Addressing Exchange Rate Uncertainty in Operational Hedging: A Comparison of Three Risk Measures

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Abstract

Operational hedging has emerged as an important strategy to manage risk and optimize profit in the global supply chain; however, there has been little effort so far to understand the impact of the risk framework chosen by the manager on the optimal strategy. In this paper, we focus on exchange rate risk as the key source of profit uncertainty faced by a US multinational with a US plant and a foreign plant, and investigate the impact on the optimal strategy of different models to capture the decision-maker's risk exposure. We provide theoretical insights for a broad class of risk measures and compare three models, based on standard deviation, shortfall and value-at-risk, respectively, through extensive numerical experiments using both simulated and historical exchange rate data. Our empirical study suggests that, although shortfall has become a popular risk measure in finance, it exhibits lackluster performance in operational hedging.

Keywords: risk measures, operational hedging, shortfall.

1 Introduction

"A large US automotive manufacturer with several key models manufactured exclusively in Europe suffered a loss of more than half a billion dollars in 2004 at its European operations. Two-third of this loss was blamed on the US dollar's fall, which hampered the company's non-US cost and US revenue dynamics." (Mahidhar 2006) This assessment in a recent report by the Deloitte Research Group underlines the risks posed to global companies by the prolonged decline of the US dollar, and more generally by shifts and trends in the exchange rates dominating the economy. Exchange rate exposure affects production costs, sourcing decisions, retail prices and ultimately a company's ability to compete with other global multinationals. In this context, operational hedging "provides companies with flexibility in their supply chains, financial positions, distribution patterns and market-facing activities by allowing dynamic adjustments in the locations used to manufacture, source and sell." (Madhidhar 2006) While such flexibility has emerged as a critical tool to protect profits against exchange rate uncertainty, in conjunction with or replacement of traditional financial hedging instruments, the impact of risk modeling on the optimal strategy has received little attention in the management literature; however, aversion to risk plays an important role in the manager's decision to implement hedging techniques, and identifying the risk framework best-suited to operational hedging is a critical component of multinationals' strategies. Our purpose in this paper is to investigate the effect of various models of exchange rate uncertainty on the global logistics problem faced by a multinational with a home plant and a foreign plant and serving known demand in both the home and the foreign markets. We assume for simplicity that the home plant is located in the US.

Contributions. The major contribution of this paper is to provide the theoretical foundation necessary to compare three well-known risk measures (standard deviation, shortfall, value-at-risk) on operational hedging, and to test these three choices on extensive numerical experiments (using both simulated and historical exchange rate data), which caution the decision-maker against using shortfall in the context of exchange rate uncertainty, despite its growing popularity as a risk measure in finance.

Literature review. The potential of operational hedging was established in the early 1990s by Huchzermeier and Cohen (1993), who showed that the flexibility of an international supply chain configuration could be used to increase expected after-tax profit while reducing downside risk. Furthermore, Kogut and Kulatilaka (1994) examined the value of changing production sites when the exchange rate varies, and proved the optimality of barrier policies for the two-location production switching problem. Dasu and Li (1997) extended these results to the general multi-facility case, and described the optimal policies for convex cost functions as well as concave ones. Subsequent research has addressed additional sources of uncertainty and other types of operational hedges. For instance, Sung and Lapan (2000) incorporate competition as well as variable output to the costless switching model. Li, Porteus and Zhang (2001) investigate the benefits of using the inventory policy as an operational hedge under demand, processing time and exchange rate uncertainty. Kazaz, Dada and Moskowitz (2005) analyze the value of under-production and unfulfillment of demand in unprofitable markets as operational hedges under exchange rate and demand uncertainty. Kouvelis, Axarloglou and Sinha (2001) study as operational hedge the ability to switch between three different modes of production for foreign markets: exporting, joint ventures with local partners, and wholly-owned foreign production facilities. Harrison and Van Mieghem (1999) examine the active decision of upper management to build in excess capacity or process options as a means of enhancing expected returns. None of these papers incorporates the manager's risk preferences.

In practice, multinational corporations can use operational hedges to minimize their exposure to exchange rate risk, by incorporating performance constraints into their decision-making frameworks. Chowdhry and Howe (1999) examine the use of financial and operational hedging to address exchange rate and demand uncertainty when the company has a mean-variance objective, and provide qualitative insights into the optimal solution. Ding, Dong and Kouvelis (2004) analyze the integration of operational and financial hedging when a risk-averse firm with a mean-variance utility function invests in capacity, which it can delay allocating until the uncertainty is resolved. Van Mieghem (2006) studies operational hedging in networks of newsvendor problems and finds that risk-averse decision-makers increase capacity (safety-stock levels) when facing multiple sources of demand uncertainty, compared to risk-neutral decision-makers. While performance measures have traditionally been based on the assumed distribution of future random variables, the probabilities governing exchange rates are difficult to estimate with accuracy. Therefore, much attention has been focused among financial managers on shortfall, a more recent risk measure which was first proposed by Uryasev and Rockafellar (1999) and Bertsimas et. al. (2004) in the field of portfolio management. An appealing feature of this risk measure, at least when the underlying random process is stationary, is that it can be computed using non-parametric estimators. To the best of our knowledge, the present work is the first to investigate the performance in operational hedging of shortfall.

Outline. In Section 2, we introduce the general model of risk-averse operational hedging. Section 3 provides insights into the impact of risk aversion on the optimal strategy for two of the three risk measures under consideration: standard deviation and shortfall. Section 4 focuses on probabilistic measures of risk, which do not fit the framework in Section 2. We present the empirical study in Section 5. Finally, Section 6 contains concluding remarks.

2 Risk-averse operational hedging

2.1 The nominal model

We briefly review here the nominal model, in which the actual exchange rate is observed before the manager selects an operational strategy.

2.1.1 Formulation

The decision-maker's objective is to maximize his total profit while meeting demand in both countries; each plant has enough capacity to supply both markets and the exchange rate is known. We use the following notations:

Parameters:

d_1: demand in US market,

d_2: demand in foreign market,

c_1: unit cost of production in US plant (in US dollars),

c_2: unit cost of production in foreign plant (in foreign currency),

t_{12}: unit transportation cost from US to foreign market (in US dollars),

t_{21}: unit transportation cost from foreign to US market (in foreign currency),

k_1: fixed start-up cost to start production in US plant (in US dollars),

k_2: fixed start-up cost to start production in foreign plant (in foreign currency),

p_1: unit sales price in US market (in US dollars),

p_2: unit sales price in foreign market (in foreign currency),

γ: exchange rate of foreign currency (in US dollars),

Decision variables:

x_1: amount produced at US plant for US market,

x_2: amount produced at foreign plant for foreign market,

y_1: binary variable representing decision to start production at US plant (if closed),

y_2: binary variable representing decision to start production at foreign plant (if closed).

Because demand for each market must be met and producing more for one market than its demand is suboptimal (as the additional items would not be sold), the amounts produced at the foreign plant for the US market and at the US plant for the foreign market are equal to $d_1 - x_1$ and $d_2 - x_2$, respectively. In line with the operational hedging literature, we assume that only one plant is currently open for production and that the decision to start production at the other plant has yet to be made.

If the US plant is open, but not the foreign one, the global production planning (**US-GPP**) problem can be formulated as the following mixed-integer programming problem after a straightforward rearranging of terms:

$$\begin{aligned}
\mathbf{US-GPP}: \max \quad & [\gamma\,(c_2 + t_{21}) - c_1]\,x_1 + [c_1 + t_{12} - \gamma\,c_2]\,x_2 - \gamma\,k_2\,y_2 \\
& \qquad + [p_1 - \gamma\,(c_2 + t_{21})]\,d_1 + [\gamma\,p_2 - c_1 - t_{12}]\,d_2 && (1) \\
\text{s.t.} \quad & 0 \le x_1 \le d_1, && (2) \\
& 0 \le x_2 \le d_2, && (3) \\
& x_1 \ge d_1\,(1 - y_2), && (4) \\
& x_2 \le d_2\,y_2, && (5) \\
& y_2 \in \{0, 1\}, && (6)
\end{aligned}$$

where Equation (1) represents the profit in US dollars, Equations (2) and (3) ensure that production at each plant for each market is nonnegative, and Constraints (4) and (5) set the binary variable y_2 to 1, i.e., pay the start-up costs at the foreign plant, if the foreign plant produces any item ($d_1 - x_1 > 0$ or $x_2 > 0$.) A similar formulation exists if the foreign plant is open, but not the US one.

Remark: The profit is a linear function of the exchange rate.

2.1.2 Optimal strategy

We now review the deterministic optimal strategy for comparison purposes with the stochastic, risk-averse case. The existence of a "hysteresis band" in the presence of switching costs has been well-documented (see, e.g., Kogut and Kulatilaka 1994), and describes the situation where the fixed costs incurred to start up production at a plant introduce inertia into the optimal strategy, as the decision-maker postpones opening a plant to avoid such costs. There are four possible hysteresis types, depending on whether it ever is optimal for at least some values of the exchange rate to produce in the US plant the whole US demand but no item for the foreign market, i.e., having the US production equal to d_1 for some γ. We refer to this situation as "myopic production." The notation used for the breakpoints is summarized in Table 1. All the breakpoints can be interpreted in operational terms; for instance, γ_1 is the exchange rate for which the decision-maker becomes indifferent, from a variable cost perspective, between producing an item for the US market at the US plant and producing it at the foreign plant and shipping it to the US.

γ_1	$=$	$\dfrac{c_1}{c_2 + t_{21}}$	γ_4	$=$	$\dfrac{c_1 + t_{12}}{c_2}$
γ_2	$=$	$\dfrac{c_1 + k_1/d_1}{c_2 + t_{21}}$	γ_5	$=$	$\dfrac{c_1\, d_1 + (c_1 + t_{12})\, d_2}{(c_2 + t_{21})\, d_1 + c_2\, d_2 + k_2}$
γ_3	$=$	$\dfrac{c_1 + t_{12}}{c_2 + k_2/d_2}$	γ_6	$=$	$\dfrac{c_1\, d_1 + (c_1 + t_{12})\, d_2 + k_1}{(c_2 + t_{21})\, d_1 + c_2\, d_2}$

Table 1: The value of the breakpoints.

The four hysteresis types are:

(i) Myopic production occurs in both directions of the hysteresis band.

When γ increases, total US production increases from 0 to d_1 (at γ_2) to $d_1 + d_2$ (at γ_4.)

When γ decreases, total US production decreases from $d_1 + d_2$ to d_1 (at γ_3) to 0 (at γ_1.)

This case arises for:

$$\gamma_2 < \gamma_4 \text{ and } \gamma_3 > \gamma_1. \tag{7}$$

(ii) Myopic production never occurs.

When γ increases, total US production increases from 0 to $d_1 + d_2$ (at γ_6.)

