Designing and Pricing Menus of Extended Warranty Contracts

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Abstract

Extended warranties provide "piece of mind" to a consumer in that product failures which occur after the base warranty expires are rectified at little or no cost. They also provide an additional source of revenue for manufacturers or third party providers, such as retailers or insurance providers. In this paper, we analyze a number of extended warranty contracts which differ in design, including restrictions on deferrals and renewals. With the use of dynamic programming, we compute the optimal strategy for a consumer with perfect information and determine the optimal pricing policy for the provider given the consumer's risk characterization. We also provide insight into when different contracts should be issued. Finally, we illustrate how profits can be dramatically increased by offering menus of warranty contracts, as opposed to stand alone contracts, with the use of integer programming. Surprisingly, risk-taking consumers provide the greatest benefit to offering menus. These insights can help a company develop a comprehensive warranty planning strategy for given products or product lines.

1 Introduction

The purchase of a product is generally accompanied by a (base) warranty. This warranty usually covers all repair costs until its expiration. An extended warranty (EW) offers the opportunity for a consumer to extend coverage after the base warranty (BW) expires. This EW is obtained at some cost either at the point of sale or some time later, such as when the BW expires. The EW can be an additional source of revenue for a manufacturer or the sole source of revenue for a warranty provider, such as a retailer or insurance provider. For the manufacturer, the warranty also provides an opportunity to maintain the relationship with the consumer. For the consumer, an EW eases concerns about product failures after the expiration of the base warranty. Clearly, a consumer weigh the benefit of an EW against its cost. Similarly, designing and pricing an EW can greatly impact a company's profitability and help define its consumer base. Thus, it is important that a company consider these issues during the strategic planning stage of new product development.

One difficulty with analyzing EWs is that they are so diverse. That is, companies can offer a "menu" of EWs to the public. For example, the time at which a warranty is offered; the length of warranty coverage; the number of times a warranty can be renewed; and the maximum age at which a warranty can be purchased are parameters which can be varied. Furthermore, the logistics of how a warranty contract is executed – such as whether the consumer must bring the product to a repair shop or whether service is provided in home – can vary tremendously. This makes comparing different warranty contracts and determining optimal consumer policies challenging. Adding to this difficulty is the fact that newer contracts are seemingly always being engineered, such as refundable policies.

There is an extensive amount of literature in the field of warranty analysis, including the analysis of extended warranties. (See Blischke and Murthy [3] and the Office of Fair Trading [19] survey for a general overview of research in extended warranty analysis.) Much of this work is concerned with the marketing aspects of EWs (see, for example, Bryant and Gerner [5], Day and Fox [8], Fox and Day [9], and Padmanabhan and Rao [21]).

In this paper, we examine the consumer's optimal replacement and extended warranty purchase policy, given his or her risk characteristics, in addition to the optimal price and warranty design that should be offered by the manufacturer or third party provider for a given consumer. This information is then utilized to design warranty menus for a population of consumers. Thus, our research touches on a number of topics addressed individually in the literature, including policies, pricing, and menus.

Mitra and Patankar [16] investigate two renewable warranty programs (linear pro-rata and lump sum payouts) and consider the effect of each program on market share and warranty cost. Patankar and Mitra [22] develop a multicriterion model for a renewable warranty where the customer has the option to renew the warranty in the case that the product did not fail during the base warranty period. Yadav et al. [25] implement service contracts which share savings between the warranty provider and its customers using historical data and simulation to identify the best warranty contracts. Lam and Lam [14] obtain optimal consumer and provider policies for an extended warranty in which the customer has the choice to renew (k-renewal policy) or not to renew (r-repair policy) at the end of the base warranty.

Chen and Ross [6] explain that extended warranties tend to be expensive because the provider wants to recover base warranty period coverage costs from intense users. Chun and Tang [7] consider the optimal price for a free-replacement warranty given a warranty period based on producer and customer risk preferences. Brooks and White [4] study pricing for the option to delay the purchase of an EW noting that asymmetric information and the time value of money are critical factors. They propose an option pricing model such that the consumer's risk preference need not be known. Jack and Murthy [13] simultaneously price a product and extended warranty coverage at the time of purchase.

Padmanabhan [20] shows that menus of extended warranties can be utilized to satisfy consumers with different usage patterns. Lutz and Padmanabhan [15] analyze offering different warranties to different consumers depending on how they value products. Partial warranties (low quality) are offered to the low valuation customer and full warranties (high quality) are offered to the high valuation customer. Hollis [11] investigates how third party insurer competition may lead to further differentiation in warranty offerings.

In this paper, we make four contributions to the vast literature in extended warranty analysis. First, in Section 2, we model and analyze a consumer's optimal policy – whether to take out extended warranties over time or not – for a number of warranty designs using dynamic programming. In this analysis, we define consumers according to their risk characteristics in that risk-averse consumers are more likely to take out a warranty and will, in general, pay more for coverage when compared to risk-taking consumers. (This work greatly extends ideas first posed in Jack et al. [12].) We review the literature which addresses a consumer's risk tolerance in extended warranty analysis in this section. Second, in Section 3, we extend this analysis to determine the optimal pricing of warranties by the manufacturer, or a third-party provider, using a game theoretic approach by repeatedly solving the proposed dynamic programs. Third, in Section 4.2, we design and price a menu of warranty contracts in order to address a population of consumers with heterogeneous risk characteristics with the use of integer programming. The dynamic programming solutions defined for consumer and pricing policies are used as input to that problem. Fourth, in Section 4.3, we gain insight into a warranty provider's strategies (designs and prices) with respect to different consumer populations by performing extensive sensitivity analysis on risk preference distributions, product failure rates, repair costs, and EW prices. We also offer suggestions for future research in Section 5.

2 Individual Consumer Policies for Varying EWs

The extended warranty policies provided on the market today differ greatly. For example, they may differ in a variety of conditions and/or costs. We propose dynamic programming (DP) to analyze the problem of determining an optimal policy for a given class of warranties. DP is a versatile method that is ideal for solving sequential or multi-stage decision problems. We are concerned with modeling a consumer's behavior in periodically choosing between keeping or replacing an asset and if a consumer decides to keep the product, whether an EW should be purchased or not.

2.1 General Assumptions and Notation

We make the following assumptions for all models:

1. All prices and costs are assumed to be fixed over the horizon. This includes repair costs, purchase costs, and warranty costs. This assumption is made for the sake of deriving optimal policies. Relaxing this assumption does not prohibit solution of any of the presented models.

2. A base or extended warranty covers the cost of all repairs, including any logistical costs involved.

3. Repair costs are the same regardless of the age of the asset.

4. Repairs occur instantaneously and are minimal such that the product is returned to the same condition as when the failure occurred.

5. All costs, including extended warranty and repair, are non-negative.

While we provide a number of dynamic programming formulations, the following notation is utilized in all formulations. An asset is tracked according to its age n with its maximum allowable age N. That is, a product must be replaced when it reaches age N, if not sooner. The amount of warranty coverage remaining is w with W representing the number of periods of coverage in the base warranty and W_i defining the number of periods of coverage for extended warranty i.

We assume fixed costs for purchasing or repairing a product, defined as C_p and C_r, respectively. The cost to purchase an extended warranty, C_e, is also fixed. The periodic discount factor is α.

The expected number of failures in the following period for an n-period old product is defined as $M(n, n+1)$. For example, if the product's failure rate is defined by a Weibull distribution, then under the minimal repair assumption, the sequence of item failures follows a non-homogeneous Poisson process and the expected number of failures $M(n, n+1)$ is defined as:

$$M(n, n+1) = \left(\frac{n+1}{\theta}\right)^{\beta} - \left(\frac{n}{\theta}\right)^{\beta}, \tag{2.1}$$

where parameters of θ and β are defined by the Weibull distribution. Thus, for an asset retained from age n to $n+1$, the expected repair costs are $C_r(M(n, n+1))$.

In defining our optimal consumer strategies, we minimize expected, discounted costs. However, minimizing expected costs only captures the tendencies of risk-neutral consumers. It should be clear that risk-averse or risk-taking consumers will have greatly different tendencies.

There are numerous ways in which to capture the risk characteristics of consumers in our analysis, most notably through the use of utility functions. Chun and Tang [7] use an exponential utility function to model risk aversion when maximizing profits for the producers, $U(Y) = -e^{-aY}$, and customers $U(Y) = -e^{bY}$, a and $b \geq 0$, where a and b are the risk parameters representing the producer's and consumer's risk preference, respectively, and Y is the monetary asset. Murthy and Asgharizadeh [17] use a utility function associated with wealth ω, $U(\omega) = (1 - e^{\beta\omega})/\beta$, where $\beta = 0$ models a risk-neutral consumer and an increasing β models increasing risk-aversion. They maximize the utility function in order to obtain the optimal consumer policy in a maintenance service operation. Lutz and Padmanabhan [15] assume that the utility function of a risk-averse consumer is increasing and concave. The utility function is defined by the product price, the product quality, the monetary value of a working product, and the product warranty. Hollis [11] utilizes a similar utility function. The consumer makes a decision that maximizes his or her expected utility. Ritchken and Tapiero [24] approximate a risk-averse utility function with a quadratic function. Given a set of warranty policies, the policy is chosen that minimizes the consumer's expected disutility function. Padmanabhan and Rao [21] maximize an expected utility function $U_i(x) = x^{\delta_i}$, $0 < \delta \leq 1$ where x is consumer wealth and $\delta < 1$ defines risk-averse and $\delta = 1$ defines risk-neutral consumers. Baker [1] and Jack and Murthy [13] use a disutility function, $U(x) = \frac{e^{\eta x}-1}{\eta}, \eta \geq 0$, where x is an expenditure to capture the consumer's risk attitude. The level of risk-aversion increases as η increases, and $\eta = 0$ corresponds to the risk-neutral consumer.

We assume that risk-averse consumers are more willing to purchase extended warranties, and often for higher prices, when compared to risk-neutral or risk-taking consumers. Conversely, risk-taking consumers will generally not pay for warranty coverage. To capture these tendencies, we re-define our expected cost of repair as a risk-adjusted expected cost of repair $C_r(M'(n, n+1))$, where:

$$M'(n, n+1) = \sum_{j=0}^{\infty} P_n(j) * j^{\gamma}, \tag{2.2}$$

and $P_n(j)$ is the probability of j failures for an asset used between age n and $n+1$. The parameter γ allows for different definitions of risk aversion, such that $\gamma < 1$ is risk-taking, $\gamma > 1$ is risk-averse, and $\gamma = 1$ returns to the traditional definition of expected value for risk-neutral. This follows the concept of Baker [1] in modeling disutility with higher expected costs when assuming higher levels of risk aversion.

Note that numerically, we can evaluate $M'(n, n+1)$ as

$$M'(n, n+1) = \sum_{j=0}^{j|P_n(j)<\epsilon} P_n(j) * j^{\gamma}, \tag{2.3}$$

where ϵ is defined as some value approaching zero (i.e., 0.0001).

Figure 1 illustrates the value of $M'(n, n+1)$ over the life of an asset for different risk parameters γ. The curves are representative of other utility functions in the literature, in that the functions are increasing in age, but the slopes are decreasing. Furthermore, $\gamma > 1.0$ results in higher than expected costs, while $\gamma < 1.0$ yields lower costs. As will be seen later, this results in predictable behavior for consumers with varying risk tolerances.

Given these assumptions and notation, we consider four different extended warranty contracts in the following sections. These include unrestricted, nondeferrable, nonrenewable, and both nondeferrable and nonrenewable contracts, which are examined from the perspective of both the consumer and provider.

2.2 Unrestricted Warranty

After the base warranty expires, we assume that the warranty provider offers an extended warranty to the consumer, providing free repairs for a specified period of time. The consumer can purchase the extended

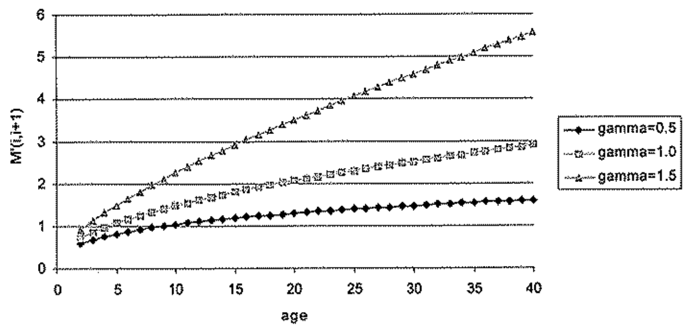

Figure 1: Risk adjusted expected number of failures according to the age of the product.

warranty at any time over the horizon, at the same price, and the warranty provider is responsible for repairing the product at all times during the warranty period.

This case is described by the network in Figure 2. A node, labeled (n, w), represents the state of the system, defined as the age of the asset n and remaining periods of warranty coverage w. An arc represents the consumer's feasible decisions for each system state. As shown in the network, when a warranty is active, $w > 0$, the consumer has no decision to make. When a warranty has expired, $w = 0$, the consumer may keep or replace the product. If the product is kept, the consumer must choose whether to purchase an extended warranty or not. (The figure labels decisions as EW (keep product and purchase an extended warranty), K (keep product without warranty), and R (replace product).)

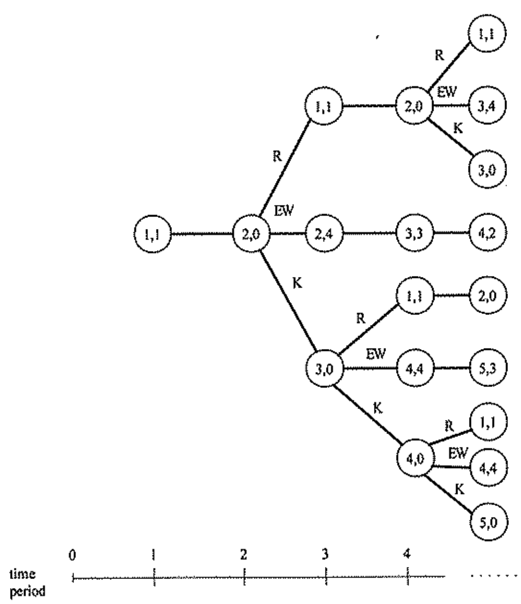

Figure 2: Network of decisions for unrestricted warranty assuming a base warranty of two periods and extended warranty of five periods, available at any period.

Formally, define the state of the system as (n, w) where n is the age of the asset and w is the remaining amount of warranty coverage. Further define $v_t(n, w)$ as the cost-to-go function, or the minimum discounted, expected cost given an asset in state (n, w) and making optimal decisions in time t through the horizon T. Assuming only one warranty is available, defined by length W_1 (W is the length of the base warranty), the value of $v_t(n, w)$ is computed as:

$$v_t(n,w) = \alpha v_{t+1}(n+1, w-1), \; n \geq 0, \quad 1 \leq w \leq \max(W, W_1) \tag{2.4}$$

$$v_t(n,0) = \min \left\{ \begin{array}{l} EW : C_e + \alpha v_{t+1}(n+1, W_1 - 1) \\ K : C_r M'(n, n+1) + \alpha v_{t+1}(n+1, 0) \\ R : C_p + \alpha v_{t+1}(1, W-1) \end{array} \right\}, \quad n < N \tag{2.5}$$

$$v_t(N,w) = C_p + \alpha v_{t+1}(1, W-1), \quad 0 \leq w \leq \max(W, W_1) \tag{2.6}$$

$$v_T(n,w) = 0, \quad \forall n, w \tag{2.7}$$

If the warranty is active, such that $w > 0$, then Equation (2.4) merely defines the transition from state (n, w) to $(n+1, w-1)$ as there are no costs incurred by the consumer and the asset ages one period. Equation (2.6) defines that the product must be replaced if it reaches its maximum age N. Note that we assume no salvage value of a replaced product, although this may be easily incorporated.

The interesting case is when the product has not reached its maximum age and the warranty expires, defined by Equation (2.5). In this case, the consumer has three choices: keep the product and purchase an extended warranty (EW); keep the product without the warranty (K); or replace the product with a new one (R). If EW is chosen, the cost of the warranty is paid and no further costs are incurred over the W_1 periods. If the product is kept without a warranty, the expected repair costs are paid and the same set of decisions are faced in the following period. (Recall that expected repair costs are risk-adjusted.) Finally, if the product is replaced, a purchase cost is paid and the base warranty period begins, providing coverage for W periods. The minimum of these decisions is chosen according to Equation (2.5).

For the infinite horizon case with fixed costs, we solve the recursion using a value iteration algorithm (Putermann [23]) using Equation (2.7) for the initialization. For the finite horizon case, we can solve using non-stationary costs if desired. The terminal condition for the finite horizon problem is also given in Equation (2.7).

As n can take on values of 1,2,...,N and w can take on $0, 1, ..., W_{\max}$, where $W_{\max} = \max(W, W_1)$, the maximum number of states in any time period is $N(W_{\max} + 1)$. At most three decisions are evaluated for a given node. Thus, over T periods of study, the DP can be solved in $O(TN(W_{\max} + 1))$ time.

