowtimes and* $-\gamma < \beta_2 < 1-\gamma$ *both* Ω *and* $\overline{\Omega}$ *are supermodular games and thus there exists at least one equilibrium for each. Moreover, the equilibrium is unique in both games if* $g/t \geq \frac{\alpha}{\Gamma(1/\alpha)}$ *and* $v/t \geq \frac{\alpha}{\Gamma(1/\alpha)}$.

Proof. See the appendix. □

Note that Γ is the Gamma function and for $\alpha \geq 1$, $\frac{\alpha}{\Gamma(1/\alpha)} \leq 1$ implying that the equilibrium is unique for relatively higher values of g and v with respect to the marginal tardiness penalty t. It should be underlined that this is a sufficient condition and does not imply multiple equilibria for other cases. Lemma 5 indicates that all the results of previous sections are valid under this setting, which will be further investigated using numerical results next.

6.1. Numerical Analysis

Since the capacity decision is modeled as a variate of the flowtime distribution, it is very difficult if not impossible to generate closed form solutions in equilibrium analysis for even distributions that are simple in structure. For this reason we cannot observe and measure the impact of certain system parameters on equilibrium behaviors and system efficiency analytically. We conduct a numerical analysis that employs quadratic cost functions and Weibull flowtime distribution to generate insights that we could not obtain from the theoretical analysis. For the numerical analysis, we consider two special cases of Weibull Belief Function which are namely, Exponential ($\alpha = 1$) and Rayleigh ($\alpha = 2$) distributions. Following parameter value ranges are employed for each case.

$$\begin{array}{ll} cd = \{0.2, 1\} & \mu_o = 1 \\ \gamma = \{0, 0.1, 0.3, 0.5, 0.7, 0.9, 1\} & t = \{0.25, 0.5, 1, 2\} \\ g = \{1, 2, 4, 8\} & v = \{1, 2, 4, 8\} \end{array}$$

where μ_o denotes the expected completion time of the project based on nominal capacity, θ_o observed *a priori*. For Weibull distribution our capacity model implies the following relation between μ and θ in general:

$$\mu = \frac{1}{\alpha\theta}\Gamma(1/\alpha)$$

Based on the foregoing equality, for $\alpha = 1$ and $\alpha = 2$, θ_o is 1 and 1.57 respectively. Notice that the coefficient of variation for a given shape parameter is constant and decreases in α in Weibull distributions. Therefore an increase in capacity parameter θ results in decrease in both mean and standard deviation. Specifically, in our examples note that the coefficient of variation for Exponential distribution ($\alpha = 1$) is almost four times larger than Raleigh Distribution ($\alpha = 2$). We consider two different values for cd. The smaller value represents the case with tight customer preferences (1/5 of the mean) while the larger one is for relatively loose due date cases (equal to mean).

Our numerical analysis is composed of 1762 problems that encompass all combinations of the foregoing parameter values. For each problem we compute the equilibrium decisions using the Iterated Deletion of Dominated Strategies Algorithm. We measure the gap between competitive and coordinated system via dividing the difference between the equilibrium outcome of the pre-coordination game and the system optimal solution by the system optimal solution. Table 1 gives the summary statistics for all solutions.