When γ decreases, total US production decreases from $d_1 + d_2$ to 0 (at γ_5.)

This case arises for:

$$\gamma_2 \geq \gamma_4 \text{ and } \gamma_3 \leq \gamma_1. \tag{8}$$

(iii) Myopic production occurs only when the US plant is open and the exchange rate decreases. When the foreign plant is open, we observe an all-or-nothing situation, where a plant produces for both markets or none.

When γ increases, total US production increases from 0 to $d_1 + d_2$ (at γ_6.)

When γ decreases, total US production decreases from $d_1 + d_2$ to d_1 (at γ_5) to 0 (at γ_1.)

This case arises for:

$$\gamma_2 \geq \gamma_4 \text{ and } \gamma_3 > \gamma_1. \tag{9}$$

(iv) Myopic production occurs only when the foreign plant is open and the exchange rate increases. When the US plant is open, we observe an all-or-nothing situation, where a plant produces for both markets or none.

When γ increases, total US production increases from 0 to d_1 (at γ_2) to $d_1 + d_2$ (at γ_4.)

When γ decreases, total US production decreases from $d_1 + d_2$ to 0 (at γ_5.)

This case arises for:

$$\gamma_2 < \gamma_4 \text{ and } \gamma_3 \leq \gamma_1. \tag{10}$$

2.2 Measuring risk

2.2.1 Preliminaries

In practice, the exchange rate at the end of the time horizon is not known when the manager decides on the production plan; hence, the profit realized by the multinational becomes a random variable, which must be quantified in numerical terms in order to solve the global production planning problem. The concerns of the decision-maker are twofold: he seeks to maximize his monetary gains and to minimize his exposure to risk. It is rarely possible to achieve those two objectives simultaneously; hence, most managers focus on maximizing a measure of their profit while imposing a bound on the level of risk that they are willing to tolerate.

The computation of the values taken by the risk and return measures may require the precise knowledge of the underlying exchange rate distribution, e.g., when return is measured by the decision-maker's expected utility. This is unappealing for real-life implementation because the probabilities and values taken by the future exchange rate are difficult to estimate accurately. Therefore, we focus on three specific risk measures that allow the manager to solve the global production planning problem under limited information: standard deviation, shortfall and value-at-risk (also called profit-at-risk). To measure return we focus on the *expected value* of the profit, as this is the metric most commonly used in practice. In the remainder of the paper, we will denote by $\overline{\gamma}$ the expected value of the exchange rate.

2.2.2 Properties

We denote by ρ the risk measure and τ the maximum level of risk allowed by the decision-maker. We also denote by S the feasible set of the operational-hedging problem, i.e., Constraints (2)-(6), and by $a(x,y)$ and $b(x,y)$ the parts of the revenue that do not and do depend on the exchange rate uncertainty, respectively. (In other words, we write the profit as $a(x,y)+\gamma\, b(x,y)$.) The risk-averse global production planning (**RA-GPP**) problem can then be formulated as:

$$\begin{aligned} \mathbf{RA-GPP}: \max \quad & a(x,y)+\overline{\gamma}\, b(x,y) \\ \text{s.t.} \quad & \rho[a(x,y)+\gamma\, b(x,y)] \leq \tau, \\ & (x,y) \in S, \end{aligned} \tag{11}$$

where a and b are linear functions of the decision variables. In order to rewrite Problem (11) in a more tractable manner, we need to investigate the impact of adding and multiplying random variables by a scalar on the risk measure. As standard deviation and shortfall have similar impacts, we will analyze these two measures in this section; Section 4 focuses on profit-at-risk.

Adding a constant amount to the profit changes neither its standard deviation nor its shortfall (as measured by the expected profit minus the tail conditional expectation for a pre-specified quantile, see Bertsimas et. al. 2001):

$$\rho[a+X] = \rho[X]\ \forall a,\ \forall X\, r.v. \tag{12}$$

Furthermore, standard deviation and shortfall both satisfy the assumption of *positive homogeneity*:

$$\rho[bX] = b\,\rho[X]\ \forall b>0,\ \forall X\, r.v. \tag{13}$$

2.3 The Model

We now reformulate Problem (11) in a tractable manner. First, we need the following lemma.

Lemma 1 *The value associated with the decision-maker's risk is:*

$$\rho[a(x,y)+\gamma\, b(x,y)] = \max\{b(x,y)\,\rho(\gamma),\ -b(x,y)\,\rho(-\gamma)\}, \tag{14}$$

where ρ is the standard deviation of the exchange rate or its shortfall.

Proof: Immediate from the definition of standard deviation and shortfall. □

Theorem 2 presents the risk-averse global production planning problem. We observe that introducing risk aversion through either standard deviation or shortfall amounts to introducing bounds on the revenue realized in the foreign market, and distinguishing between the cases whether the appreciation of the foreign currency increases or decreases revenue.

Theorem 2 *The risk-averse global production planning problem (11) can be rewritten as a single* mixed-integer programming *problem, where the part of the profit subject to exchange rate uncertainty is bounded:*

$$\begin{aligned} \mathbf{RA-GPP}: \max \quad & a(x,y) + b(x,y)\,\overline{\gamma} \\ s.t. \quad & -\frac{\tau}{\rho(-\gamma)} \leq b(x,y) \leq \frac{\tau}{\rho(\gamma)}, \\ & (x,y) \in S. \end{aligned} \tag{15}$$

Proof: Follows from distinguishing between $b(x,y) \geq 0$ and $b(x,y) \leq 0$ in Problem (11) and invoking Lemma 1. □

Remark: A key consequence of Theorem 2 (using the fact that Problem (15) becomes a linear programming problem when the foreign facility is opened, and hence has an optimal solution at a corner point of the feasible set) is that, provided that $\tau \neq 0$, it is *never optimal* to choose the production plan in order to *cancel out* the effect of the exchange rate on the profit, when the decision-maker measures his profit by its expected value and uses standard deviation or shortfall to measure risk.

For shortness sake, we only provide below the explicit formulations when the US plant is already open. The case when the foreign plant is open can be derived in a similar manner.

Corollary 3 (US plant open)

(i) When the US plant is already open and the manager opens the foreign plant, the optimal allo-

cation is obtained by solving the linear programming problem*:*

$$\begin{aligned}
\mathbf{US-GPP:}\ & -\overline{\gamma}\,k_2 + [p_1 - \overline{\gamma}\,(c_2+t_{21})]\,d_1 + [\overline{\gamma}\,p_2 - c_1 - t_{12}]\,d_2 + \\
\max\ & [\overline{\gamma}\,(c_2+t_{21}) - c_1]\,x_1 + [c_1 + t_{12} - \overline{\gamma}\,c_2]\,x_2 \\
s.t.\ & k_2 + (c_2+t_{21})\,d_1 - p_2\,d_2 - \frac{\tau}{\rho(-\gamma)} \leq (c_2+t_{21})\,x_1 - c_2\,x_2 \leq k_2 + (c_2+t_{21})\,d_1 - p_2\,d_2 + \frac{\tau}{\rho(\gamma)}, \\
& 0 \leq x_1 \leq d_1, \\
& 0 \leq x_2 \leq d_2.
\end{aligned} \tag{16}$$

(ii) If $\frac{\tau}{\rho(\gamma)} < p_2\,d_2$*, keeping the foreign facility closed does not satisfy the risk constraint and the optimal solution to the risk-averse production planning problem has been found in (i).*

(iii) If $\frac{\tau}{\rho(\gamma)} \geq p_2\,d_2$*, the optimal solution to the risk-averse production planning problem is found by comparing the higher objective found in (i) with the profit* $[p_1 - c_1]\,d_1 + [\overline{\gamma}\,p_2 - c_1 - t_{12}]\,d_2$*, achieved when the foreign plant remains closed.*

Proof: This is a straightforward application of Theorem 2 where we inject $y_2 = 1$ (foreign plant open) and replace $a(x,y)$ and $b(x,y)$ by their exact expressions as a function of the decision variables and cost parameters. □

Section 3 investigates conditions for the risk-averse production problem to be feasible, as well as the impact of the risk threshold on the operational hedge.

3 The impact of risk on operational hedging

In this section, we discuss the impact of the decision-maker's risk preferences on his operational hedging strategy. Sections 3.1 and 3.2 present our insights when risk is measured by the shortfall of the profit and its standard deviation, respectively. We present the standard deviation case last as it can be derived as a simplification of the shortfall case, where we replace $\rho(\gamma)$ and $\rho(-\gamma)$ by the standard deviation σ of the exchange rate.

3.1 Shortfall

We focus our attention on the case where the US facility is already open; the case where the foreign facility is already open is similar and left to the reader.

Theorem 4 links the choice of the risk threshold τ with the feasibility of the global production planning problem and ties it to the decision of opening the foreign plant. We use the following notation:

$$\begin{aligned}
\tau_0 &= \rho(\gamma)\, p_2\, d_2, && (17)\\
\tau_1^- &= \rho(-\gamma)[k_2 - p_2\, d_2], && (18)\\
\tau_1^+ &= \rho(\gamma)\,[p_2\, d_2 - k_2 - (c_2 + t_{21})\, d_1 - c_2\, d_2]\,, && (19)
\end{aligned}$$

with $\rho(\gamma) = \overline{\gamma} - E[\gamma|\gamma \leq q_\alpha(\gamma)]$ where $q_\alpha(\gamma)$ is the α-quantile of the random variable γ ($\alpha \in (0,1)$). Note that $\tau_1^+ < \tau_0$. As mentioned above, the US plant alone cannot satisfy the risk constraint when $\tau/\rho(\gamma)$ is smaller than $p_2\, d_2$; this corresponds to $\tau < \tau_0$. We will see in Theorem 4 that τ_1^- and τ_1^+ represent risk thresholds that play a fundamental role in understanding the options available to the risk-averse decision-maker.

Theorem 4 (Risk impact on decision of opening foreign plant)
(i) If $\tau < \max(\tau_1^-, \tau_1^+)$, opening the foreign facility does not allow the decision-maker to meet the risk constraint. It follows that if $\tau < \min\{\tau_0, \max(\tau_1^-, \tau_1^+)\}$, the problem is infeasible.
(ii) If $\min\{\tau_0, \max(\tau_1^-, \tau_1^+)\} \leq \tau < \max(\tau_0, \tau_1^-, \tau_1^+)$, the decision of opening or keeping the foreign facility closed is dictated by feasibility (risk) considerations rather than the goal of maximizing expected profit.