Example 1: Unrestricted Warranty

Consider a \$500 product with a base warranty of 2 years and a maximum useful life of 20 years. The product's failure rate is defined by a Weibull distribution with mean 2 ($\mu = \theta\Gamma\left[1 + (1/\beta)\right]$ with $\beta = 1.5$ and $\theta = 2.215$). Under the minimal repair assumption, the sequence of item failures follows a non-homogeneous Poisson process as given in Equation (2.1) which is modified according to risk levels as in Equation (2.2). We vary the extended warranty price from \$50 to \$250 and the repair cost for the consumer from \$50 to \$100. The extended warranty provides two years of coverage. A periodic discount factor of 0.90, which translates to an interest rate of about 11.1% per period, is assumed with annual periods. Table 1 illustrates the different strategies for three levels of risk according to Equation (2.3) with $\gamma = 0.5, 1.0$, and 1.5. Recall that a value of 0.5 represents a risk-taking consumer, 1.0 is risk-neutral, and 1.5 is risk-averse. A strategy is defined by the triple (x, y, z) where x is the first period in which an extended warranty is purchased, y is the last period in which an extended warranty is purchased, and z is the age at which the product is replaced. Note that an extended warranty is purchased in all periods between x and y. If x and y are defined as "–," no extended warranty is purchased.

Table 1: Optimal strategy for unrestricted warranty case.

C_r:	$50			$75			$100		
C_e	γ=0.5	γ=1.0	γ=1.5	γ=0.5	γ=1.0	γ=1.5	γ=0.5	γ=1.0	γ=1.5
$50	2,18,20	2,18,20	2,18,20	2,18,20	2,18,20	2,18,20	2,18,20	2,18,20	2,18,20
$75	6,18,20	4,18,20	2,18,20	2,18,20	2,18,20	2,18,20	2,18,20	2,18,20	2,18,20
$100	12,18,20	6,18,20	2,18,20	4,18,20	2,18,20	2,18,20	4,18,20	2,18,20	2,18,20
$125	–,–,20	8,18,20	4,18,20	6,18,20	4,18,20	2,18,20	6,18,20	2,18,20	2,18,20
$150	–,–,20	12,18,20	6,18,20	12,18,20	6,18,20	4,18,20	8,18,20	2,18,20	2,18,20
$175	–,–,20	16,18,20	8,18,20	18,18,20	8,18,20	4,18,20	12,18,20	4,18,20	2,18,20
$200	–,–,20	–,–,20	10,18,20	–,–,20	10,18,20	6,18,20	16,18,20	6,18,20	3,17,19
$225	–,–,20	–,–,20	12,18,20	–,–,20	12,18,20	6,18,20	–,–,20	6,18,20	4,18,20
$250	–,–,20	–,–,20	–,–,13	–,–,20	–,–,15	8,18,20	–,–,20	8,18,20	4,18,20
$275	–,–,20	–,–,20	–,–,13	–,–,20	–,–,15	–,–,13	–,–,20	10,18,20	5,17,19
$300	–,–,20	–,–,20	–,–,13	–,–,20	–,–,15	–,–,13	–,–,20	–,–,11	6,18,20
$325	–,–,20	–,–,20	–,–,13	–,–,20	–,–,15	–,–,13	–,–,20	–,–,11	7,17,19
$350	–,–,20	–,–,20	–,–,13	–,–,20	–,–,15	–,–,13	–,–,20	–,–,11	–,–,7

Consider the case where C_r = $50. In this case, the risk-taking consumer only purchases an extended warranty at costs of $50, $75, and $100. For the $100 warranty, the first extended warranty is not purchased until age 12, while it is purchased at age 2 for the $50 price. These policies contrast greatly with the risk-neutral and risk-averse consumer. The risk-neutral consumer purchases warranties up to a price of $175, delaying the purchase of the first EW with the increase in price. The risk-averse consumer is willing to spend $225 for a warranty and for higher warranty costs, will replace the product sooner (age 13). Similar trends can be seen when the repair costs increase.

The trends from the previous example are not unexpected, as one would expect a consumer with a lower risk tolerance to (1) pay more for a warranty; (2) to purchase a warranty sooner; and (3) to keep the warranty longer (or replace the product sooner). It is also expected that extended warranties are purchased and replacements are earlier as the cost of repair increases. These characteristics are important to note when designing and pricing warranty contracts.

When examining the solutions, all consumers, regardless of risk tolerance, follow a similar strategy: if an extended warranty is purchased, it is renewed until the product is replaced. We formalize this in the following theorem.

Theorem 1 *If purchasing an EW is optimal in state $(n,0)$, $M'(n,n+1) \leq \alpha^m M'(n+m,n+m+1), \forall n$ and m, and $0 < \alpha \leq 1$, then an EW is purchased or the asset is replaced in any state $(n',0)$, $n' > n$.*

Proof: If purchasing an EW in state $(n,0)$ is optimal, then it is cheaper than the other options of repair:

$$C_e + \alpha v_{t+1}(n+1, W_1 - 1) \leq C_r M'(n, n+1) + \alpha v_{t+1}(n+1, 0), \tag{2.8}$$

replacement:

$$C_e + \alpha v_{t+1}(n+1, W_1 - 1) \leq C_p + \alpha v_{t+1}(1, W-1), \tag{2.9}$$

and delaying the purchase of an EW:

$$C_e + \alpha v_{t+1}(n+1, W_1 - 1) \leq C_r M'(n, n+1) + \alpha C_e + \alpha^{W_1+1} v_{t+W_1+1}(n + W_1 + 1, 0). \tag{2.10}$$

Note that $C_e + \alpha v_{t+1}(n+1, W_1 - 1)$ is defined as:

$$C_e + \alpha v_{t+1}(n+1, W_1 - 1) = C_e + \min \begin{cases} \alpha^{W_1}\left[C_r M'(n+W_1, n+W_1+1) + \alpha v_{t+W_1+1}(n+W_1+1, 0)\right] \\ \alpha^{W_1}\left[C_e + \alpha v_{t+2W_1}(n+2W_1, 0)\right] \\ \alpha^{W_1}\left[C_p + \alpha v_{t+W_1+1}(1, W-1)\right] \end{cases}$$

when considering the options after the EW expires at time $t + W_1$.

Consider the first of these, which is to keep the asset without an EW. Compared to Equation (2.10):

$$C_e + \alpha^{W_1} C_r M'(n+W_1, n+W_1+1) + \alpha^{W_1+1} v_{t+W_1+1}(n+W_1+1, 0) < C_r M'(n, n+1) + \alpha C_e + \alpha^{W_1+1} v_{t+W_1+1}(n+W_1+1, 0)$$

which reduces to:

$$C_e(1-\alpha) < C_r\left(M'(n, n+1) - \alpha_1^W M'(n+W_1, n+W_1+1)\right)$$

As the left hand side of the equation is positive and the right hand side is negative, this is never true. Thus, the optimal decision at time $n + W_1$ cannot be to keep the asset without a warranty. Rather, the optimal decision must be to purchase another EW or replace the asset, proving the theorem. Note that this policy is optimal for any values of W and W_1.

Related to the Theorem 1, it should be clear that a consumer that purchases an extended warranty will retain the product at least as long (and probably longer) than the consumer that does not purchase an extended warranty. We show this in the following corollary.

Corollary 1.1 *The optimal replacement age N' for a consumer that purchases an extended warranty is greater than or equal to the optimal replacement age N^* when no extended warranty is offered to the same consumer.*

Proof: Given the optimal replacement age N^*, for any age $n < N^*$ after the base warranty expires, it is clear that the option to keep the product is cheaper than replacement, or:

$$C_r M'(n, n+1) + \alpha v_{t+1}(n+1, 0) \leq C_p + \alpha v_{t+1}(1, W-1), \tag{2.11}$$

If an EW is offered to the consumer and they purchase it, this implies that:

$$C_e + \alpha v_{t+1}(n+1, W_1 - 1) \leq C_r M'(n, n+1) + \alpha v_{t+1}(n+1, 0), \tag{2.12}$$

Considering Equations (2.11) and (2.12), we obtain:

$$C_e + \alpha v_{t+1}(n+1, W_1 - 1) \leq C_p + \alpha v_{t+1}(1, W-1), \tag{2.13}$$

It should be clear from Equation (2.13) that $N' \geq N^*$ as the extended warranty option cost is always less than or equal to the replacement cost if the option to keep the product without an EW is also less than the option to replace the product. Thus, if it is cheaper to keep the product when no warranty is offered, it is also cheaper to keep the product when an EW is available.

These consumer tendencies clearly have implications on the design of warranty contracts, because the liability for the provider increases with the age of the product (as the failure rate is generally increasing in the age of the product). That is, while the extended warranties may be beneficial to the consumer and allow the consumer to retain the product longer, the consumer's purchase policy may not be profitable to the provider. Thus, we examine a number of different designs, and eventually examine pricing, in the following sections.

2.3 Nondeferrable Warranty

In the unrestricted warranty analysis, many of the consumers delayed purchasing the extended warranty until the product was older, as this was when costs were expected to rise. A company may curb this practice by allowing a consumer to only defer the purchase of a warranty for a certain number of periods and if the consumer does not purchase the extended warranty within the deferrable period, the chance to extend the warranty coverage is no longer available.

This case can be described by the network in Figure 3. As before, a node represents the state of the system (n, w), defined as the age of the asset n and remaining warranty coverage w. The parameter F defines the number of periods in which a consumer can defer purchasing an extended warranty. This is tracked using w, which now ranges from $W, W-1, \ldots, 0, \ldots, -F, -F-1$ in that if F periods pass in which no extended warranty is purchased, it is no longer an option. Thus, only two arcs emanate from a node $(n, -F-1)$, relating to the options to replace the product or keep it without a warranty.

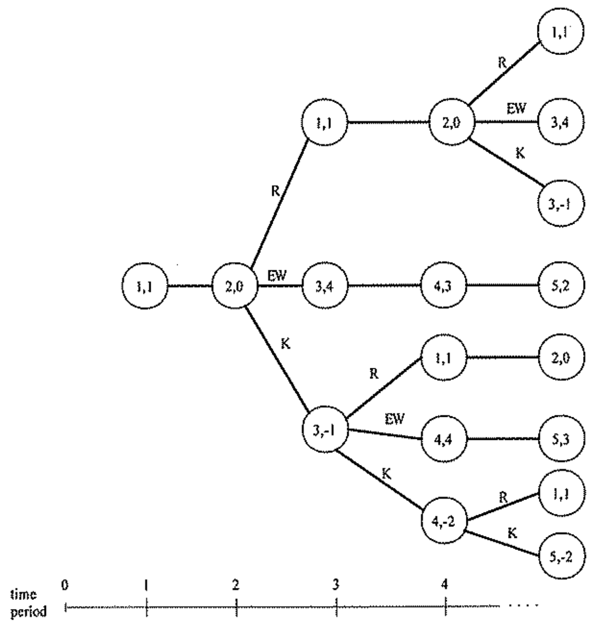

Figure 3: Network of decisions for a two-period base warranty and five-period extended warranty that can be deferred for at most two periods.

The dynamic programming recursion follows similarly to the unrestricted warranty case as defined in Equations (2.4) through (2.7). The only difference is that the options to purchase an extended warranty (EW), keep the asset without a warranty (K), or replace the product (R), which were available for all feasible states $(n, 0)$ as defined in Equation (2.5), are now valid for states $(n,0), (n,-1), \ldots (n,-F)$. For state $(n, -F-1)$, the following choices are evaluated:

$$v_t(n, -F-1) = \min \left\{ \begin{array}{l} K: C_r M'(n, n+1) + \alpha v_{t+1}(n+1, -F-1) \\ R: C_p + \alpha v_{t+1}(1, W-1) \end{array} \right\}, \quad \forall n < N \qquad (2.14)$$

as the ability to purchase an extended warranty is no longer available.

Algorithmically, the number of possible nodes in the representative network has increased to $N(W_{max} + F + 2)$, as w can vary from the maximum of W and W_1 to $-F-1$. The maximum number of decisions at a given node remains three such that over T periods of study, the DP can be solved in $O(TN(W_{max}+F+2))$ time.

Example 2: Nondeferrable Warranty

This example utilizes the data from Example 1 but the consumer is only allowed to delay the purchase of an extended warranty for one time period after the base warranty expires. If more than one period passes, the extended warranty is not offered again.

The optimal strategies for the consumer are given in Table 2, following the same (x, y, z) notation previously defined. Comparing these results to those of Table 1, it is clear that this warranty design has a drastic impact on consumer strategies. These can be summarized, when comparing the new design to the base case, as: (1) consumers will not purchase higher priced extended warranties; (2) consumers will purchase extended warranties sooner (earlier product lives), which is by design; and (3) consumers purchase extended warranties for longer time periods as they continue to renew the policies. In fact, the optimal policies produced in Table 2 follow those in the unrestricted warranty analysis in that once an extended warranty is procured, it is renewed until the product is replaced.

Table 2: Optimal strategy for nondeferrable warranties.

C_r :	$50			$75			$100		
C_e	γ=0.5	γ=1.0	γ=1.5	γ=0.5	γ=1.0	γ=1.5	γ=0.5	γ=1.0	γ=1.5
$50	2,18,20	2,18,20	2,18,20	2,18,20	2,18,20	2,18,20	2,18,20	2,18,20	2,18,20
$75	3,17,20	2,18,20	2,18,20	2,18,20	2,18,20	2,18,20	2,18,20	2,18,20	2,18,20
$100	–,–,20	3,17,19	2,18,20	3,17,20	2,18,20	2,18,20	2,18,20	2,18,20	2,18,20
$125	–,–,20	–,–,20	3,17,19	3,17,20	3,17,19	2,18,20	2,18,20	2,18,20	2,18,20
$150	–,–,20	–,–,20	3,17,19	–,–,20	3,17,19	3,17,19	3,17,19	2,18,20	2,18,20
$175	–,–,20	–,–,20	3,17,19	–,–,20	3,17,19	3,17,19	3,17,20	3,17,19	2,18,20
$200	–,–,20	–,–,20	–,–,14	–,–,20	–,–,15	3,17,19	–,–,20	3,17,19	3,17,19
$225	–,–,20	–,–,20	–,–,14	–,–,20	–,–,15	3,17,19	–,–,20	3,17,19	3,17,19
$250	–,–,20	–,–,20	–,–,14	–,–,20	–,–,15	–,–,9	–,–,20	–,–,11	3,17,19
$275	–,–,20	–,–,20	–,–,14	–,–,20	–,–,15	–,–,9	–,–,20	–,–,11	3,17,19
$300	–,–,20	–,–,20	–,–,14	–,–,20	–,–,15	–,–,9	–,–,20	–,–,11	3,17,19
$325	–,–,20	–,–,20	–,–,14	–,–,20	–,–,15	–,–,9	–,–,20	–,–,11	–,–,7

There is an interesting tradeoff in potential revenues when comparing the unrestricted and nondeferrable cases. For a given extended warranty price and repair cost, the nondeferrable warranty can provide higher revenues by forcing the consumer to purchase the extended warranty earlier and renew it over a longer period of time. Considering $C_e = \$175$ and $C_r = \$50$, the risk-averse consumer purchases the first warranty at age 8 in the unrestricted case but at age 3 in the nondeferrable case, as the design allows only one period of delay. This drastically increases the revenues for the provider.

However, an increase in revenues is not guaranteed as a consumer may no longer purchase an extended warranty under the restricted design. Consider a slight increase in the extended warranty price such that $C_e = \$200$ and $C_r = \$50$. In this example, the consumer purchases the first warranty at age 10 in the unrestricted case while not purchasing any extended warranties in the nondeferrable case. In this case, restricting the options to the consumer actually reduces the revenues for the provider. Thus, it is clear that both warranty contract designs and prices must considered in tandem in order to optimize profits for the provider.

The optimal strategy outlined in Theorem 1 holds for the nondeferrable extended warranty design. That is, if the EW is purchased in state $(n, 0)$, the consumer always purchases an EW or replaces the product in state $(n', 0)$, where $n' > n$. This is true because if an extended warranty is purchased within the allowable period of deferral, it is always available for renewal. However, if the extended warranty is not purchased in that period, the option is taken away.

2.4 Nonrenewable Warranty

One method in which to limit the exposure of insuring older products with increasing failure rates with age is to limit the number of times an EW can be renewed. Here, we assume that once a product reaches a certain age N_1, a consumer cannot purchase an EW. The representative decision network follows as previously in that the EW option is not feasible when a product reaches age N_1. Under this warranty contract, when a state $(n, 0)$, $n > N_1$, is reached, the extended warranty is no longer available for purchase. The age limit N_1 is specified in the contract.

The formulation follows directly from the unrestricted case with the only difference being the parameter at which the extended warranty option is no longer feasible, altering Equation (2.5) for $n > N_1$. In the nonrenewable case, N_1 defines the highest age at which an extended warranty can be purchased:

$$v_t(n,0) = \min \left\{ \begin{array}{l} K : C_r M'(n, n+1) + \alpha v_{t+1}(n+1, 0) \\ R : C_p + \alpha v_{t+1}(1, W-1) \end{array} \right\} \quad \forall n > N_1 \tag{2.15}$$

The worst case run time does not change from the unrestricted case, remaining $O(TN(W_{max} + 1))$.

Example 3: Nonrenewable Warranty

We continue analyzing the data from Example 1 but do not allow the purchase of a warranty beyond a product age of 7. The consumer's optimal strategies for different repair costs and extended warranty prices are given in Table 3.

Table 3: Optimal strategy for nonrenewable warranty.