		$cd = 0.2$				$cd = 1$			
	γ	10th Percentile	Median	90th Percent.	Max	10th Percent.	Median	90th Percent.	Max
$\alpha = 1$	0	1.96%	7.39%	25.03%	43.59%	0.80%	3.11%	11.47%	21.14%
	0.1	1.79%	6.19%	18.58%	29.91%	0.76%	2.89%	9.39%	16.63%
	0.3	1.76%	4.50%	10.04%	14.02%	1.13%	2.70%	6.15%	9.39%
	0.5	2.87%	5.27%	7.79%	8.54%	2.25%	4.60%	6.88%	7.76%
	0.7	4.27%	9.14%	14.11%	16.03%	3.67%	9.05%	15.28%	17.91%
	0.9	7.17%	18.78%	37.23%	46.87%	6.32%	17.98%	40.04%	52.81%
	1	19.27%	27.83%	71.52%	110.13%	8.20%	25.81%	72.31%	116.19%
$\alpha = 2$	0	3.81%	14.15%	49.85%	89.67%	1.02%	2.65%	7.64%	8.81%
	0.1	2.80%	10.89%	34.63%	55.20%	0.61%	1.99%	7.32%	8.53%
	0.3	1.75%	6.18%	16.26%	22.22%	0.44%	1.21%	3.97%	6.47%
	0.5	1.04%	3.35%	7.24%	8.65%	0.20%	0.77%	2.44%	6.47%
	0.7	0.74%	2.23%	4.98%	7.29%	0.05%	0.44%	2.37%	4.76%
	0.9	0.49%	1.97%	7.36%	13.55%	0.02%	0.44%	1.25%	4.58%
	1	0.59%	2.35%	9.82%	18.78%	0.02%	0.33%	1.78%	3.40%

Table 1 Percentage gap between equilibrium outcome and system optimal

Our results indicate that the gap between the competitive and optimal solution is higher for lower values of α and cd suggesting that the degrading impact of competition on system efficiency is more prevalent for tight due dates and high uncertainty. These are the circumstances where a coordination mechanism is especially beneficial. It is clear from the results that the difference in deviations drops significantly for relatively loose delivery constraints for $\alpha = 2$, whereas the impact is not obvious for the exponential distribution case. We attribute this observation to the fact that exponential distribution has a constant failure rate while Raleigh is an increasing failure rate (IFR) distribution[1]. Basically in the latter case, a more relaxed delivery constraint will decrease the relative impact of the expected tardiness penalty on the overall costs

[1]For any IFR distribution, the ratio $f(x)/\overline{F}(x)$ (known as the *failure rate*) increases in x.

more significantly. Consequently, once the tardiness penalty becomes less relevant, there is no reason to expect a significant deviation from optimal solution since the division costs primarily differ from the integrated cost function over their margins on tardiness penalties in the pre-coordination game.

The relation between γ and the size of the gap is context specific. For the exponential case, the percentage deviation increases as the distance between γ and 1/2 grows. Moreover, it is more substantial for higher values of γ where the responsibility for the tardiness penalty is higher for marketing. Interestingly, the reverse occurs for the Raleigh distribution where the gap decreases in γ in general. We suspect that this phenomenon is also related to the level of uncertainty implied by the flowtime distribution. Marginal increase in capacity for the exponential case means a higher rate of decrease in variation with respect to the $\alpha = 2$ case. In this situation the system efficiency is more sensitive to efforts of engineering rather than marketing. Clearly for higher γ, engineering will have less incentives to provide the much needed efforts to improve capacity. The results suggest that the merits of coordination are more obvious when marketing carries the much of the burden related to tardiness under flowtimes with high variations that can be reduced by improvements in capacity. When the improvement in capacity do not contribute significant reductions to variation then efforts of marketing become relatively more weighty and thus the coordination is more needed if engineering has the higher responsibility on the tardiness in the decentralized environment.

The numerical results show that the gap between competitive and coordinated systems increases in t. Clearly, this implies that the system degradation will be more obvious as the relative weight of the tardiness penalty increases. Specifically, we have empirically deduced that, for a given γ, the gap decreases in general (not monotonically) in the value of the following function for all cases:

$$\Phi = \left((\gamma - 0.5)\left(\frac{\gamma}{g} - \frac{1-\gamma}{v}\right) - \gamma(1-\gamma)\right)t$$

Figure 1 illustrates this pattern. Notice that for $\gamma > 0.5$, Φ decreases in g and increases in v. The reverse is true for $\gamma < 0.5$. The underlying intuition is that for high values of γ and small values of g, the tardiness component becomes more influential on the decisions of marketing while its margin is not aligned with the firm's. Eventually the gap grows. Similar analogy applies to engineering under lower γ and v. In general the gap is more sensitive to these parameters for values of γ closer to 0 and 1.

