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Abstract

Due to demanding and unstable business environments, companies must be able to quickly react to disturbances from outside sources. Many supply chain (SC) planning models exist which involve proactive measures to mitigate effects due to SC uncertainty and therefore a certain level of prior investment. However, a reactive supply chain may be able to avoid these upfront costs. In this research, supply uncertainty with regards to lead time is investigated. The lead time distribution, for each supplier in a multi-echelon SC, is dependent upon the current level of bottleneck within the supplier (light, normal or congested). For each of these three levels, two distributions of lead time are investigated, beta and normal. The two types of distributions are considered separately and chance constrained programming is used to solve for the optimal supplier set while minimizing cost to the entire SC. The result is a SC which, by re-optimizing at each echelon, can exhibit lower overall cost and increased on-time performance.

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Contents

1 Introduction

In its simplest representation, a supply chain is composed of a forward flow of material and a backward flow of information [13]. A disruption in one of the links of a SC can have a devastating impact on the others [1]. Supply chain disruptions can be categorized broadly as uncertainty in the demand (downstream) side or supply (upstream) side and can stem from a wide range of sources: natural disasters, labor disputes or strikes, logistic problems and quality issues to name a few [2] [16] [4]. The root cause can be either internal or external to the company. The consequences of a disruption can have an extensive financial impact on companies. This research develops a methodology for a flexible SC which avoids the costs associated with robust planning under uncertainty and allows firms to react swiftly and effectively. By optimizing the ordering policy at each echelon, a disruption in the SC can be overcome and the SC re-configured with minimal cost while still maintaining the due date.

The supply leadtime is what governs the uncertainty and policies are developed for many different levels of reliability. Two distributions of leadtime are considered: normal and beta. For each of these distributions, the disparity between suppliers is varied and the problem of determining ordering quantities is formulated as a chance constrained program. The outputs yield many useful managerial insights. The results lead to a SC which can exhibit lower overall costs, improved on-time shipment performance and lower total inventory than an uncontrolled SC.

2 Literature Review

Supplier selection based upon unreliable suppliers has been addressed significantly in the literature. Tomlin [19] provided an overview of this literature. This research combines three issues typically addressed independently in literature: 1)suppliers are categorized into a certain state at each time period, 2)stochastic leadtimes are considered and 3)a multi-echelon SC is studied. The leadtimes considered in most state-dependent leadtime literature are discrete.

Ozekici and Parlar [14] characterized a process by assuming that either the full order amount or nothing is received. The determination of which state will occur is probabilistically based upon the state of the system that is observed at that time. Dada et al. [3] characterized a supplier state by the output of its production. This state is a function of the amount ordered and its reliability. In this situation, a supplier delivered the minimum of either the full amount ordered and the output of its production based upon its state. The tradeoff between cost and reliability was examined by maximizing expected profit. The authors concluded that cost takes precedence. In particular, if one supplier is inactive, all more expensive suppliers will also be inactive. Tomlin and Snyder [20] also examined an unreliable supplier situation. In their paper, the supplier is either active or inactive. In the active state, there are a finite number of threat levels and the probabilities of transitioning between threat levels and the probabilities of transitioning from a threat level to an inactive state are known a priori. Ordering policies were developed based upon the current supplier state. When the two supplier case is examined, only one supplier is unreliable, however, significantly less expensive that the reliable supplier. With uncapacitated suppliers, single sourcing was found to be optimal. In Dada et. al., [3], the authors assigned a ship quantity equal to either the

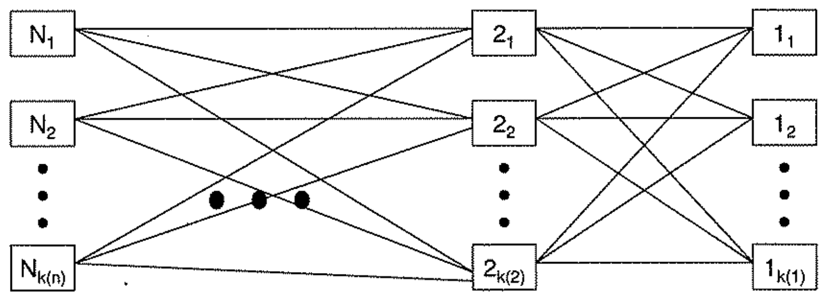


Figure 1: Multi-Echelon, Multi-Stage Supply Chain

full amount or some lower value based upon the suppliers expected production output.

The literature on stochastic leadtimes is also broad. One early work by Feeney and Sherbrooke [7] considered a process in which demands form a compound Poisson process with i.i.d. leadtimes. In contrast to Feeney and Sherbrooke [7], Song [17] investigated the effect of leadtime uncertainty on the base stock level and overall cost in a system using a sequential supply process where leadtimes are non-independent. Their results showed that when leadtimes are exponential, larger variability yields higher optimal costs and for a general distribution, bounds are established. Svoronos and Zipkin [18] study stochastic leadtimes in an exogenous process, meaning that the system behavior is independent of the demands and orders in the inventory system.

The research here differs from the literature in three ways. First, in this research, leadtimes are state-dependent and stochastic within each state. Most state-dependent literature discussed above considers a deterministic leadtime. Secondly, order quantities for a particular echelon are optimized over the current state of all of its upstream suppliers in the SC. And finally, this research considers a multi-echelon situation in which all suppliers are unreliable in their leadtime to some extent. The unreliability is defined in terms of the leadtime. This information is gained from historical data. Determining supplier state through the Bayesian method is intended to yield a more accurate description of a suppliers readiness and is unlike any of the literature discussed. Furthermore, by segmenting the stochastic leadtime by the supplier's state, a fairly good representation of anticipated delivery date should be achievable since historical supplier data is used.

3 Supply Chain Environment

This research considers a multi-echelon SC as shown in Figure 1. There are N echelons and $k(i)$, $i \in N$, suppliers (or stages) in each echelon. Binary decisions are made at each stage regarding supplier choice. For example, when an order enters stage 1_1, $k(2)$ binary decisions are made for the suppliers in echelon 2, representing the binary decisions of whether or not to purchase from each of the $k(2)$ suppliers in echelon 2.

In this research, the ordering decisions are based on two parameters: the current state of the upstream supplier(s) and their respective leadtime distribution. The state refers to the level of congestion seen at the supplier. Therefore, due to the dynamic nature of the supplier states and distributions, these ordering decisions are not made for the entire SC at once, but rather, they are made sequentially. For example, at time $t = 1$, an order is only placed initially with the supplier(s) in echelon 2. After that order is placed, some time has elapsed and the states of the remaining upstream suppliers may have changed, which may change the ordering decisions of the upstream supplier(s). A new supplier subset and order quantities are determined from the suppliers in echelon 3. This process is repeated until the orders are placed with the suppliers in echelon N.

Three possible states for a supplier are introduced here: light, normal and congested and this state of the supplier is determined from information which is shared between buyer and seller. The reader is referred to [5] for the details of the supplier state determination methodology. A downstream supplier routinely maintains and updates distributions of actual and quoted leadtimes for each of its upstream suppliers, along with the supplier state at that time. Then, when an order needs to be placed, the supplier state is determined, and the buyer can formulate their own expected leadtime for delivery from each upstream suppliers.

3.1 Supplier Selection and Ordering Process

An order from an end customer contains three variables: price, delivery time and quantity. For this incoming order to the SC, a specific amount of material is needed from each SC member in the chain (note that the quantity may be zero for some suppliers). Knowing the state of each supplier, the subsets of suppliers from which to purchase can be determined. The process is as follows:

1. At time t, information is shared between suppliers to determine the state of each supplier ij, $i \in N$, $j \in k(i)$. The state may be light, normal or congested.
2. The expected leadtime, conditioned upon the supplier state, is calculated using historical data.
3. The upstream supplier(s), quotes a leadtime, lt_{ijt}.
4. Decision regarding order quantity from each supplier are made using a chance constrained program described in Section 4.

4 Ordering Quantities Under Uncertainty: A Synchronized Supply Chain

When making the supplier selection decisions, the objective is to minimize the total cost of the entire SC while trying to deliver on-time with a certain service level. The uncertainty in the leadtime is what drives the problem. The problem is formulated as a chance constrained program (CCP) and is discussed below. The results will yield order quantities for a synchronized SC (SSC) which optimizes order quantities under uncertainty and minimize cost. The supplier choice variables used in this research are defined in Table 1.

4.1 General CCP Model

An N-member supply chain with $k(i)$, $i \in N$ suppliers in each stage is considered in this research (refer to Figure 1). The chance constrained program model in general is:

Table 1: Supplier Choice Variables

c_{ij}	cost per unit of supplier j in stage i, $i \in n$, $j \in k(i)$
e_{ij}	penalty cost per day of supplier j in stage i, $i \in n$, $j \in k(i)$
dt_{ij}	quoted leadtime (days per unit) of supplier j in stage i, $i \in n$, $j \in k(i)$
Q_i	total number of units needed from echelon i, $i \in n$
L_t	remaining leadtime at time t
m_{ijt}	state of supplier j in echelon $i$at time t, $i \in n$, $j \in k(i)$
q_{ij}	order quantity of supplier j in echelon i, $i \in n$, $j \in k(i)$
α	service level
γ_{ij_t}	random variable of expected delivery time (days per unit) of supplier j in echelon i at time t (from historical data), $i \in n$, $j \in k(i)$

$$\text{minimize} \sum_{i=1}^{N} \sum_{j \in k(i)} c_{ij} q_{ij} \tag{1}$$

subject to

$$P((\gamma_{ij} q_{ij} \leq dt_{ij}) | m_{ij}) \geq \alpha_i \; i = 1, ..., N; \forall j \in k(i) \tag{2}$$

$$\sum_{i=1}^{N} (\sum_{j \in k(i)} q_{ij}) \leq Q_i \tag{3}$$

$$q_{ij} \geq 0 \text{ i=1,...,N } \forall j \in k(i) \tag{4}$$

The objective function minimizes ordering cost. Constraint 2 stipulates that the probability of an on-time actual delivery ($\gamma_{ij} | m_{ij} \leq dt_{ij}$), by each supplier $ik(i)$ in their respective state $m_{ik(i)}$, meets or exceeds the service level. Constraint 3 ensures that the proper amount of material is purchased and the final constraint ensures nonnegativity of the order quantity.

Constraint 2 requires that the distribution of delivery time be known. In this research two distributions are considered: normal and beta. Using historical data fitted to a normal distribution, the parameters (mean and variance) are easily computed. However, for a beta distribution, the mean and variance are not as clear [22]. The parameter development for this distribution are discussed in Appendix i.

4.2 Modeling Parameters and Assumptions

The actual SC used for testing is a three echelon SC with 2 suppliers in echelons 2 and 3 and one supplier in echelon 1, as shown in Figure 2. The following assumptions are also made:
A1: The supply leadtimes of upstream suppliers are independent, identically distributed random variables.

A2: The distribution describing the leadtimes will be dependent upon the state of the supplier. Three states are considered: light, normal and congested.

A3: Demand is normally distributed.

In each echelon, the more reliable supplier charges a higher price for their product. Three ratios of cost are investigated (2:1, 3:1 and 4:1).

A4: For comparison purposes, one supplier will be more reliable than the other. This reliability is given within the parameters of the leadtime distribution.

A5: When the leadtime is considered to be normally distributed, with a specified mean (μ) and variance (σ^2), the reliability is given in terms of the standard deviation (i.e. more reliable supplier has a lower standard deviation).

A6: When leadtimes are beta distributed, with a specified alpha (α) and beta (β) value, the reliability is given in terms of the alpha-parameter value.

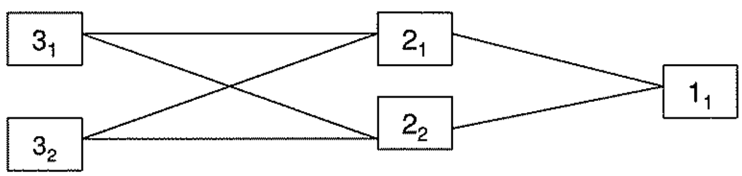


Figure 2: Multi-Echelon SC Used for Testing

The 3 echelon SC shown in Figure 2 was simulated and the CCP solved at echelons 1 and 2 to determine the optimal quantities to be ordered from suppliers 2_1, 2_2, 3_1 and 3_3. The CCP was solved assuming first a normal and then a beta distribution of leadtime for Constraint 2. The specific parameters and results are discussed in Section 4.3 for the normal and 5.2 for the beta distribution.

4.3 Normal Distribution of Leadtime: Parameters and Results

The chance constrained program was modeled in AMPL for both the normal distribution and the beta distributions. The supply chain consisted of 3 echelons with 2 suppliers in echelon 3 and echelon 2 and one end retailer as seen in Figure 2.

4.3.1 Modeling Parameters and Scenarios

The trials for the normal distribution were determined by keeping the state-dependent variances of suppliers 2_1 and 3_1 constant within a state but different for each state so that those two suppliers each had three variances (one for light, normal and congested). A variance ratio (VR) is introduced here to find the variance of suppliers 2_2 and 3_2. For each trial, the VR of suppliers within an echelon, $\sigma^2_{2_2} : \sigma^2_{2_1}$ and $\sigma^2_{3_2} : \sigma^2_{3_1}$, was incremented from 1.0 to 3.4 in 0.1 increments yielding 25 scenarios.

Each of the 4 suppliers can be in one of 3 stages (light, normal congested) or 81 possible system states, which were tested over the 25 scenarios yielding 2025 different scenarios. Each of these 2025 scenarios was optimized over three distinct cost ratios:

2:1, 3:1 and 4:1, where the more reliable supplier in each echelon was the more expensive supplier.