C_r:	$50			$75			$100		
C_e	γ=0.5	γ=1.0	γ=1.5	γ=0.5	γ=1.0	γ=1.5	γ=0.5	γ=1.0	γ=1.5
$50	3,7,20	3,7,18	3,7,9	3,7,20	3,7,9	2,6,8	3,7,11	2,6,8	2,6,8
$75	5,7,20	3,7,20	3,7,10	3,7,20	3,7,11	3,7,9	3,7,13	3,7,9	2,6,8
$100	–,–,20	5,7,20	3,7,11	3,7,20	3,7,12	3,7,9	3,7,15	3,7,9	2,6,8
$125	–,–,20	–,–,20	5,7,12	7,7,20	3,7,13	3,7,9	3,7,17	3,7,9	3,7,9
$150	–,–,20	–,–,20	7,7,12	–,–,20	5,7,14	3,7,9	5,7,19	3,7,9	3,7,9
$175	–,–,20	–,–,20	7,7,13	–,–,20	7,7,14	5,7,9	7,7,20	5,7,10	3,7,9
$200	–,–,20	–,–,20	–,–,13	–,–,20	–,–,15	5,7,9	–,–,20	5,7,10	3,7,9
$225	–,–,20	–,–,20	–,–,13	–,–,20	–,–,15	7,7,9	–,–,20	7,7,11	3,7,9
$250	–,–,20	–,–,20	–,–,13	–,–,20	–,–,15	7,7,9	–,–,20	–,–,11	5,7,9
$275	–,–,20	–,–,20	–,–,13	–,–,20	–,–,15	–,–,9	–,–,20	–,–,11	5,7,9
$300	–,–,20	–,–,20	–,–,13	–,–,20	–,–,15	–,–,9	–,–,20	–,–,11	7,7,9
$325	–,–,20	–,–,20	–,–,13	–,–,20	–,–,15	–,–,9	–,–,20	–,–,11	–,–,7

Compared to the unrestricted case, the consumer is forced to replace the product earlier because the extended warranty is not available in later years. From Table 3, the highest price that can be offered is $75,

\$100, and \$175 for a risk-taking consumer, a risk-neutral consumer, and a risk-averse consumer, respectively, for a repair cost of \$50. These prices are drastically lower than the unrestricted case but similar to the nondeferrable case.

Interestingly, the nonrenewable extended warranty design is the first case in which the optimal strategy outlined in Theorem 1 is not followed. That is, even if an extended warranty is purchased at some state $(n, 0)$, the product may be retained without warranty at some state $(n', 0)$, $n' > n$. This is because the option to purchase the EW may not be available. In addition to Theorem 1 not holding, the results in Table 4 are contrary to Corollary 1.1 in that products are retained for longer periods when no extended warranties are purchased.

2.5 Nonrenewable and Nondeferrable Warranty

The nondeferrable warranty limits how long a consumer can wait to purchase an extended warranty while a nonrenewable warranty limits how long they can keep a product under warranty. Each design combats the possibility of lost revenues – the first due to consumers taking on the (low) risk of failures themselves and the second due to the higher expected repair costs for older products. Here, we combine these design features into one EW contract. Specifically, in this case, we assume that the extended warranty is only offered when the base warranty expires and it cannot be renewed.

The decision network for this situation does not vary significantly from Figures 2 and 3. As with the nondeferrable warranty, a state is reached when the extended warranty is no longer available. Thus, arcs representing the EW option are removed when a state $(n, 0)$ is reached, where $n > W$, as the the extended warranty is only offered at state $(n, 0)$ when $n = W$.

This logic transfers directly to the recursion in Equation (2.15) for the nonrenewable case merely altered as follows:

$$v_t(n,0) = \min \left\{ \begin{array}{l} K : C_r M'(n, n+1) + \alpha v_{t+1}(n+1, 0) \\ R : C_p + \alpha v_{t+1}(1, W-1) \end{array} \right\}, \quad \forall n > W \tag{2.16}$$

The other equations in the recursion follow as before. As with the unrestricted case, this can be solved in $O(TN(W_{max} + 1))$ time as the maximum number of nodes in a given period and the maximum number of decisions for a node have not changed in this formulation.

Example 4: Nonrenewable and Nondeferrable Warranty

This example again follows directly from Example 1. However, the extended warranty is only offered when the base warranty expires and it may not be renewed. The results are given in Table 4.

Table 4: Optimal strategy for nonrenewable and nondeferrable warranty.

C_r:	\$50			\$75			\$100		
C_e	γ=0.5	γ=1.0	γ=1.5	γ=0.5	γ=1.0	γ=1.5	γ=0.5	γ=1.0	γ=1.5
\$50	2,2,20	2,2,20	2,2,12	2,2,20	2,2,13	2,2,8	2,2,17	2,2,9	2,2,6
\$75	–,–,20	–,–,20	2,2,12	2,2,20	2,2,14	2,2,8	2,2,18	2,2,10	2,2,6
\$100	–,–,20	–,–,20	–,–,13	–,–,20	2,2,14	2,2,9	2,2,19	2,2,10	2,2,6
\$125	–,–,20	–,–,20	–,–,13	–,–,20	–,–,15	2,2,9	–,–,20	2,2,11	2,2,7
\$150	–,–,20	–,–,20	–,–,13	–,–,20	–,–,15	–,–,9	–,–,20	2,2,11	2,2,7
\$175	–,–,20	–,–,20	–,–,13	–,–,20	–,–,15	–,–,9	–,–,20	–,–,11	2,2,7
\$200	–,–,20	–,–,20	–,–,13	–,–,20	–,–,15	–,–,9	–,–,20	–,–,11	–,–,7

This warranty has two effects on consumer strategies. First, as the options available to the consumer are severely limited, they will not pay high prices to take out extended warranties. For example, in the unrestricted case with $C_r = \$50$, consumers would spend up to \$100, \$175, and \$225, for an extended warranty depending on their risk tolerance. However, the maximum prices paid in this restricted case are \$50, \$50, and \$75, respectively. Second, especially in the case of high repair costs, the product is replaced earlier when compared to the previous cases. For example, the risk-averse consumer generally retains the product until it is 19 or 20 periods old when extended warranty coverage is available, but the replacement age plummets to 6 or 7 periods in this case. Thus, even though money is lost in terms of warranty revenue, there is an increase in replacement costs, which also benefits the manufacturer, although not the third party provider.

As with the nonrenewable case, Theorem 1 does not hold because of the EW cannot be renewed. This actually occurs in every situation in which an extended warranty is purchased at time period 2, as shown in Table 4. In this situation, the problem follows more of a traditional equipment replacement problem (see, for example, Oakford et al. (1990) [18]) in that the decision to replace or retain the product must consider the tradeoff of high replacement costs, which increase with shorter replacement ages, and high maintenance costs, which increase with higher replacement ages.

3 EW Pricing for an Individual Consumer

We now turn our attention to the warranty provider, whether an original equipment manufacturer or a third party provider, and examine optimal pricing strategies for the different extended warranty contract designs. If we assume that the warranty provider knows the optimal consumer strategy, then we can utilize the developed dynamic programming models to solve for the optimal price. This problem can be described by Stackelberg game, as utilized in Murthy and Asgharizadeh [17] to maximize expected profit.

The players in a Stackelberg game are the leader and follower. For this problem, the leader is the warranty provider, whose goal is to maximize profits, and the follower is the consumer, whose goal is to minimize costs which are adjusted according to his or her level of risk aversion. First, an extended warranty provider offers a warranty contract and price to the consumer, assuming that the provider knows the consumer's optimal strategy. Under this assumption, the provider can determine an expected profit for the given contract and price. Each price (for a given contract) results in a different optimal strategy for the consumer and thus, the provider can search over all prices for a given contract to determine the price that maximizes profits. Note that we assume complete information in that the consumer knows the failure rate of the product and repair costs while the provider knows the consumer's risk tolerance.

In searching for the optimal price, we maximize annualized profits. This is because different prices may result in different consumer strategies, including replacing the product at different ages. Thus, maximizing annualized profits allows for a fair comparison among different strategies which may have different product life cycle lengths.

Note that profits are determined differently for an original manufacturer and a third party provider. Both entities receive revenues for selling extended warranty contracts. The difference between this revenue and the expected warranty costs defines the expected profits over the length of the warranty contract. (We assume that the repair cost for the extended warranty provider, $\delta_1 C_r$ or $\delta_2 C_r$, is generally lower than what the consumer pays, C_r.) The manufacturer is responsible for BW costs, unlike a third party provider, and the manufacturer also receives revenue from replacements. We assume some profit margin (δ) to determine what percentage of the revenue is actual profit. While we realize that a retailer serving as a third party provider may receive revenue (and thus profit) for a replacement, we assume this is negligible compared to the extended warranty revenue.

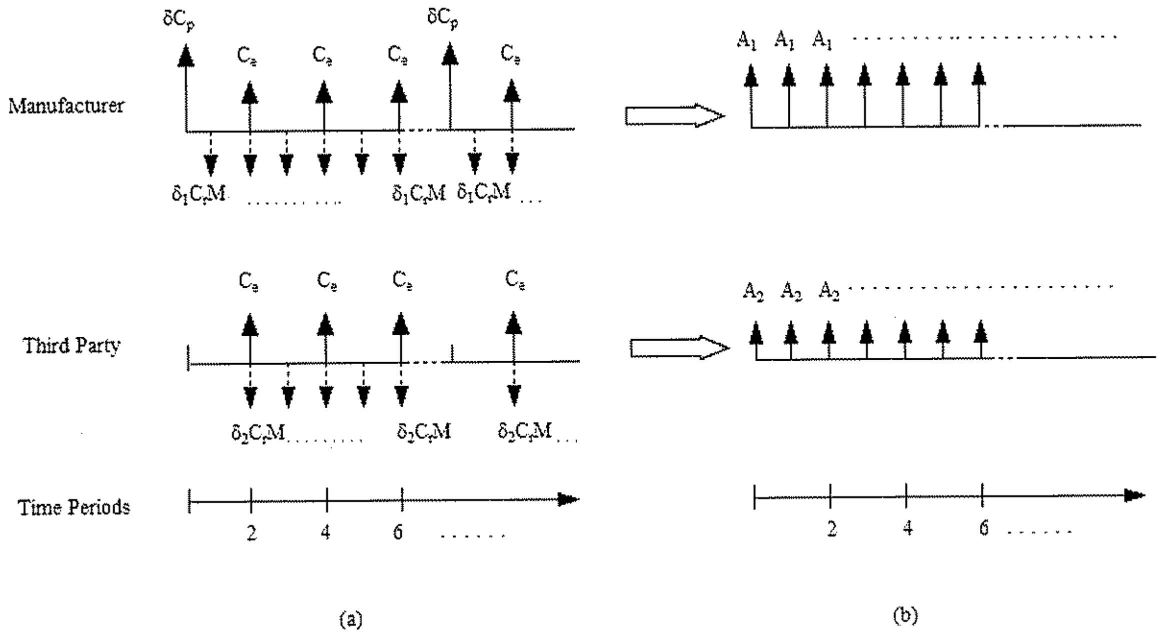

Figure 4: Income flow chart (a) realized cash flow and (b) annualized net cash flow for manufacturer and third party provider.

Figure 4 illustrates the cash flows from the perspective of a manufacturer and a third party provider with two year base and extended warranties. As shown in the figure, we convert these to annualized flows with an appropriate interest rate to determine the optimal price. Note that the cash flows are assumed to occur at the beginning of each period.

3.1 Unrestricted Warranty

Following our Stackelberg game, our approach to find the optimal price for a given contract is merely to repeatedly solve the appropriate dynamic programming formulation for various C_e values. Once the consumer's optimal strategy is revealed (by the dynamic program), the annual profits for the provider can be calculated. The price that maximizes annualized profits is then chosen.

Figures 5 and 6 illustrate the annualized profits from the perspective of the manufacturer and the third party provider, respectively, at various extended warranty costs (C_e) for the three consumer risk profiles (γ). The graphs were generated by solving the dynamic programming recursion, defined for the unrestricted case in Equations (2.4) through (2.7). The solutions define the optimal consumer policies which in turn define the providers net cash flow stream over time, which was annualized. As noted earlier, a third-party provider does not receive income for replacements.

The data from Example 1 was used assuming that providers pay $\delta_1 = 80\%$ of the repair cost if the product is under warranty. Also, the net profit for a replacement was assumed to be $\delta = 15\%$ of the replacement cost, $C_p = \$500$, in this example, for purposes of computing the annualized profits. Note that the consumer's optimal policy is unaffected by whether the warranty is offered from a manufacturer or third party provider, as the EW price and coverage is the same to the consumer.

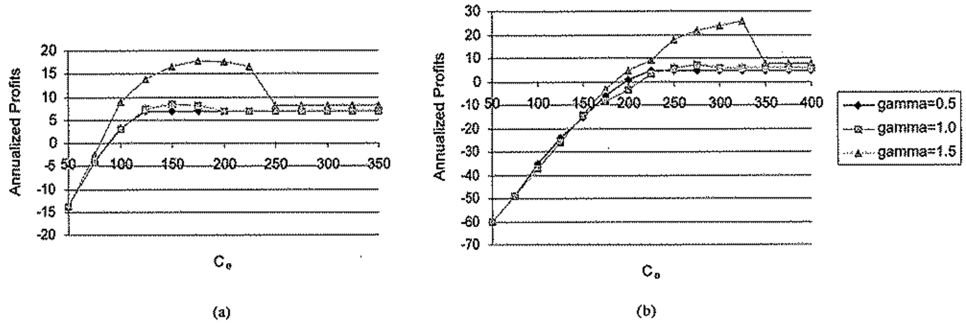

Figure 5: Manufacturer's expected annualized profits for the unrestricted warranty with C_r of (a) \$50 and (b) \$100.

For the manufacturer, in Figure 5, the optimal price for a risk-averse consumer ($\gamma = 1.5$) is \$175 for $C_r = \$50$ and \$325 for $C_r = \$100$. This defines respective annualized profits of \$17.74 and \$25.39. For the risk-neutral consumer ($\gamma = 1.0$), the respective optimal prices are \$150 and \$275, but define significantly lower annualized profits of \$8.50 and \$6.87. For the risk-taking consumer ($\gamma = 0.5$), the manufacturer does not make any profit from the sale of extended warranties, regardless of the repair cost.

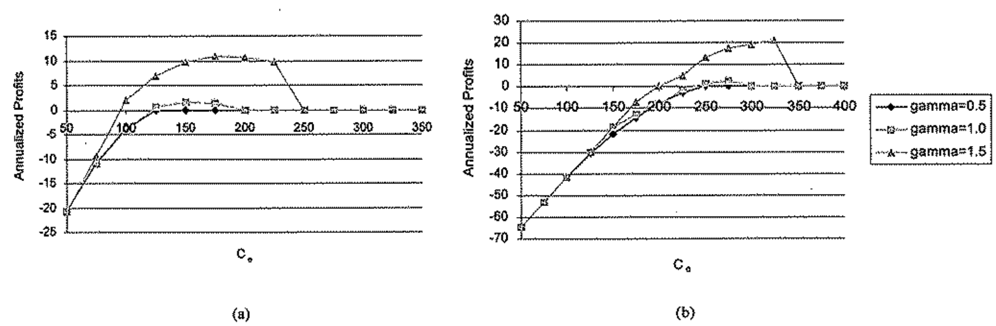

Figure 6: Third party's expected annualized profits for the unrestricted warranty with C_r of (a) \$50 and (b) \$100.

The third party provider achieves similar results to the manufacturer in that the optimal prices are also \$175 and \$325 for the risk-averse and risk-neutral consumers while no profit is made on risk-taking consumers. The third party provider must price the extended warranty at \$125 ($C_r = \$50$) or \$250 ($C_r = \$100$) in order to breakeven with risk-taking consumers.

There are a number of interesting observations to be made from these results. First, regardless of the provider, only the risk-averse consumer provides significant profits. This is not overly surprising, as these consumers are willing to purchase extended warranties earlier and at higher prices. Second, as the repair cost increases, the ability to profit increases dramatically (nearly doubling in our example for the third-party provider) as consumers are more likely to purchase warranties at higher prices. Third, with the increase in the repair cost, the ability to lose money also dramatically increases if the extended warranty is priced too low. For the manufacturer, the breakeven price moves from just over \$75 to over \$175 with the \$50 increase in repair costs assuming the consumer is risk-averse. For the third party provider, this breakeven price moves from just over $75 to $200. Fourth, examining the figures, the pricing window for making profits is quite tight, especially for third party providers, and this window decreases with an increase in repair cost. For the low repair cost, the third party provider enjoys profits for prices between $100 and $225, moving between $225 and $325 for higher priced repairs. Of course, these profits are buffered by replacement revenues for the manufacturer.

Recall that the product in this example carries a $500 purchase price. This example helps explain why extended warranty prices, especially those offered by third-party providers, can cost roughly 50% of the value of the product. The fact is, if the extended warranties are priced too low, the provider is at great risk of loss. Because of this, they must rely on the consumer being risk-averse in order to sell the warranties and generate profits.

3.2 Nondeferrable Warranty

A nondeferrable warranty limits the number of periods a consumer can wait to purchase an extended warranty after the base warranty expires. Figures 7 and 8, generated similarly to our previous examples, illustrate that the profit curves for both the manufacturer and third party provider follow similarly to the unrestricted extended warranty. However, it is clear that the potential for higher profits has increased while the pricing window has decreased.