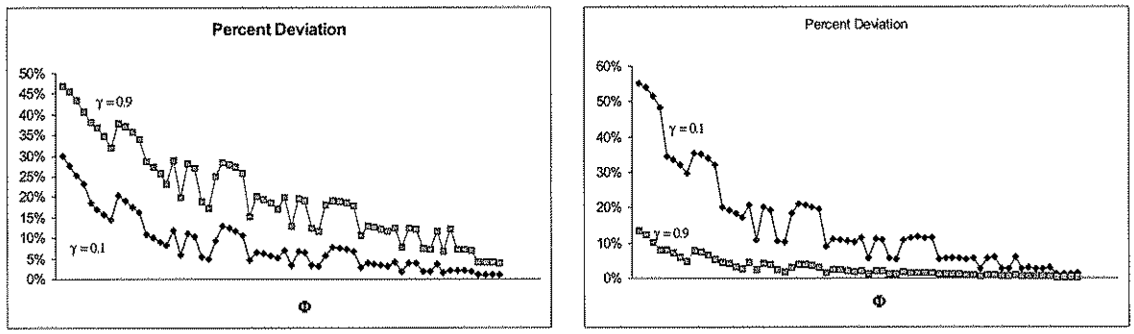

Figure 1.a. $\alpha = 1, cd = 0.2$ **Figure 1.b.** $\alpha = 2, cd = 0.2$

Figure 1. Change in the gap between equilibrium pre-coordinated and post-coordinated system costs

Since the expected division costs in pre-coordination game are positive and their total is always less than the optimal system cost, the firm can always devise a contract that is acceptable for both parties given that $0 < \gamma < 1$. Notice that for $\beta_2 = 1 - \gamma$ all system costs are directed to marketing whereas engineering pays for everything when $\beta_2 = \gamma$. Minimum (maximum) β_2 that marketing (engineering) voluntarily accepts depends on the cost parameters along with the division's share on the tardiness penalty. The relation between the minimum acceptable β_2 for marketing and Φ is illustrated in Figure 2 for both flowtime distributions and $\gamma = 0.1, 0.9$. Observe from Theorem 5 that for $\gamma = 0.1$, $-0.1 < \beta_2 < 0.9$ and for $\gamma = 0.9$, $-0.9 < \beta_2 < 0.1$. Since the marketing's share in system cost increases in β_2, a higher minimum acceptable β_2 implies that marketing will expect a lower share in system costs in the coordinating contract. Clearly higher γ will lead to higher β_2. The graph in Figure 2 indicates that it is relatively easier to motivate marketing for coordination in the exponential distribution case when γ is large. Reverse is true for $\gamma < 0.5$. We have observed the opposite for engineering and its maximum acceptable β_2 (Figure 3). This observation is consistent with data in Table 1 indicating that for large (small) values of γ, in the Exponential (Raleigh) distribution case, the pre-coordination game outcome deviates from the optimal significantly while much of the burden is bore by marketing (engineering).