Proof: Let $K = k_2 + (c_2 + t_{21})\, d_1 - p_2\, d_2$. The feasible set of (US-GPP) (Problem (16)) is nonempty and bounded; hence, by strong duality, (US-GPP) is equivalent to (dropping the constant term in the objective):

$$\begin{aligned}
\min \quad & \left(K + \frac{\tau}{\rho(\gamma)}\right) \alpha^+ - \left(K - \frac{\tau}{\rho(-\gamma)}\right) \alpha^- + d_1\, \beta_1 + d_2\, \beta_2 \\
\text{s.t.} \quad & (c_2 + t_{21})(\alpha^+ - \alpha^-) + \beta_1 \geq \overline{\gamma}(c_2 + t_{21}) - c_1, \\
& -c_2(\alpha^+ - \alpha^-) + \beta_2 \geq c_1 + t_{12} - \overline{\gamma}\, c_2, \\
& \alpha^+,\, \alpha^-,\, \beta_1,\, \beta_2 \geq 0,
\end{aligned} \tag{20}$$

or, using that $\beta_1 = \max\{0, \overline{\gamma}(c_2 + t_{21}) - c_1 - (c_2 + t_{21})(\alpha^+ - \alpha^-)\}$ and $\beta_2 = \max\{0, c_1 + t_{12} - \overline{\gamma}\, c_2 + c_2(\alpha^+ - \alpha^-)\}$, and using that $\alpha^+ - \alpha^- = \alpha$, $\alpha^+ = \max(0, \alpha)$ and $\alpha^- = \max(0, -\alpha)$:

$$\begin{aligned}
\min \quad & \left[\tau \left(\frac{\max(0,\alpha)}{\rho(\gamma)} + \frac{\max(0,-\alpha)}{\rho(-\gamma)}\right) + K\, \alpha + d_1\, (c_2 + t_{21}) \cdot \max\left\{0, \overline{\gamma} - \frac{c_1}{c_2 + t_{21}} - \alpha\right\}\right. \\
& \left. + d_2\, c_2 \cdot \max\left\{0, \frac{c_1 + t_{12}}{c_2} - \overline{\gamma} + \alpha\right\}\right].
\end{aligned} \tag{21}$$

Problem (21) is an unconstrained piecewise linear convex problem; hence, it has a finite minimum if and only if the function has nonpositive slope when $\alpha \to -\infty$ and nonnegative slope when $\alpha \to \infty$, i.e.:

$$-\frac{\tau}{\rho(-\gamma)} + k_2 - p_2\, d_2 \le 0, \tag{22}$$

and

$$\frac{\tau}{\rho(\gamma)} + k_2 + (c_2 + t_{21})\, d_1 - p_2\, d_2 + c_2\, d_2 \ge 0. \tag{23}$$

If (and only if) either Condition (22) or Condition (23) is not satisfied, Problem (21) has an unbounded optimal cost, which makes its dual, (US-GPP), infeasible (it cannot be unbounded since its feasible set is bounded). In other words, (US-GPP) is infeasible if and only if:

$$k_2 - p_2\, d_2 > \frac{\tau}{\rho(-\gamma)} \text{ or } k_2 + (c_2 + t_{21})\, d_1 - p_2\, d_2 + c_2\, d_2 < -\frac{\tau}{\rho(\gamma)}, \tag{24}$$

which can be written as:

$$\tau < \tau_1^- \text{ or } \tau < \tau_1^+. \tag{25}$$

This means that the risk-averse production planning problem with foreign facility opened is infeasible if and only if $\tau < \max(\tau_1^-, \tau_1^+)$. Finally, the master risk-averse production planning problem (including the opening decision) is infeasible if and only if the US facility alone cannot meet the risk constraint ($\tau < \tau_0$) and (US-GPP) is infeasible as well. This proves (i). (ii) follows from the observation that both the US facility alone and (US-GPP) can meet the risk constraint for $\tau > \max(\tau_0, \tau_1^-, \tau_1^+)$. □

We now investigate the impact of the decision-maker's risk tolerance on the optimal profit (Theorem 5) and allocation (Theorem 6), and draw managerial insights based on our findings. We assume that the decision-maker can satisfy the risk constraint by opening the foreign facility; otherwise, the problem is either infeasible or the US plant meets both markets' demand while keeping profit volatility low.

The key insight of Theorem 5 is that the optimal profit is not concave in the risk tolerance. This has important implications in practical examples, as the decision-maker might be able to increase his profit significantly by accepting a moderate increase in risk.

Theorem 5 (Optimal profit as a function of risk) *The optimal profit is piecewise linear in the risk tolerance; specifically, it is piecewise concave, but not necessarily concave in τ.*

Proof: If (US-GPP) is feasible, then the minimum of the objective in Equation (21), which is a linear function in the risk tolerance τ, is reached at one of the breakpoints, i.e., $\delta = 0$, $\delta = \overline{\gamma} - \gamma_1$ or $\delta = \overline{\gamma} - \gamma_4$, where γ_1 and γ_4 were defined in Table 1. Therefore, the optimal objective of (US-GPP) is concave in τ. Finally, the optimal objective in the case where only the US facility is open is constant in τ, provided that this operating situation is feasible in terms of risk. The optimal objective for the whole problem is the maximum between these objectives (when the operating conditions meet the risk constraint). $\square$

Theorem 6 (Optimal allocation as a function of risk)
(i) The optimal allocation is piecewise linear in τ.
(ii) When the optimal allocation varies with the risk tolerance, one plant produces either all or nothing for its own market, and the production of the other plant for its own market is linear in the risk tolerance.

Proof: Because (US-GPP) is a linear programming problem, we can use constraint splitting to rewrite the problem under the following equivalent form, for the optimal non-negative shadow prices α^+ and α^-:

$$\begin{aligned} (\alpha^+ - \alpha^-) K + \left(\frac{\alpha^+}{\rho(\gamma)} + \frac{\alpha^-}{\rho(-\gamma)}\right)\tau + \max \quad & A(x) + [\overline{\gamma} - \alpha^+ + \alpha^-] B(x) \\ \text{s.t.} \quad & 0 \leq x_1 \leq d_1, \\ & 0 \leq x_2 \leq d_2, \end{aligned} \tag{26}$$

where $A(x)$ and $B(x)$ are linear in x_1 and x_2 and represent the components of the expected profit that do not depend, respectively depend, on the exchange rate. Problem (26) is equivalent to the deterministic model (when the manager evaluates the optimal profit with the foreign facility open) for a known exchange rate of $\pi(\gamma) - \alpha^+ + \alpha^-$. The optimal solution is found by enumerating the four extreme points ($x_1 = 0$, $x_1 = d_1$, $x_2 = 0$, $x_2 = d_2$, in conjunction with $B(x) = K + \tau/\rho(\gamma)$ or $B(x) = K - \tau/\rho(-\gamma)$). $\square$

Remarks:

- If $\alpha^+ > 0$ at optimality, then $\alpha^- = 0$ (the constraint $B(x) \geq K - \tau/\rho(-\gamma)$ is not binding) and the risk aversion of the decision-maker has the high-level effect of *decreasing* the value of the exchange rate (from $\overline{\gamma}$ to $\overline{\gamma} - \alpha^+$) to one of the *breakpoints* of the deterministic model; this is the only way to have at optimality x_1 that is neither 0 nor d_1 nor $d_1 + d_2$. Similarly, if $\alpha^- > 0$ at optimality, the risk aversion of the decision-maker has the high-level effect of *increasing* the value of the exchange rate (from $\overline{\gamma}$ to $\overline{\gamma} + \alpha^-$).

- The profit is locally *convex* in the risk tolerance in the region for which it becomes optimal to open the foreign facility, i.e., increasing the risk level τ *increases* the slope of the profit function around that point. This suggests that a strategic decision such as opening facilities can have a significant and even counterintuitive impact on profit for the risk-averse decision-maker.

The main managerial insight we draw from these observations is that understanding the *specific operating mode* (facility open or closed) that is optimal for the risk level considered is critical to risk-averse operational hedging; for some risk values, moderate increases in the threshold would yield significant increases in the measured profit (due to local convexity). In those areas, the manager should evaluate the strategic outcome that drives optimality and consider increasing his risk level to take advantage of the local convexity of the profit curve.

3.2 Standard Deviation

In this section, we illustrate the approach when risk is measured by the standard deviation of the profit. In particular, we have: $\rho(\gamma) = \rho(-\gamma)$, which simplifies the exposition. We will use the notation $\rho(\gamma) = \sigma$. The following results follow directly from Section 3.1; hence, we state them without proof.

- Keeping the foreign plant closed is feasible if and only if $\tau/\sigma \geq p_2\, d_2$, in which case the profit is equal to $[p_1 - c_1]d_1 + [\overline{\gamma}\, p_2 - c_1 - t_{12}]\, d_2$.

- The risk-averse production planning problem when the foreign facility is open can be reformulated as, with $R_a = [p_1 - \overline{\gamma}(c_2 + t_{21})]\, d_1 + [\overline{\gamma}\, p_2 - (c_1 + t_{12})]\, d_2 - \overline{\gamma}\, k_2$ the total revenue generated by the multinational for the expected exchange rate in the "altruistic" configuration where the US plant serves the foreign market and the foreign plant (upon opening) serves the US market, and with $C_{a2} = (c_2 + t_{21})\, d_1 - p_2\, d_2 + k_2$ the part of the cost (the opposite of the revenue) incurred in the foreign market in the "altruistic" configuration:

$$
\begin{aligned}
R_a + \max \quad & [\overline{\gamma}\,(c_2 + t_{21}) - c_1]\,x_1 + [c_1 + t_{12} - \overline{\gamma}\,c_2]\,x_2 \\
\text{s.t.} \quad & -\tfrac{\tau}{\sigma} + C_{a2} \le (c_2 + t_{21})\,x_1 - c_2\,x_2 \le \tfrac{\tau}{\sigma} + C_{a2}, \\
& 0 \le x_1 \le d_1, \\
& 0 \le x_2 \le d_2.
\end{aligned}
\tag{27}
$$

The reader can check using the dual formulation of Problem (27), specifically:

$$
\begin{aligned}
\min_{\alpha} \frac{\tau}{\sigma}|\alpha| + [(c_2 + t_{21})\,d_1 - p_2\,d_2 + k_2]\,\alpha + d_1\,(c_2 + t_{21})\,\max(0, \overline{\gamma} - \alpha - \gamma_1) \\
+ d_2\,c_2\,\max(0, \alpha - \overline{\gamma} + \gamma_4),
\end{aligned}
\tag{28}
$$

with γ_1 and γ_4 defined in Table 1, that Problem (27) is feasible if and only if $\tau \ge \max(\tau_1^-,\, \tau_1^+)$ with τ_1^- and τ_1^+ defined in Equations (18) and (19), respectively.