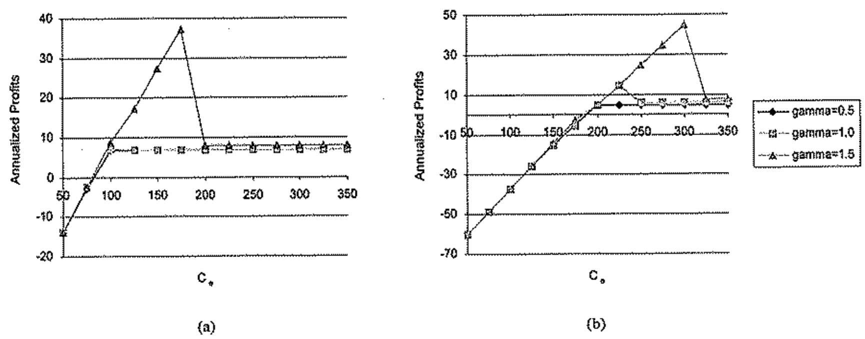

Figure 7: Manufacturer's expected annualized profits for nondeferrable warranty with C_r of (a) $50 and (b) $100.

This extended warranty leads to dramatically increased profits for the third party provider if the consumer is either risk-averse or risk-neutral, as no profits are generated (again) from the risk-taking consumer. The third party provider achieves profits of $30 from a price of $175 and $40 from a price of $300 for low and high repair costs, respectively. The first profit is nearly triple from a similar price for low repair costs in the unrestricted case while nearly double for the high repair cost case. Additionally, while the risk-neutral consumer provided a small profit at the low repair cost case for the unrestricted warranty and no profit at the high repair cost case, those roles reverse for the nondeferrable warranty. The profits for the manufacturer follow similarly to the third party provider with the extra boost from replacements. It should also be noted that the potential losses from warranties being priced too low are the same as in the unrestricted case.

This example clearly illustrates that a warranty provider can generate significantly higher profits from both the *design* and *pricing* of the extended warranty. By limiting the consumer's options (number of periods

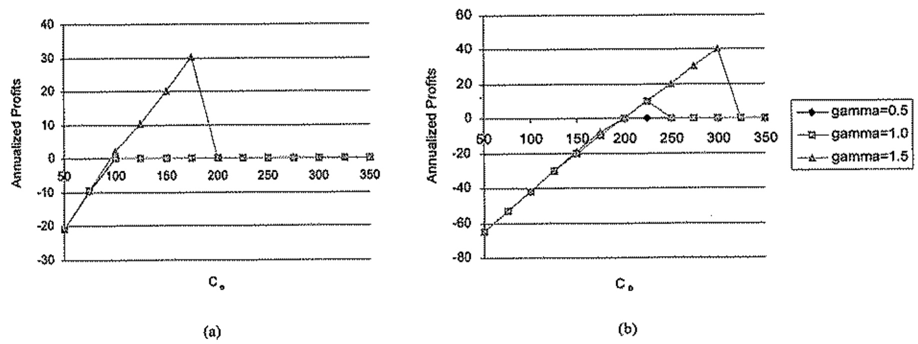

Figure 8: Third party's expected annualized profits for nondeferrable warranty with C_r of (a) \$50 and (b) \$100.

to defer a warranty in this case), the providers can increase profits at similar prices when compared to the unrestricted warranty in which the consumer has complete freedom.

3.3 Nonrenewable Warranty

Unlike the nondeferrable warranty, which limits the consumer's options near the beginning of a product's life, the nonrenewable warranty limits options towards the end of the product's life, depending on the age limit.

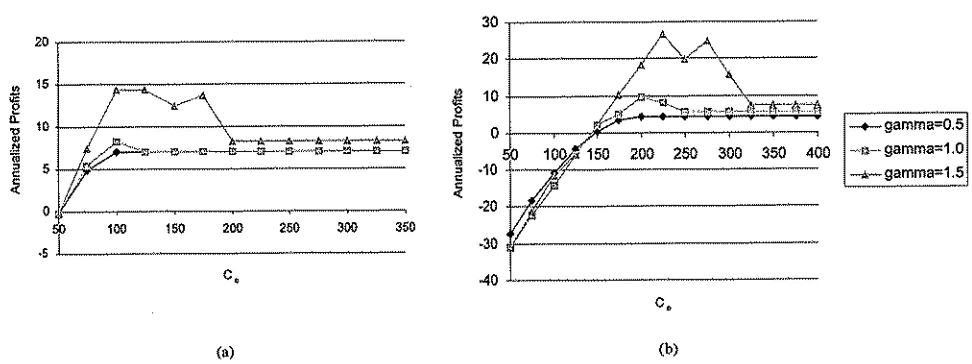

Figure 9: Manufacturer's expected annualized profits for nonrenewable warranty with C_r of (a) \$50 and (b) \$100.

Figures 9 and 10 provide respective solutions for the manufacturer and third party provider for repair costs of \$50 and \$100. The optimal price for the third party provider is \$125 and \$225 for repair costs of \$50 and \$100, respectively, for a risk-averse consumer. The manufacturer achieves optimal profits at \$100 and \$225 prices, respectively. The profits are similar to the unrestricted case, but interestingly, the potential losses here are drastically smaller than the previous two cases. In fact, the manufacturer does not lose money with the \$50 price in the \$50 repair cost case.

It should be noted that the profit curves are no longer concave for the nonrenewable case. This can be

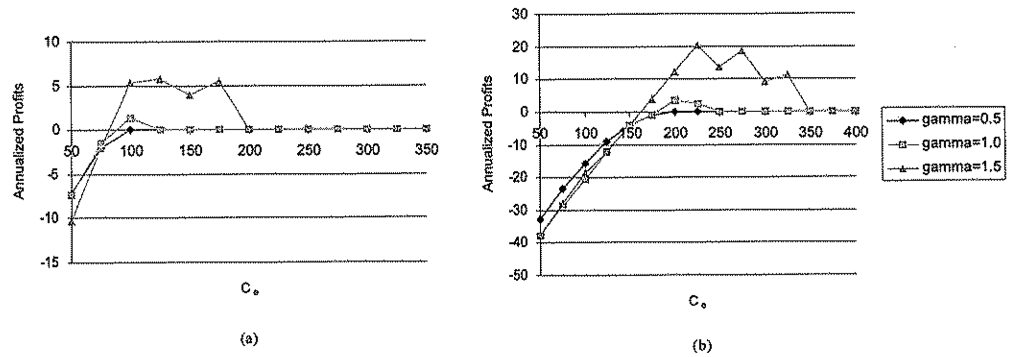

Figure 10: Third party's expected annualized profits for nonrenewable warranty with C_r of (a) \$50 and (b) \$100.

attributed to the age limit, which is essentially a terminal condition in the dynamic program that introduces end-of-study effects such that policies are not consistent for all prices because options are not always available to the consumer.

3.4 Nonrenewable and Nondeferrable Warranty

The results of the combined warranty designs are somewhat predictable, as seen Figures 11 and 12: (1) potential profits are drastically reduced; (2) potential losses are drastically reduced; and (3) the pricing window for profits is shortened significantly. Thus, the design combines results from previous designs.

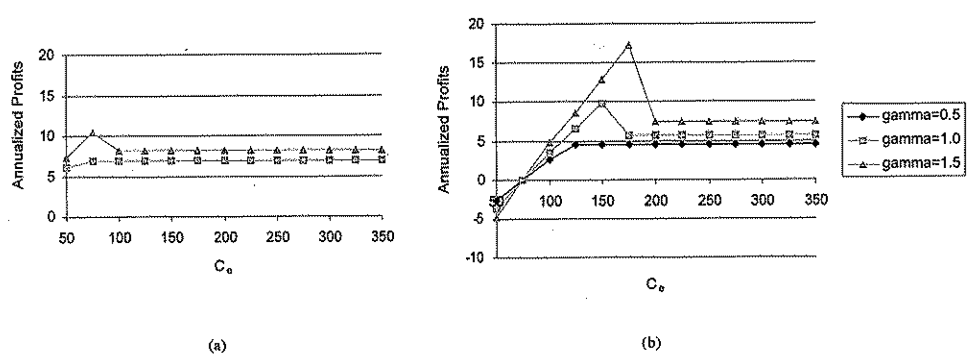

Figure 11: Manufacturer's expected annualized profits for nonrenewable and nondeferrable warranty with C_r of (a) \$50 and (b) \$100.

Examining the figures, the low repair cost cases show barely any profit potential while profits for the high repair cost case are only achieved between \$125 and \$175 for the third party provider and the risk-averse consumer. This leads to profits of just under \$10, which is roughly half of those achieved in the unrestricted and one-quarter of those in the nondeferrable warranty. However, the greatest potential loss for the third party provider is just over \$10, while it reaches nearly \$70 in the previous warranty designs. This is not unexpected due to the limitations on the consumer at both the beginning and end of the product's life.

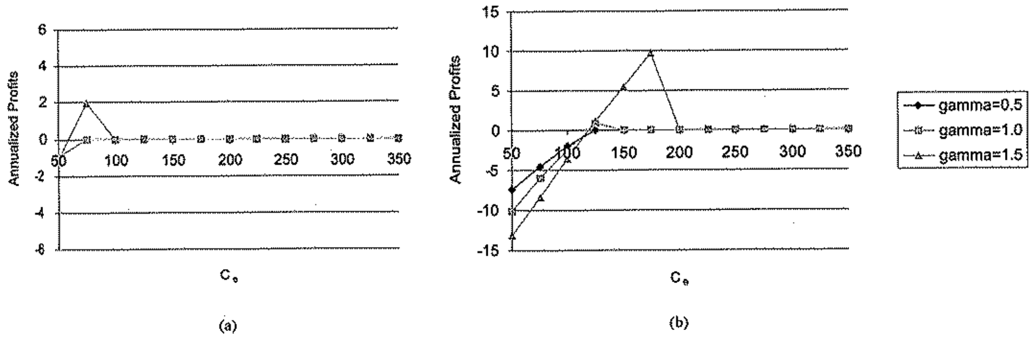

Figure 12: Third party's expected annualized profits for nonrenewable and nondeferrable warranty with C_r of (a) \$50 and (b) \$100.

3.5 Summary of EW Pricing

The results from the preceding pricing analyses for the third party provider are summarized in Table 5. It is clear from the table that risk-taking consumers do not provide any profit and, as expected, the optimal price increases in the level of risk-aversion.

Table 5: Summary of EWs price providing the highest profit third party providers assuming C_r = \$100 for each warranty type.

Warranty type	risk level	EW price
Unrestricted	risk-taking	not offer
Unrestricted	risk-neutral	\$275
Unrestricted	risk-averse	\$325
Nondeferrable	risk-taking	not offer
Nondeferrable	risk-neutral	\$225
Nondeferrable	risk-averse	\$300
Nonrenewable	risk-taking	not offer
Nonrenewable	risk-neutral	\$200
Nonrenewable	risk-averse	\$225
Nonrenewable and Nondeferrable	risk-taking	not offer
Nonrenewable and Nondeferrable	risk-neutral	\$125
Nonrenewable and Nondeferrable	risk-averse	\$175

It should be noted that these four designs truly represent a broad range of extended warranty designs. The unrestricted warranty gives the consumer complete freedom on renewals and replacement times. The nondeferrable warranty limits the consumer at the beginning of the life of the product while the nonrenewable warranty limits options at the end of the life. These limits are combined in the final design. It is clear that the provider must consider the warranty design and price in order to maximize profits. While we concentrated our efforts on the pricing here, we address the design issues (i.e., optimal age limit for nonrenewable designs) in the following sections.

4 EW Pricing for a Population of Consumers

We now turn our attention to pricing a warranty contract, or set of contracts, to a population of consumers with different risk tolerances, as opposed to targeting a specific risk category. We assume that the distribution of a population's level of risk, defined by γ is known, such as given in the distribution curve in Figure 13.

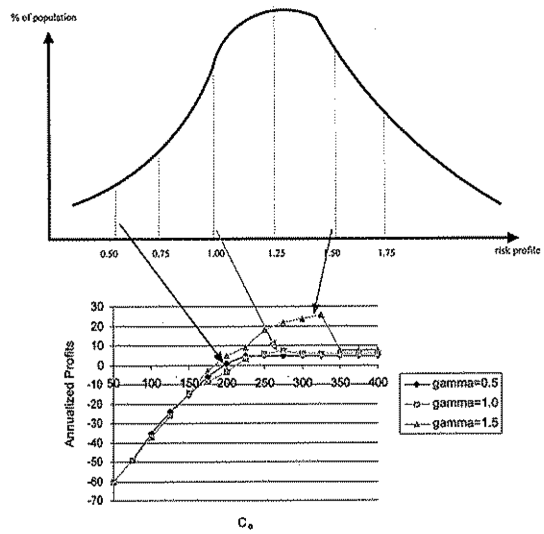

Figure 13: Relationship between a population's risk tolerance distribution and annual profit.

The figure also illustrates our overall approach: discretize the population according to risk aversion γ. For each level, repeatedly solve the associated dynamic program for over all possible prices C_e and determine the optimal price for the provider. We illustrate in the following two sections.

4.1 Pricing a Single Warranty Offering

Here we assume that only one warranty contract (at one price) can be offered to an entire consumer population. Therefore, the extended warranty provider must identify the extended warranty policy and price that maximizes the profit for the entire population.

To determine the optimal warranty design and price, we discretize the population according to values of γ. Given the population, define s_γ as the number of consumers with risk tolerance γ. For each value of discretized γ, solve the dynamic program for a given warranty design. For example, the recursion in Equations (2.4) through (2.7) is solved for the unrestricted warranty to determine the optimal price given that the consumer minimizes their expected, discounted, risk-adjusted cost. The annualized profit from the solution of this problem is $\pi_{i\gamma}$, where i defines the warranty design and price. It is assumed that the population has been discretized into m different risk levels and there are n possible warranty designs.

Now, define p_i as a binary variable equal to one when a given design for a given price (i) is chosen, otherwise is zero. To find the optimal price, solve the integer program:

$$\max \sum_{i=1}^{n} \sum_{\gamma=1}^{m} s_\gamma \pi_{i\gamma} p_i$$

$$\begin{aligned}
\text{s.t} \quad \sum_{i=1}^{n} p_i &= 1 && (4.17)\\
p_i &\in \{0,1\} \quad \forall i = 1,2,\ldots,n && (4.18)
\end{aligned}$$

Solving for p_i identifies the design and price i that maximizes profits for a given population, with the assumption that only one design is offered. Given the profit for each design and price combination for each risk level from the dynamic programs, the integer program identifies the most profitable for the population. We illustrate solutions to this problem after the next section.

4.2 Designing and Pricing a Menu of Warranty Offerings

We relax our restriction on the number of extended warranties offered to the population in that a provider may choose to offer a menu of extended warranties. Here, the objective of the provider is to maximize profits by designing a menu in order to attract multiple consumer types and increase revenues. The integer program for this problem is defined as:

$$\max \sum_{i=1}^{n} \sum_{\gamma=1}^{m} s_\gamma \pi_{i\gamma} p_{i\gamma}$$

$$\begin{aligned}
\text{s.t} \quad \sum_{i=1}^{n} p_{i\gamma} &= 1 \quad \forall \gamma = 1,2,\ldots,m && (4.19)\\
c_{i\gamma} p_{i\gamma} &\leq c_{j\gamma} k_j \quad \forall i = 1,2,\ldots,n, j = 1,2,\ldots,n, \gamma = 1,2,\ldots,m && (4.20)\\
k_i &\leq 1 + (1 - p_{i\gamma})M \quad \forall i = 1,2,\ldots,n, \gamma = 1,2,\ldots,m && (4.21)\\
p_{i\gamma} &\in \{0,1\} \quad \forall i = 1,2,\ldots,n, \gamma = 1,2,\ldots,m && (4.22)
\end{aligned}$$

The objective function maximizes the profits from the derived warranty contract menu, defined by $p_{i\gamma}$. The binary variable $p_{i\gamma}$ defines the warranty contract (design and price) targeted for the population with risk level γ. The value of s_γ defines the population (slice) with risk level γ while $\pi_{i\gamma}$ defines the per unit profit derived from the population with risk level γ for warranty design (and price) i. As previously, it is assumed that the population has been discretized into m different risk levels and there are n possible warranty designs.

Constraint (4.19) ensures that only one warranty design is chosen for each consumer group γ. The parameter $c_{i\gamma}$ defines the annualized cost (risk adjusted) for consumer type γ given warranty design i. The variable k_i is used in Constraints (4.20) and (4.21) to ensure that a consumer prefers the warranty offered to their respective group γ and not another warranty on the menu. These are often referred to as incentive compatibility and participation constraints in the warranty literature. In Equation (4.21), M refers to a large number (as in the big M method).

To illustrate how these constraints work, assume there are two warranty designs available ($i = 1,2$) and two consumer classes ($\gamma = 1,2$). Further assume that consumer type 1 prefers warranty design 2 (due to cost) and consumer type 2 prefers warranty design 1. This defines:

$$c_{21} \leq c_{11}$$
$$c_{12} \leq c_{22}$$

Note that the menu options are to offer design 1, design 2, or designs 1 and 2 simultaneously. If the designs are offered separately, there is no potential problem as a consumer has only one choice. However, a potential problem arises if the profit for design 1 is the highest for consumer 1 and the profit for design 2 is highest for consumer 2. If this is the case, the optimal solution from an unconstrained profit perspective would define $p_{11} = 1, p_{21} = 0, p_{12} = 0$, and $p_{22} = 1$. This in turn would define $k_1 = k_2 = 1$ in Constraints (4.21). Thus, Constraints (4.20) would follow as:

$$c_{11}(1) \leq c_{21}(1)$$
$$c_{21}(0) \leq c_{11}(1)$$
$$c_{12}(0) \leq c_{22}(1)$$
$$c_{22}(1) \leq c_{12}(1)$$

As we previously defined $c_{21} \leq c_{11}$ due to consumer preferences, this solution is clearly infeasible. Thus, Constraints (4.20) and (4.21) ensure that the optimal solution according to profit aligns with the consumer's optimal preferences according to cost. Thus, in this example, designs 1 and 2 are not offered simultaneously.