4.3.2 Results

The total order quantity for each echelon was set to 100. Note that in the results, the order quantities discussed and shown on the following graphs are the quantities to be ordered from that particular supplier (not the quantities that a particular supplier orders). Each of the 81 states were numbered and the listing of this numbering scheme is given in Table 6. The graphical results for the 25 VRs for each supplier are split into 4 graphs for readability. Figure 3 shows the quantities that are ordered from supplier 3_1. The states along the horizontal axis are referenced to Table 6. In particular, states 19 through 27, 46 through 54, and 73 through 81, are states when supplier 3_2 is in a congested state, which explains the larger order quantities from the more reliable supplier at these times. This pattern is repeated, although exaggerated, for the other 2 cost ratios. In particular, the order quantity of 3_1 increases as that echelon becomes more congested and with the most congestion (state 81), the most material is ordered from the more reliable supplier and the percentage is increases with in increasing VR. The ordering quantities for supplier 3_1 under a cost ratio of 3:1 and 4:1 follow a similar pattern and are shown in Appendix III in Figures 15 and 16 for the 3:1 cost ratio and Figures 17 and 18 for the 4:1 cost ratio. The ordering patterns from supplier 2_1 are shown in Figure 4 for a cost ratio of 2:1. As seen in the graphs, the behavior of echelon 2 is different than echelon 1. Significantly more material is always ordered from the more reliable supplier (2_1) each time the less reliable supplier is in a congested state. However, this quantity remains the same throughout all the states, so that for example, 77% is always ordered from the 2_1 every time 2_2 becomes congested and this remains constant for each state the 2_2 is congested. Similar patterns are seen for the remaining two cost ratios of 3:1 and 4:1. The order quantity graphs for supplier 2_1 for a cost ratio of 3:1 are given in Figures 19 through 20 in Appendix III and in Figures 21 through 22 for the cost ratio of 4:1.

Typically, an increase in congestion yielded an increase in ordering more from the more reliable supplier. However, this was not always the case. Table 2 illustrates the order quantity ratio of the more reliable to less reliable supplier. For a cost ratio of 3:1, echelon 2 orders a greater quantity from the more reliable supplier while echelon 3 is reversed. This reverses for the cost ratios of 2:1 and 4:1 where the less reliable supplier has a higher order quantity in echelon 2 and a lower order quantity in echelon 3.

Table 2: Mean and Variance of Ordering Ratios for Each Echelon

	Cost Ratio 2:1		Cost Ratio 3:1		Cost Ratio 4:1	
	1_2:1_1	2_2:2_1	1_2:1_1	2_2:2_1	1_2:1_1	2_2:2_1
μ	0.988	0.9843	1.25	0.8932	0.913	0.923
σ^2	0.109	0.023	0.109	0.044	0.057	0.035

The standard deviation of cost is, on average, $2.33 higher with the higher cost ratio. When the more reliable supplier costs 4 times more than the less reliable, the costs increase by an average of $6.48, however the standard deviation decreases with the higher cost ratio by an average of $0.54.

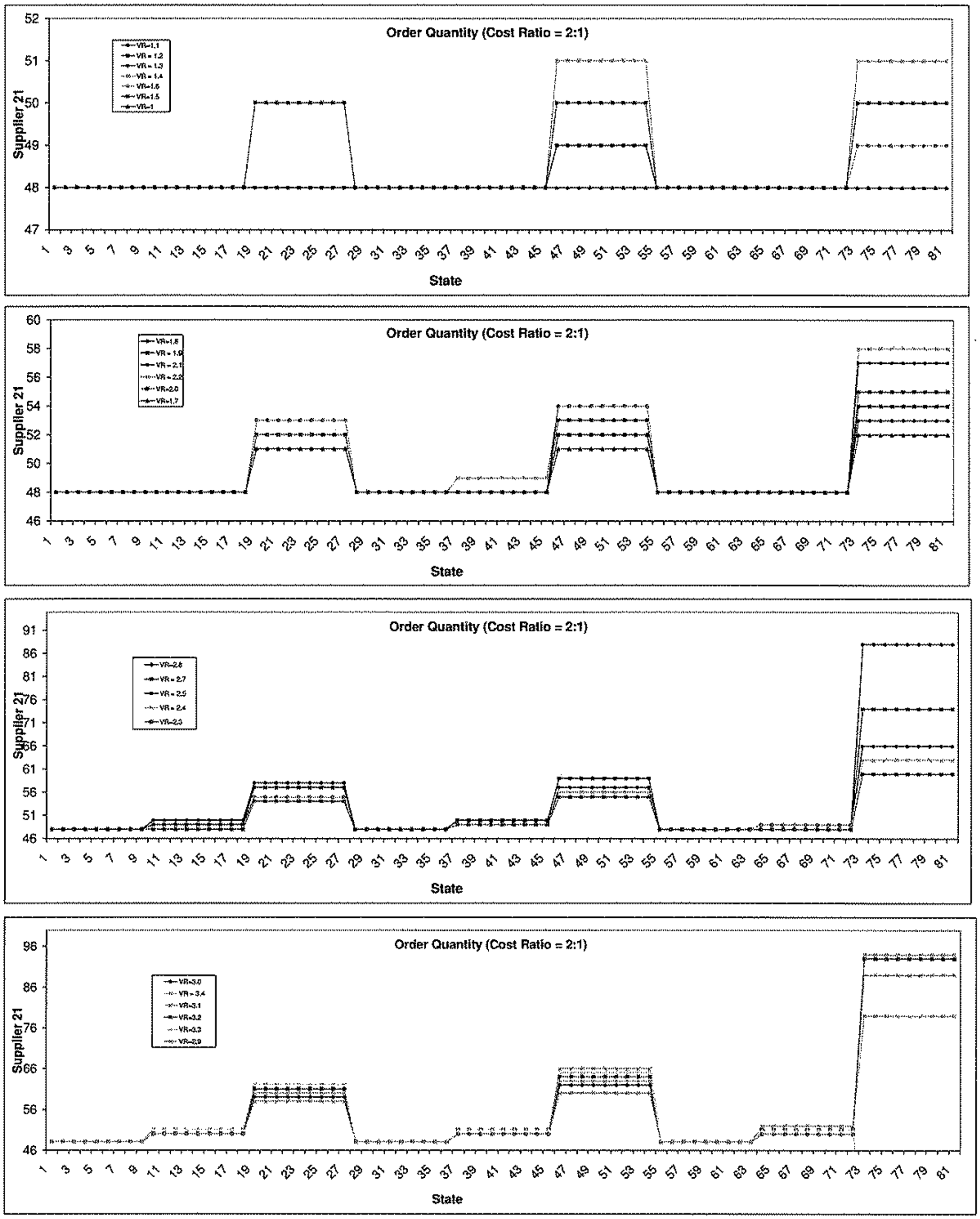


Figure 3: Ordering Quantities from Supplier 3_1 (Cost Ratio=2:1;Normal Dist)

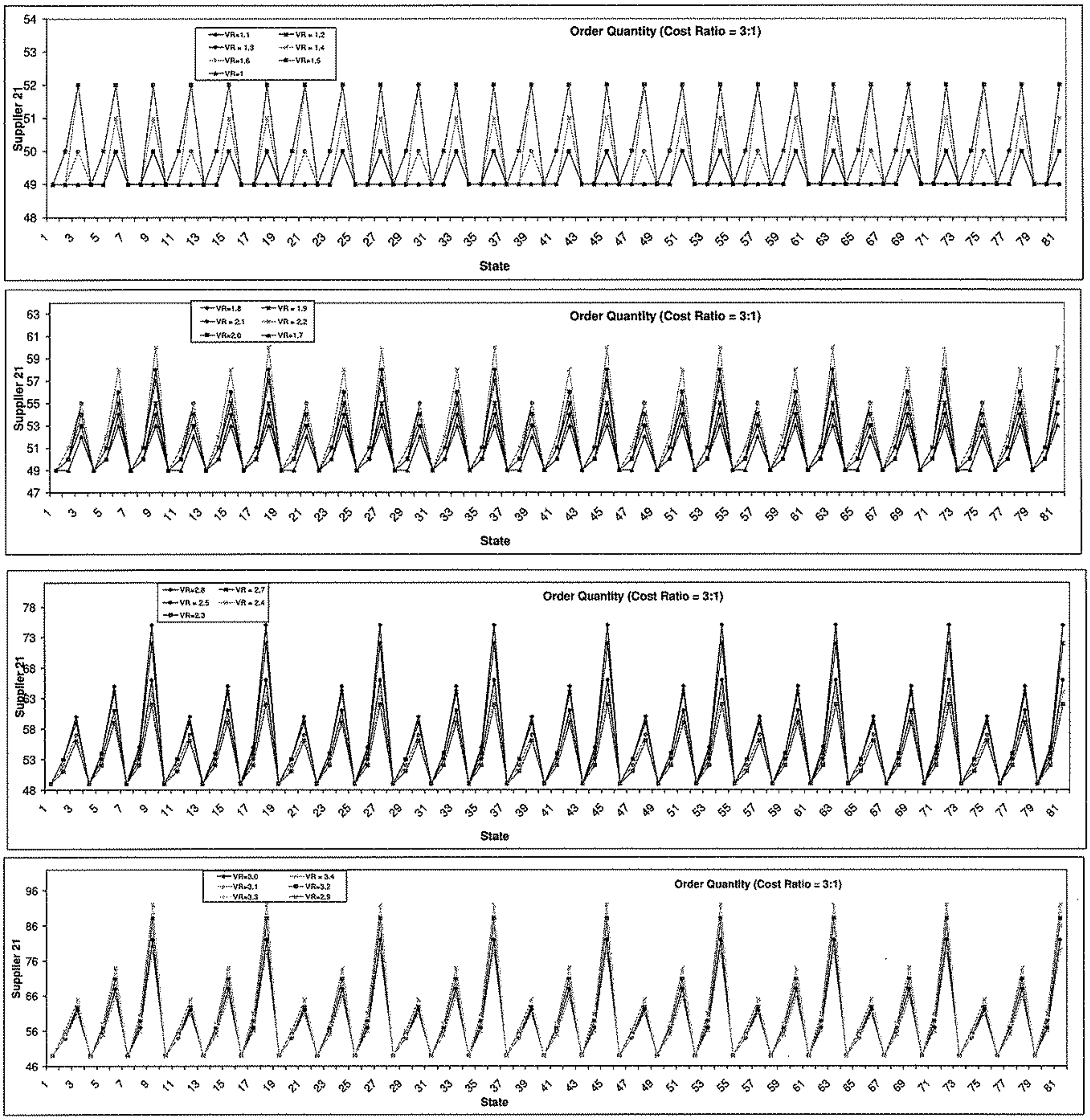


Figure 4: Ordering Quantities from Supplier 2_1 (Cost Ratio=2:1;Normal Dist)

4.3.3 Managerial Insights

Some managerial insights are gained from these results. First, an increase in cost of a more reliable supplier (2_1 and 3_1) does not necessarily increase the total supplychain cost, however, the discrepancy in reliability within an echelon is directly proportional to the total SC cost and variance. However, when the reliability discrepancy is greater than 3.3, the cost and variance are inversely proportional to the variance ratio for a cost ratio of 2:1. The reason for this is because, at a cost ratio of 2:1, the order quantity from the reliable supplier exceeds that of the less reliable supplier 63.89% of the time when the variance ratio is 3.3 and 3.4 while for the remainder of the variance ratios, those order quantities exceed the less reliable supplier only 55.56% of the time. It is interesting to note that on average, suppliers 2_1 and 3_1 have a larger order quantity than their less reliable competitor 58.33% and 66.67% of the time when the cost ratios are 3:1 and 4:1, respectively.

4.4 Beta Distributed Leadtime: Testing and Results

4.4.1 Modeling Parameters and Scenarios

For a beta distribution of leadtime, the triangular approximation was used, however this is only valid for $\beta \geq \alpha > 1$. The parameters used for suppliers 1_1 and 2_1 are shown in Table 3.

Table 3: Beta Distribution Parameters Used In This Research

	Supplier 2_1			Supplier 3_1		
	light	normal	congested	light	normal	congested
β	1.1	1.6	2	1.3	1.7	2.2
α	5	5	5	5	5	5

These values satisfy the requirements for the triangular approximation and are all positive skewed. The β parameter was held constant at 5 and the α parameter was increased. This results in an increase of the mean. Thus, as congestion increased (light to congested state) the α parameter was increased. A constant β and increasing α changes the shape and make the curve less right-skewed (approaching a normal distribution) while changing beta values tends to change the peak height.

To determine the scenarios, the distributions for 2_1 and 3_1 were held constant and are shown in Table 3. Similar to the VRs in the normal distribution trials, here, an alpha ratio (AR) is introduced. To determine the beta parameters for the other 2 suppliers, a ratio of alpha values is used. In particular ratios of: α_{2_2}:α_{2_1} and α_{3_2}:α_{3_1} were incremented in 0.2 steps up through a 2.3 ratio. In all this yields 567 trials and each trial was run again over the three cost ratios of 2:1, 3:1 and 4:1.