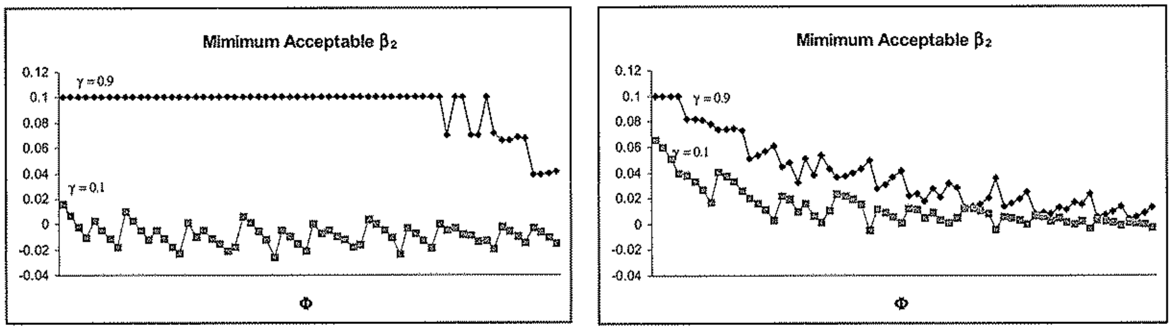

Figure 2.a. $\alpha = 1, cd = 0.2$ **Figure 2.b.** $\alpha = 2, cd = 0.2$

Figure 2. Minimum acceptable β_2 values for marketing

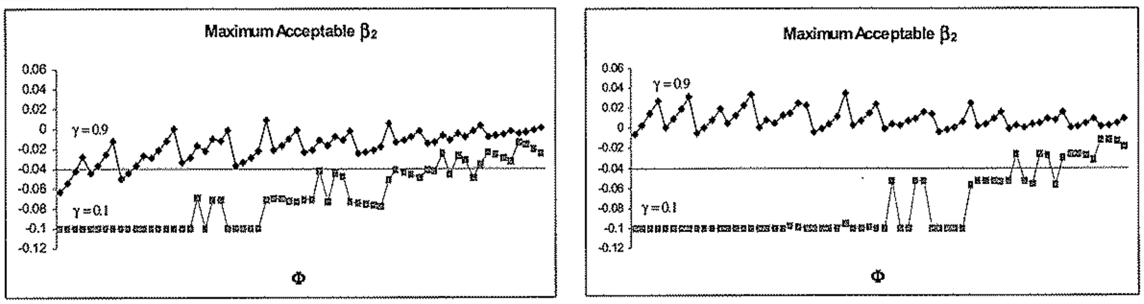

Figure 3.a. $\alpha = 1, cd = 0.2$ **Figure 3.b.** $\alpha = 2, cd = 0.2$

Figure 3. Maximum acceptable β_2 values for engineering

7. Conclusions

In this paper, we propose an incentive scheme to coordinate the due-date quotation decision in an internal market with players representing the marketing and engineering divisions of an engineering-to-order company. We consider divisional cost structure that satisfies a few mild assumptions. We first analyze the centralized model where the due-date quotation and the capacity utilization decisions are jointly given. We then investigate the decentralized case, in which the engineering and marketing divisions are considered as independent decision makers; marketing decides what due date to quote to the customer while engineering sets the capacity. Both parties are responsible for the late-delivery penalties. We model due-date quotation as a Nash game in which the players announce their own decisions simultaneously based on their local cost structures. We observe that the Nash equilibrium decisions never optimize the firm's problem due to externalities. We propose a set of coordination schemes regulating the allotment of the revenue that are composed of transfer payments based on cost elements within the system. By employing these transfer payments, it is possible to achieve the coordinated solution as the incentives to deviate from the system optimal are eliminated, and thus, the system optimal solution coincides with the Nash equilibrium.

In our analysis, the existence of Nash equilibrium is established due to supermodularity. Our analysis shows that supermodularity is guaranteed as long as the players' cost functions are continuous, and the capacity adjustment/flowtimes satisfy the *stochastic ordering* property. In order to establish the uniqueness of the equilibrium, a specific cost structure and flowtime distribution must be considered. We illustrate our approach by a case study using quadratic costs and Weibull-distributed flowtime. We deduce conditions for the existence of equilibrium and its uniqueness under this setting. We investigate the relation among various system parameters, and the level of degradation due to marketing-engineering competition. Our numerical analysis reveals that while the competition degrades the system efficiency, however, the extent of the efficiency loss is context specific.

The transfer payment that coordinates the due date and capacity decisions essentially defines a cost-sharing scheme. As the transfer payment is defined quite generally and it only need to satisfy rather mild conditions (Theorem 5), it allows for an infinite number of coordinating schemes between the divisions. We discuss extensions of our approach to more general settings to handle make-to-order systems with steady-state characteristics.