4.3 Menu Solutions for Various Populations

Table 6 presents warranty menus that result from the solution of both integer programs presented in the previous two sections for various population assumptions. The underlying dynamic programs were solved assuming failures according to a Weibull distribution with $\theta = 2.215$ and $\beta = 1.5$, a repair cost of $C_r = \$100$, a maximum EW price of \$400 (for any design), the maximum age of product is 10 years, the purchase price is \$1000, and from the perspective of a third party provider.

In this example, we assume there are 12 warranty designs available for the menu with a range of prices from \$50 to \$400 in increments of \$25. The EW policies are defined in the table as:

- UN_2 and UN_3: 2- or 3-period unrestricted EWs, respectively.
- ND1_2 and ND1_3: 2- or 3-period EWs nondeferrable after one period, respectively.
- ND2_2 and ND2_3: 2- or 3-period EWs nondeferrable after two periods, respectively.
- NR5_2 and NR5_3: 2- or 3-period EWs nonrenewable after age 5, respectively.
- NR6_2 and NR6_3: 2- or 3-period EWs nonrenewable after age 6, respectively.
- NRND_2 and NRND_3: 2- or 3-period nonrenewable and nondeferrable EWs, respectively.

Furthermore, there are three consumer types: $\gamma = 0.50$ (risk-taking), 1.00 (risk-neutral), 1.50 (risk-averse). Columns 2 through 4 give the percentage of the population defined by γ. (Note that the values in columns 2 through 4 of the table were used as the s_γ in the integer programs. To truly gauge profits, one would require a population estimate of each consumer type. That is, this example assumes a consumer population of 100.) In total, the solution to 27 different population distributions are provided. Cases 1 through 3 assume a homogeneous population, cases 4 through 18 assume a population of two consumer types, and the remaining nine cases assume a mixture of all three types. In all, we solved 180 dynamic programs and 54 integer programs to develop the results in Table 6. The integer programs generally took between 20 minutes and 20 hours to solve using CPLEX version 9.1.0 on a desktop PC with a 758 MHz processor and 384 MB of RAM. The dynamic programs were implemented in C++ using Bloodshed Dev-C++ version 4.9.9.2. The solution times of the dynamic programs were negligible.

For the first three solutions in Table 6 (homogeneous populations), the risk-taking and risk-neutral solutions have the same design (ND1_2) at prices of \$150 and \$175, respectively. The profit is \$5474 for

Table 6: Third party provider menus with $\theta = 2.215, \beta = 1.5, C_R = \$100$, and a \$400 EW price limit.

<table>
<tr><th rowspan="2">Case</th><th colspan="3">Percentage γ</th><th rowspan="2">Single EW</th><th rowspan="2">Profit</th><th colspan="3">EW menu</th><th rowspan="2">Profit</th><th>Increase</th></tr>
<tr><th>0.5</th><th>1.0</th><th>1.5</th><th>0.5</th><th>1.0</th><th>1.5</th><th>(%)</th></tr>
<tr><td>1</td><td>100</td><td>0</td><td>0</td><td>ND1_2 $150</td><td>$1473</td><td></td><td></td><td></td><td></td><td></td></tr>
<tr><td>2</td><td>0</td><td>100</td><td>0</td><td>ND1_2 $175</td><td>$3162</td><td></td><td></td><td></td><td></td><td></td></tr>
<tr><td>3</td><td>0</td><td>0</td><td>100</td><td>ND1_3 $400</td><td>$5474</td><td></td><td></td><td></td><td></td><td></td></tr>
<tr><td>4</td><td>90</td><td>10</td><td>0</td><td></td><td></td><td colspan="2">ND1_2 $150</td><td></td><td>$1538</td><td>0.0</td></tr>
<tr><td>5</td><td>70</td><td>30</td><td>0</td><td></td><td></td><td colspan="2">ND1_2 $150</td><td></td><td>$1669</td><td>0.0</td></tr>
<tr><td>6</td><td>50</td><td>50</td><td>0</td><td>ND1_2 $150</td><td>$1800</td><td>NRND_3 $175</td><td>ND1_2 $175</td><td></td><td>$2008</td><td>11.56</td></tr>
<tr><td>7</td><td>90</td><td>0</td><td>10</td><td></td><td></td><td>ND1_2 $150</td><td></td><td>ND1_2 $150</td><td>$1538</td><td>0.0</td></tr>
<tr><td>8</td><td>70</td><td>0</td><td>30</td><td>ND1_2 $150</td><td>$1669</td><td>NRND_3 $175</td><td></td><td>ND1_2 $225</td><td>$2167</td><td>29.85</td></tr>
<tr><td>9</td><td>50</td><td>0</td><td>50</td><td>ND1_3 $400</td><td>$2737</td><td>NRND_3 $175</td><td></td><td>ND1_2 $225</td><td>$3043</td><td>11.18</td></tr>
<tr><td>10</td><td>0</td><td>90</td><td>10</td><td></td><td></td><td></td><td colspan="2">ND1_2 $175</td><td>$3162</td><td>0.0</td></tr>
<tr><td>11</td><td>0</td><td>70</td><td>30</td><td></td><td></td><td></td><td colspan="2">ND1_2 $200</td><td>$3364</td><td>0.0</td></tr>
<tr><td>12</td><td>0</td><td>50</td><td>50</td><td></td><td></td><td></td><td colspan="2">ND1_2 $200</td><td>$3602</td><td>0.0</td></tr>
<tr><td>13</td><td>10</td><td>90</td><td>0</td><td>ND1_2 $175</td><td>$2846</td><td>NRND_3 $175</td><td>ND1_2 $175</td><td></td><td>$2931</td><td>3.00</td></tr>
<tr><td>14</td><td>30</td><td>70</td><td>0</td><td>ND1_2 $175</td><td>$2213</td><td>NRND_3 $175</td><td>ND1_2 $175</td><td></td><td>$2469</td><td>11.56</td></tr>
<tr><td>15</td><td>0</td><td>10</td><td>90</td><td></td><td></td><td></td><td>-</td><td>ND1_3 $400</td><td>$4926</td><td>0.0</td></tr>
<tr><td>16</td><td>0</td><td>30</td><td>70</td><td>ND1_2 $200</td><td>$3840</td><td></td><td>NRND_3 $225</td><td>ND1_2 $225</td><td>$4126</td><td>7.46</td></tr>
<tr><td>17</td><td>10</td><td>0</td><td>90</td><td></td><td></td><td>-</td><td></td><td>ND1_3 $400</td><td>$4927</td><td>0.0</td></tr>
<tr><td>18</td><td>30</td><td>0</td><td>70</td><td>ND1_3 $400</td><td>$3832</td><td>NRND_3 $175</td><td></td><td>ND1_2 $225</td><td>$3919</td><td>2.28</td></tr>
<tr><td>19</td><td>80</td><td>10</td><td>10</td><td></td><td></td><td colspan="3">ND1_2 $150</td><td>$1604</td><td>0.0</td></tr>
<tr><td>20</td><td>60</td><td>20</td><td>20</td><td>ND1_2 $150</td><td>$1734</td><td>NRND_3 $175</td><td colspan="2">ND1_2 $175</td><td>$1777</td><td>2.44</td></tr>
<tr><td>21</td><td>40</td><td>30</td><td>30</td><td>ND1_2 $200</td><td>$2161</td><td>NRND_3 $175</td><td colspan="2">ND1_2 $175</td><td>$2238</td><td>3.57</td></tr>
<tr><td>22</td><td>10</td><td>10</td><td>80</td><td></td><td></td><td>-</td><td>-</td><td>ND1_3 $400</td><td>$4379</td><td>0.0</td></tr>
<tr><td>23</td><td>20</td><td>20</td><td>60</td><td>ND1_3 $400</td><td>$3284</td><td colspan="2">NRND_3 $175</td><td>ND1_2 $225</td><td>$3481</td><td>5.99</td></tr>
<tr><td>24</td><td>30</td><td>30</td><td>40</td><td>ND1_2 $200</td><td>$2581</td><td colspan="2">NRND_3 $175</td><td>ND1_2 $225</td><td>$2605</td><td>0.93</td></tr>
<tr><td>25</td><td>10</td><td>80</td><td>10</td><td>ND1_2 $175</td><td>$2846</td><td>NRND_3 $175</td><td colspan="2">ND1_2 $175</td><td>$2931</td><td>3.00</td></tr>
<tr><td>26</td><td>20</td><td>60</td><td>20</td><td>ND1_2 $200</td><td>$2644</td><td>NRND_3 $175</td><td colspan="2">ND1_2 $175</td><td>$2700</td><td>2.14</td></tr>
<tr><td>27</td><td>30</td><td>40</td><td>30</td><td>ND1_2 $200</td><td>$2462</td><td>NRND_3 $175</td><td colspan="2">ND1_2 $175</td><td>$2469</td><td>0.30</td></tr>
</table>

the risk-averse population, 73.12 percent higher than the risk-neutral consumer and 42.25 percent over the risk-taking consumer, with a similar warranty contract that runs for 3 periods.

The remaining 24 cases illustrate the benefit of offering a menu of warranty contracts. Of the 15 cases with two consumer types, seven increase profits by offering multiple warranty contracts. The percentage increase in profit over offering a single warranty contract varies from 2.28 percent to nearly 30 percent for the seven cases. The increases in profits are not as drastic for the final nine cases, but seven of the nine are improved through the offering of menus.

It is interesting to note that risk-neutral and risk-averse consumers prefer the nondeferrable warranties while risk-taking consumers prefer nonrenewable and nondeferrable warranties. This may be the result of risk-averse consumers wanting to purchase warranties for longer periods of time. As expected, the risk-taking consumers pay considerably less, on average, for the warranty contracts. When these menus are compared to the offering of a single warranty contract to the population (column 5), the single contract is generally targeted to the risk-neutral and risk-averse consumers.

Table 7 attempts to generalize the results from the above example by grouping the results according to populations, with L representing the percentage of consumers between 10-30, M between 40-60, and H between 70-90. It is hard to generalize results from a single example, but it appears that the benefits (increased profits) of offering menus is more likely to occur if there is a risk-taking population. Specifically, the largest increases in profits occur when there is a medium or large risk-taking population or a small (or no) risk-averse population. This makes some sense because it is unlikely that a risk-averse or risk-neutral consumer would desire the same contract as a risk-taking consumer. As the risk-averse consumer is clearly better for generating profits, it is clear that a menu would be required in order to entice a risk-taking consumer to purchase an EW with a mixed population. On average, the menus do not add much value if there is no, or a low, risk-taking population.

Table 7: Aggregate results for third party provider menus with $\theta = 2.215, \beta = 1.5$, and $C_R = \$100$.

Percentage γ			Menus	Average Profit
0.5	1.0	1.5	Offered	Increase (%)
H	L	0	0/2	0.00
H	0	L	1/2	14.93
H	L	L	0/1	0.00
M	M	0	1/1	11.56
M	0	M	1/1	11.18
M	L	L	2/2	3.01
L	L	H	0/1	0.00
L	L	M	2/2	3.46
L	M	L	2/2	1.22
L	H	L	1/1	3.00
L	H	0	2/2	7.28
L	0	H	1/2	1.14
0	H	L	0/2	0.00
0	M	M	0/1	0.00
0	L	H	1/2	3.73

Assuming repair costs can be accurately estimated, it should be clear that there are two risks involved when designing and pricing EWs: (1) underestimating the failure rate of the product which may lead to EWs that are priced too low to cover costs; and (2) incorrectly estimating the distribution of a population's risk preference such that potential profits are lost. Thus, we performed extensive sensitivity analysis on the the failure rate, the repair cost, and the EW price limit. All the results are shown in the Appendix (19 tables in all).

Table 8 summarizes the conclusions from the sensitivity analysis according to failure rates and costs. For the data in the table, the low failure rate refers to a $\beta = 1.5$ and $\theta = 2.215$ for the Weibull distribution and the high failure rate assumes $\beta = 2.0$ and $\theta = 2.257$; a low repair cost is \$100 and a high repair cost is \$150; and the EW price limit varies from \$200 to no limit.

Table 8: Summary of 15 scenarios (27 cases each) with varying input parameters.

Failure rate/Repair cost/Price limit	Menus Offered (out of 24)	Average Profit Increase (%) *if* Menu Offered
Low/Low/No limit	13	6.25 (0.30-27.91)
Low/Low/\$400	14	6.80 (0.30-29.85)
Low/Low/\$200	12	6.67 (0.30-11.56)
High/Low/No limit	8	8.48 (0.02-25.91)
High/Low/\$600	5	12.78 (3.84-18.23)
High/Low/\$400	8	10.25 (0.98-25.63)
High/Low/\$200	0	0.00
Low/High/No limit	5	6.27 (1.65-17.47)
Low/High/\$600	5	6.27 (1.65-17.47)
Low/High/\$400	8	7.65 (1.17-19.43)
Low/High/\$200	0	0.00
High/High/No limit	2	6.00 (1.18-10.81)
High/High/\$600	5	3.93 (0.08-10.42)
High/High/\$400	13	10.28 (0.79-37.51)
High/High/\$200	0	0.00

The results in Table 8 are interesting because it appears that the potential to offer a menu of extended warranty contracts increases when the failure rate and repair cost are both low. However, the potential to increase profits over the offering of a single contract seems stronger when the failure rate is high – especially when the repair cost is low. When one examines the range of profit increases, it is clear that double digit percentage increases are possible for all combinations, depending on the population's risk tolerance distribution. Finally, it is clear that the opportunity to offer a menu of contracts requires flexibility in pricing in that the limit cannot be too low, or they will not be offered.

We further summarize the sensitivity results in Table 9 according to the distribution of the population. According to the results, the likelihood of offering a menu of contracts increases when the risk-taking population level is medium or high – especially when the risk-averse population is not large. Similarly, menus are generally not offered when there is no risk-taking population or there is a high distribution of risk-neutral or risk-averse consumers.

Examining the increase in profits when menus are offered, it is clear that the largest increases result from having medium or high risk-taking populations. These populations create the largest averages and greatest potential for profit increases, with highs near 40%. Similarly, of the nine cases with a low or no risk-taking

Table 9: Summary of 15 scenarios (27 cases each) with varying consumer populations.

Percentage γ			Menus	Average Profit Increase (%)
0.5	1.0	1.5	Offered	*if* Menu Offered
H	L	0	8/30	7.77 (0.08-16.67)
H	0	L	12/30	17.34 (4.51-29.85)
H	L	L	5/15	18.80 (5.76-25.91)
M	M	0	4/15	18.05 (11.56-37.51)
M	0	M	8/15	8.48 (0.79-20.32)
M	L	L	11/30	4.94 (1.45-17.40)
L	L	H	1/15	3.20 (3.20-3.20)
L	L	M	13/30	3.03 (0.93-8.52)
L	M	L	9/30	2.52 (0.02-9.68)
L	H	L	4/15	2.88 (2.51-3.00)
L	H	0	8/30	8.09 (3.00-16.73)
L	0	H	7/30	3.38 (0.98-8.71)
0	H	L	0/30	0.00
0	M	M	1/15	1.45 (1.45-1.45)
0	L	H	7/30	5.48 (2.30-10.98)

population, only two increase profits more than 5%.

It should be noted that over all of these scenarios, the majority of contracts offered, either separately or via a menu, are either nondeferrable or nonrenewable and nondeferrable. As seen in the Appendix, there are situations in which strictly nonrenewable contract were offered to risk-averse consumers.

In total, over 400 scenarios have been examined with differing population distributions, failure rates, repair costs, and price limits. Yet, it seems fairly clear that the opportunity to offer, and profit from, menus of warranty contracts relies on the population having a risk-taking segment. If the population is defined as risk-neutral or risk-averse, then a single warranty contract may suffice. This is a somewhat surprising result, as it would seem unlikely that a marketing strategy would be aimed at risk-taking consumers (as they provide the least profit). However, the benefit of a warranty contract menu is to attract less likely consumers, i.e. risk-takers, as risk-averse consumers are already more likely to purchase an extended warranty.

5 Conclusions and Directions for Future Research

This paper presents dynamic programming formulations to determine optimal consumer strategies when extended warranties are offered after the base warranty expires. Specifically, unrestricted, nondeferrable, nonrenewable, and both nondeferrable and nonrenewable warranty designs are analyzed assuming the consumer's risk preference (taking, neutral or averse), the product's failure rate, repair costs, and extended warranty prices are known. This information is then used in a game theoretic framework to determine optimal pricing strategies from the viewpoint of a manufacturer or third party provider. It is shown that consumers prefer nondeferrable or unrestricted contracts while providers prefer nonrenewable or combined (nondeferrable and nonrenewable) policies, as they limit the exposure to high repair costs. Also, it is clear that risk-averse consumers provide significantly greater profit potential to other types of consumers. This helps explain why extended warranties tend to be relatively expensive when compared to the product's purchase price (nearly 50% in many cases).

This paper also presents an integer programming approach, which utilizes input information from the dynamic programming solutions, to design a menu of warranty contracts for a consumer population of heterogeneous risk preferences. An analysis of over 400 test problems showed the potential to increase profits by as much as 38% when compared to offering only one warranty contract to the public. While risk-averse consumers provide the greatest amount of profit, profit increases from menu offerings tend to be the result of providing contracts that appeal to less likely, risk-taking consumers. This is an interesting insight as one would not normally expect to increase potential profits from the risk-taking segment of the population.