4.4.2 Results

Compared to the normal distribution, the results in the beta distribution are not as intuitive. On the graphs shown below, the order quantities for the 7 ARs were split over 2 graphs in each figure for readability. The order quantity results for the beta distribution at a cost ratio of 2:1 are shown in the graphs in Figures 5 and 6

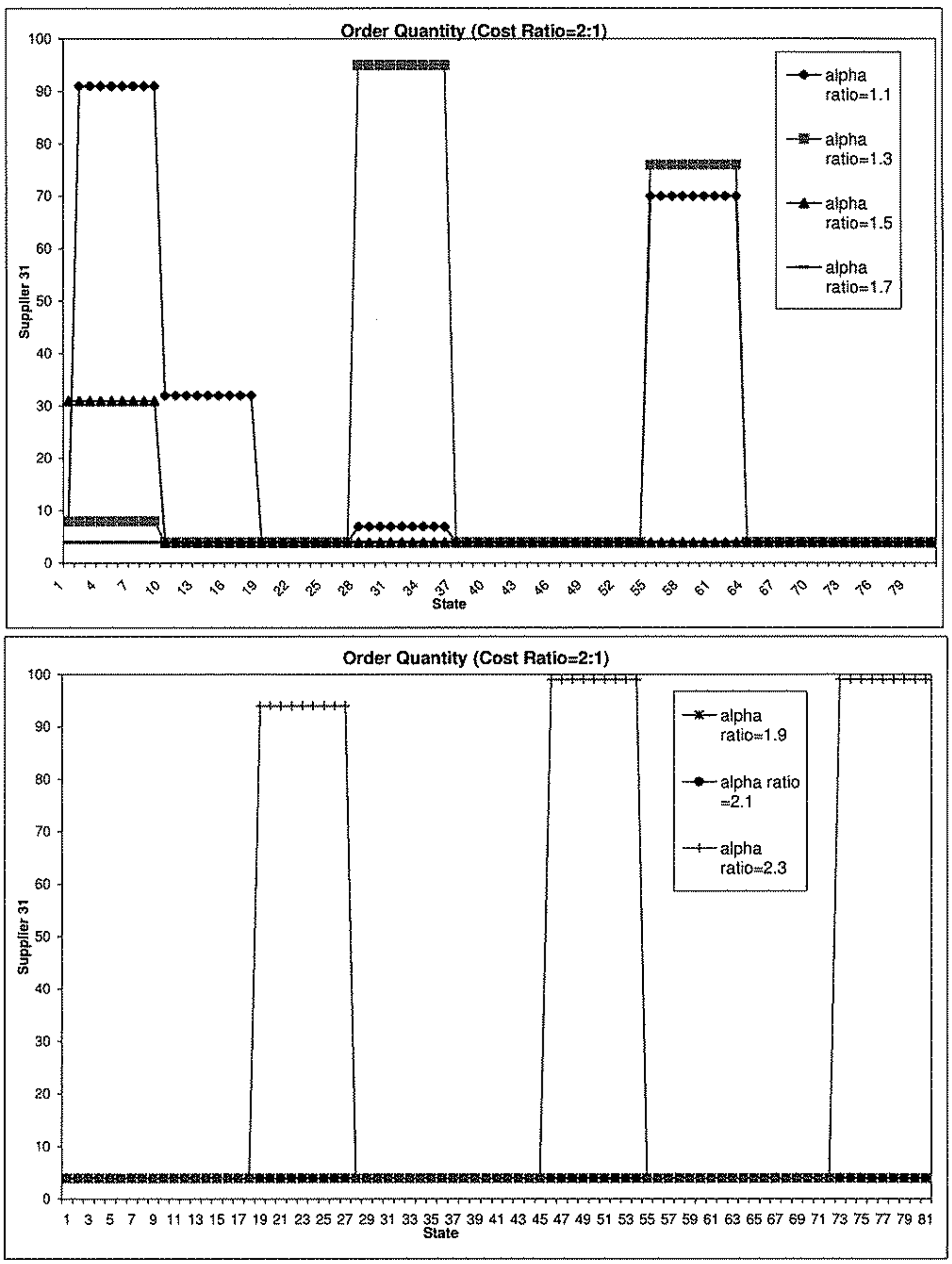


Figure 5: Ordering Quantities from Supplier $3_1(CostRatio = 2:1; BetaDist)$

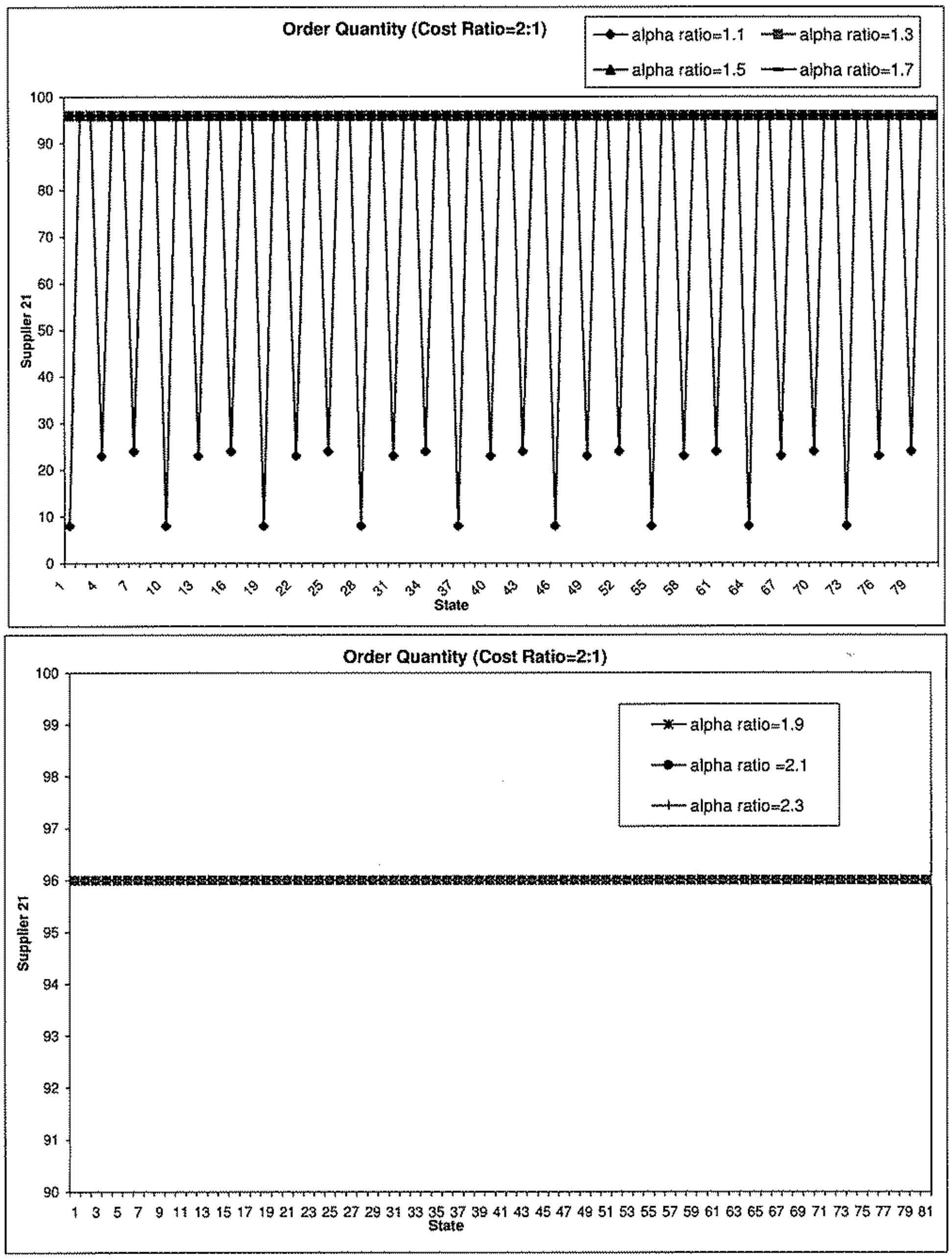


Figure 6: Ordering Quantities from Supplier $2_1(CostRatio = 2:1; BetaDist)$

for suppliers 3_1 and 2_1 respectively. By comparing these two figures, it can be seen that as the discrepancy between suppliers increases (AR increases), the more reliable supplier (3_1) had an ordering percentage of at least 50% when both 3_1 and 3_2 were within a congestion level of each other (ie. one was not congested and the other light). Furthermore, this only held true for either low ARs (1.1 and 1.3) or a high AR (2.3). For echelon 2, which depends on the state of both echelons 3 and 2, only when AR=1.1 did the less reliable supplier have an ordering percentage greater than 4%. In particular, this occurred only for AR=1.1 and only when the less reliable supplier was in a light state. The remaining cost ratios showed a similar pattern and are given in Appendix III in Figures 23 through 27.

4.4.3 Managerial Insight

The managerial insight here is much different than the normal distribution. The reliability seems to be the overriding factor when splitting an order. For any AR greater than 1.1, the reliable supplier had the majority of the order in echelon 2. For echelon 2, the more reliable supplier had the majority less frequently than its competitor, but again, this occurred only for small alpha values (1.1, 1.3 and 1.5). A conclusion that can be drawn here is that the AR effects the ordering split more than the VR.

5 Comparing the SSC to an Uncontrolled SC

In order to illustrate the benefit of using the SSC type of ordering system, an uncontrolled model (UCM) was used as a baseline. The UCM utilizes the same SC configuration however, no chance constraint programming was used and no control was added. The UCM differs from the SSC in the following ways:

- Each supplier only orders from the least expensive upstream supplier.
- Leadtimes are stochastic but not state-dependent
- Order quantity is determined using the modified base-stock policy for stochastic leadtimes, without any CCP optimization.

The assumptions are as follows:

A1: Each supplier will only order from the least expensive upstream supplier. This assumption creates a 3-echelon serial supply chain for this research.

A2: In this research the less reliable, and less expensive suppliers are 2_2 and 3_2.

A3: The leadtime distributions of the suppliers in the UCM are also normal- or beta-distributed, however, since there is no CCP optimization, there are no state-dependent distributions and only one distribution of leadtime is used. The development of the parameters is given in Appendix X. In general, however, since the distribution of this leadtime is taken from all of the historical data (light, normal and congested states) the distribution of this leadtime should have a much larger variance than any of the state-dependent distributions.

A4: The ordering policy used for the suppliers in the UCM is the modified base-stock policy for stochastic leadtimes. The base stock level is calculated using

$$y^* = \mu_j \tau_j + z_\alpha \sigma \sqrt{\tau_j} \tag{5}$$

where,

μ=mean of demand

$z_\alpha = F^{-1}(\alpha)$

and the order quantity is determined by

$$\text{Order quantity} = [y^* - il + q]^+ \tag{6}$$

Since no CCP optimization takes place in the UCM, 100% of the order quantity is placed with the least expensive upstream supplier.

These parameters are used to characterize the UCM. The results and comparisons between the uncontrolled and controlled model are discussed next.

5.1 UCM-SSC Comparison: Normal Distribution

For the normal demand situation, a scenario consisted of 2 factors: variance ratio and cost ratio. The controlled system and uncontrolled system were simulated and compared for 3 responses: overall lateness, cost and order quantity.

Table 4 is a summary of the average lateness results. In this table, the average lateness over all the variance ratios for each of the suppliers is shown. As expected, the less reliable suppliers (2_2 and 3_2) delivered late more frequently than their more reliable competitor. Supplier 2_2 was late 15.61% of the time, on average while 2_1 averaged only 5.2%. For echelon 3, the less reliable supplier was late 13.32% on average while supplier 3_1 was late only 4.46%. However, what is also evident is that the timeliness (decrease in lateness) decreases down the chain, exhibiting some of the same tendencies as the bullwhip effect. This is seen in Table 4, where, for each cost ratio in the controlled system the on-time percentage in every instance was greater for the upstream supplier. Figures 7 illustrates the difference in lateness of an uncontrolled and controlled system.