APPENDIX

The appendix include the proofs for all lemmas introduced in the paper. The description of the algorithm for *iterated deletion of dominated strategies* mentioned in Section 4.2 is presented at the end.

PROOF OF LEMMA 1. From integration by partition we can rewrite the expected tardiness cost in (1) as follows:

$$E[\mathcal{T}([c-dd]^+)] = \int_{dd}^{\infty} \mathcal{T}(x-dd)f(x|\theta)dx = \int_{dd}^{\infty} \frac{\partial \mathcal{T}(x-dd)}{\partial x}\overline{F}(x|\theta)dx$$

and thus,

$$\frac{\partial E[\mathcal{T}([c-dd]^+)]}{\partial \theta} = \int_{dd}^{\infty} \mathcal{T}^{(x)}(x-dd)\frac{\partial \overline{F}(x|\theta)}{\partial \theta}dx$$

From Assumption 1, we know that the tardiness cost increases in the amount of lateness. Since the lateness itself increases in the job completion time, from the *chain rule* in differentiation

$$\mathcal{T}^{(x)}(x-dd)=\frac{\partial y}{\partial x}\cdot\frac{\partial \mathcal{T}(y)}{\partial y}$$

where $y=x-dd$. From Assumption 2, $\partial\mathcal{T}(y)/\partial y>0$. Also observe that $\partial y/\partial x=1$ indicating that $\mathcal{T}^{(x)}(x-dd)>0$. Note that, the stochastic dominance rule requires that $\partial\overline{F}(x|\theta)/\partial\theta<0$. Hence, the right hand side in the foregoing equation is but an integral of all negative numbers and therefore it must return a negative value for all $dd>0$ implying that expected tardiness cost decreases in θ. □

PROOF OF LEMMA 2. First assume that Let (dd_1^q,θ_1^q) be the strategy pair at any equilibrium for Ω and (dd_2^q,θ_2^q) another strategy pair at another equilibrium for the same game. Without loss of generality suppose $dd_1^q>dd_2^q$. Then, since the cost functions are convex, we know that $r_{\mathrm{e}}^{(dd)}<0$ and $r_{\mathrm{m}}^{(\theta)}<0$ and thus $\theta_1^q<\theta_2^q$.

i) When $r_{\mathrm{e}}^{(dd)}>-1$, $dd_1^q-dd_2^q>\theta_2^q-\theta_1^q$. However, since $r_{\mathrm{m}}^{(\theta)}>-1$, $dd_1^q-dd_2^q<\theta_2^q-\theta_1^q$ must also be true. Clearly, this is a contradiction. Hence, the equilibrium must be unique.

ii) When $r_{\mathrm{e}}^{(dd)}<-1$, $dd_1^q-dd_2^q<\theta_2^q-\theta_1^q$. However, since $r_{\mathrm{m}}^{(\theta)}<-1$, $dd_1^q-dd_2^q>\theta_2^q-\theta_1^q$ must also be true. Clearly, this is a contradiction. Hence, the equilibrium must be unique. □

PROOF OF LEMMA 3. Suppose $dd_1^q>dd_2^q$ then $\theta_1^q>\theta_2^q$. We know from Theorem 3 that Ω is a supermodular game. In this case, the supermodularity implies that the following inequality must hold

$$H_{\mathrm{m}}(dd_1^q,\theta_1^q)+H_{\mathrm{m}}(dd_2^q,\theta_2^q)\geq H_{\mathrm{m}}(dd_1^q,\theta_2^q)+H_{\mathrm{m}}(dd_2^q,\theta_1^q)$$