Remark: The choice of the risk measure will naturally affect feasibility. For instance, if the distribution of the exchange rate is symmetric, $\rho(\gamma) = \rho(-\gamma)$ when risk is measured by the shortfall, and the condition $\tau \ge \max(\tau^-,\, \tau^+)$ (shared by both risk measures to have the opening of the foreign facility meet the risk constraint) becomes $\tau \ge \sigma\, t$ and $\tau \ge \rho(\gamma)\, t$ in the standard deviation and shortfall cases, respectively, with $t = \max(k_2 - p_2\, d_2,\, p_2\, d_2 - k_2 - (c_2 + t_{21})\, d_1 - c_2\, d_2)$. The shortfall model allows greater flexibility through the choice of the quantile parameter while obtaining similar qualitative insights as in the standard deviation case; the flip side of that coin, however, is that the quantile must be chosen carefully to avoid over-conservatism. In practice it is often best to compute a *family* of optimal allocations indexed by the quantile, for the decision-maker to choose from based on his individual preferences.

The remainder of this section is devoted to the following two points: (i) understanding when the nominal production plan described in Section 2 remains optimal in the presence of risk considerations, and (ii) illustrating how the optimal allocation and profit evolve with the decision-maker's risk tolerance.

Optimality of nominal solution:

To study the values of $(\overline{\gamma}, \tau/\sigma)$ for which risk does not change the optimal solution, we compute the risk associated with the deterministic solution for each production mode, and checking whether it satisfies the risk constraint. The nominal solution is optimal if and only if τ/σ exceeds a threshold.

Since we only analyze here the case where the US plant is already open (the case where the foreign plant is open is similar), we can group the four hysteresis types described in Section 2.1 as follows:

- Three production modes, depending on whether total US production is equal to 0, d_1 or $d_1 + d_2$ (for $\gamma \leq \gamma_1$, $\gamma_1 < \gamma \leq \gamma_3$ and $\gamma > \gamma_3$, respectively). This case arises when $\gamma_3 > \gamma_1$. In turn, this means that we will have three thresholds for τ/σ.

- Two production modes, depending on whether total US production is equal to 0 or $d_1 + d_2$ (for $\gamma \leq \gamma_5$ and $\gamma > \gamma_5$, respectively). This case arises when $\gamma_3 \leq \gamma_1$. We will then have two thresholds for τ/σ.

The threshold is equal to the absolute value of one of the following three parameters: (a) the profit in the foreign market of the foreign-plant-produces-all configuration when the foreign plant is opened, (b) the profit in the foreign market of the myopic configuration when the foreign plant is opened, and (c) the revenue in the foreign market. (In the case with two production modes, only cases (a) and (c) occur.)

Dependence of profit and production plan on risk:

Because our purpose here is simply to illustrate the dependence of the decision-maker's strategy on his risk tolerance, we only point out notable properties of the optimal solution and profit when $\tau_1^+ < 0$. Their counterparts when $\tau_1^+ > 0$ can be derived in a similar manner. We highlight the potential appeal of an "open plant without producing there" strategy for the risk-averse decision-maker, when keeping the foreign facility closed does not satisfy the risk constraint but generates higher expected profits. We also emphasize that the decision to open the foreign plant is not, in general, a threshold policy in τ/σ, as this decision is motivated by two possible factors, constraint feasibility and expected profit maximization, which drive the manager's strategy for two potentially disjoint regions of values taken by τ/σ.

We know that the optimal profit when the foreign plant is open (Equation (28)) is a piecewise linear convex function to be minimized, with a finite minimum for $\tau \geq \tau_1^-$, and we find its optimal by comparing the values taken by the function for the three possible breakpoints: $\alpha = 0$, $\alpha = \overline{\gamma} - \gamma_1$ and $\alpha = \overline{\gamma} - \gamma_4$. The optimal solution is found by complementarity slackness.

<u>Case $\overline{\gamma} < \gamma_1$:</u>

Figure 1 shows the optimal profit when $\overline{\gamma} < \gamma_1$ and $\tau_1^- > 0$, where we introduce the notation: $\tau_2 = \sigma\,[k_2 - (p_2 - c_2)\,d_2] = \tau_1^- + c_2\,d_2\,\sigma$, in addition to the notation introduced previously.

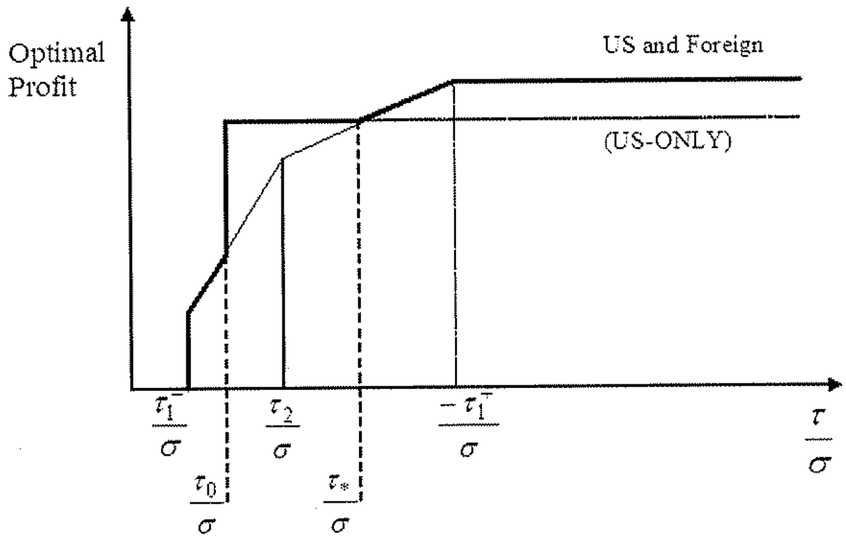

Figure 1: Optimal profit when $\overline{\gamma} < \gamma_1$.

We distinguish between three cases to analyze the optimal profit *when* the manager opens the foreign facility:

$\mathbf{max(0, \tau_1^-) \leq \tau < \tau_2}$**:** The US plant produces for the US market ($x_1 = d_1$), while the amount produced by the foreign plant for its own market increases *linearly* from $\max\left(0, -\tau_1^-/[\sigma\, c_2]\right)$ to d_2 as τ increases from $\max(0, \tau_1^-)$ to τ_2 ($x_2 = [\tau - \tau_1^-]/(\sigma\, c_2)$). The slope of the profit when the foreign plant is open is $\gamma_4 - \overline{\gamma}$.

$\tau_2 \leq \tau < -\tau_1^+$**:** The foreign plant produces for the foreign market ($x_2 = d_2$), while the amount produced by the US plant for its own market decreases from d_1 to 0 ($x_1 = d_1 - [\tau + \tau_1^+]/[\sigma\,(c_2 + t_{21})]$). The slope of the profit when the foreign plant is open is $\gamma_1 - \overline{\gamma}$. (Note that $\gamma_1 < \gamma_4$.)

$\tau > -\tau_1^+$**:** The foreign plant produces for both the US and the foreign markets ($x_1 = 0$, $x_2 = d_2$.) The slope of the profit when the foreign plant is open is zero.

To find the optimal operational hedge, we must then compare the optimal profit when the foreign plant is open with that when the US plant is operating alone, taking into account that using only the US plant is feasible, from a risk perspective, if and only if $\tau \geq \tau_0$. Figure (1) shows an example where $\tau_1^- < \tau_0 < \tau_2$ and $\overline{\gamma} < \gamma_5$ (besides $\overline{\gamma} < \gamma_1$). The most important observation we draw from Figure 1 is that *the decision to open the foreign plant is <u>not</u> a threshold-type policy* with respect to the manager's risk tolerance: here, it is optimal to open the foreign plant for low and high levels of risk tolerance, but not for medium levels. Two opposite effects explain this situation:

1. for low levels of risk tolerance, the decision-maker would benefit (in terms of return) from not opening the foreign facility, but operating the US plant alone violates the risk constraint; hence, the optimal strategy is solely driven by the manager's *risk aversion* and the resulting feasibility or infeasibility of the operating modes,

2. the decision-maker switches to the profit-maximizing strategy of keeping the foreign facility closed as soon as the risk constraint is satisfied and this strategy becomes feasible; he later opens the foreign facility when the risk tolerance is large enough and the strategy is *more profitable*.

Case $\gamma_1 < \overline{\gamma} \leq \gamma_4$:

Similarly, we can show that, under the same assumption that $\tau_1^- > 0$ as before (so that $\tau_2 > 0$ as well), the production plan as a function of risk, when the foreign plant is opened, is given by: (i) for $\tau_1^- \leq \tau \leq \tau_2$, the US plant produces for the US market ($x_1 = d_1$) and the amount produced by the foreign plant for its own market increases from 0 to d_2; the profit slope is $\gamma_4 - \overline{\gamma}$, (ii) for $\tau > \tau_2$, the US plant produces for the US market ($x_1 = d_1$) and the foreign plant produces for the foreign market ($x_2 = d_2$); the profit slope is zero.

Case $\overline{\gamma} > \gamma_4$:

It is always optimal to have the US facility produce for both the US market and the foreign market. Note that while we assume $\tau \geq \max(0, \tau_1^-)$, so that it is feasible from a risk perspective to open the foreign plant, we do not assume $\tau \geq \tau_0$. In particular, when $p_2\, d_2 > k_2$, $\tau_1^- < \tau_0$ and it is optimal for $\tau_1^- \leq \tau < \tau_0$ to incur startup costs at the foreign plant *without operating it* to bring the risk within bounds despite the profit loss, because the standard deviation of the profit is decreased by opening the foreign plant. Again, this behavior, which is plainly suboptimal from a profit perspective, is dictated by the need to keep risk within bounds, i.e., the need for a strategy with moderate (or at least tolerable) risk.

In practice, if the decision-maker truly wanted to open the foreign plant without operating it, he would maintain production at minimum levels, as is often done in assembly plants with high stopping costs when inventory vastly exceeds demand. This strategy would, however, be much more difficult to justify in the context of exchange rate uncertainty than in a personnel situation, as the standard deviation of the profit is much more difficult to visualize than immediate costs such as layoff indemnities or penalties for violating unionized labor contracts, and investors and management would question the decision-maker's risk tolerance. Therefore, the manager must be aware that the situation where a mathematical model recommends to open but not use a plant may arise, and possibly consider increasing his risk tolerance when it does. This suggests that some values of the risk tolerance parameter are *undesirable* in operational hedging, in contrast with mean-variance models in portfolio management, where no portfolio on the efficient frontier is strictly more desirable than another (lower return is counterbalanced by lower risk), and asset allocations evolve without jumps.

4 Profit-at-Risk

Another well-known measure of risk is profit-at-risk, i.e., a quantile of the profit distribution; for instance, there is (by definition) only a 5% chance that the profit will fall below the 95% Profit-at-Risk. This risk metric does not fall within the framework developed in Sections 2 and 3, as it does not satisfy the additivity axiom posited in Equation (12): adding a constant amount to a random variable modifies the probability of exceeding a threshold. In this section, we analyze how the choice of this risk measure impacts the manager's optimal strategy.