Table 4: Average Lateness (Normally Distributed LT)

	Controlled System (%)				Uncontrolled System	
Cost Ratio	Supp 2_2	Supp 2_1	Supp 3_2	Supp 3_1	Supp 2_2	Supp 3_2
2:1	14.38	5.47	12.17	4.49	20.75	24.52
3:1	16.26	4.74	13.81	4.41	20.75	24.52
4:1	16.18	5.39	13.97	4.49	20.75	24.52

Figure 7(a) compares the difference in lateness between the uncontrolled and controlled system for only the unreliable suppliers (2_2 and 3_2). The points below the horizontal axis are where the uncontrolled system outperformed the controlled system in terms of timeliness of delivery. What is evident is that as the variance ratio increases, the reduction in lateness also increases with a controlled system. Essentially, controlling the system indicates that timeliness benefits are proportional to the discrepancy in reliability (VR). Note that supplier 3_2 increases the percentage of on-time shipments 87.5% of the time with a controlled system. Figure 7(b) incorporates both suppliers

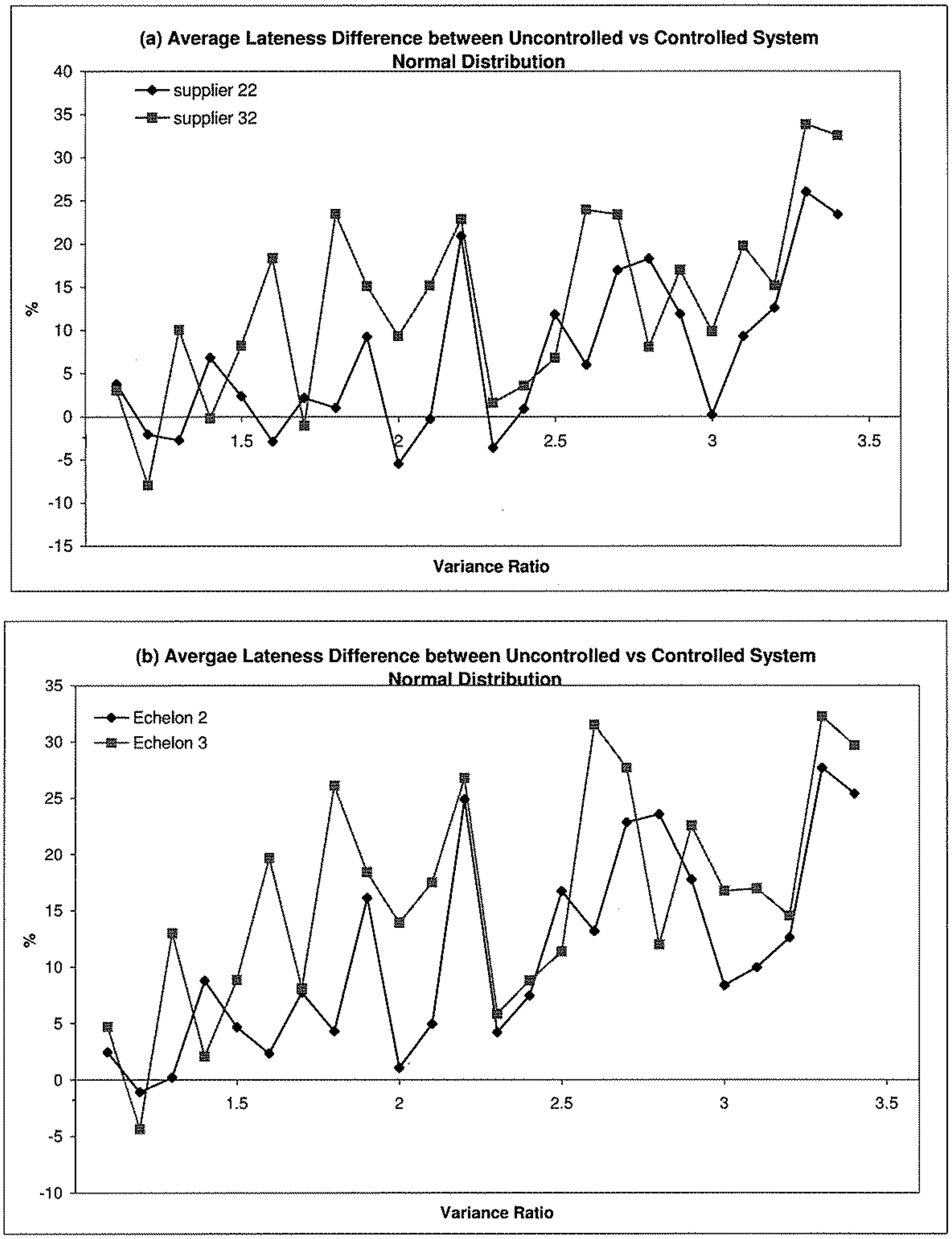


Figure 7: Average Lateness in the SSC (Normal Dist.)

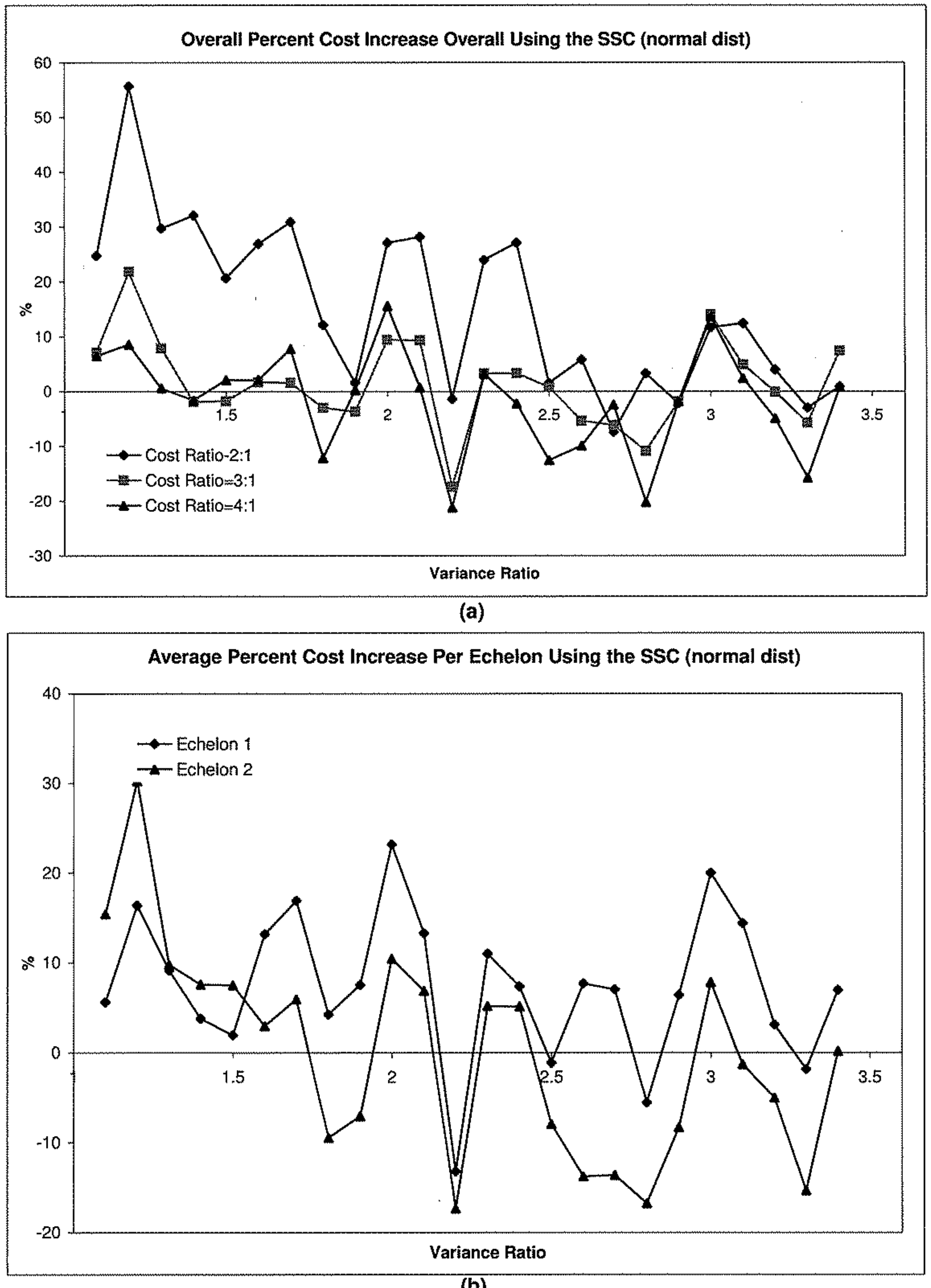


Figure 8: Average Cost in the SSC (Normal Dist.)

for each echelon. Again, with the exception of a variance ratio of 1.2, the controlled system studied always increased average timeliness. For supplier 2_2, lateness is reduced by 6.72% with the controlled system and 13.03% for supplier 3_2.

Figure 8 provides the cost results of the simulation for the controlled and uncontrolled systems. In Figure 8(a), it is shown that the controlled system exhibits a lower overall cost than the uncontrolled system at 22 instances when the variance ratio is high (over 1.4) and the cost ratio is both 3:1 and 4:1. For the lower cost ratio of 2:1, the controlled system is only less expensive for variance ratios of 2.2, 2.7, 2.9 and 3.0. In Figure 8(b) the overall cost is divided into an echelon-based cost. It shows that about 90% of the time, echelon 2 sees a greater reduction or smaller increase in cost than echelon 1.

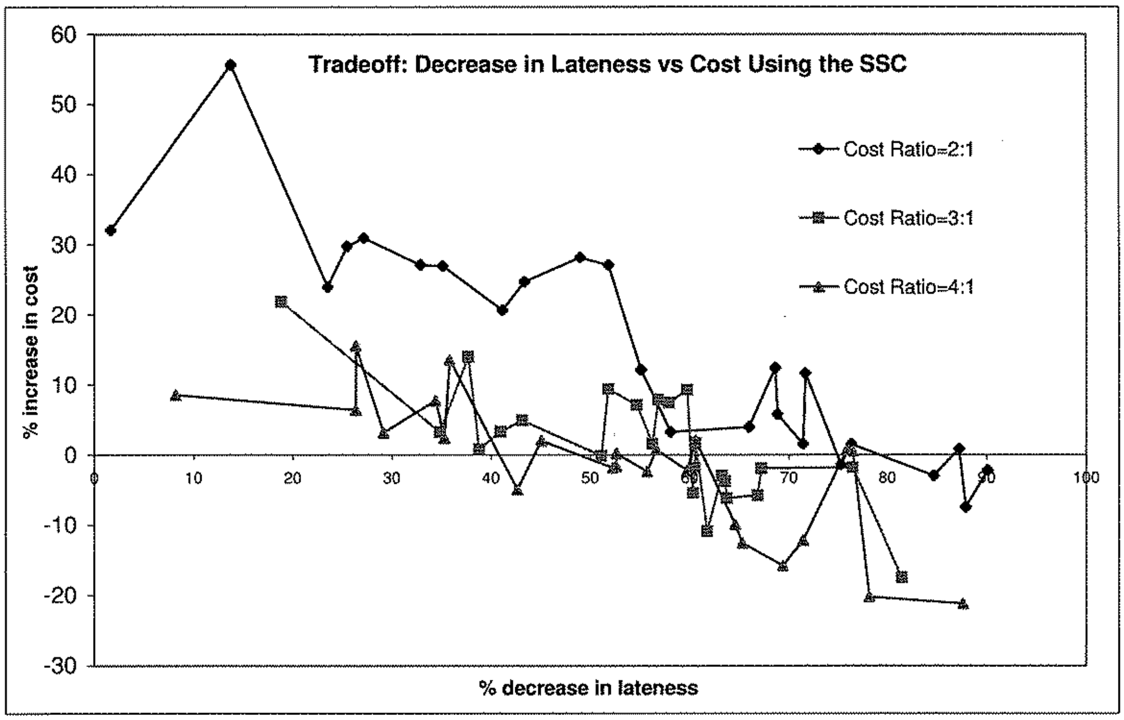


Figure 9: Tradeoff Curve of Average Lateness With Cost for SSC Model (Normal Dist.)

The tradeoff between a reduction in average lateness and increase in cost is shown in Figure 9. The cost is inversely proportional to the desired decrease in average lateness. The reason for this, however can be explained by the difference in ordering patterns between the two systems. The mean and standard deviation of incoming demands for echelons 2 and 3 are shown in Figures 10(a) and (b) respectively. The mean and standard deviation of the incoming demand are significantly larger in the uncontrolled system. The average demand was higher in the uncontrolled system 75%

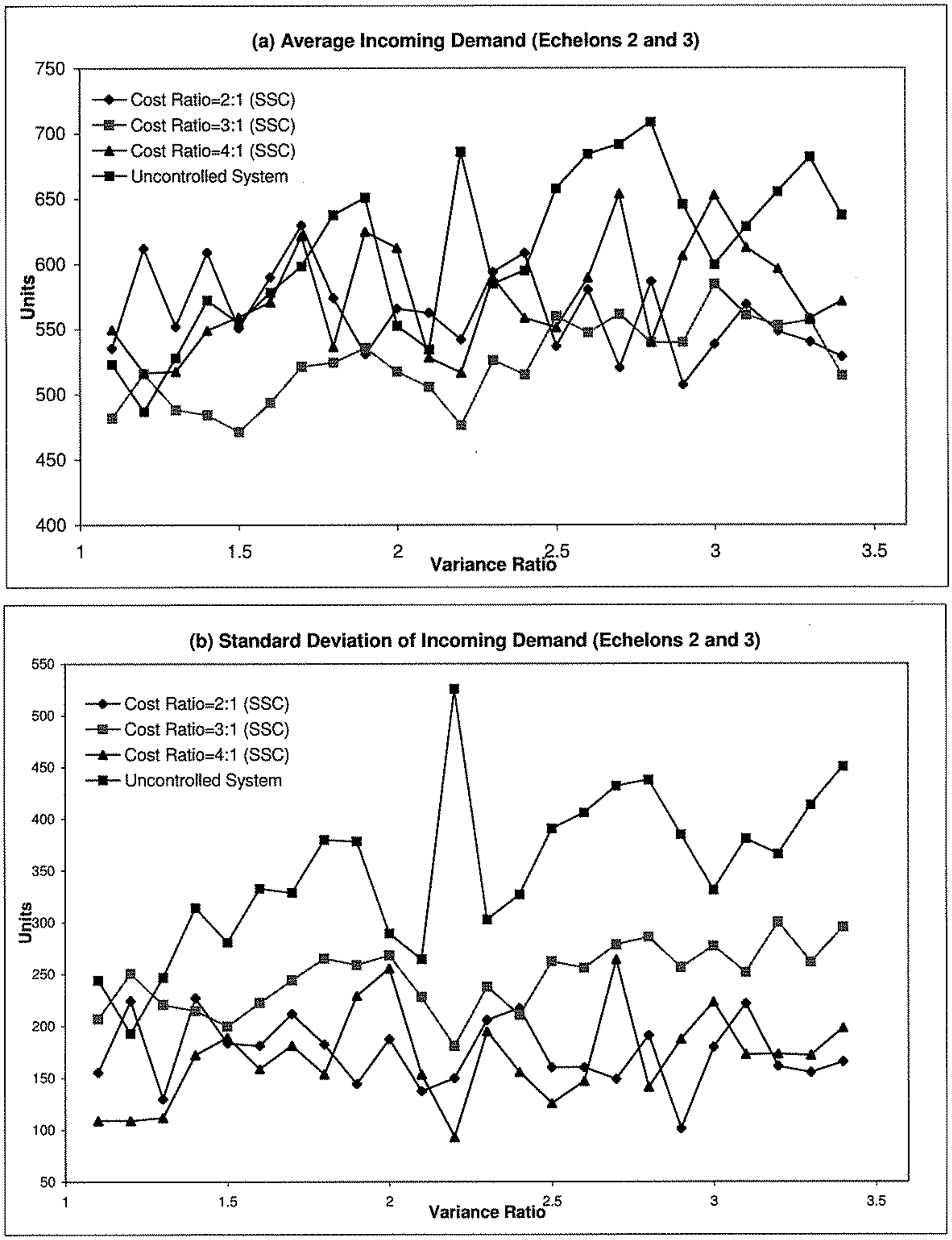


Figure 10: Mean and Standard Deviation of Order Quantity (Normal Dist.)