To see that the foregoing equality cannot hold observe that dd_1^q and dd_2^q are best responses of marketing to θ_1^q and θ_2^q. Therefore $H_{\mathrm{m}}(dd_1^q,\theta_1^q)<H_{\mathrm{m}}(dd_1^q,\theta_2^q)$ and $H_{\mathrm{m}}(dd_2^q,\theta_2^q)<H_{\mathrm{m}}(dd_2^q,\theta_1^q)$ must hold. This contradicts with the foregoing inequality. Therefore, if $dd_1^q>dd_2^q$ then $\theta_1^q<\theta_2^q$. In this case, from supermodularity

$$H_{\mathrm{m}}(dd_1^q,\theta_2^q)+H_{\mathrm{m}}(dd_2^q,\theta_1^q)\geq H_{\mathrm{m}}(dd_1^q,\theta_1^q)+H_{\mathrm{m}}(dd_2^q,\theta_2^q)$$

which is indeed true because of the same reason explained above. □

PROOF OF LEMMA 4. Let H_{m}^{*} and H_{e}^{*} denote the optimal expected costs and dd^{*} and θ^{*} the optimal decisions for marketing and engineering given θ and dd respectively. From envelop theorem

$$\frac{\partial H_{\mathrm{m}}^{*}}{\partial \theta} = \int_{dd^{*}}^{\infty} T^{(x)}(x-dd^{*})\overline{F}^{(\theta)}(x|\theta)dx$$

$$\frac{\partial H_{\mathrm{m}}^{*}}{\partial \theta} = \int_{dd^{o}}^{\infty} T^{(dd)}(x-dd)f^{(\theta)}(x|\theta^{*})dx$$

We know from Theorem 1 that both foregoing functions return a negative value implying that optimal cost for a player decreases in the decision variable of the other player. Since in the highest equilibrium for a player the other player's decision is smallest, its expected cost must be highest. This completes the proof. □

PROOF OF LEMMA 5. To see that both games are supermodular, it is sufficient to establish that $H_{\mathrm{m}}^{(dd,\theta)}$, $H_{\mathrm{e}}^{(dd,\theta)}$,$T_{\mathrm{m}}^{(dd,\theta)}$ and $T_{\mathrm{e}}^{(dd,\theta)}$ are all positive. With the quadratic cost and Weibull distribution, in Ω

$$H_{\mathrm{m}}^{(dd,\theta)} = \frac{\gamma}{1-\gamma}H_{\mathrm{e}}^{(dd,\theta)} = 2\gamma t\int_{dd}^{\infty}\frac{x}{\theta}f(x|\theta)dx$$

Clearly for $\gamma \leq 1$, $H_{\mathrm{m}}^{(dd,\theta)}$ and $H_{\mathrm{e}}^{(dd,\theta)}$ are positive. In $\overline{\Omega}$

$$T_{\mathrm{m}}^{(dd,\theta)} = \frac{\gamma+\beta_2}{1-\gamma-\beta_2}T_{\mathrm{e}}^{(dd,\theta)} = 2(\gamma+\beta_2)t\int_{dd}^{\infty}\frac{x}{\theta}f(x|\theta)dx$$

The equalities indicate that for $\gamma \leq 1$ and $-\gamma < \beta_2 < 1-\gamma$, $T_{\mathrm{m}}^{(dd,\theta)}$ and $T_{\mathrm{e}}^{(dd,\theta)}$ are positive. Also observe that in Ω

$$r_{\mathrm{m}}^{(\theta)} = \frac{-\gamma t\int_{dd}^{\infty}\frac{x}{\theta}f(x|\theta)dx}{g+\gamma t\overline{F}(dd|\theta)} \text{ and } r_{\mathrm{e}}^{(dd)} = \frac{-(1-\gamma)t\int_{dd}^{\infty}\frac{x}{\theta}f(x|\theta)dx}{v+(1-\gamma)t\frac{1}{\theta^2}\int_{dd}^{\infty}(2x-dd)\left(\overline{F}(x|\theta)+xf(x|\theta)\right)dx}$$

First notice that both right hand sides return negative values. It is straightforward to see that

$$\int_{dd}^{\infty} \frac{x}{\theta} f(x|\theta)dx \leq \frac{\mu}{\theta} \text{ for } dd \geq 0.$$