While there has been significant research in warranty, and extended warranty, analysis, it is believed that the frameworks (dynamic and integer programming) presented here provide a basis for continued research. First, while we analyze a number of warranty contracts here, there are obviously numerous others to pursue. Of recent interest is the ability to pay consumers (either fully or partially) at the end of a warranty cycle if the warranty is not utilized. This would require that failures be tracked in the state space (if dynamic programming is utilized), which may pose computational difficulties. However, this additional information may allow failure rate estimates to be updated (i.e., in a Bayesian framework).

Second, we assumed constant costs and prices to facilitate finding optimal policies, but it would be interesting to analyze policies where the prices can be changed systematically. This may be the result of changing failure rates with time due to technological change. Interestingly, technological change could be modeled as improving failure rates with new releases over time or as degrading failure rates due to added complexities of products (which may provide more utility to the user). These more complicated scenarios, especially those which explicitly address technological change, provide significant avenues for future research. As technological change is tied to the design process and new product innovation, it is critical to a company's marketing strategy. Thus, warranty designs and offerings should be considered concurrently with the design cycle.

Finally, another computational challenge of interest is to include usage in the model - not just the product's age. This is critical for products such as automobiles where warranties are designed around the age and cumulative utilization of the vehicle. If usage for a consumer is highly variable, this can lead to complications in a dynamic programming formulation where this must be tracked. However, as noted in the replacement analysis literature (see Bethuyne [2] or Hartman [10], for example), this is a critical input variable.

6 Acknowledgments

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7 Appendix

This section provides the individual sensitivity analysis results on the failure rate ($\beta = 1.5, \theta = 2.215$ and $\beta = 2.0, \theta = 2.257$), the repair cost (\$100 and \$150), and the EW price limit (no price limit, \$600, \$400, and \$200) on different consumer populations as shown in Tables 10 through 24. These results are summarized in Tables 25 through 28 according to consumer populations and EW price limits for different combinations of the failure rate and the repair cost.

Table 10: Third party provider menus with $\theta = 2.215, \beta = 1.5, C_R = \$100$, and no price limit.

Case	Percentage γ			Single EW	Profit	EW menu			Profit	Increase
	0.5	1.0	1.5			0.5	1.0	1.5		(%)
1	100	0	0	ND1_2 $150	$1473					
2	0	100	0	ND1_2 $175	$3162					
3	0	0	100	ND2_3 $450	$5647					
4	90	10	0			ND1_2 $150			$1538	
5	70	30	0			ND1_2 $150			$1669	
6	50	50	0	ND1_2 $150	$1800	NRND_3 $175	ND1_2 $175		$2008	11.56
7	90	0	10			ND1_2 $150		ND1_2 $150	$1538	
8	70	0	30	ND2_3 $450	$1694	NRND_3 $175		ND1_2 $225	$2167	27.91
9	50	0	50	ND2_3 $450	$2824	NRND_3 $175		ND1_2 $225	$3043	7.77
10	0	90	10				ND1_2 $175		$3162	
11	0	70	30				ND1_2 $200		$3364	
12	0	50	50				ND1_2 $200		$3602	
13	10	90	0	ND1_2 $175	$2846	NRND_3 $175	ND1_2 $175		$2931	3.00
14	30	70	0	ND1_2 $175	$2213	NRND_3 $175	ND1_2 $175		$2469	11.56
15	0	10	90				-	ND2_3 $450	$5802	
16	0	30	70	ND2_3 $450	$3953		NRND_3 $225	ND1_2 $225	$4126	4.39
17	10	0	90			-		NRND2_3 $450	$5082	
18	30	0	70			-		NRND2_3 $450	$3953	
19	80	10	10			ND1_2 $150			$1604	
20	60	20	20	ND1_2 $150	$1734	NRND_3 $175	ND1_2 $175		$1777	2.44
21	40	30	30	ND1_2 $200	$2161	NRND_3 $175	ND1_2 $175		$2238	3.57
22	10	10	80			-	-	ND2_3 $450	$4518	
23	20	20	60	ND2_3 $450	$3388	NRND_3 $175		ND1_2 $225	$3481	2.74
24	30	30	40	ND1_2 $200	$2581	NRND_3 $175		ND1_2 $225	$2605	0.93
25	10	80	10	ND1_2 $175	$2846	NRND_3 $175	ND1_2 $175		$2931	3.00
26	20	60	20	ND1_2 $200	$2644	NRND_3 $175	ND1_2 $175		$2700	2.14
27	30	40	30	ND1_2 $200	$2462	NRND_3 $175	ND1_2 $175		$2469	0.30

Table 11: Third party provider menus with $\theta = 2.215, \beta = 1.5, C_R = \$100$, and a \$400 price limit.

Case	Percentage γ			Single EW	Profit	EW menu			Profit	Increase
	0.5	1.0	1.5			0.5	1.0	1.5		(%)
1	100	0	0	ND1_2 $150	$1473					
2	0	100	0	ND1_2 $175	$3162					
3	0	0	100	ND1_3 $400	$5474					
4	90	10	0			ND1_2 $150			$1538	
5	70	30	0			ND1_2 $150			$1669	
6	50	50	0	ND1_2 $150	$1800	NRND_3 $175	ND1_2 $175		$2008	11.56
7	90	0	10			ND1_2 $150		ND1_2 $150	$1538	
8	70	0	30	ND1_2 $150	$1669	NRND_3 $175		ND1_2 $225	$2167	29.85
9	50	0	50	ND1_3 $400	$2737	NRND_3 $175		ND1_2 $225	$3043	11.18
10	0	90	10				ND1_2 $175		$3162	
11	0	70	30				ND1_2 $200		$3364	
12	0	50	50				ND1_2 $200		$3602	
13	10	90	0	ND1_2 $175	$2846	NRND_3 $175	ND1_2 $175		$2931	3.00
14	30	70	0	ND1_2 $175	$2213	NRND_3 $175	ND1_2 $175		$2469	11.56
15	0	10	90				-	ND1_3 $400	$4926	
16	0	30	70	ND1_2 $200	$3840		NRND_3 $225	ND1_2 $225	$4126	7.46
17	10	0	90			-		ND1_3 $400	$4927	
18	30	0	70	ND1_3 $400	$3832	NRND_3 $175		ND1_2 $225	$3919	2.28
19	80	10	10			ND1_2 $150			$1604	
20	60	20	20	ND1_2 $150	$1734	NRND_3 $175	ND1_2 $175		$1777	2.44
21	40	30	30	ND1_2 $200	$2161	NRND_3 $175	ND1_2 $175		$2238	3.57
22	10	10	80			-	-	ND1_3 $400	$4379	
23	20	20	60	ND1_3 $400	$3284	NRND_3 $175		ND1_2 $225	$3481	5.99
24	30	30	40	ND1_2 $200	$2581	NRND_3 $175		ND1_2 $225	$2605	0.93
25	10	80	10	ND1_2 $175	$2846	NRND_3 $175	ND1_2 $175		$2931	3.00
26	20	60	20	ND1_2 $200	$2644	NRND_3 $175	ND1_2 $175		$2700	2.14
27	30	40	30	ND1_2 $200	$2462	NRND_3 $175	ND1_2 $175		$2469	0.30

Table 12: Third party provider menu with $\theta = 2.215, \beta = 1.5, C_R = \$100$, and a \$200 price limit.

Case	Percentage γ			Single EW	Profit	EW menu			Profit	Increase
	0.5	1.0	1.5			0.5	1.0	1.5		(%)
1	100	0	0	ND1_2 $150	$1473					
2	0	100	0	ND1_2 $175	$3162					
3	0	0	100	ND1_2 $200	$4197					
4	90	10	0			ND1_2 $150			$1538	
5	70	30	0			ND1_2 $150			$1669	
6	50	50	0	ND1_2 $150	$1800	NRND_3 $175	ND1_2 $175		$2008	11.56
7	90	0	10			ND1_2 $150		ND1_2 $150	$1538	
8	70	0	30	ND1_2 $150	$1669	NRND_3 $175		ND1_2 $200	$1856	11.22
9	50	0	50	ND1_2 $200	$2099	NRND_3 $175		ND1_2 $200	$2525	20.32
10	0	90	10				ND1_2 $175		$3162	
11	0	70	30				ND1_2 $200		$3364	
12	0	50	50				ND1_2 $200		$3602	
13	10	90	0	ND1_2 $175	$2846	NRND_3 $175	ND1_2 $175		$2931	3.00
14	30	70	0	ND1_2 $175	$2213	NRND_3 $175	ND1_2 $175		$2469	11.56
15	0	10	90				-	ND1_2 $200	$4078	
16	0	30	70				-	ND1_2 $200	$3840	
17	10	0	90	ND1_2 $200	$3777	NRND_3 $175		ND1_2 $200	$3862	2.26
18	30	0	70	ND1_2 $200	$2938	NRND_3 $175		ND1_2 $200	$3194	8.71
19	80	10	10			ND1_2 $150			$1604	
20	60	20	20	ND1_2 $150	$1734	NRND_3 $175	ND1_2 $175		$1777	2.44
21	40	30	30	ND1_2 $200	$2161	NRND_3 $175	ND1_2 $175		$2238	3.57
22	10	10	80			-	-	ND1_2 $200	$3658	
23	20	20	60			-	-	ND1_2 $200	$3120	
24	30	30	40			-	-	ND1_2 $200	$2581	
25	10	80	10	ND1_2 $175	$2846	NRND_3 $175	ND1_2 $175		$2931	3.00
26	20	60	20	ND1_2 $200	$2644	NRND_3 $175	ND1_2 $175		$2700	2.14
27	30	40	30	ND1_2 $200	$2462	NRND_3 $175	ND1_2 $175		$2469	0.30

Table 13: Third party provider menus with $\theta = 2.257, \beta = 2.0, C_R = \$100$, and no price limit.

Case	Percentage γ			Single EW	Profit	EW menu			Profit	Increase
	0.5	1.0	1.5			0.5	1.0	1.5		(%)
1	100	0	0	ND1_2 $250	$1153					
2	0	100	0	ND1_3 $575	$6219					
3	0	0	100	ND1_3 $825	$11935					
4	90	10	0	ND1_2 $250	$1231	NRND_3 $250	ND1_3 $500		$1290	4.83
5	70	30	0	ND1_3 $575	$1866	NRND_2 $150	ND1_3 $550		$2072	11.08
6	50	50	0			-	ND1_3 $575		$3110	
7	90	0	10	ND1_2 $250	$1231	ND1_3 $575		NR5_3 $475	$1455	18.23
8	70	0	30			-		ND1_3 $825	$3581	
9	50	0	50			-		ND1_3 $825	$5968	
10	0	90	10				ND1_3 $575		$6219	
11	0	70	30				ND1_3 $575		$6219	
12	0	50	50	ND1_3 $575	$6219		ND2_3 $650	ND1_2 $425	$6309	1.45
13	10	90	0			-	ND1_3 $575		$5597	
14	30	70	0			-	ND1_3 $575		$4353	
15	0	10	90				-	ND1_3 $825	$10742	
16	0	30	70				-	ND1_3 $825	$8355	
17	10	0	90			-		ND1_3 $825	$10742	
18	30	0	70			-		ND1_3 $825	$8355	
19	80	10	10	ND1_2 $250	$1308	NRND_3 $250	ND1_3 $500		$1647	25.91
20	60	20	20	ND1_3 $575	$2487	NRND_2 $150	ND2_3 $625	ND1_3 $575	$2608	4.86
21	40	30	30	ND1_3 $575	$3731	-	ND2_3 $650	ND1_2 $425	$3785	1.45
22	10	10	80			-	-	ND1_3 $825	$9548	
23	20	20	60			-	-	ND1_3 $825	$7161	
24	30	30	40			-	-	ND1_3 $825	$4774	
25	10	80	10			-	ND1_3 $575		$5597	
26	20	60	20			-	ND1_3 $575		$4975	
27	30	40	30	ND1_3 $575	$4353	-	ND2_3 $650	ND1_2 $425	$4354	0.02

Table 14: Third party provider menus with $\theta = 2.257, \beta = 2.0, C_R = \$100$, and a \$600 price limit.

Case	Percentage γ			Single EW	Profit	EW menu			Profit	Increase
	0.5	1.0	1.5			0.5	1.0	1.5		(%)
1	100	0	0	ND1_2 \$250	\$1153					
2	0	100	0	ND1_3 \$575	\$6219					
3	0	0	100	ND1_2 \$575	\$11824					
4	90	10	0	ND1_2 \$250	\$1231	NRND_3 \$250	ND1_3 \$500		\$1290	4.83
5	70	30	0	ND1_3 \$575	\$1866	NRND_2 \$150	ND1_3 \$550		\$2072	11.08
6	50	50	0			-	ND1_3 \$575		\$3110	
7	90	0	10	ND1_2 \$250	\$1231	ND1_3 \$575		NR5_3 \$475	\$1455	18.23
8	70	0	30			-		ND1_2 \$575	\$3547	
9	50	0	50			-		ND1_2 \$575	\$5912	
10	0	90	10				ND1_3 \$575		\$6219	
11	0	70	30				ND1_3 \$575		\$6219	
12	0	50	50				ND1_3 \$575		\$6219	
13	10	90	0			-	ND1_3 \$575		\$5597	
14	30	70	0			-	ND1_3 \$575		\$4353	
15	0	10	90				-	ND1_2 \$575	\$10642	
16	0	30	70				-	ND1_2 \$575	\$8277	
17	10	0	90			-		ND1_2 \$575	\$10642	
18	30	0	70			-		ND1_2 \$575	\$8277	
19	80	10	10	ND1_2 \$250	\$1308	NRND_3 \$250	ND1_3 \$500		\$1647	25.91
20	60	20	20	ND1_3 \$575	\$2487	NRND_2 \$150	ND1_3 \$550		\$2583	3.84
21	40	30	30				ND1_2 \$575		\$4354	
22	10	10	80			-	-	ND1_2 \$575	\$9459	
23	20	20	60			-	-	ND1_2 \$575	\$7094	
24	30	30	40			-	-	ND1_2 \$575	\$4730	
25	10	80	10			-	ND1_3 \$575		\$5597	
26	20	60	20			-	ND1_3 \$575		\$4975	
27	30	40	30			-	ND1_3 \$575		\$4353	

Table 15: Third party menus with $\theta = 2.257, \beta = 2.0, C_R = \$100$, and a \$400 price limit.

Case	Percentage γ			Single EW	Profit	EW menu			Profit	Increase
	0.5	1.0	1.5			0.5	1.0	1.5		(%)
1	100	0	0	ND1_2 $250	$1153					
2	0	100	0	ND1_2 $400	$6117					
3	0	0	100	NR5_2 $400	$6138					
4	90	10	0	ND1_2 $250	$1231	NRND_3 $250	ND1_2 $350		$1288	4.69
5	70	30	0	ND1_2 $400	$1835	NRND_3 $250	ND1_2 $350		$1999	8.94
6	50	50	0			-	ND1_2 $400		$3059	
7	90	0	10	ND1_2 $250	$1231	NRND_3 $250		ND1_2 $375	$1370	11.31
8	70	0	30	NR5_2 $400	$1841	NRND_3 $250		ND1_2 $375	$2244	21.85
9	50	0	50	NR5_2 $400	$3069	NRND_2 $150		NR5_2 $400	$3339	8.80
10	0	90	10				ND1_2 $400		$6117	
11	0	70	30				ND1_2 $400		$6117	
12	0	50	50				ND1_2 $400		$6117	
13	10	90	0			-	ND1_2 $400		$5505	
14	30	70	0			-	ND1_2 $400		$4282	
15	0	10	90				ND1_2 $400		$6117	
16	0	30	70				ND1_2 $400		$6117	
17	10	0	90	NR5_2 $400	$5524	NRND_2 $150		NR5_2 $400	$5578	0.98
18	30	0	70	NR5_2 $400	$4297	NRND_2 $150		NR5_2 $400	$4459	3.77
19	80	10	10	ND1_2 $250	$1308	NRND_3 $250	ND1_2 $350		$1644	25.63
20	60	20	20				ND1_2 $400		$2447	
21	40	30	30				ND1_2 $400		$3670	
22	10	10	80				ND1_2 $400		$5505	
23	20	20	60				ND1_2 $400		$4894	
24	30	30	40				ND1_2 $400		$4282	
25	10	80	10				ND1_2 $400		$5504	
26	20	60	20				ND1_2 $400		$4894	
27	30	40	30				ND1_2 $400		$4282	

Table 16: Third party provider menus with $\theta = 2.257, \beta = 2.0, C_R = \$100$, and a \$200 price limit.