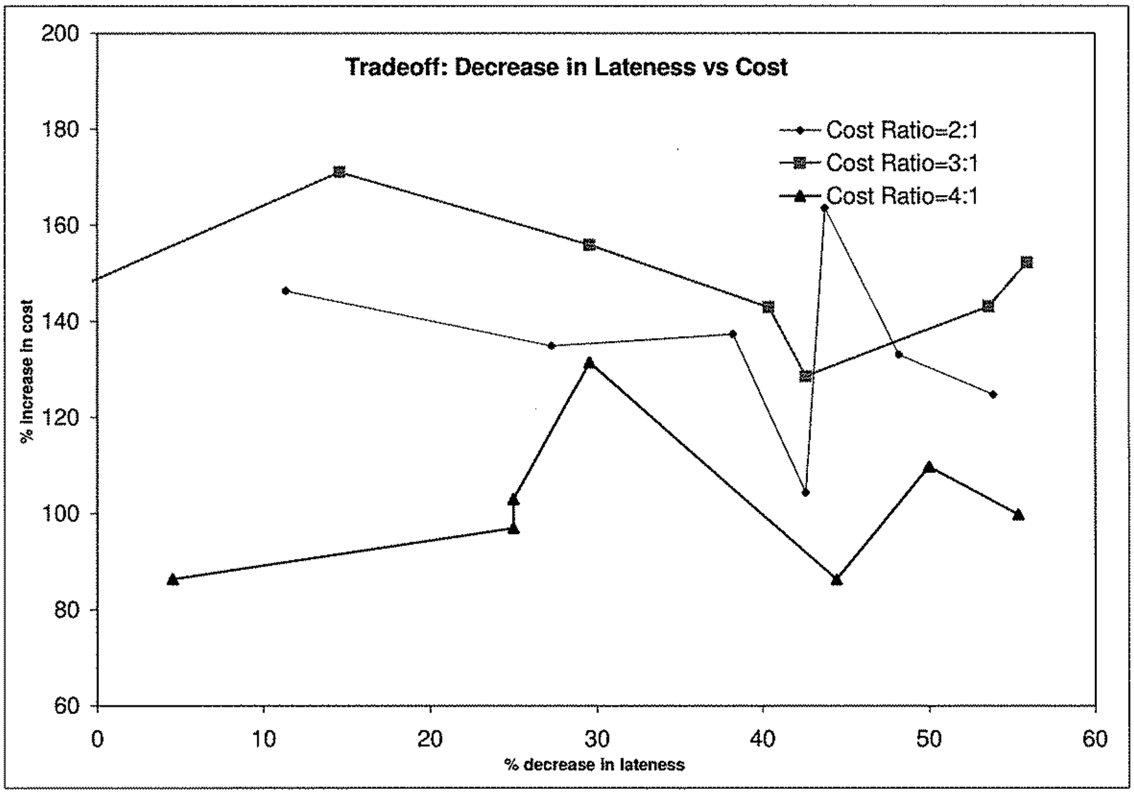


Figure 11: Tradeoff Curve With Average Lateness and Cost for the SSC Model (Beta Dist.)

of the time while the standard deviation was higher 97.2% of the time. Figures 8 and 9 shows using a controlled system that optimizes order quantity with each new demand can result in a cost savings and increase in efficiency.

5.2 UCM-SSC Comparison: Beta Distributed Leadtime

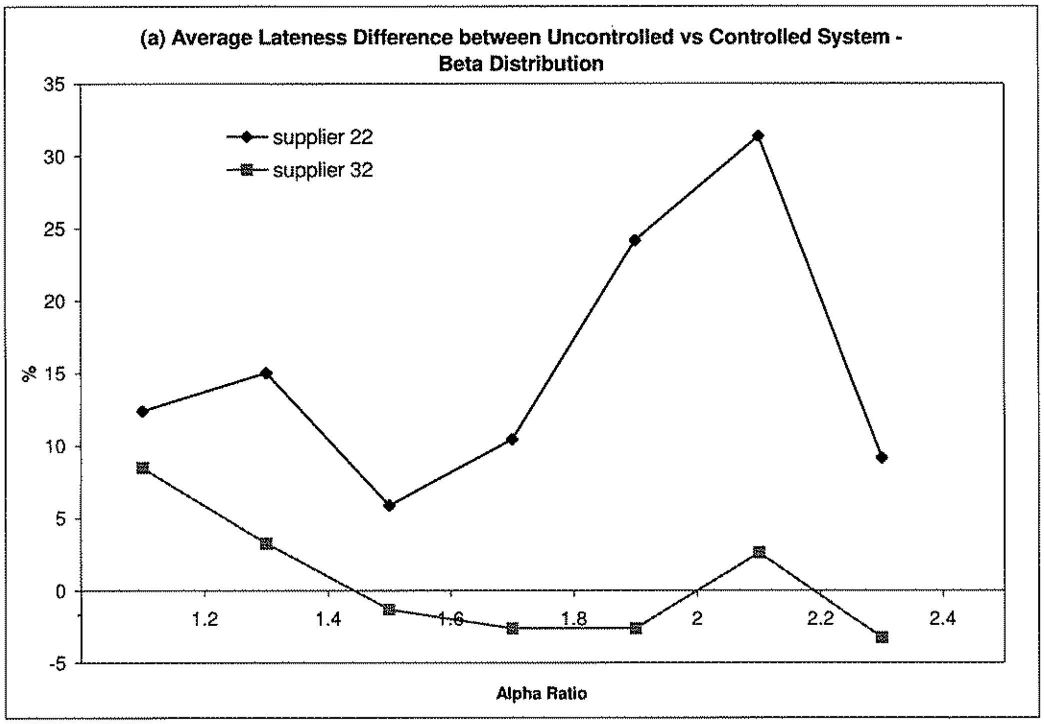
There were 7 scenarios of the alpha parameter ratios (AR) tested over 3 cost ratios. As in the normal distribution simulation, the controlled system and uncontrolled system were simulated and compared for 3 responses: overall lateness, cost and order quantity. Again, the more reliable suppliers were kept at 3_1 and 2_1.

The average lateness and cost data is summarized in Table 5. In this table, it is

Table 5: Average Lateness (Beta Distributed LT)

	Controlled System (%)				Uncontrolled System	
Cost Ratio	22	21	32	31	22	32
2:1	14.85	15.41	19.05	13.45	17.09	34.2
3:1	15.97	17.09	19.05	13.73	17.09	34.2
4:1	16.81	17.93	18.77	12.89	17.09	34.2

shown that the less reliable supplier outperformed the more reliable supplier in terms



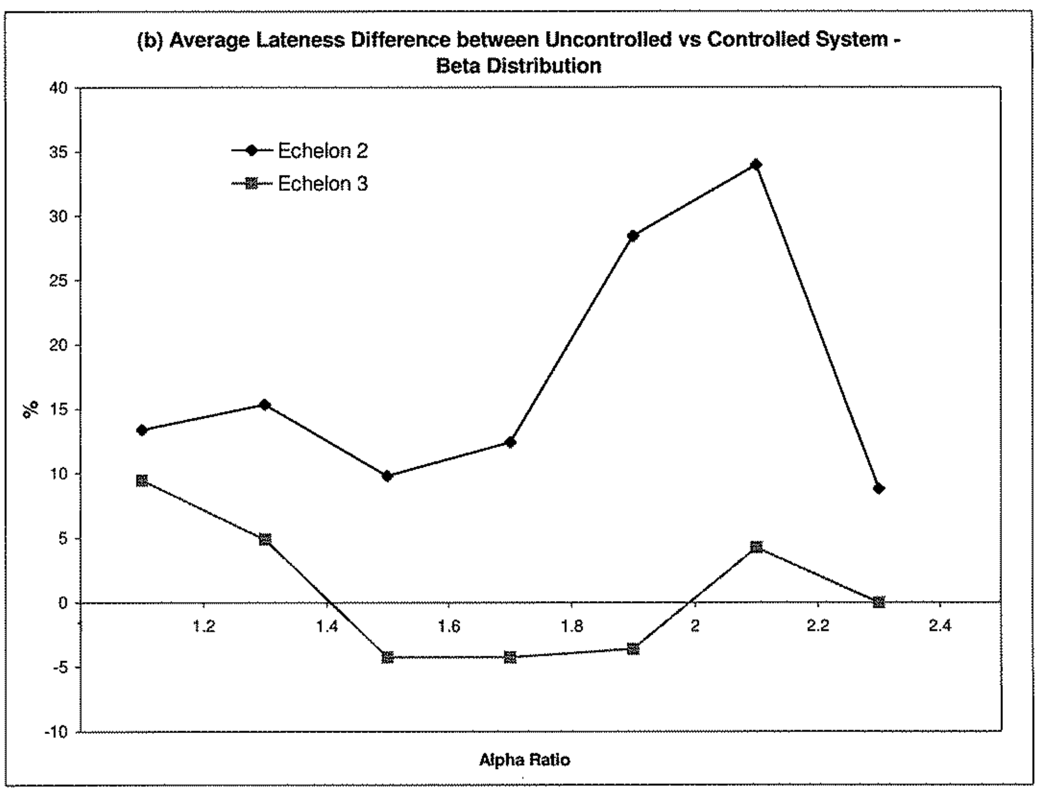


Figure 12: Average Lateness Between Uncontrolled and SSC Models(Beta Dist.)

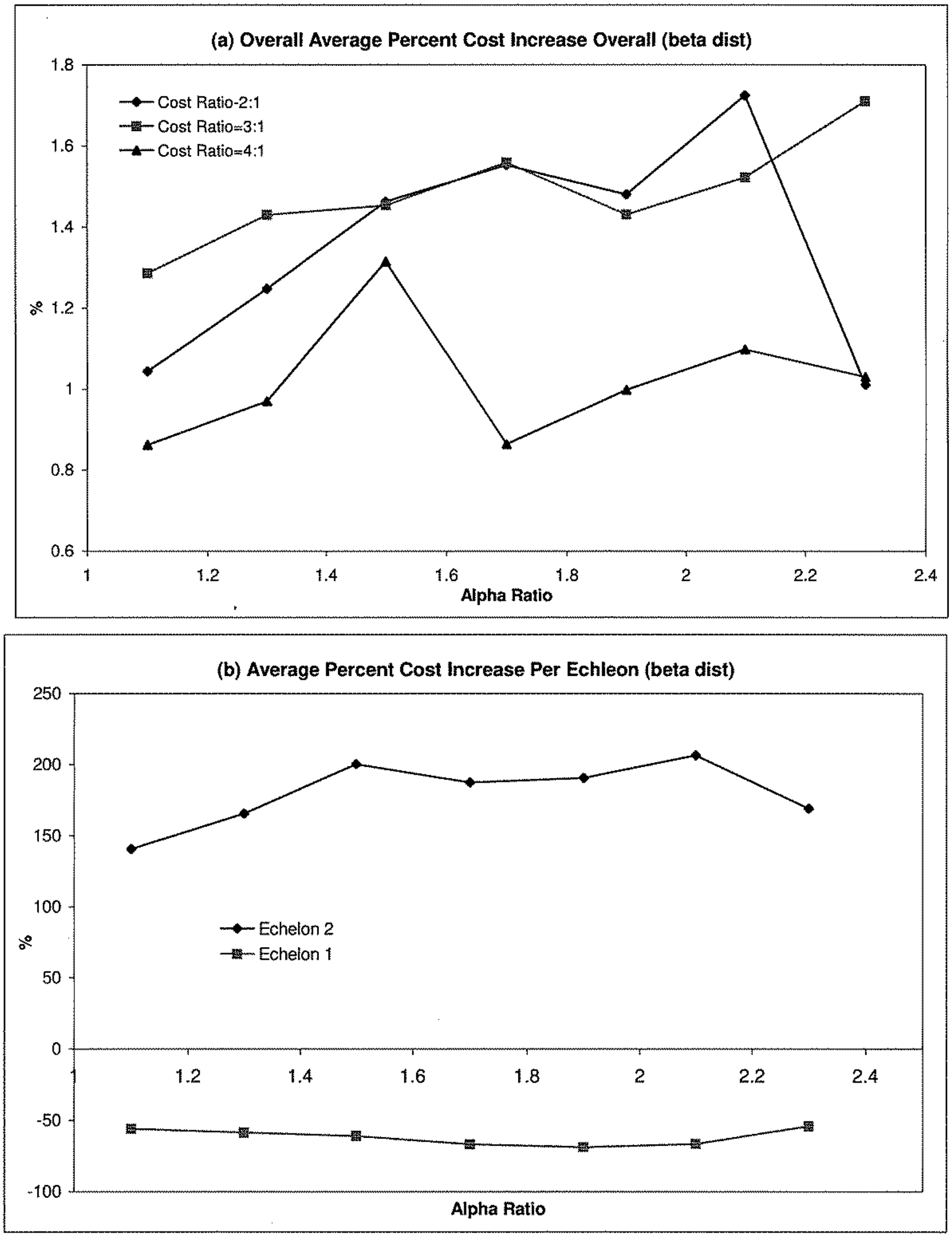


Figure 13: Average Cost of the SSC (Beta Dist.)