From (14), $\mu/\theta = (1/\alpha)\Gamma(\,1/\alpha)$ implying that if both g/t and v/t are greater than or equal to $(1/\alpha)\Gamma(\,1/\alpha)$, $-1 < r_{\mathrm{m}}^{(\theta)} < 0$ and $-1 < r_{\mathrm{e}}^{(dd)} < 0$. Finally, from Lemma 2 we can conclude that equilibrium in Ω is unique. With straightforward analysis we can show that in $\overline{\Omega}$,

$$\overline{r}_{\mathrm{m}}^{(\theta)} = \frac{-(\gamma+\beta_2)t\int_{dd}^{\infty}\frac{x}{\theta}f(x|\theta)dx}{g+(\gamma+\beta_2)t\overline{F}(dd|\theta)} \text{ and}$$

$$\overline{r}_{\mathrm{e}}^{(dd)} = \frac{-(1-\gamma-\beta_2)t\int_{dd}^{\infty}\frac{x}{\theta}f(x|\theta)dx}{v+(1-\gamma-\beta_2)t\frac{1}{\theta^2}\int_{dd}^{\infty}(2x-dd)\big(\overline{F}(x|\theta)+xf(x|\theta)\big)dx}$$

Obviously for $-\gamma < \beta_2 < 1-\gamma$ both right hand sides return a value in $(-1,0)$. Hence equilibrium in $\overline{\Omega}$ must be unique as well. □

Iterated Deletion of Dominated Strategies:

We first present the process and then make the necessary proofs showing that it converges to highest and smallest equilibrium point(s).

Step 1. Start with $\widehat{dd}_1 = cd$ as the marketing strategy and compute the best response for engineering. Let $\widehat{\theta}_1$ denote the best response for engineering (EBR) to cd. Update the engineering strategy space as $(\theta_o, \widehat{\theta}_1)$ and delete all other possibilities. Next, compute $\widehat{dd}_2$ which is marketing's best response (MBR) to $\widehat{\theta}_1$. Update marketing strategy space with $(\widehat{dd}_1, M_{\mathrm{m}})$ by deleting all other possibilities. Continue with this fashion until the computations converge to an equilibrium point, say, (dd_1^q, θ_1^q). This is the smallest (highest) equilibrium for marketing (engineering). In other words, there can be no other equilibrium with smaller dd and higher θ. Consequently, at the end of this stage, the strategy spaces for marketing and engineering are (θ_o, θ_1^q) and (dd_1^q, M_{m}) respectively.

Step 2. Repeat the same procedure described in step 1 and delete dominated strategies, this time starting with $\theta = \theta_o$ and computing the MBR first. If, at any iteration, the current MBR is less than or equal to dd_1^q, or the current EBR is greater than or equal to θ_1^q go to Step 4. Otherwise, continue until the iteration converges to (dd_2^q, θ_2^q). This is the highest (smallest) equilibrium for marketing (engineering).

Step 3. If $(dd_1^q, \theta_1^q) = (dd_2^q, \theta_2^q)$ then go to Step 4. Otherwise, there are at least two equilibrium points and thus the equilibrium is not unique. STOP.

Step 4. The equilibrium, (dd_1^q, θ_1^q), is unique. STOP.

Next we show that Steps 1 and 2 converge to smallest and highest equilibria. First consider Step 1. Suppose we are at the nth iteration in Step 1. At this stage, $\widehat{\theta}_n$ is EBR to $\widehat{dd}_n$. Let $\widehat{dd}_{n+1}$ be the MBR to $\widehat{\theta}_n$. To prove that Step 1 converges to (dd_1^q, θ_1^q), it is sufficient to show that 1) $\widehat{dd}_{n+1} > \widehat{dd}_n$ and $\widehat{\theta}_{n+1} < \widehat{\theta}_n$ and, 2) there cannot exist an equilibrium, (dd^q, θ^q) where $\widehat{dd}_{n+1} > dd^q \geq \widehat{dd}_n$ and $\widehat{\theta}_{n+1} < \theta^q \leq \widehat{\theta}_n$ for all n.