The decision-maker protects his profit against exchange rate volatility by enforcing that the profit will fall below a threshold T with probability at most ϵ. (ϵ should of course be small, and is assumed such that $F^{-1}(\epsilon) < 0 < F^{-1}(1-\epsilon)$. For instance, if the distribution is symmetric, $F^{-1}(0.5) = 0$ and we simply assume $\epsilon < 0.5$.) This constraint is called the *profit-at-risk constraint*. The problem is stated as:

$$\begin{aligned} \max \quad & a(x,y) + \overline{\gamma}\, b(x,y) \\ \text{s.t.} \quad & Pr[a(x,y) + \gamma\, b(x,y) \leq T] \leq \epsilon, \\ & (x,y) \in S. \end{aligned} \tag{29}$$

Note that in this framework, it is least constraining to have T as *small* as possible, since a small threshold decreases the probability that the revenue falls below that value.

Theorem 7 *The profit-at-risk global production planning problem (29) can be formulated as a* mixed-integer programming problem:

$$
\begin{aligned}
\mathbf{PR-GPP}: \max \quad & a(x,y) + b(x,y)\,\overline{\gamma} \\
s.t. \quad & -\frac{a(x,y)-T}{F^{-1}(1-\epsilon)} \le b(x,y) \le \frac{a(x,y)-T}{-F^{-1}(\epsilon)}, \\
& (x,y) \in S,
\end{aligned}
\tag{30}
$$

Proof: Is an immediate reformulation of Problem (29) upon distinguishing between $b(x,y) \ge 0$ and $b(x,y) \le 0$. □

We observe strong similarities between the profit-at-risk models (PR-GPP) on one hand and the risk-averse models (RA-GPP) on the other hand. In particular, the risk level τ has been replaced by the difference $a(x,y) - T$, and the parameters $\rho(\gamma)$ and $\rho(-\gamma)$ have been replaced by $-F^{-1}(\epsilon)$ and $F^{-1}(1-\epsilon)$, respectively. A major difference is that the ratios $\tau/\rho(\gamma)$ and $\tau/\rho(-\gamma)$ have been replaced by *decision-dependent* quantities, due to the term in $a(x,y)$. Hence, T is harder to fine-tune because it must be selected in the context of actual production allocations and their impact on the part of the decision-maker's profit that does not depend on the exchange rate.

Let again $R_a = [p_1 - (c_2 + t_{21})\,\overline{\gamma}]\,d_1 + [\pi(\gamma)\,p_2 - (c_1 + t_{12})]\,d_2 - \overline{\gamma}\,k_2$ the total revenue computed for the nominal exchange rate $\overline{\gamma}$, $C_{a2} = (c_2 + t_{21})\,d_1 - p_2\,d_2 + k_2$ the cost incurred in the foreign market, and $R_{a1} = p_1\,d_1 - (c_1 + t_{12})\,d_2$ the revenue incurred in the US market, all in the altruistic configuration. Furthermore, we define the function G by, for all $0 \le \epsilon \le 1$:

$$
G(\epsilon) = -C_{a2}\,F^{-1}(\epsilon) + d_1\,(c_2 + t_{21})\,\max\{0, F^{-1}(\epsilon) - \gamma_1\} + d_2\,c_2\,\max\{0, \gamma_4 - F^{-1}(\epsilon)\}. \tag{31}
$$

Theorem 8 investigates the values of the threshold T for which the risk-averse production planning problem is feasible, as a function of the probability level ϵ. This helps emphasize the influence of various cost parameters on system performance, as well as provide some guidelines regarding the selection of T and ϵ for practical implementations.

Theorem 8 (Feasibility)

(i) (US-ONLY) is feasible if and only if:

$$
T \le (p_1 - c_1)\,d_1 - (c_1 + t_{12})\,d_2 + p_2\,d_2\,F^{-1}(\epsilon). \tag{32}
$$

(ii) The production planning problem with the foreign facility open is feasible if and only if:

$$T \leq R_{a1} + \min\{G(\epsilon), G(1-\epsilon)\}. \tag{33}$$

Proof: If the foreign facility is opened, the optimal strategy is found by solving the following linear programming problem:

$$\begin{aligned}
R_a + \max \quad & [-c_1 + \overline{\gamma}\,(c_2 + t_{21})]\, x_1 + [(c_1 + t_{12}) - \overline{\gamma}\, c_2]\, x_2 \\
\text{s.t.} \quad & \left[c_2 + t_{21} - \frac{c_1}{F^{-1}(\epsilon)}\right] x_1 + \left[-c_2 + \frac{c_1 + t_{12}}{F^{-1}(\epsilon)}\right] x_2 \leq C_{a2} + \frac{T - R_{a1}}{F^{-1}(\epsilon)}, \\
& \left[c_2 + t_{21} - \frac{c_1}{F^{-1}(1-\epsilon)}\right] x_1 + \left[-c_2 + \frac{c_1 + t_{12}}{F^{-1}(1-\epsilon)}\right] x_2 \geq C_{a2} + \frac{T - R_{a1}}{F^{-1}(1-\epsilon)}, \\
& 0 \leq x_1 \leq d_1, \\
& 0 \leq x_2 \leq d_2,
\end{aligned} \tag{34}$$

or, equivalently, its dual counterpart:

$$\begin{aligned}
R_a + \min \quad & \left[C_{a2} + \frac{T - R_{a1}}{F^{-1}(\epsilon)}\right] \alpha^+ - \left[C_{a2} + \frac{T - R_{a1}}{F^{-1}(1-\epsilon)}\right] \alpha^- + d_1\, \beta_1 + d_2\, \beta_2 \\
\text{s.t.} \quad & \left[c_2 + t_{21} - \frac{c_1}{F^{-1}(\epsilon)}\right] \alpha^+ - \left[c_2 + t_{21} - \frac{c_1}{F^{-1}(1-\epsilon)}\right] \alpha^- + \beta_1 \geq -c_1 + \overline{\gamma}\,(c_2 + t_{21}), \\
& \left[-c_2 + \frac{c_1 + t_{12}}{F^{-1}(\epsilon)}\right] \alpha^+ - \left[-c_2 + \frac{c_1 + t_{12}}{F^{-1}(1-\epsilon)}\right] \alpha^- + \beta_2 \geq (c_1 + t_{12}) - \overline{\gamma}\, c_2, \\
& \alpha^+,\, \alpha^-,\, \beta_1,\, \beta_2 \geq 0.
\end{aligned} \tag{35}$$

Two cases may arise: either both Problems (34) and (35) have a finite optimum, or Problem (34) is infeasible and Problem (35) has unbounded objective. (Problem (34) cannot have unbounded objective, since its feasible set is bounded.) To analyze the infeasibility of Problem (34), i.e., the impossibility of meeting the risk constraint by opening the foreign facility, we study Problem (35) in more details by rewriting it under the equivalent, piecewise linear form:

$$\begin{aligned}
R_a + \min \quad & \left[C_{a2} + \frac{T - R_{a1}}{F^{-1}(\epsilon)}\right] \alpha^+ - \left[C_{a2} + \frac{T - R_{a1}}{F^{-1}(1-\epsilon)}\right] \alpha^- \\
& + d_1\,(c_2 + t_{21}) \max\left\{0, -\gamma_1 + \overline{\gamma} - \left[1 - \frac{\gamma_1}{F^{-1}(\epsilon)}\right] \alpha^+ + \left[1 - \frac{\gamma_1}{F^{-1}(1-\epsilon)}\right] \alpha^-\right\} \\
& + d_2\, c_2 \max\left\{0, \gamma_4 - \overline{\gamma} - \left[\frac{\gamma_4}{F^{-1}(\epsilon)} - 1\right] \alpha^+ + \left[\frac{\gamma_4}{F^{-1}(1-\epsilon)} - 1\right] \alpha^-\right\} \\
\text{s.t.} \quad & \alpha^+,\, \alpha^- \geq 0,
\end{aligned} \tag{36}$$

which becomes after introducing the notations $\lambda = \alpha^+ - \alpha^-$ and $\mu = -\frac{\alpha^+}{F^{-1}(\epsilon)} + \frac{\alpha^-}{F^{-1}(1-\epsilon)}$ (a well-defined change of variables because $F^{-1}(\epsilon) \neq F^{-1}(1-\epsilon)$):

$$\begin{aligned} R_a + \min \quad & C_{a2}\,\lambda - (T - R_{a1})\,\mu + d_1\,(c_2 + t_{21})\,\max\{0, \overline{\gamma} - [\lambda + \gamma_1\,(1+\mu)]\} \\ & \qquad\qquad\qquad\qquad + d_2\,c_2\,\max\{0, -\overline{\gamma} + \lambda + \gamma_4\,(1+\mu)\} \\ \text{s.t.} \quad & -F^{-1}(1-\epsilon)\,\mu \leq \lambda \leq -F^{-1}(\epsilon)\,\mu. \end{aligned} \tag{37}$$

Note that we must have $\mu \geq 0$ since $F^{-1}(\epsilon) < 0 < F^{-1}(1-\epsilon)$. Problem (37) is feasible if and only if the slope of the objective is nonnegative when we follow the extreme rays $\lambda = -F^{-1}(\epsilon)\,\mu$ and $\lambda = -F^{-1}(1-\epsilon)\,\mu$ when $\mu \to \infty$. We conclude by noting that the slope of the objective (in μ) when $\lambda = -F^{-1}(\epsilon)\,\mu$ is: $-C_{a2}\,F^{-1}(\epsilon) - (T - R_{a1}) + d_1\,(c_2 + t_{21})\,\max\{0, F^{-1}(\epsilon) - \gamma_1\} + d_2\,c_2\,\max\{0, \gamma_4 - F^{-1}(\epsilon) = G(\epsilon)$. Similarly, the slope of the objective (in μ) when $\lambda = -F^{-1}(1-\epsilon)\,\mu$ is: $G(1-\epsilon)$.
$\square$

Remark: As is clear from Formulation (36), key to the change in production plan as $F^{-1}(\epsilon)$ and $F^{-1}(1-\epsilon)$ vary is their position with respect to γ_1 and γ_4. Another important observation is that the impact on the multinational's performance of the threshold parameter T, when the foreign plant is open, is entirely captured by the *difference* between the threshold T and the revenue R_{a1} generated in the US market by the altruistic policy.

Finally, we analyze how the adoption of a probability-based model to capture risk impacts the optimal production plan. The fact that the manager must fine-tune two parameters, T and ϵ, once the risk and return measures have been chosen, creates new flexibility in allowing the manager more control over the regions for which a specific production strategy is optimal (e.g., the US plant produces for its own market and the upper risk constraint is tight), but it also introduces more complexity in the problem.