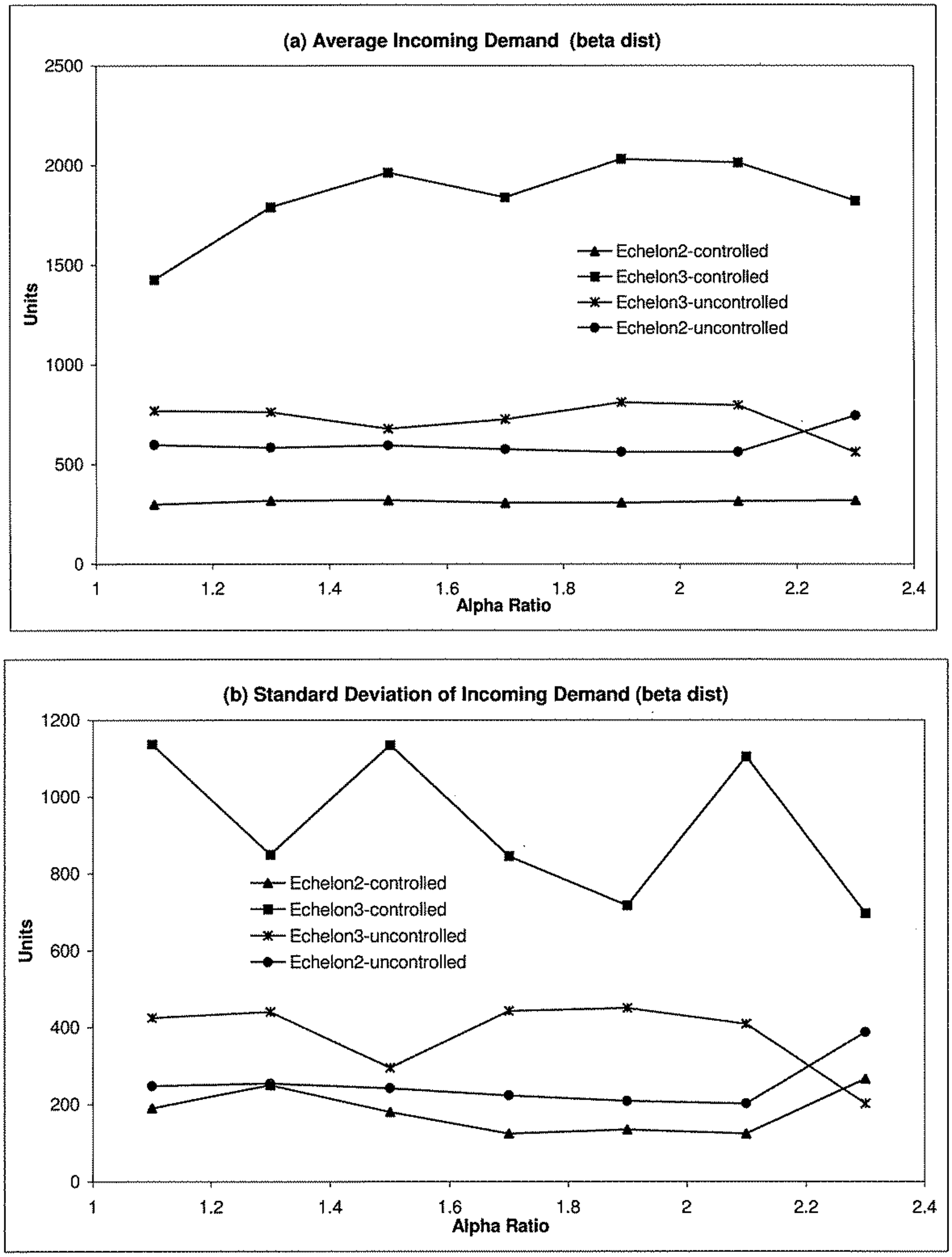


Figure 14: Ordering Patterns-Mean and Standard Deviation (Beta Dist.)

of timeliness of delivery in echelon 2, composed of suppliers 2_1 and 2_2 (although only by an average of 0.93%), whereas in echelon 3, composed of suppliers 3_1 and 3_2, the situation is reversed. In this latter situation, in contrast to the normal distribution, the average lateness either decreases or remains stationary as the material travels downstream, exhibiting the opposite of the bullwhip effect. In Figures 12(a) and (b), the difference in lateness of an uncontrolled and controlled system is shown.

In Figure 12(a), the contrast between the two unreliable suppliers is shown while the bottom graph includes both suppliers for each echelon. With an increase in the alpha ratio (which essentially increases the mean), lateness in the controlled system approaches the uncontrolled system (the difference approaches zero) for supplier 3_2.

Figure 12(a) shows the percent change in average lateness for the 2 unreliable suppliers (2_2 and 3_2). For three alpha ratios, the uncontrolled system outperforms the controlled system in terms of timeliness of delivery by about 1% for supplier 3_2. Supplier 2_2 increases its timeliness for each alpha ratio. The overall delivery performance of each echelon is shown in Figure 12(b). This figure indicates that, by using the SSC, lateness is reduced by approximately 16% for echelon 2 however, it is only reduced by an average of about 3% in echelon 3.

The average percent of cost increase needed to control the system and each echelon is shown in Figures 13(a) and (b) respectively. Figure 13 illustrates that, unlike the normal distribution results, the controlled system exhibits a higher overall cost by about 120% on average. However, as seen in Figure 13(b), this cost increase is all absorbed by echelon 2, while echelon 1, actually sees a decrease in cost. The reason for this though, can be seen in the ordering patterns. Unlike the normal distribution, the increase or decrease is not shared between the two suppliers in each echelon. Here, the differences in the two suppliers is investigated. The mean and standard deviation of incoming demand for echelons 2 and 3 are shown in Figures 14(a) and (b) respectively. The ordering into echelon 2 is controlled much better than echelon 3, thus the reason for the cost increase seen by echelon 2. Again, a tradeoff curve is shown in Figure 11. For the beta distribution, timeliness can be increased up to about 55% for approximately the same cost increase (about 140%). However, the minimum that must be spent to increase the timeliness by any amount is 86%.

5.3 Managerial Insights

Normal Distribution

In this research a 3 echelon SC was used, with 2 stages in the 2 upstream echelons. Two suppliers in each echelon exhibited some level of disparity in their reliability of delivery timeliness, denoted by the variance ratio (VR). By controlling the SC using a SSC, Table 4 indicates that the percentage of lateness can be reduced. In particular, the VR and the amount of lateness are directly proportional. That is, as the VR increases (the disparity increases), the percentage of lateness reduction is also increased. Additionally, the cost increase by using the SSC is inversely proportional to the VR. As the disparity increases, it costs less, on average, to reduce lateness and this cost is primarily absorbed by the upstream echelon (echelon 1). In certain instances of VR, as seen Figure 8(b), echelon 2 actually exhibits a cost savings. Lastly, the cost increase is the highest when the cost ratio between the more and less reliable supplier is the lowest (2:1).

Beta Distribution
Controlling a system using the SSC, when leadtimes are beta-distributed, was shown in Table 5 to reduce the amount of average lateness in the system. The amount of the average lateness was reduced significantly more for the downstream supplier than the upstream supplier (16% versus 3%). The cost increase for implementing this timeliness was much higher for the beta than the normal distribution with an average of about 120%, although it is payed entirely by echelon 2 while echelon 1 reduces its cost. In practice, some sort of balance would have to be made between the two echelons to even out the cost increase. The disparity between suppliers, the AR, is not directly related to the amount of average lateness reduction as seen in Figure 12.

6 Conclusion

This research presents an architecture for a synchronized supply chain capable of reacting quickly and effectively enabling decision making under uncertainty. A 3-echelon SC was tested. States for each supplier were initially generated and the system optimized. With each new incoming demand, the system was re-optimized and performance of leadtime, cost and order quantities were used as measures of performance. This again, was simulated for each of the 24 VRs and 7 ARs (each tested at the 3 cost levels) for a simulation period of 50 incoming demands. They were compared to an uncontrolled system which only ever ordered from the less expensive supplier. Results show that overall, leadtime performance is increased with a controlled system, with a cost change varying from -21.16% to 55.69% for the normal distribution and always increasing from 86.29% to 171.08% for the beta distribution.

Current work includes SC control using techniques of supervisory control for Petri nets. Future work includes an extension of this to incorporate dynamic control of the SC.

APPENDIX I
Beta Distribution Parameters Using a Modified PERT Analysis

Two of the historical data items maintained are the quoted leadtime and actual leadtime. This information can be sorted by state and for each state the distribution of actual leadtime is determined.

PERT was introduced by the Department of the Navy [22] in 1958 and is used to estimate activity task durations. Since that time, although many improvements have been made upon the initial estimates, most of the operations community still uses the traditional formulas [10].

The Extended Swanson-Megill (ES-M) [12] parameter estimations are used in this research for the beta distribution and are given as:

$$\widehat{\mu} = 0.400x(0.50) + 0300[x(0.10) + x(0.90)] \quad (7)$$

$$\widehat{\sigma^2} = 0.400[x(0.50) - \widehat{\mu}]^2 + 0.300([x(0.10) - \widehat{\mu}]^2 + [x(0.90) - \widehat{\mu}]^2) \quad (8)$$

Other approximations include the Extended Pearson-Tukey (EP-T) [15], Troutt Formula for Mean [21], Farnum-Stanton Formulas [6] and Golenko-Ginzburg Formulas [8]. In particular, the EP-T was recommended due a much closer approximation of the variance, however, the ES-M use estimates of the 0.10, 0.50 and 0.90 fractiles which have been shown to be more reliable than those used in EP-T (0.05, 0.50 and 0.95). The reason is that reliability for assessing a 5% and 95% fractile is not as good as 10% and 90% since the former deals with very rare occurrences [9]. Lau et al. [10] developed estimates for 5 and 7 fractiles, however, in a later paper, Lau and Lau [11] made their initial estimates more simple by using only three fractiles. They suggest that more than 3 fractiles may be a significant shortcoming of the original model.

Both the EP-T estimate and the estimate by Lau and Lau [11] employed the 5% and 95% fractiles which have been shown to have a low reliability. For that reason, this research employs the ES-M method.

APPENDIX II
Supplier State Designation

Table 6:

State	2_1	2_2	3_1	3_2	State	2_1	2_2	3_1	3_2	State	2_1	2_2	3_1	3_2
1	L	L	L	L	28	N	L	L	L	55	C	L	L	L
2	L	L	L	N	29	N	L	L	N	56	C	L	L	N
3	L	L	L	C	30	N	L	L	C	57	C	L	L	C
4	L	L	N	L	31	N	L	N	L	58	C	L	N	L
5	L	L	N	N	32	N	L	N	N	59	C	L	N	N
6	L	L	N	C	33	N	L	N	C	60	C	L	N	C
7	L	L	C	L	34	N	L	C	L	61	C	L	C	L
8	L	L	C	N	35	N	L	C	N	62	C	L	C	N
9	L	L	C	C	36	N	L	C	C	63	C	L	C	C
10	L	N	L	L	37	N	N	L	L	64	C	N	L	L
11	L	N	L	N	38	N	N	L	N	65	C	N	L	N
12	L	N	L	C	39	N	N	L	C	66	C	N	L	C
13	L	N	N	L	40	N	N	N	L	67	C	N	N	L
14	L	N	N	N	41	N	N	N	N	68	C	N	N	N
15	L	N	N	C	42	N	N	N	C	69	C	N	N	C
16	L	N	C	L	43	N	N	C	L	70	C	N	C	L
17	L	N	C	N	44	N	N	C	N	71	C	N	C	N
18	L	N	C	C	45	N	N	C	C	72	C	N	C	C
19	L	C	L	L	46	N	C	L	L	73	C	C	L	L
20	L	C	L	N	47	N	C	L	N	74	C	C	L	N
21	L	C	L	C	48	N	C	L	C	75	C	C	L	C
22	L	C	N	L	49	N	C	N	L	76	C	C	N	L
23	L	C	N	N	50	N	C	N	N	77	C	C	N	N
24	L	C	N	C	51	N	C	N	C	78	C	C	N	C
25	L	C	C	L	52	N	C	C	L	79	C	C	C	L
26	L	C	C	N	53	N	C	C	N	80	C	C	C	N
27	L	C	C	C	54	N	C	C	C	81	C	C	C	C