Case	Percentage γ			Single EW	Profit	EW menu			Profit	Increase (%)
	0.5	1.0	1.5			0.5	1.0	1.5		
1	100	0	0	NRND_2 $150	$540					
2	0	100	0	NRND_2 $200	$1408					
3	0	0	100	NRND_2 $200	$1958					
4	90	10	0			NRND_2 $150			$551	
5	70	30	0			NRND_2 $150			$572	
6	50	50	0			NRND_2 $200			$704	
7	90	0	10			NRND_2 $150		NRND_2 $150	$572	
8	70	0	30			NRND_2 $150		NRND_2 $150	$636	
9	50	0	50			NRND_2 $200		NRND_2 $200	$979	
10	0	90	10				NRND_2 $200		$1463	
11	0	70	30				NRND_2 $200		$1573	
12	0	50	50				NRND_2 $200		$1683	
13	10	90	0			-	NRND_2 $200		$1267	
14	30	70	0			-	NRND_2 $200		$986	
15	0	10	90				NRND_2 $200		$1903	
16	0	30	70				NRND_2 $200		$1793	
17	10	0	90			NRND_2 $200		NRND_2 $200	$1762	
18	30	0	70			NRND_2 $200		NRND_2 $200	$1371	
19	80	10	10			NRND_2 $150			$583	
20	60	20	20			-	NRND_2 $200		$673	
21	40	30	30			-	NRND_2 $200		$1010	
22	10	10	80			-	NRND_2 $200		$1707	
23	20	20	60			-	NRND_2 $200		$1456	
24	30	30	40			-	NRND_2 $200		$1206	
25	10	80	10			-	NRND_2 $200		$1322	
26	20	60	20			-	NRND_2 $200		$1236	
27	30	40	30			-	NRND_2 $200		$1151	

Table 17: Third party provider menus with $\theta = 2.215, \beta = 1.5, C_R = \$150$, and no price limit.

Case	Percentage γ			Single EW	Profit	EW menu			Profit	Increase
	0.5	1.0	1.5			0.5	1.0	1.5		(%)
1	100	0	0	ND1_2 $225	$2350					
2	0	100	0	ND1_2 $300	$4795					
3	0	0	100	ND1_2 $425	$8871					
4	90	10	0			ND1_2 $225			$2434	
5	70	30	0			ND1_2 $225			$2602	
6	50	50	0			ND1_2 $225			$2770	
7	90	0	10			ND1_2 $225		ND1_2 $225	$2434	
8	70	0	30	ND1_2 $425	$2661	NRND_2 $175		ND1_2 $350	$3126	17.47
9	50	0	50	ND1_2 $425	$4436	NRND_2 $175		ND1_2 $350	$4624	4.24
10	0	90	10				ND1_2 $300		$4945	
11	0	70	30				ND1_2 $300		$5245	
12	0	50	50				ND1_3 $575		$5546	
13	10	90	0			-	ND1_2 $300		$4316	
14	30	70	0			-	ND1_2 $300		$3357	
15	0	10	90				-	ND1_2 $425	$7984	
16	0	30	70	ND1_2 $425	$6210		NR6_3 $500	ND1_2 $350	$6532	5.19
17	10	0	90			-		ND1_2 $425	$7984	
18	30	0	70			-		ND1_2 $425	$6210	
19	80	10	10				ND1_2 $225		$2518	
20	60	20	20				ND1_2 $225		$2685	
21	40	30	30			-	ND1_2 $300		$3327	
22	10	10	80			-	-	ND1_2 $425	$7097	
23	20	20	60	ND1_2 $425	$5323	-	ND1_2 $250	NR6_3 $500	$5470	2.78
24	30	30	40	ND1_2 $300	$3957		NR6_3 $500	ND1_2 $350	$4022	1.65
25	10	80	10			-	ND1_2 $300		$4466	
26	20	60	20			-	ND1_2 $300		$4136	
27	30	40	30			-	ND1_2 $300		$3807	

Table 18: Third party provider menus with $\theta = 2.215, \beta = 1.5, C_R = \$150$, and a \$600 price limit.

<table>
<tr><th rowspan="2">Case</th><th colspan="3">Percentage γ</th><th rowspan="2">Single EW</th><th rowspan="2">Profit</th><th colspan="3">EW menu</th><th rowspan="2">Profit</th><th rowspan="2">Increase (%)</th></tr>
<tr><th>0.5</th><th>1.0</th><th>1.5</th><th>0.5</th><th>1.0</th><th>1.5</th></tr>
<tr><td>1</td><td>100</td><td>0</td><td>0</td><td>ND1_2 $225</td><td>$2350</td><td></td><td></td><td></td><td></td><td></td></tr>
<tr><td>2</td><td>0</td><td>100</td><td>0</td><td>ND1_2 $300</td><td>$4795</td><td></td><td></td><td></td><td></td><td></td></tr>
<tr><td>3</td><td>0</td><td>0</td><td>100</td><td>ND1_2 $425</td><td>$8871</td><td></td><td></td><td></td><td></td><td></td></tr>
<tr><td>4</td><td>90</td><td>10</td><td>0</td><td></td><td></td><td colspan="2">ND1_2 $225</td><td></td><td>$2434</td><td></td></tr>
<tr><td>5</td><td>70</td><td>30</td><td>0</td><td></td><td></td><td colspan="2">ND1_2 $225</td><td></td><td>$2602</td><td></td></tr>
<tr><td>6</td><td>50</td><td>50</td><td>0</td><td></td><td></td><td colspan="2">ND1_2 $225</td><td></td><td>$2770</td><td></td></tr>
<tr><td>7</td><td>90</td><td>0</td><td>10</td><td></td><td></td><td>ND1_2 $225</td><td></td><td>ND1_2 $225</td><td>$2434</td><td></td></tr>
<tr><td>8</td><td>70</td><td>0</td><td>30</td><td>ND1_2 $425</td><td>$2661</td><td>NRND_2 $175</td><td></td><td>ND1_2 $350</td><td>$3126</td><td>17.47</td></tr>
<tr><td>9</td><td>50</td><td>0</td><td>50</td><td>ND1_2 $425</td><td>$4436</td><td>NRND_2 $175</td><td></td><td>ND1_2 $350</td><td>$4624</td><td>4.24</td></tr>
<tr><td>10</td><td>0</td><td>90</td><td>10</td><td></td><td></td><td></td><td colspan="2">ND1_2 $300</td><td>$4945</td><td></td></tr>
<tr><td>11</td><td>0</td><td>70</td><td>30</td><td></td><td></td><td></td><td colspan="2">ND1_2 $300</td><td>$5245</td><td></td></tr>
<tr><td>12</td><td>0</td><td>50</td><td>50</td><td></td><td></td><td></td><td colspan="2">ND1_3 $575</td><td>$5546</td><td></td></tr>
<tr><td>13</td><td>10</td><td>90</td><td>0</td><td></td><td></td><td>-</td><td>ND1_2 $300</td><td></td><td>$4316</td><td></td></tr>
<tr><td>14</td><td>30</td><td>70</td><td>0</td><td></td><td></td><td>-</td><td>ND1_2 $300</td><td></td><td>$3357</td><td></td></tr>
<tr><td>15</td><td>0</td><td>10</td><td>90</td><td></td><td></td><td></td><td>-</td><td>ND1_2 $425</td><td>$7984</td><td></td></tr>
<tr><td>16</td><td>0</td><td>30</td><td>70</td><td>ND1_2 $425</td><td>$6210</td><td></td><td>NR6_3 $500</td><td>ND1_2 $350</td><td>$6532</td><td>5.19</td></tr>
<tr><td>17</td><td>10</td><td>0</td><td>90</td><td></td><td></td><td>-</td><td></td><td>ND1_2 $425</td><td>$7984</td><td></td></tr>
<tr><td>18</td><td>30</td><td>0</td><td>70</td><td></td><td></td><td>-</td><td></td><td>ND1_2 $425</td><td>$6210</td><td></td></tr>
<tr><td>19</td><td>80</td><td>10</td><td>10</td><td></td><td></td><td colspan="3">ND1_2 $225</td><td>$2518</td><td></td></tr>
<tr><td>20</td><td>60</td><td>20</td><td>20</td><td></td><td></td><td colspan="3">ND1_2 $225</td><td>$2685</td><td></td></tr>
<tr><td>21</td><td>40</td><td>30</td><td>30</td><td></td><td></td><td>-</td><td colspan="2">ND1_2 $300</td><td>$3327</td><td></td></tr>
<tr><td>22</td><td>10</td><td>10</td><td>80</td><td></td><td></td><td>-</td><td>-</td><td>ND1_2 $425</td><td>$7097</td><td></td></tr>
<tr><td>23</td><td>20</td><td>20</td><td>60</td><td>ND1_2 $425</td><td>$5322</td><td>-</td><td>ND1_2 $250</td><td>NR6_3 $500</td><td>$5470</td><td>2.78</td></tr>
<tr><td>24</td><td>30</td><td>30</td><td>40</td><td>ND1_2 $300</td><td>$3957</td><td></td><td>NR6_3 $500</td><td>ND1_2 $350</td><td>$4022</td><td>1.65</td></tr>
<tr><td>25</td><td>10</td><td>80</td><td>10</td><td></td><td></td><td>-</td><td colspan="2">ND1_2 $300</td><td>$4466</td><td></td></tr>
<tr><td>26</td><td>20</td><td>60</td><td>20</td><td></td><td></td><td>-</td><td colspan="2">ND1_2 $300</td><td>$4136</td><td></td></tr>
<tr><td>27</td><td>30</td><td>40</td><td>30</td><td></td><td></td><td>-</td><td colspan="2">ND1_2 $300</td><td>$3807</td><td></td></tr>
</table>

Table 19: Third party provider menus with $\theta = 2.215, \beta = 1.5, C_R = \$150$, and a \$400 price limit.

<table>
<tr><th rowspan="2">Case</th><th colspan="3">Percentage γ</th><th rowspan="2">Single EW</th><th rowspan="2">Profit</th><th colspan="3">EW menu</th><th rowspan="2">Profit</th><th>Increase</th></tr>
<tr><th>0.5</th><th>1.0</th><th>1.5</th><th>0.5</th><th>1.0</th><th>1.5</th><th>(%)</th></tr>
<tr><td>1</td><td>100</td><td>0</td><td>0</td><td>ND1_2 $225</td><td>$2350</td><td></td><td></td><td></td><td></td><td></td></tr>
<tr><td>2</td><td>0</td><td>100</td><td>0</td><td>ND1_2 $300</td><td>$4795</td><td></td><td></td><td></td><td></td><td></td></tr>
<tr><td>3</td><td>0</td><td>0</td><td>100</td><td>ND1_2 $350</td><td>$8367</td><td></td><td></td><td></td><td></td><td></td></tr>
<tr><td>4</td><td>90</td><td>10</td><td>0</td><td></td><td></td><td colspan="2">ND1_2 $225</td><td></td><td>$2434</td><td></td></tr>
<tr><td>5</td><td>70</td><td>30</td><td>0</td><td></td><td></td><td colspan="2">ND1_2 $225</td><td></td><td>$2602</td><td></td></tr>
<tr><td>6</td><td>50</td><td>50</td><td>0</td><td></td><td></td><td colspan="2">ND1_2 $225</td><td></td><td>$2770</td><td></td></tr>
<tr><td>7</td><td>90</td><td>0</td><td>10</td><td></td><td></td><td>ND1_2 $225</td><td></td><td>ND1_2 $225</td><td>$2434</td><td></td></tr>
<tr><td>8</td><td>70</td><td>0</td><td>30</td><td>ND1_2 $325</td><td>$2618</td><td>NRND_2 $175</td><td></td><td>ND1_2 $350</td><td>$3126</td><td>19.43</td></tr>
<tr><td>9</td><td>50</td><td>0</td><td>50</td><td>ND1_2 $350</td><td>$4184</td><td>NRND_2 $175</td><td></td><td>ND1_2 $350</td><td>$4624</td><td>10.52</td></tr>
<tr><td>10</td><td>0</td><td>90</td><td>10</td><td></td><td></td><td></td><td colspan="2">ND1_2 $300</td><td>$4945</td><td></td></tr>
<tr><td>11</td><td>0</td><td>70</td><td>30</td><td></td><td></td><td></td><td colspan="2">ND1_2 $300</td><td>$5245</td><td></td></tr>
<tr><td>12</td><td>0</td><td>50</td><td>50</td><td></td><td></td><td></td><td colspan="2">ND1_2 $300</td><td>$5546</td><td></td></tr>
<tr><td>13</td><td>10</td><td>90</td><td>0</td><td></td><td></td><td>-</td><td>ND1_2 $300</td><td></td><td>$4316</td><td></td></tr>
<tr><td>14</td><td>30</td><td>70</td><td>0</td><td></td><td></td><td>-</td><td>ND1_2 $300</td><td></td><td>$3357</td><td></td></tr>
<tr><td>15</td><td>0</td><td>10</td><td>90</td><td>ND1_2 $350</td><td>$7530</td><td></td><td>NRND_3 $325</td><td>ND1_2 $350</td><td>$7745</td><td>2.85</td></tr>
<tr><td>16</td><td>0</td><td>30</td><td>70</td><td>ND1_2 $350</td><td>$5857</td><td></td><td>NRND_3 $325</td><td>ND1_2 $350</td><td>$6500</td><td>10.98</td></tr>
<tr><td>17</td><td>10</td><td>0</td><td>90</td><td>ND1_2 $350</td><td>$7530</td><td>NRND_2 $175</td><td></td><td>ND1_2 $350</td><td>$7618</td><td>1.17</td></tr>
<tr><td>18</td><td>30</td><td>0</td><td>70</td><td>ND1_2 $350</td><td>$5857</td><td>NRND_2 $175</td><td></td><td>ND1_2 $350</td><td>$6122</td><td>4.51</td></tr>
<tr><td>19</td><td>80</td><td>10</td><td>10</td><td></td><td></td><td colspan="3">ND1_2 $225</td><td>$2518</td><td></td></tr>
<tr><td>20</td><td>60</td><td>20</td><td>20</td><td></td><td></td><td colspan="3">ND1_2 $225</td><td>$2686</td><td></td></tr>
<tr><td>21</td><td>40</td><td>30</td><td>30</td><td></td><td></td><td>-</td><td colspan="2">ND1_2 $300</td><td>$3327</td><td></td></tr>
<tr><td>22</td><td>10</td><td>10</td><td>80</td><td>ND1_2 $350</td><td>$6694</td><td>-</td><td>NRND_3 $325</td><td>ND1_2 $350</td><td>$6908</td><td>3.20</td></tr>
<tr><td>23</td><td>20</td><td>20</td><td>60</td><td>ND1_2 $350</td><td>$5020</td><td>-</td><td>NRND_3 $325</td><td>ND1_2 $350</td><td>$5448</td><td>8.52</td></tr>
<tr><td>24</td><td>30</td><td>30</td><td>40</td><td></td><td></td><td colspan="3">ND1_2 $300</td><td>$3957</td><td></td></tr>
<tr><td>25</td><td>10</td><td>80</td><td>10</td><td></td><td></td><td colspan="3">ND1_2 $300</td><td>$4466</td><td></td></tr>
<tr><td>26</td><td>20</td><td>60</td><td>20</td><td></td><td></td><td colspan="3">ND1_2 $300</td><td>$4136</td><td></td></tr>
<tr><td>27</td><td>30</td><td>40</td><td>30</td><td></td><td></td><td colspan="3">ND1_2 $300</td><td>$3807</td><td></td></tr>
</table>

Table 20: Third party provider menus with $\theta = 2.215, \beta = 1.5, C_R = \$150$, and a \$200 price limit.

Case	Percentage γ			Single EW	Profit	EW menu			Profit	Increase
	0.5	1.0	1.5			0.5	1.0	1.5		(%)
1	100	0	0	ND1_2 $200	$2154					
2	0	100	0	ND1_2 $200	$2154					
3	0	0	100	NR6_2 $200	$2414					
4	90	10	0			ND1_2 $200			$2154	
5	70	30	0			ND1_2 $200			$2154	
6	50	50	0			ND1_2 $200			$2154	
7	90	0	10			ND1_2 $200		ND1_2 $200	$2154	
8	70	0	30			ND1_2 $200		ND1_2 $200	$2154	
9	50	0	50			ND1_2 $200		ND1_2 $200	$2154	
10	0	90	10				ND1_2 $200		$2154	
11	0	70	30				NR6_2 $200		$2202	
12	0	50	50				NR6_2 $200		$2263	
13	10	90	0			ND1_2 $200			$2154	
14	30	70	0			ND1_2 $200			$2154	
15	0	10	90				NR6_2 $200		$2384	
16	0	30	70				NR6_2 $200		$2323	
17	10	0	90			NR6_2 $200		NR6_2 $200	$2261	
18	30	0	70			ND1_2 $200		ND1_2 $200	$2154	
19	80	10	10			ND1_2 $200			$2154	
20	60	20	20			ND1_2 $200			$2154	
21	40	30	30			ND1_2 $200			$2154	
22	10	10	80			NR6_2 $200			$2230.8	
23	20	20	60			ND1_2 $200			$2154	
24	30	30	40			ND1_2 $200			$2154	
25	10	80	10			ND1_2 $200			$2154	
26	20	60	20			ND1_2 $200			$2154	
27	30	40	30			ND1_2 $200			$2154	

Table 21: Third party provider menus with $\theta = 2.257, \beta = 2.0, C_R = \$150$, and no price limit.