Theorem 9 (Optimal solution)
(i) The optimal production plan is piecewise linear in τ. The coefficients of the linear pieces depend on $F^{-1}(\epsilon)$ or $F^{-1}(1-\epsilon)$.
(ii) The transition points in the different regimes of the production plan (e.g., when x_1 increases to d_1 and cannot increase further) depend on $F^{-1}(\epsilon)$ and $F^{-1}(1-\epsilon)$, and as a result of the manager's choice of risk parameters.
(iii) One plant always produces all or nothing of the demand in its own market.

Proof: At optimality, either the production plan does not depend on risk (the nominal solution satisfies the risk constraint or the foreign facility remains closed), in which case the results are trivial, or one of the constraints $b(x,y) = \frac{a(x,y)-T}{-F^{-1}(\epsilon)}$ and $b(x,y) = -\frac{a(x,y)-T}{F^{-1}(1-\epsilon)}$ is binding. Injecting the expression of $a(x,y)$ and $b(x,y)$ allows us to conclude following straightforward algebraic manipulations. $\square$

Remark: We have seen that the production planning problem studied in Section 3 when the manager decides to open the foreign facility can be interpreted as a deterministic problem with a modified exchange rate linear in the marginal price the manager would pay to relax the risk constraints. (We have also seen that, in order to have total US production differ from 0, d_1 and $d_1 + d_2$, the modified exchange rate must be one of the breakpoints in the deterministic model.) In contrast, the probability-based model can be reformulated, again using the optimal nonnegative shadow prices α^+ and α^- for the risk constraints:

$$\begin{aligned}
\left[\frac{\alpha^+}{F^{-1}(\epsilon)} - \frac{\alpha^-}{F^{-1}(1-\epsilon)}\right] T + \max \quad & \left[1 - \frac{\alpha^+}{F^{-1}(\epsilon)} + \frac{\alpha^-}{F^{-1}(1-\epsilon)}\right] A(x) + [\overline{\gamma} - \alpha^+ + \alpha^-]\, B(x) \\
\text{s.t.} \quad & 0 \leq x_1 \leq d_1, \\
& 0 \leq x_2 \leq d_2,
\end{aligned} \tag{38}$$

Risk, measured by the probability the revenue will fall below a threshold, now affects the portion of the revenue that is US-based, i.e., does not depend on the exchange rate, as well as the portion that is foreign-based. The equivalent nominal exchange rate in this context would be: $[\pi(\gamma) - \alpha^+ + \alpha^-]/[1 - \frac{\alpha^+}{F^{-1}(\epsilon)} + \frac{\alpha^-}{F^{-1}(1-\epsilon)}]$, which is no longer linear in the dual variables. In this model, risk is not solely captured by the value of the shadow prices; the (manager-selected) value of $F^{-1}(\epsilon)$ and $F^{-1}(1-\epsilon)$ also have a *direct* impact on the nominal exchange rate.

The probability-driven model has appealing properties, but also raises implementation challenges, in particular in the selection of the parameters, and it is not clear that this approach outperforms the simpler framework studied in Sections 2 and 3. The purpose of Section 5 is to provide some insights into the comparative merits of the models we have presented so far by conducting a numerical study using simulated and historical data.

5 Empirical study

5.1 Purpose

In this section, we analyze how the decision maker's choice of exchange rate model and performance measure affects the optimal allocation through extensive numerical experiments. In particular, we investigate the following issues:

- How does shortfall perform compared to the traditional variance- and probability-based risk measures?
- How important to overall performance is it to assume the correct model for the exchange rate?
- Do these insights hold for the historical data available?

Researchers have traditionally assumed that exchange rates follow either a Geometric Brownian Motion (Huchzermeier and Cohen 1993, Dasu and Li 1997, Kouvelis, Axarloglou and Sinha 2001) or a mean-reverting (Ornstein-Uhlenbeck) Brownian Motion (Kogut and Kulatilaka 1994) with known distribution parameters. Therefore, we will focus our attention on these processes using simulated data from both Geometric Brownian Motions and mean-reverting Brownian Motions for a wide array of parameters, as well as historical exchange rates in ranges around the breakpoints identified in the theoretical part of our work. Our analysis compares the optimal strategies of seven types of risk-averse decision-makers, whose characteristics (the exchange rate process they assume and the risk measure they consider) are summarized in Table 2.

	Exchange rate process	Risk measure
1	Geometric BM	Variance
2	Geometric BM	Probability
3	Mean-reverting BM	Variance
4	Mean-reverting BM	Probability
5	Normal (i.i.d.)	Variance
6	Normal (i.i.d.)	Probability
7	Data only	Shortfall

Table 2: Characteristics of the decision-makers.

5.2 Research Methodology for Simulated Data

Our methodology in simulating the data can be summarized as follows:

1. Generate n=8 or n=20 sequential exchange rates (to analyze the impact of the size of the data set).

2. Compute the parameter values for each exchange rate model based on this data (for shortfall, we use a simple linear regression model to generate the residuals.)

3. Find the optimal allocation for each decision-maker, based on his exchange rate model and performance measure.

4. Generate 100 "next period" exchange rate instances.

5. Compute the profit that would be realized for each of the 100 "next period" exchange rate instances and for each of the allocations generated in step 3.

6. Create an Excel graph to evaluate the results from the simulation.

7. Repeat the process for each set of parameters presented in Tables 3 and 4 below for a Geometric Brownian motion, with (a) drift= .1 and σ = .1 and (b) drift= .05 and σ = .1, and a mean-reverting Brownian motion (with reverting factor λ given in Table 4 and $\sigma = 2 \cdot \bar{\gamma}$, respectively).

The second column in Tables 3 and 4 indicates the breakpoint that we are investigating for that run. The last simulated data point must be close to the breakpoint for the simulation to be of any interest; otherwise, there is no uncertainty on the strategy that will be optimal once the exchange rate is realized in the next period. α in the shortfall column refers to the specific quantile considered.

5.3 Simulation Results

As a large number of scenarios were evaluated in the numerical study, we chose a limited number of instances to present the general results that are supported by the entire, wider collection of simulation runs. Figures 2 and 3 present the profits when the exchange rates were generated using a Geometric Brownian motion for the breakpoints 0.66 and 1.32, respectively; therefore, the standard deviation, shortfall and profit-at-risk measured for the correct assumption of Geometric Brownian motion serve as benchmarks to compare the results obtained assuming a mean-reverting Brownian motion or a Normal distribution. Figure 4 presents the results when the exchange rates were generated using a mean-reverting Brownian motion. Similarly, the metrics (standard deviation,

		Probability		Variance	Shortfall	
Index	$\approx n^{\text{th}}\ \gamma$	T	ϵ	τ	α	τ
1	0.66	4000	0.1	10	0.1	30
2	0.66	4000	0.2	10	0.2	30
3	0.66	4000	0.3	10	0.3	30
4	0.66	4250	0.1	25	0.1	75
5	0.66	4250	0.2	25	0.2	75
6	0.66	4250	0.3	25	0.3	75
7	0.66	4350	0.1	50	0.1	150
8	0.66	4350	0.2	50	0.2	150
9	0.66	4350	0.3	50	0.3	150
10	0.66	4400	0.1	100	0.1	300
11	0.66	4400	0.2	100	0.2	300
12	0.66	4400	0.3	100	0.3	300
1	1.32	5000	0.1	100	0.1	300
2	1.32	5000	0.2	100	0.2	300
3	1.32	5000	0.3	100	0.3	300
4	1.32	4750	0.1	250	0.1	750
5	1.32	4750	0.2	250	0.2	750
6	1.32	4750	0.3	250	0.3	750
7	1.32	4600	0.1	500	0.1	1500
8	1.32	4600	0.2	500	0.2	1500
9	1.32	4600	0.3	500	0.3	1500
10	1.32	4500	0.1	1000	0.1	3000
11	1.32	4500	0.2	1000	0.2	3000
12	1.32	4500	0.3	1000	0.3	3000

Table 3: Parameters for simulation runs using Geometric Brownian motions, for breakpoints 0.66 and 1.32.

shortfall, profit-at-risk) computed with the correct process assumption will serve as benchmark to compare the other results. In the title of the graphs, the first number refers to the breakpoint and the second number to the index of the simulation run, which refers to either Table 2 or 3. The upper half of the graphs corresponds to runs with 8 data points and the lower half with 20 data points.

Once the allocation is determined, the profit is linear in the realized exchange rate, which is indeed what we observe in the graphs. A line with a slope steeper, in absolute value, than the absolute value of the benchmark indicates that the performance measure did not constrain the production allocation sufficiently to meet the targeted risk level. This is because the slope is the revenue generated in the foreign market, which is bounded by τ/σ when the manager makes the correct assumption on the process. Similarly, a line with a slope gentler than the benchmark

Index	$\bar{\gamma}$	λ	Probability T	ϵ	Variance τ	Shortfall α	τ
1	0.66	0.3	4000	0.1	10	0.1	30
2	0.66	0.7	4000	0.2	10	0.2	30
3	0.66	1.0	4000	0.3	10	0.3	30
4	0.66	0.3	4250	0.1	25	0.1	75
5	0.66	0.7	4250	0.2	25	0.2	75
6	0.66	1.0	4250	0.3	25	0.3	75
7	0.66	0.3	4350	0.1	50	0.1	150
8	0.66	0.7	4350	0.2	50	0.2	150
9	0.66	1.0	4350	0.3	50	0.3	150
10	0.66	0.3	4400	0.1	100	0.1	300
11	0.66	0.7	4400	0.2	100	0.2	300
12	0.66	1.0	4400	0.3	100	0.3	300
1	1.32	0.3	5000	0.1	100	0.1	300
2	1.32	0.7	5000	0.2	100	0.2	300
3	1.32	1.0	5000	0.3	100	0.3	300
4	1.32	0.3	4750	0.1	250	0.1	750
5	1.32	0.7	4750	0.2	250	0.2	750
6	1.32	1.0	4750	0.3	250	0.3	750
7	1.32	0.3	4600	0.1	500	0.1	1500
8	1.32	0.7	4600	0.2	500	0.2	1500
9	1.32	1.0	4600	0.3	500	0.3	1500
10	1.32	0.3	4500	0.1	1000	0.1	3000
11	1.32	0.7	4500	0.2	1000	0.2	3000
12	1.32	1.0	4500	0.3	1000	0.3	3000

Table 4: Parameters for simulation runs using mean-reverting Brownian motions, for breakpoints 0.66 and 1.32.

(in absolute value) indicates that this performance measure overly constrained the production allocation to meet the targeted risk level. Steep slopes in absolute value also indicate high profit volatility. In this context, we interpret robustness as relative closeness to the benchmark and make the following observations:

1. The number of historical data points used ($n = 8$ or $n = 20$) can have a significant impact on the profit when the true exchange rate process obeys a Geometric Brownian motion. For instance, GBMStd coincides with NormStd in the upper right panel in Figure 2 and with GOUStd in the lower right panel, which indicates that these assumptions yield the same production plan. The number of data points, however, does not have any significant effect on the metrics when the true exchange rate process is mean-reverting (Figure 4).