First assume that $\widehat{dd}_{n-1} < \widehat{dd}_n$ and $\widehat{\theta}_{n-1} > \widehat{\theta}_n$. Suppose now $\widehat{dd}_{n+1} < \widehat{dd}_n$. Recall that $\widehat{dd}_{n+1}$ is the MBR to $\widehat{\theta}_n$. From supermodularity, the following inequality must hold:

$$H_{\mathrm{m}}(\widehat{dd}_n, \widehat{\theta}_{n-1}) + H_{\mathrm{m}}(\widehat{dd}_{n+1}, \widehat{\theta}_n) \geq H_{\mathrm{m}}(\widehat{dd}_n, \widehat{\theta}_n) + H_{\mathrm{m}}(\widehat{dd}_{n+1}, \widehat{\theta}_{n-1})$$

However since $\widehat{dd}_n$ and $\widehat{dd}_{n+1}$ are MBR to $\widehat{\theta}_{n-1}$ and $\widehat{\theta}_n$ respectively, the foregoing inequality cannot be true implying that $\widehat{dd}_{n+1}$ must be greater than $\widehat{dd}_n$. Next suppose that $\widehat{\theta}_{n+1} > \widehat{\theta}_n$. Supermodularity requires that

$$H_{\mathrm{e}}(\widehat{dd}_n, \widehat{\theta}_n) + H_{\mathrm{e}}(\widehat{dd}_{n+1}, \widehat{\theta}_{n+1}) \geq H_{\mathrm{e}}(\widehat{dd}_n, \widehat{\theta}_{n+1}) + H_{\mathrm{e}}(\widehat{dd}_{n+1}, \widehat{\theta}_n)$$

Since $\widehat{\theta}_n$ and $\widehat{\theta}_{n+1}$ are EBR to $\widehat{dd}_n$ and $\widehat{dd}_{n+1}$, the foregoing inequality cannot hold either implying that $\widehat{\theta}_{n+1} < \widehat{\theta}_n$. Consequently, we conclude that if $\widehat{dd}_{n-1} < \widehat{dd}_n$ and $\widehat{\theta}_{n-1} > \widehat{\theta}_n$ then $\widehat{dd}_n < \widehat{dd}_{n+1}$ and $\widehat{\theta}_n > \widehat{\theta}_{n+1}$. Observe that $\widehat{dd}_1 = cd$ and from Theorem 2, $\widehat{\theta}_1 > \theta_o$. Once again, from Theorem 2, $\widehat{dd}_2 > \widehat{dd}_1$ since $\widehat{dd}_1 = cd$. From supermodularity we can conclude that $\widehat{\theta}_2$, which is the EBR to $\widehat{dd}_2$, is less than $\widehat{\theta}_1$. Finally, since $\widehat{dd}_1 < \widehat{dd}_2$ and $\widehat{\theta}_1 > \widehat{\theta}_2$, the conditions given in part 1 must be true for all following iterations, *i.e.*, $\widehat{dd}_{n+1} > \widehat{dd}_n$ and $\widehat{\theta}_{n+1} < \widehat{\theta}_n$ for all n.

To complete the proof for the second part, first observe that there cannot be an equilibrium capacity level that is greater than $\widehat{\theta}_1$ since the following cannot be true for $cd < dd^q$

$$H_{\mathrm{e}}(cd, \widehat{\theta}_1) + H_{\mathrm{e}}(dd^q, \theta^q) \geq H_{\mathrm{e}}(cd, \theta^q) + H_{\mathrm{e}}(dd^q, \widehat{\theta}_1)$$

Subsequently a property of supermodularity is violated. Then, from the results of part 1 and supermodularity, dd^q cannot be smaller than $\widehat{dd}_2$ which implies that θ^q cannot be greater than $\widehat{\theta}_2$ as well. Continuing in this fashion, we conclude that there cannot exist any equilibrium point

(dd^q, θ^q) where $dd^q < dd_1^q$ and $\theta^q > \theta_1^q$. The proof for Step 2 can be carried out in similar fashion. □

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