APPENDIX III
Chance Constrained Program Results

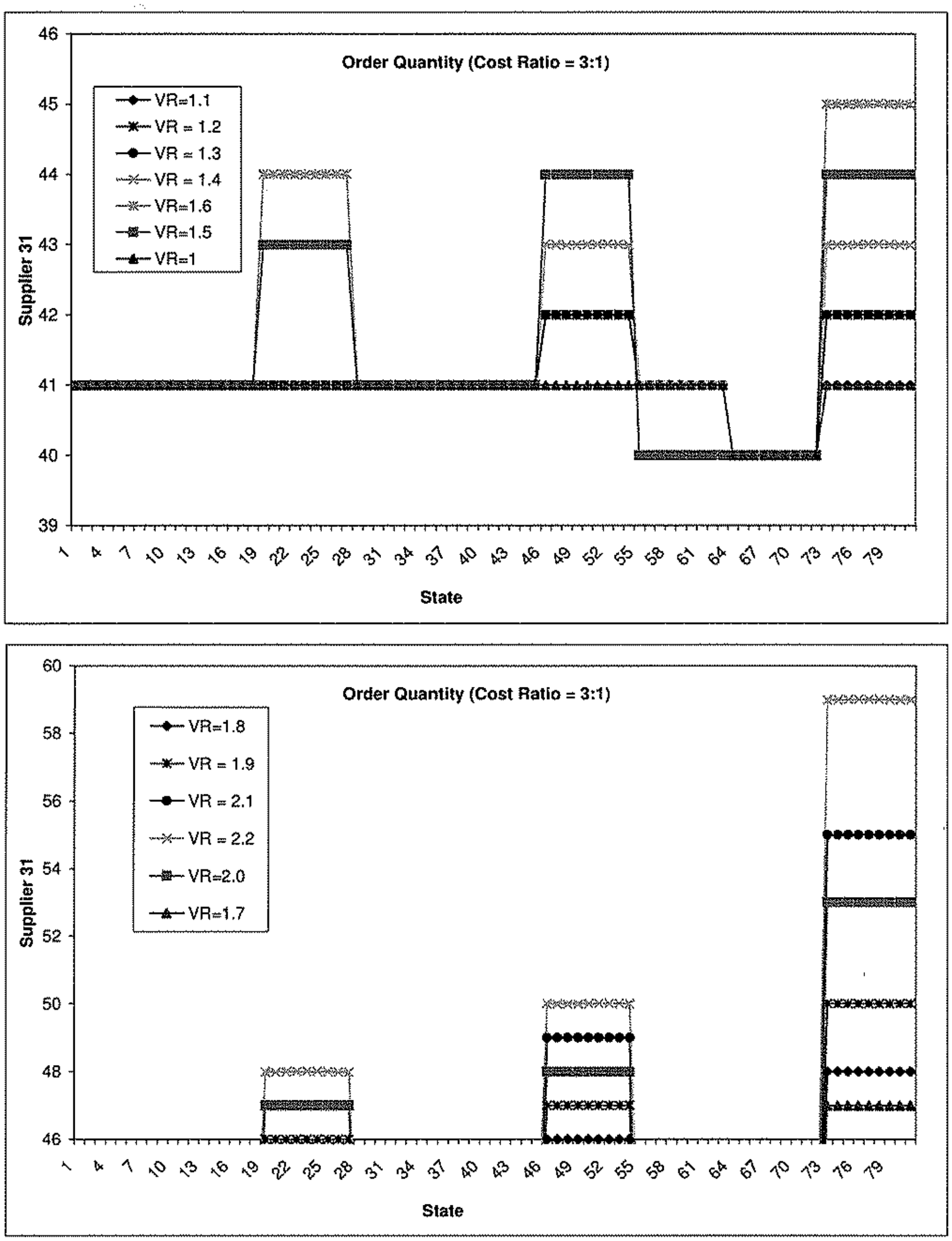
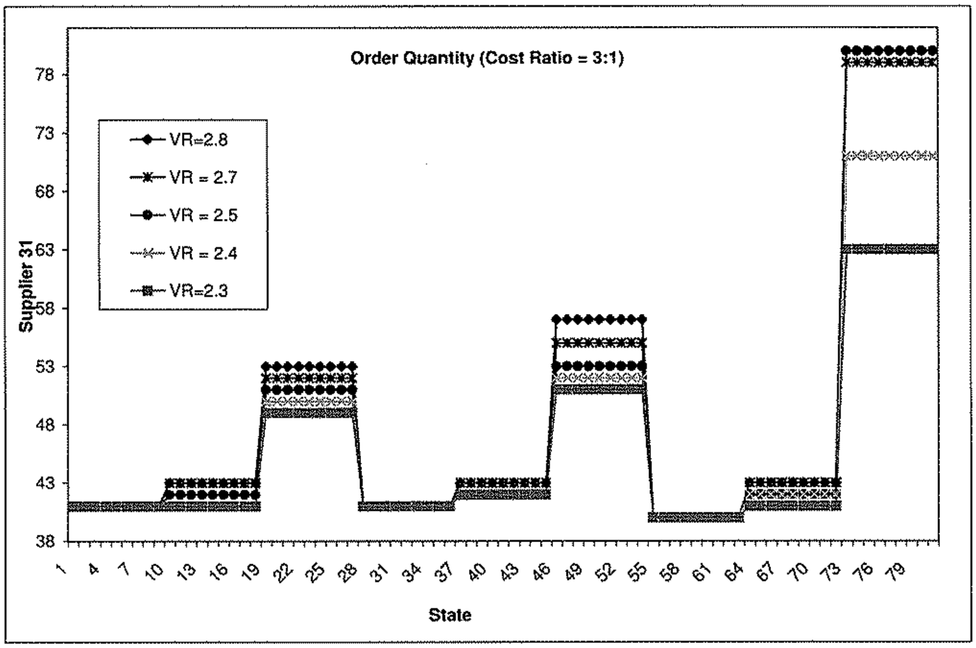


Figure 15: Ordering Quantities from Supplier 3_1 (VR=1.0 through 2.2; Cost Ratio=3:1;Normal Dist)



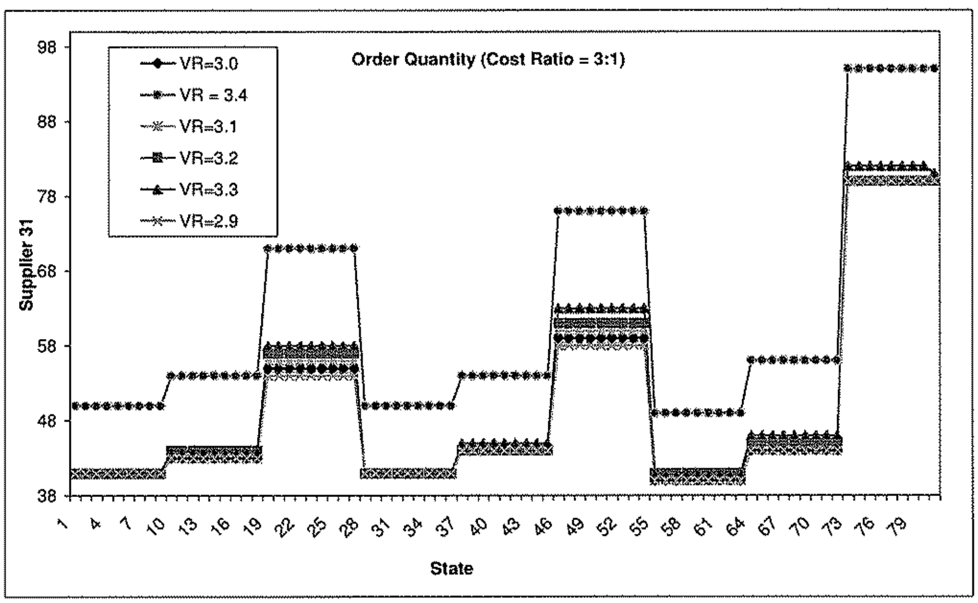


Figure 16: Ordering Quantities from Supplier 3_1 (VR=2.3 though 3.4; Cost Ratio=3:1;Normal Dist)

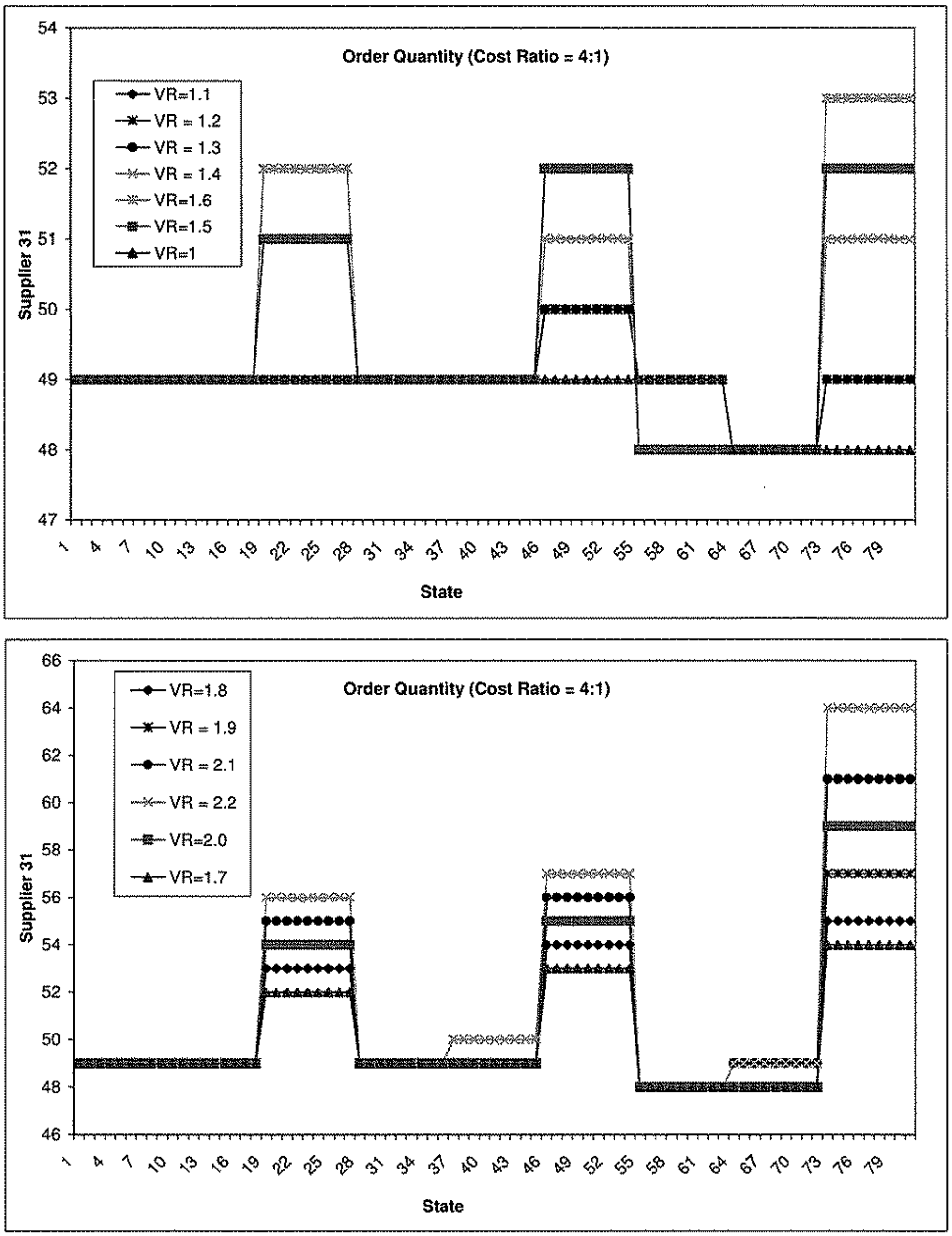
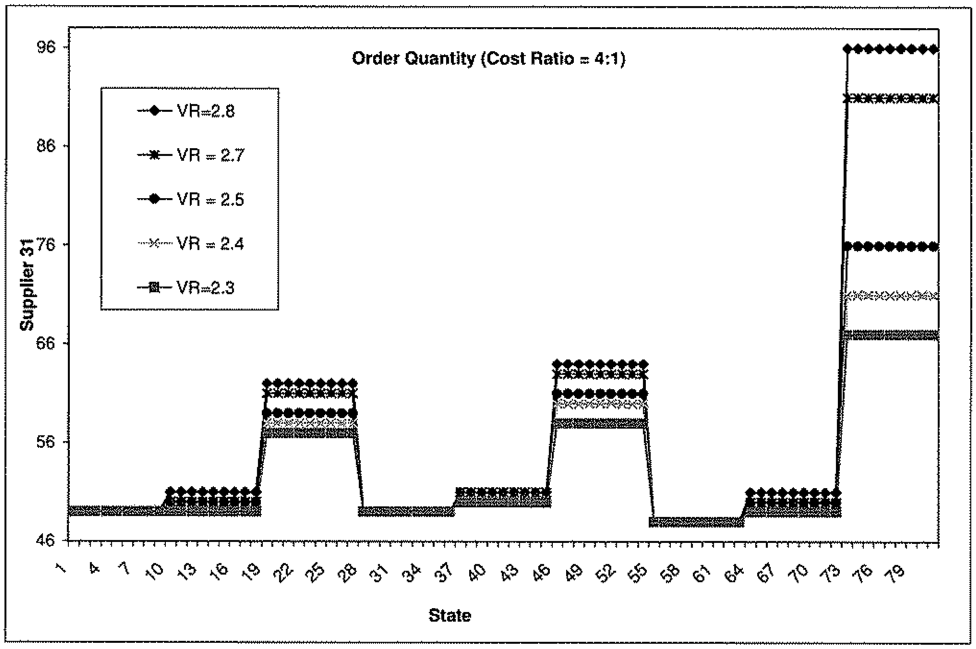


Figure 17: Ordering Quantities from Supplier 3_1 (VR=1.0 through 2.2; Cost Ratio=4:1;Normal Dist)