2. Assuming a stationary Normal distribution for the exchange rates leads to unpredictable per-

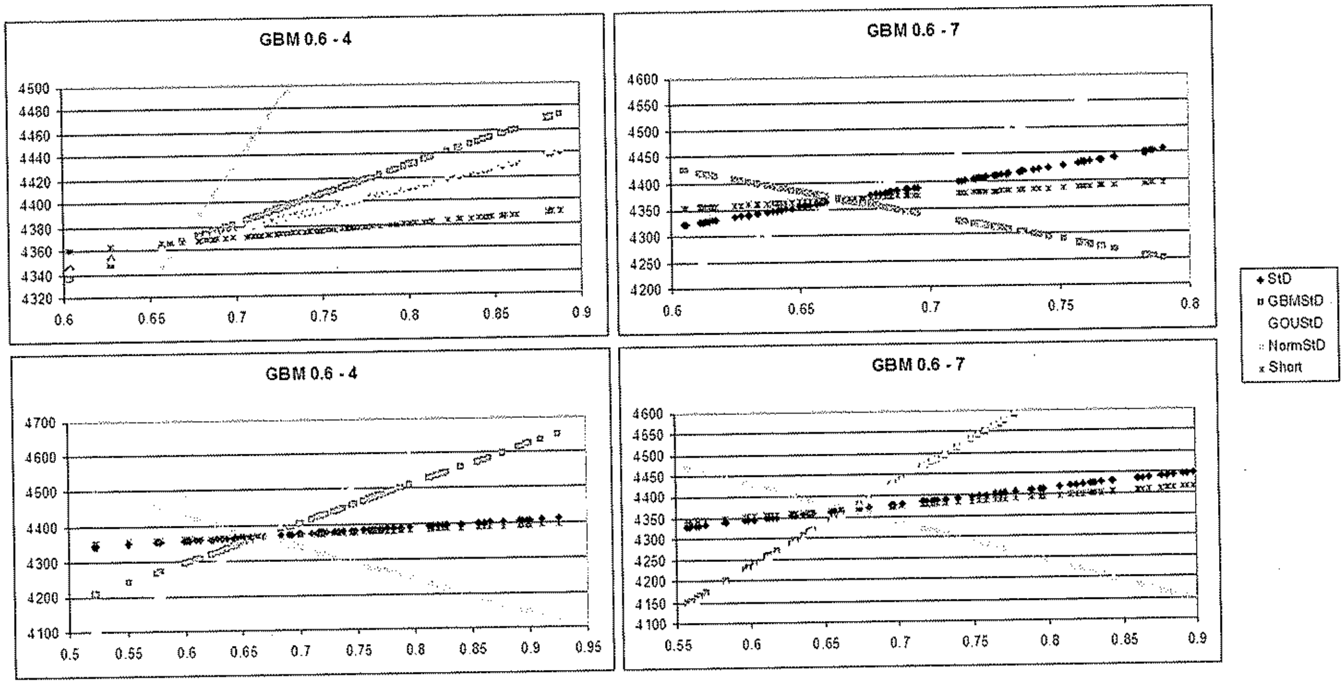

Figure 2: Results for exchange rates from a Geometric Brownian motion (breakpoint 0.66).

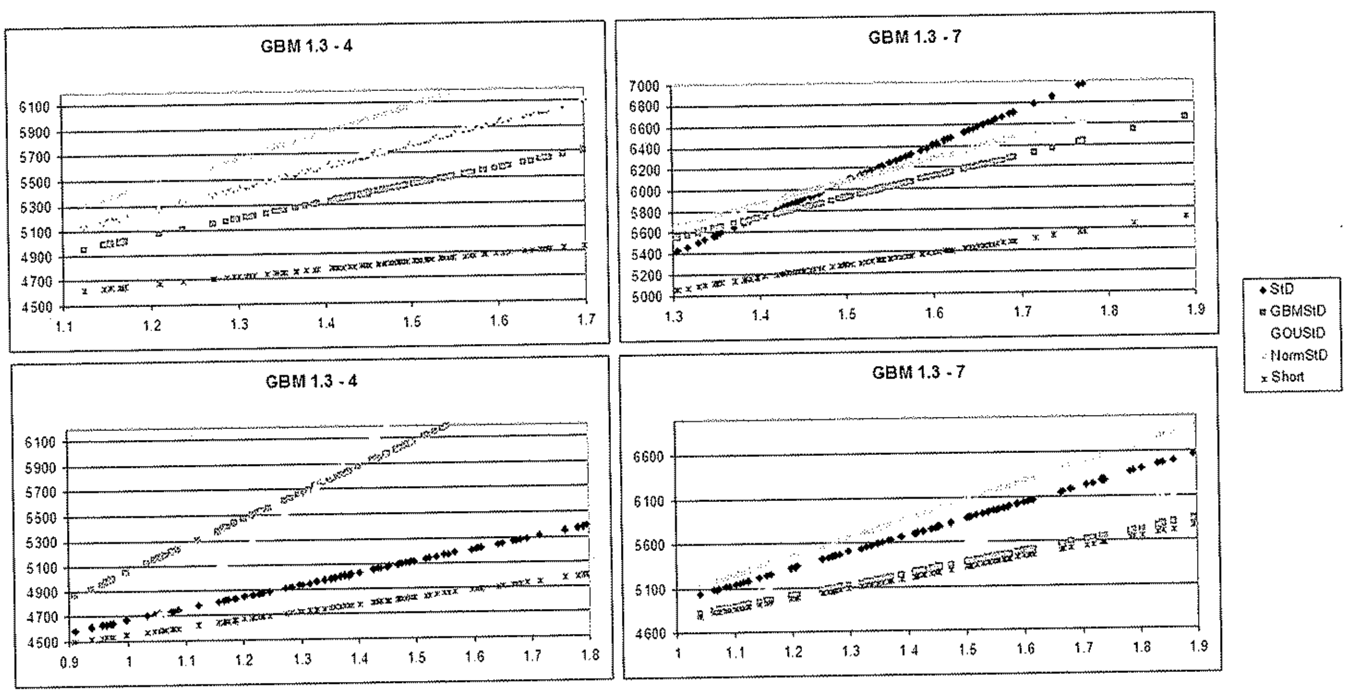

Figure 3: Results for exchange rates from a Geometric Brownian motion (breakpoint 1.32).

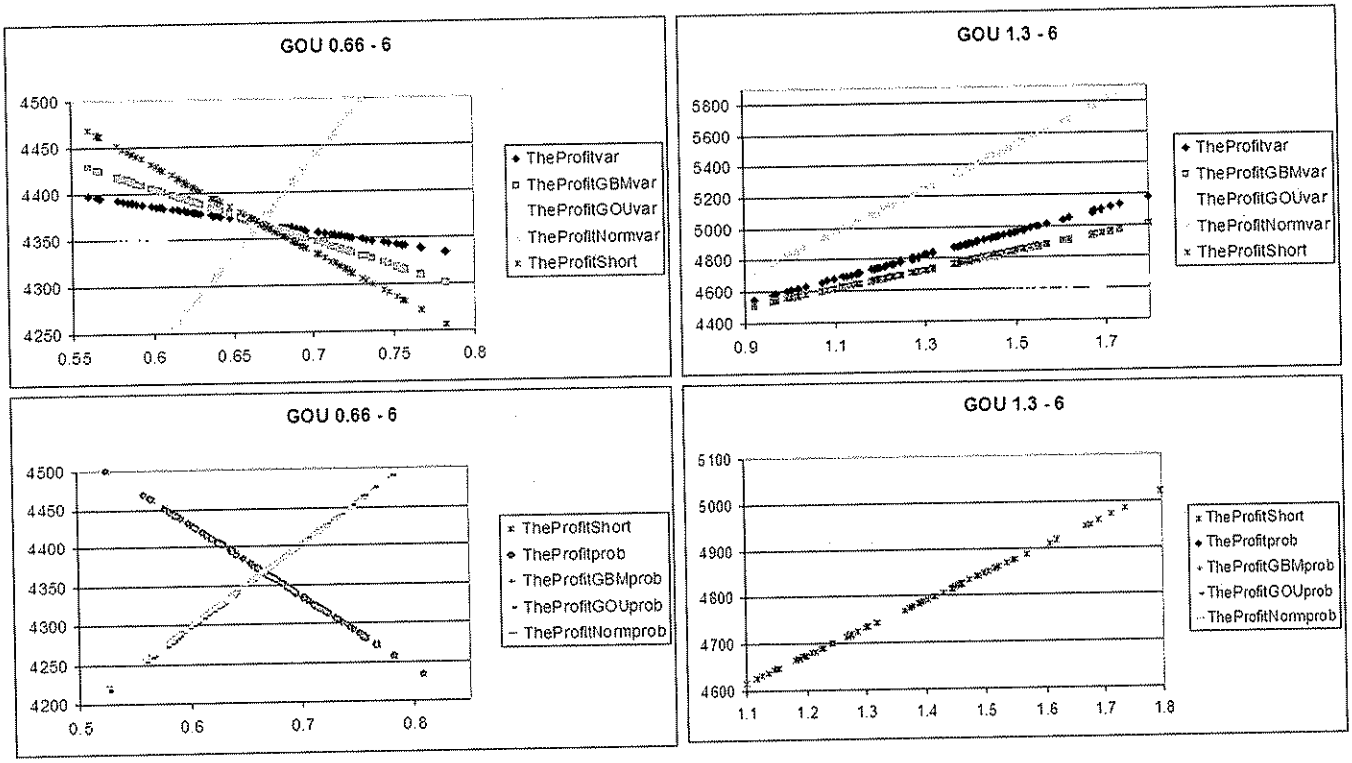

Figure 4: Results for exchange rates from a mean-reverting Brownian motion.

formance; for instance, this leads to the steepest slope in the upper left panel (simulation run 4, first breakpoint) of Figure 2, but to the same allocation as the one obtained in GBM-Std with the correct assumption of Geometric Brownian motion in the upper right panel (simulation run 6, first breakpoint).

3. Assuming the correct Brownian motion process, while obviously desirable, does not substantially improve performance, as indicated by similar slopes obtained under the assumptions of geometric and mean-reverting Brownian motions, for instance TheProfitGBMvar and TheProfitGOUvar in Figure 4.

4. In most instances, using shortfall as risk measure results in slightly over-constraining profit. This effect is more pronounced at the second breakpoint value (Figure 3), where we observe that the shortfall corresponds to the lowest plot on each of the four graphs.