Case	Percentage γ			Single EW	Profit	EW menu			Profit	Increase
	0.5	1.0	1.5			0.5	1.0	1.5		(%)
1	100	0	0	ND1_2 $375	$1729					
2	0	100	0	ND1_3 $800	$7900					
3	0	0	100	ND1_2 $725	$13252					
4	90	10	0			ND1_2 $375			$1740	
5	70	30	0			-	ND1_3 $800		$2370	
6	50	50	0			-	ND1_3 $800		$3950	
7	90	0	10			ND1_2 $375		ND1_2 $375	$1846	
8	70	0	30			-		ND1_2 $375	$3976	
9	50	0	50			-		ND1_2 $375	$6626	
10	0	90	10				ND1_3 $800		$7900	
11	0	70	30				ND1_3 $800		$7900	
12	0	50	50				ND1_3 $800		$7900	
13	10	90	0			-	ND1_3 $800		$7100	
14	30	70	0			-	ND1_3 $800		$5530	
15	0	10	90				-	ND1_2 $725	$11927	
16	0	30	70				-	ND1_2 $725	$9276	
17	10	0	90			-		ND1_2 $725	$11927	
18	30	0	70			-		NR5_2 $600	$9276	
19	80	10	10	ND1_2 $375	$1857	NRND_3 $375	NR6_3 $625	NR5_3 $575	$2057	10.81
20	60	20	20			-	ND1_3 $800		$3160	
21	40	30	30			-	ND1_3 $800		$4740	
22	10	10	80			-	-	ND1_2 $725	$10602	
23	20	20	60			-	-	ND1_2 $725	$7951	
24	30	30	40	ND1_3 $800	$5530	-	ND2_3 $875	ND1_3 $850	$5595	1.18
25	10	80	10			-	ND1_3 $800		$7110	
26	20	60	20			-	ND1_3 $800		$6320	
27	30	40	30			-	ND1_3 $800		$5530	

Table 22: Third party provider menus with $\theta = 2.257, \beta = 2.0, C_R = \$150$, and a $600 price limit.

Case	Percentage γ			Single EW	Profit	EW menu			Profit	Increase
	0.5	1.0	1.5			0.5	1.0	1.5		(%)
1	100	0	0	ND1_2 $375	$1729					
2	0	100	0	ND1_2 $550	$7545					
3	0	0	100	NR5_2 $600	$9206					
4	90	10	0			ND1_2 $375			$1740	
5	70	30	0	ND1_2 $550	2263.5	NRND_3 $375	ND1_2 $450		$2265	0.08
6	50	50	0			-	ND1_2 $550		$3773	
7	90	0	10			ND1_2 $375		ND1_2 $375	$1846	
8	70	0	30	NR5_2 $600	$2762	NRND_2 $225		NR6_2 $475	$3050	10.42
9	50	0	50			-		NR5_2 $600	$4603	
10	0	90	10				ND1_2 $550		$7545	
11	0	70	30				ND1_2 $550		$7545	
12	0	50	50				ND1_2 $550		$7545	
13	10	90	0			-	ND1_2 $550		$6791	
14	30	70	0			-	ND1_2 $550		$5282	
15	0	10	90				NR5_2 $600		$8620	
16	0	30	70	ND1_2 $550	$7545		ND2_2 $600	ND1_2 $575	$7719	2.30
17	10	0	90			-		NR5_2 $600	$8285	
18	30	0	70			-		NR5_2 $600	$6444	
19	80	10	10	ND1_2 $375	$1857	NRND_3 $375	ND1_2 $425		$1964	5.76
20	60	20	20			-	ND1_2 $550		$3018	
21	40	30	30			-	ND1_2 $550		$4527	
22	10	10	80			-	NR5_2 $600		$7699	
23	20	20	60	NR5_2 $600	$6192	-	ND2_2 $600	ND1_2 $575	$6261	1.10
24	30	30	40			-	ND1_2 $550		$5281	
25	10	80	10			-	ND1_2 $550		$6791	
26	20	60	20			-	ND1_2 $550		$6036	
27	30	40	30			-	ND1_3 $550		$5282	

Table 23: Third party provider menus with $\theta = 2.257, \beta = 2.0, C_R = \$150$, and a $400 price limit.

Case	Percentage γ			Single EW	Profit	EW menu			Profit	Increase
	0.5	1.0	1.5			0.5	1.0	1.5		(%)
1	100	0	0	ND1_2 $375	$1729					
2	0	100	0	NR5_2 $400	$3586					
3	0	0	100	NRND_2 $400	$6114					
4	90	10	0			ND1_2 $375			$1740	
5	70	30	0	ND1_2 $375	$1762	NRND_3 $375	NR5_2 $400		$2056	16.67
6	50	50	0	NRND_3 $375	$1813	NRND_3 $375	NR5_2 $400		$2493	37.51
7	90	0	10			ND1_2 $375		ND1_2 $375	$1846	
8	70	0	30	ND1_2 $375	$2079	NRND_2 $225		NR6_2 $400	$2173	4.51
9	50	0	50	NRND_2 $400	$3057	NRND_2 $225		NR6_2 $400	$3081	0.79
10	0	90	10				NR5_2 $400		$3586	
11	0	70	30				NR5_2 $400		$3586	
12	0	50	50				NRND_2 $325		$3599	
13	10	90	0	NR5_2 $400	$3227	NRND_3 $375	NR5_2 $400		$3367	4.34
14	30	70	0	NR5_2 $400	$2510	NRND_3 $375	NR5_2 $400		$2930	16.73
15	0	10	90				-	NRND_2 $400	$5503	
16	0	30	70				-	NRND_2 $400	$4280	
17	10	0	90			-		NRND_2 $400	$5503	
18	30	0	70			-		NRND_2 $400	$4280	
19	80	10	10				ND1_2 $375		$1857	
20	60	20	20	ND1_2 $375	$1984	NRND_3 $375	ND1_2 $400		$2157	8.70
21	40	30	30	NRND_2 $325	$2159	NRND_3 $375	ND1_2 $400		$2535	17.40
22	10	10	80			-	-	NRND_2 $400	$4891	
23	20	20	60	NRND_2 $400	$3668	NRND_2 $225	NR6_2 $400		$3703	1.00
24	30	30	40	NR6_2 $400	$2666	NRND_2 $225	NR6_2 $400		$2882	8.11
25	10	80	10	NR5_2 $400	$3227	NRND_2 $225	NR5_2 $400		$3308	2.51
26	20	60	20	NR5_2 $400	$2869	NRND_2 $225	NR5_2 $400		$3031	5.65
27	30	40	30	NR5_2 $400	$2510	NRND_2 $225	NR5_2 $400		$2753	9.68

Table 24: Third party provider menus with $\theta = 2.257, \beta = 2.0, C_R = \$150$, and a \$200 price limit.

<table>
<tr><th rowspan="2">Case</th><th colspan="3">Percentage γ</th><th rowspan="2">Single EW</th><th rowspan="2">Profit</th><th colspan="3">EW menu</th><th rowspan="2">Profit</th><th>Increase</th></tr>
<tr><th>0.5</th><th>1.0</th><th>1.5</th><th>0.5</th><th>1.0</th><th>1.5</th><th>(%)</th></tr>
<tr><td>1</td><td>100</td><td>0</td><td>0</td><td>NRND_2 $200</td><td>$465</td><td></td><td></td><td></td><td></td><td></td></tr>
<tr><td>2</td><td>0</td><td>100</td><td>0</td><td>NRND_2 $200</td><td>$646</td><td></td><td></td><td></td><td></td><td></td></tr>
<tr><td>3</td><td>0</td><td>0</td><td>100</td><td>NRND_2 $200</td><td>$880</td><td></td><td></td><td></td><td></td><td></td></tr>
<tr><td>4</td><td>90</td><td>10</td><td>0</td><td></td><td></td><td colspan="2">NRND_2 $200</td><td></td><td>$483</td><td></td></tr>
<tr><td>5</td><td>70</td><td>30</td><td>0</td><td></td><td></td><td colspan="2">NRND_2 $200</td><td></td><td>$519</td><td></td></tr>
<tr><td>6</td><td>50</td><td>50</td><td>0</td><td></td><td></td><td colspan="2">NRND_2 $200</td><td></td><td>$556</td><td></td></tr>
<tr><td>7</td><td>90</td><td>0</td><td>10</td><td></td><td></td><td>NRND_2 $200</td><td></td><td>NRND_2 $200</td><td>$507</td><td></td></tr>
<tr><td>8</td><td>70</td><td>0</td><td>30</td><td></td><td></td><td>NRND_2 $200</td><td></td><td>NRND_2 $200</td><td>$590</td><td></td></tr>
<tr><td>9</td><td>50</td><td>0</td><td>50</td><td></td><td></td><td>NRND_2 $200</td><td></td><td>NRND_2 $200</td><td>$673</td><td></td></tr>
<tr><td>10</td><td>0</td><td>90</td><td>10</td><td></td><td></td><td></td><td colspan="2">NRND_2 $200</td><td>$669</td><td></td></tr>
<tr><td>11</td><td>0</td><td>70</td><td>30</td><td></td><td></td><td></td><td colspan="2">NRND_2 $200</td><td>$716</td><td></td></tr>
<tr><td>12</td><td>0</td><td>50</td><td>50</td><td></td><td></td><td></td><td colspan="2">NRND_2 $200</td><td>$763</td><td></td></tr>
<tr><td>13</td><td>10</td><td>90</td><td>0</td><td></td><td></td><td colspan="2">NRND_2 $200</td><td></td><td>$628</td><td></td></tr>
<tr><td>14</td><td>30</td><td>70</td><td>0</td><td></td><td></td><td colspan="2">NRND_2 $200</td><td></td><td>$592</td><td></td></tr>
<tr><td>15</td><td>0</td><td>10</td><td>90</td><td></td><td></td><td></td><td colspan="2">NRND_2 $200</td><td>$857</td><td></td></tr>
<tr><td>16</td><td>0</td><td>30</td><td>70</td><td></td><td></td><td></td><td colspan="2">NRND_2 $200</td><td>$810</td><td></td></tr>
<tr><td>17</td><td>10</td><td>0</td><td>90</td><td></td><td></td><td>NRND_2 $200</td><td></td><td>NRND_2 $200</td><td>$839</td><td></td></tr>
<tr><td>18</td><td>30</td><td>0</td><td>70</td><td></td><td></td><td>NRND_2 $200</td><td></td><td>NRND_2 $200</td><td>$756</td><td></td></tr>
<tr><td>19</td><td>80</td><td>10</td><td>10</td><td></td><td></td><td colspan="3">NRND_2 $200</td><td>$525</td><td></td></tr>
<tr><td>20</td><td>60</td><td>20</td><td>20</td><td></td><td></td><td colspan="3">NRND_2 $200</td><td>$584</td><td></td></tr>
<tr><td>21</td><td>40</td><td>30</td><td>30</td><td></td><td></td><td colspan="3">NRND_2 $200</td><td>$644</td><td></td></tr>
<tr><td>22</td><td>10</td><td>10</td><td>80</td><td></td><td></td><td colspan="3">NRND_2 $200</td><td>$815</td><td></td></tr>
<tr><td>23</td><td>20</td><td>20</td><td>60</td><td></td><td></td><td colspan="3">NRND_2 $200</td><td>$750</td><td></td></tr>
<tr><td>24</td><td>30</td><td>30</td><td>40</td><td></td><td></td><td colspan="3">NRND_2 $200</td><td>$685</td><td></td></tr>
<tr><td>25</td><td>10</td><td>80</td><td>10</td><td></td><td></td><td colspan="3">NRND_2 $200</td><td>$651</td><td></td></tr>
<tr><td>26</td><td>20</td><td>60</td><td>20</td><td></td><td></td><td colspan="3">NRND_2 $200</td><td>$657</td><td></td></tr>
<tr><td>27</td><td>30</td><td>40</td><td>30</td><td></td><td></td><td colspan="3">NRND_2 $200</td><td>$662</td><td></td></tr>
</table>

Table 25: Summary of third party provider menus with $\theta = 2.215, \beta = 1.5$, and $C_R = \$100$.

Consumer type			No Limit		\$400 Limit		\$200 Limit	
0.5	1.0	1.5	Menus Offered	Average Profit Increase (%) *if* Menu Offered	Menus Offered	Average Profit Increase (%) *if* Menu Offered	Menus Offered	Average Profit Increase (%) *if* Menu Offered
H	L	0	0/2	0	0/2	0	0/2	0
H	0	L	1/2	27.91	1/2	29.85	1/2	11.22
H	L	L	0/1	0	0/1	0	0/1	0
M	M	0	1/1	11.56	1/1	11.56	1/1	11.56
M	0	M	1/1	7.77	1/1	11.18	1/1	20.32
M	L	L	2/2	3.01	2/2	3.01	2/2	3.01
L	L	H	0/1	0	0/1	0	0/1	0
L	L	M	2/2	1.84	2/2	3.46	0/2	0
L	M	L	2/2	1.22	2/2	1.22	2/2	1.22
L	H	L	1/1	3	1/1	3	1/1	3
L	H	0	2/2	7.28	2/2	7.28	2/2	7.28
L	0	H	0/2	0	1/2	2.28	2/2	5.48
0	H	L	0/2	0	0/2	0	0/2	0
0	M	M	0/1	0	0/1	0	0/1	0
0	L	H	1/2	4.39	1/2	7.46	0/2	0

Table 26: Summary of third party provider menus with $\theta = 2.257, \beta = 2.0$, and $C_R = \$100$.

Consumer type			No Limit		\$600 Limit		\$400 Limit		\$200 Limit	
0.5	1.0	1.5	Menus Offered	Average Profit Increase (%) *if* Menu Offered	Menus Offered	Average Profit Increase (%) *if* Menu Offered	Menus Offered	Average Profit Increase (%) *if* Menu Offered	Menus Offered	Average Profit Increase (%) *if* Menu Offered
H	L	0	2/2	7.96	2/2	7.96	2/2	6.82	0/2	0
H	0	L	1/2	18.23	1/2	18.23	2/2	16.85	0/2	0
H	L	L	1/1	25.91	1/1	25.91	1/1	25.63	0/1	0
M	M	0	0/1	0	0/1	0	0/1	0	0/1	0
M	0	M	0/1	0	0/1	0	1/1	8.8	0/1	0
M	L	L	2/2	3.15	1/2	3.84	0/2	0	0/2	0
L	L	H	0/1	0	0/1	0	0/1	0	0/1	0
L	L	M	0/2	0	0/2	0	0/2	0	0/2	0
L	M	L	1/2	0.02	0/2	0	0/2	0	0/2	0
L	H	L	0/1	0	0/1	0	0/1	0	0/1	0
L	H	0	0/2	0	0/2	0	0/2	0	0/2	0
L	0	H	0/2	0	0/2	0	2/2	2.37	0/2	0
0	H	L	0/2	0	0/2	0	0/2	0	0/2	0
0	M	M	1/1	1.45	0/1	0	0/1	0	0/1	0
0	L	H	0/2	0	0/2	0	0/2	0	0/2	0

Table 27: Summary of third party provider menus with $\theta = 2.215, \beta = 1.5$, and $C_R = \$150$.

Consumer type			No Limit		\$600 Limit		\$400 Limit		\$200 Limit	
0.5	1.0	1.5	Menus Offered	Average Profit Increase (%) *if* Menu Offered	Menus Offered	Average Profit Increase (%) *if* Menu Offered	Menus Offered	Average Profit Increase (%) *if* Menu Offered	Menus Offered	Average Profit Increase (%) *if* Menu Offered
H	L	0	0/2	0	0/2	0	0/2	0	0/2	0
H	0	L	1/2	17.47	1/2	17.47	1/2	19.43	0/2	0
H	L	L	0/1	0	0/1	0	0/1	0	0/1	0
M	M	0	0/1	0	0/1	0	0/1	0	0/1	0
M	0	M	1/1	4.24	1/1	4.24	1/1	10.52	0/1	0
M	L	L	0/2	0	0/2	0	0/2	0	0/2	0
L	L	H	0/1	0	0/1	0	1/1	3.2	0/1	0
L	L	M	2/2	2.21	2/2	2.21	1/2	8.52	0/2	0
L	M	L	0/2	0	0/2	0	0/2	0	0/2	0
L	H	L	0/1	0	0/1	0	0/1	0	0/1	0
L	H	0	0/2	0	0/2	0	0/2	0	0/2	0
L	0	H	0/2	0	0/2	0	2/2	2.84	0/2	0
0	H	L	0/2	0	0/2	0	0/2	0	0/2	0
0	M	M	0/1	0	0/1	0	0/1	0	0/1	0
0	L	H	1/2	5.19	1/2	5.19	2/2	6.91	0/2	0

Table 28: Summary of third party provider menus with $\theta = 2.257, \beta = 2.0$, and $C_R = \$150$.

Consumer type			No Limit		\$600 Limit		\$400 Limit		\$200 Limit	
0.5	1.0	1.5	Menus Offered	Average Profit Increase (%) *if* Menu Offered	Menus Offered	Average Profit Increase (%) *if* Menu Offered	Menus Offered	Average Profit Increase (%) *if* Menu Offered	Menus Offered	Average Profit Increase (%) *if* Menu Offered
H	L	0	0/2	0	1/2	0.08	1/2	16.67	0/2	0
H	0	L	0/2	0	1/2	10.42	1/2	4.51	0/2	0
H	L	L	1/1	10.81	1/1	5.76	0/1	0	0/1	0
M	M	0	0/1	0	0/1	0	1/1	37.51	0/1	0
M	0	M	0/1	0	0/1	0	1/1	0.79	0/1	0
M	L	L	0/2	0	0/2	0	2/2	13.05	0/2	0
L	L	H	0/1	0	0/1	0	0/1	0	0/1	0
L	L	M	1/2	1.18	1/2	1.1	2/2	4.56	0/2	0
L	M	L	0/2	0	0/2	0	2/2	7.66	0/2	0
L	H	L	0/1	0	0/1	0	1/1	2.51	0/1	0
L	H	0	0/2	0	0/2	0	2/2	10.53	0/2	0
L	0	H	0/2	0	0/2	0	0/2	0	0/2	0
0	H	L	0/2	0	0/2	0	0/2	0	0/2	0
0	M	M	0/1	0	0/1	0	0/1	0	0/1	0
0	L	H	0/2	0	1/2	2.3	0/2	0	0/2	0