5.4 Research Methodology for Historical Data

We now apply our methodology to actual exchange rates between the US dollar and the currencies of Brazil, Taiwan, Japan, Russia, Mexico, South Africa, Switzerland and Israel. We chose this group of foreign countries because: (i) they represent varied regions of the world, (ii) they have a strong, worldwide manufacturing presence, and (iii) their currencies show considerable variability to the US dollar (see Figure 5). In order to study the currencies near the breakpoints identified in

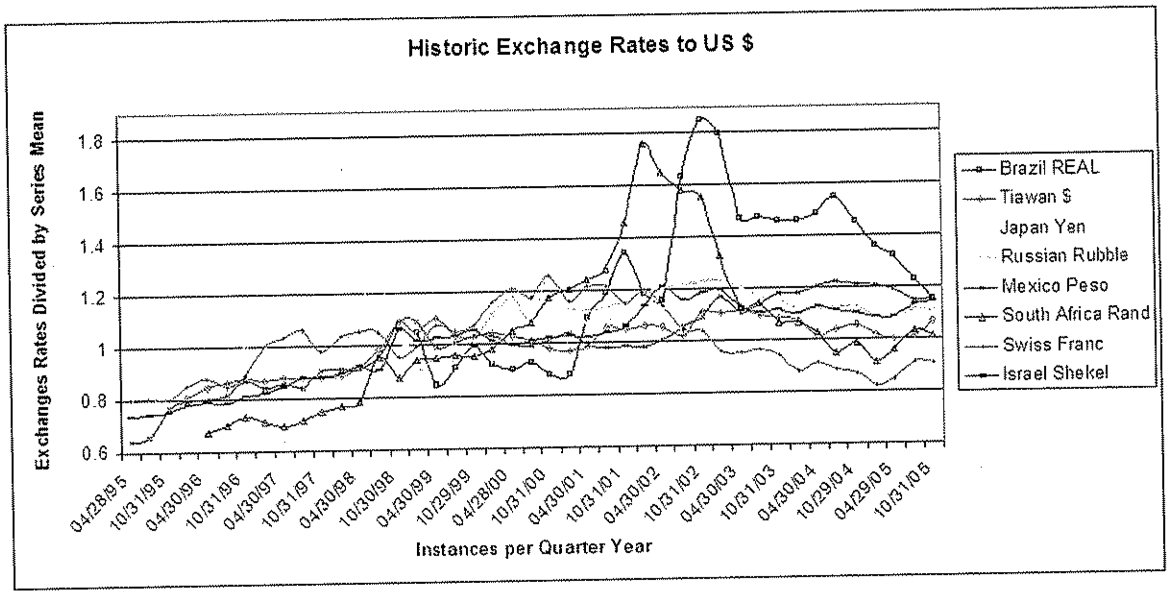

Figure 5: Normalized historical exchange rates to the US dollar.

the theoretical model, we normalize each currency data set by dividing all the instances by their arithmetic mean, and then multiplying them by each breakpoint. This yields two sets of data points per currency (one for each breakpoint). For each set, we follow the same procedure as for the simulated data to generate the optimal allocation for each decision- maker. The parameters used in this part of our empirical study are summarized in Table 5.

5.5 Results with Historical Data

Once again, we chose a limited number of representative results to present, which generalize the entire collection. From these results (see Figures 6-7), we make the following observations:

1. Generally, choosing shortfall as a risk measure constrains the profit potential more than choosing other risk measures, especially for the higher breakpoint, 1.32.

Index	Breakpoint	Probability		Variance	Shortfall	
		T	ϵ	T	α	T
1	0.66	4000	0.1	10	0.1	30
2	0.66	4250	0.1	25	0.1	75
3	0.66	4350	0.1	50	0.1	150
4	0.66	4400	0.1	100	0.1	300
5	1.32	5000	0.1	100	0.1	300
6	1.32	4750	0.1	250	0.1	750
7	1.32	4600	0.1	500	0.1	1500
8	1.32	4500	0.1	1000	0.1	3000

Table 5: Parameters in study using actual exchange rates.

2. The results when a stationary Normal distribution is assumed often differ quite noticeably from the others.

3. There is no appreciable difference in performance when we assume a Geometric Brownian motion as opposed to a mean-reverting one.

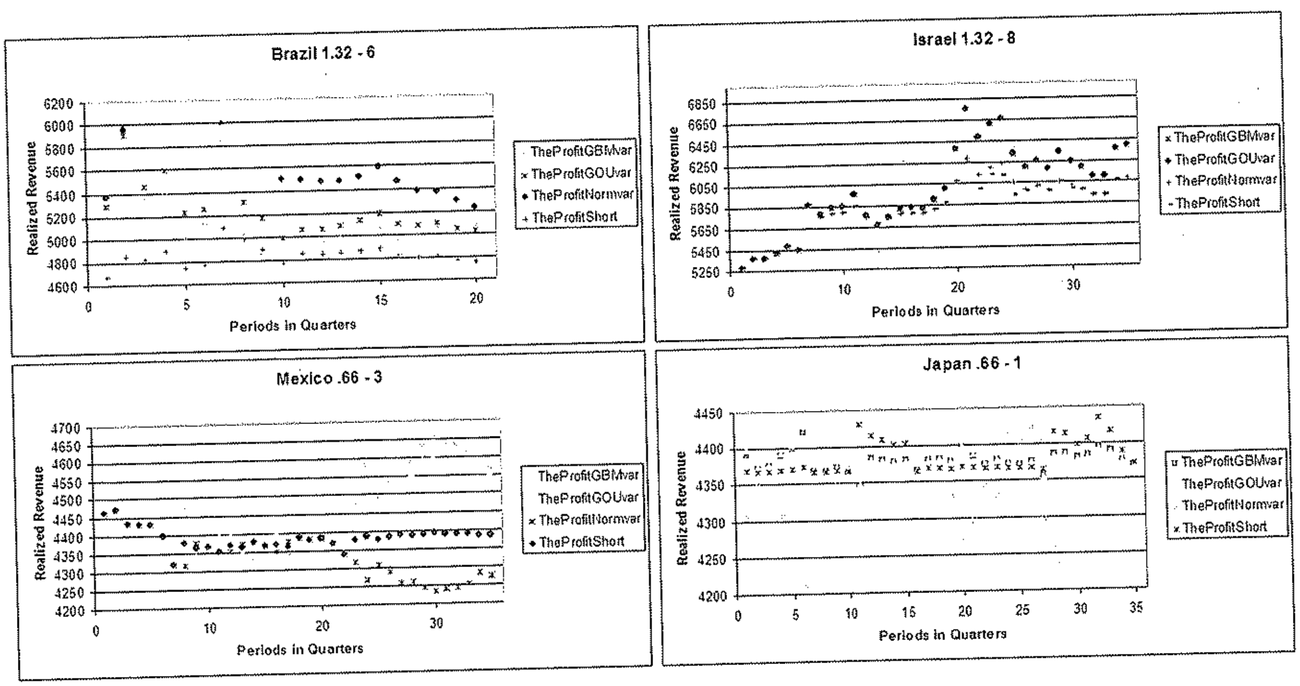

Figure 6: Results for historical exchange rates (variance model).

The conservative results associated with shortfall suggest that the use of linear-regression residuals is likely inadequate in the case of actual exchange rates. This could for instance be due to correlation

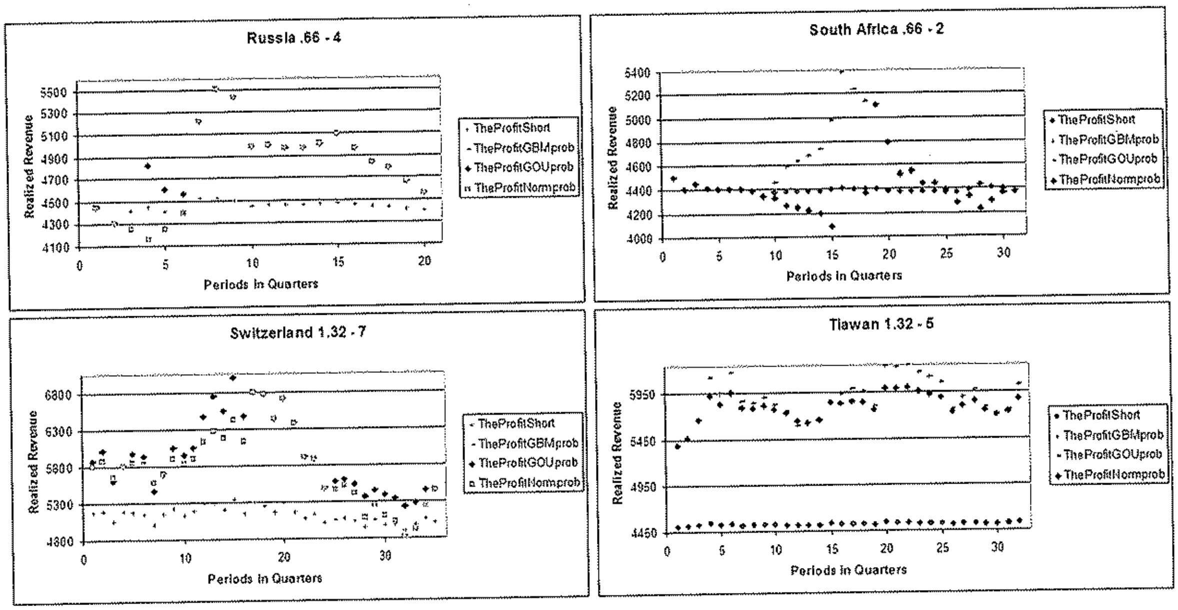

Figure 7: Results for historical exchange rates (probability model).

or time-varying effects across actual exchange rates, whereas the simulated data points are assumed to obey i.i.d. distributions.

5.6 Summary of Numerical Results

The aim of this study was to test various risk measures in a realistic implementation of operational hedging, and in particular to investigate the performance of shortfall as well as to compare its performance with that of traditional measures such as variance and probability (profit-at-risk). When we simulated exchange rates based on the stochastic processes commonly used in the literature, we found that the shortfall-based approach performed similarly to (sometimes better, sometimes worse than) the other risk measures. When we considered real exchange rates, however, the shortfall-based approach limited profit more than its variance and probability-driven counterparts. This suggests that enforcing shortfall constraints to model risk aversion might not be appropriate in operational hedging, as the resulting profit is often dominated by the profit obtained using other metrics for a wide range of values of the exchange rate.

6 Conclusions

We have integrated risk aversion to operational hedging when exchange rate is the only source of uncertainty. This has allowed us to quantify the impact of the manager's risk preferences on the optimal strategy, and to compare three well-known risk measures: variance, probability (profit-at-risk) and shortfall. We have also conducted an extensive numerical study to test our results using both simulated and actual data. While shortfall has received much attention in the finance literature and performs well for simulated values of the exchange rates, our results for historical exchange rates indicate that this concept should be used with caution in operational hedging, as it leads to very conservative results compared to other measures of risk. Further research includes extensions to facilities in several foreign countries and multi-period models.

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