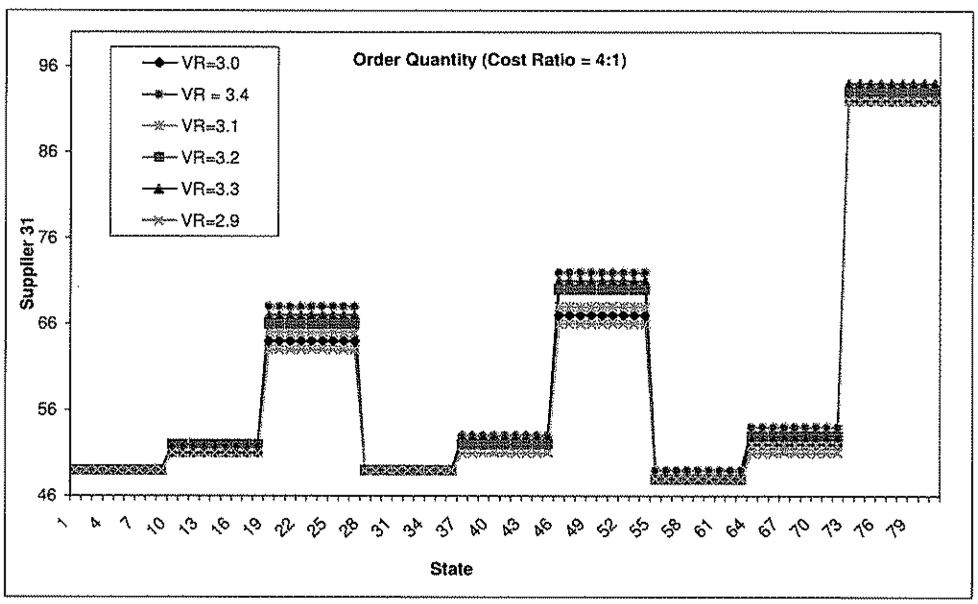


Figure 18: Ordering Quantities from Supplier 3_1 (VR=2.3 though 3.4; Cost Ratio=4:1;Normal Dist)

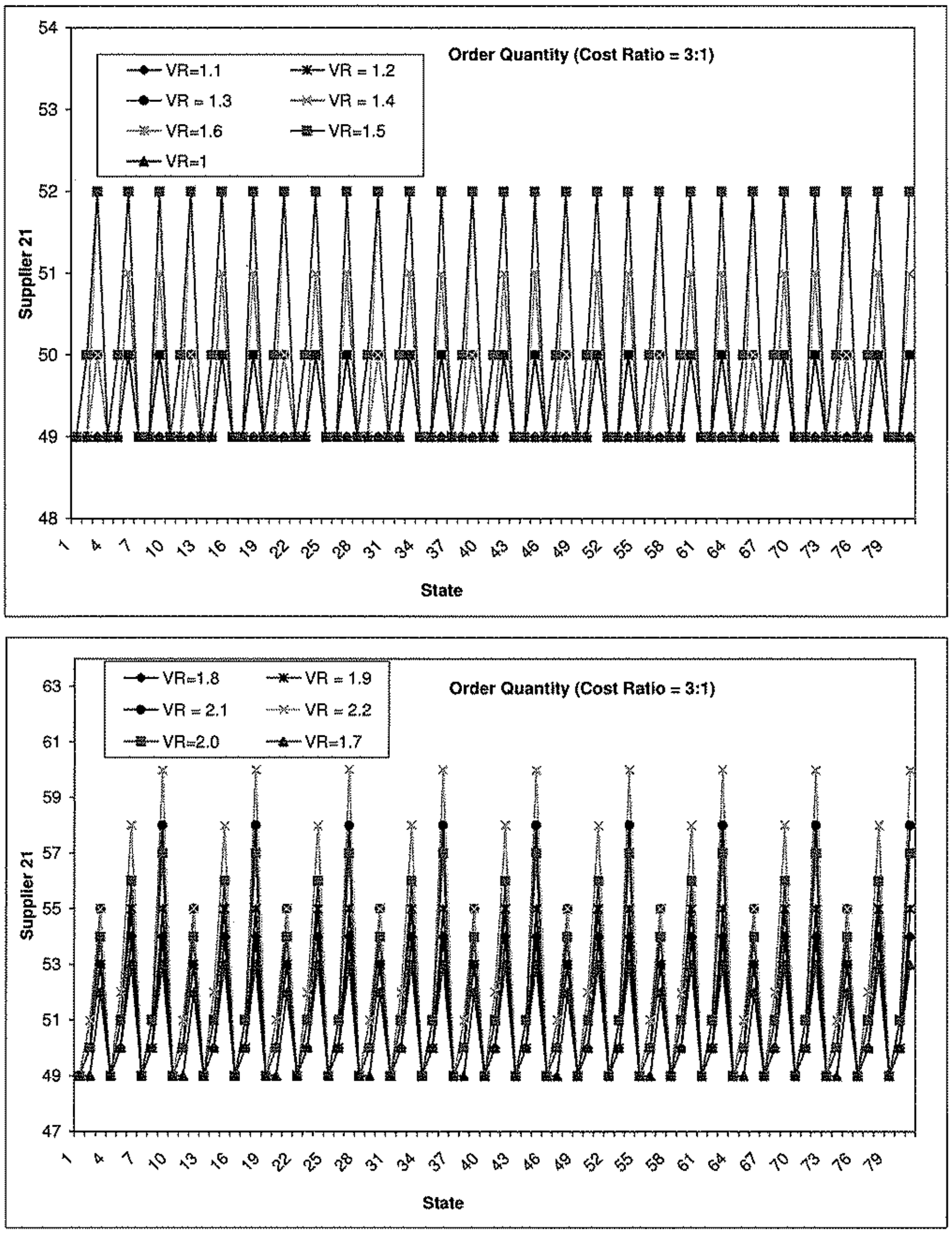


Figure 19: Ordering Quantities from Supplier 2_1 (VR=1.0 through 2.2; Cost Ratio=3:1;Normal Dist)

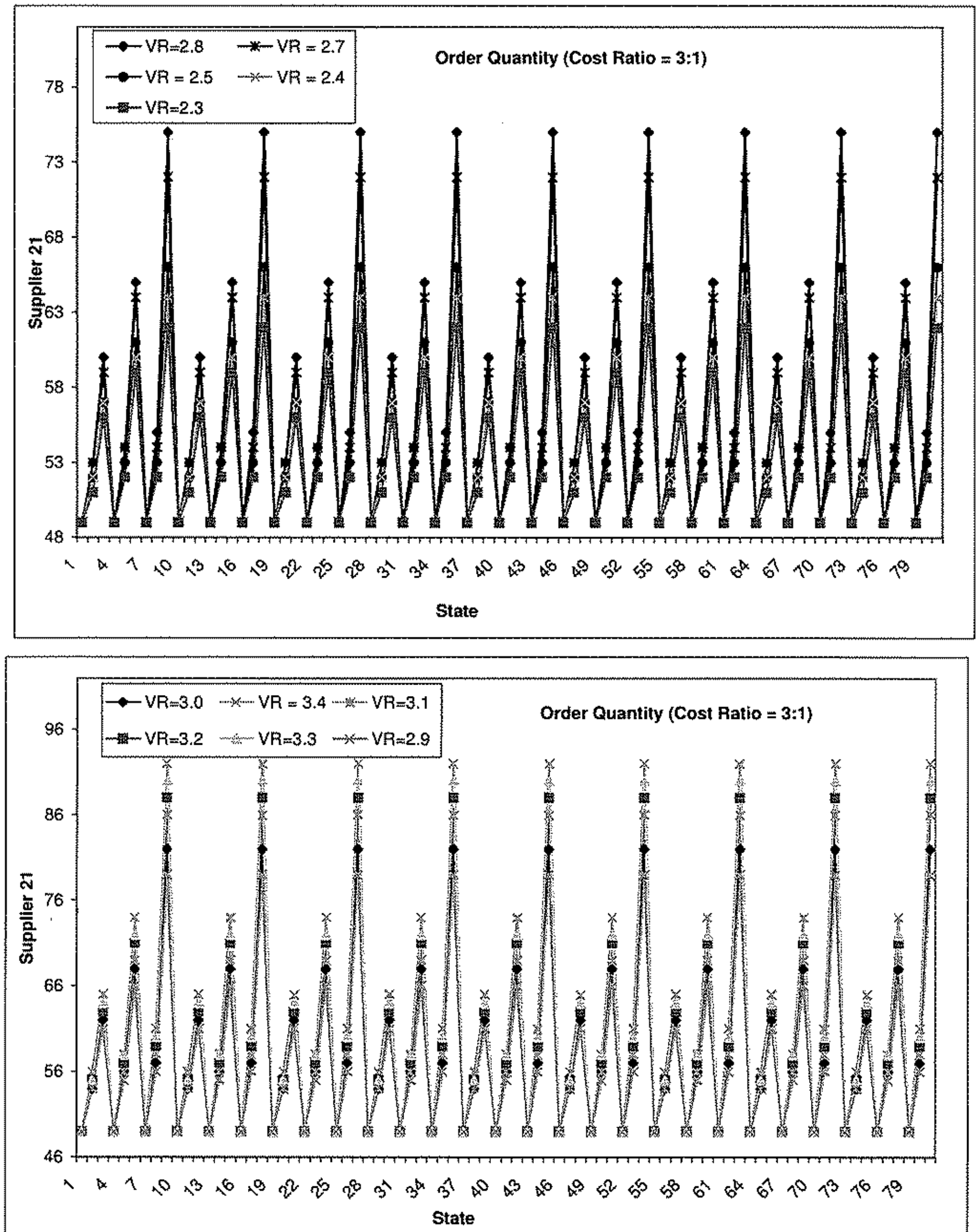


Figure 20: Ordering Quantities from Supplier 2_1 (VR=2.3 though 3.4; Cost Ratio=3:1;Normal Dist)

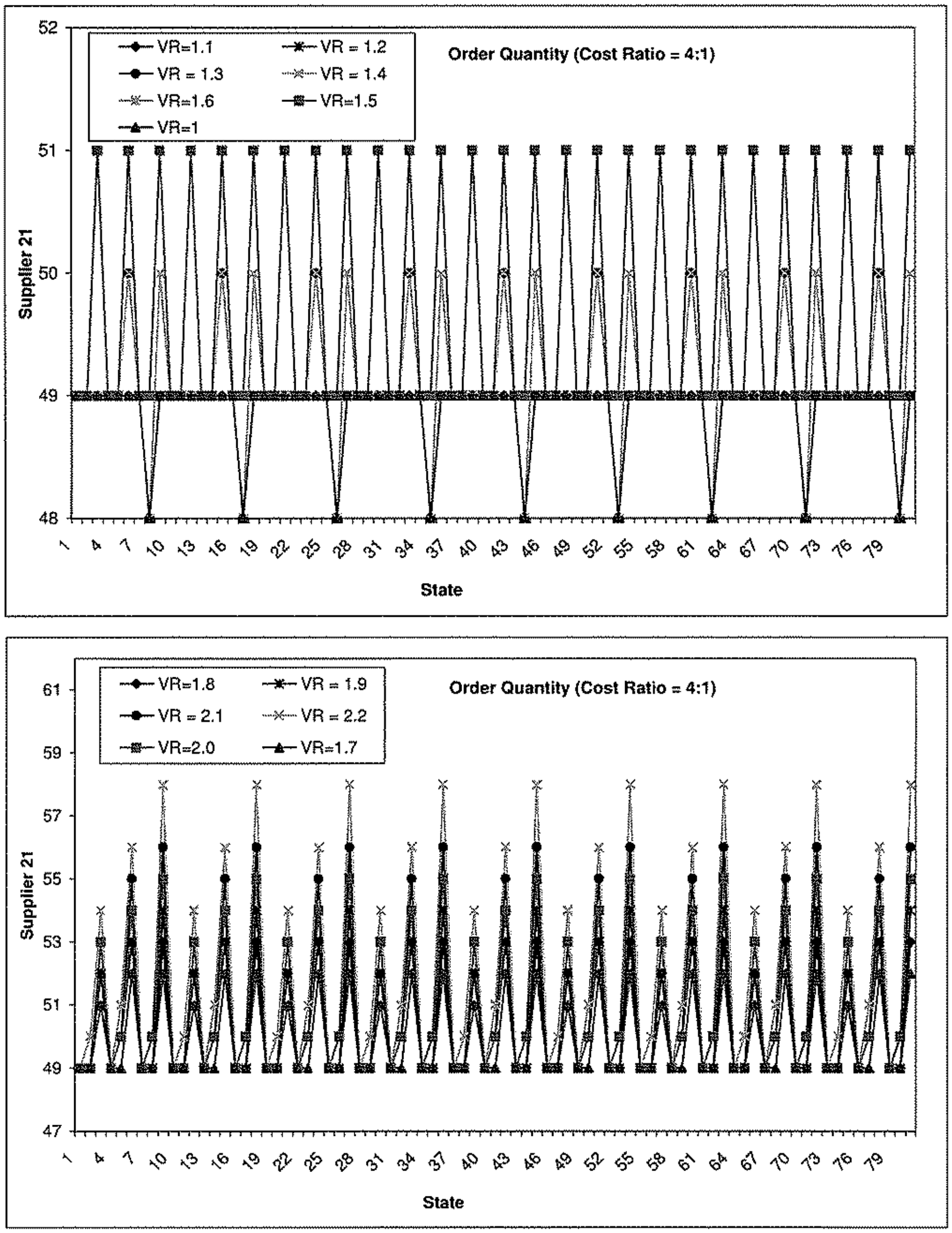


Figure 21: Ordering Quantities from Supplier 2_1 (VR=1.0 through 2.2; Cost Ratio=4:1;Normal Dist)

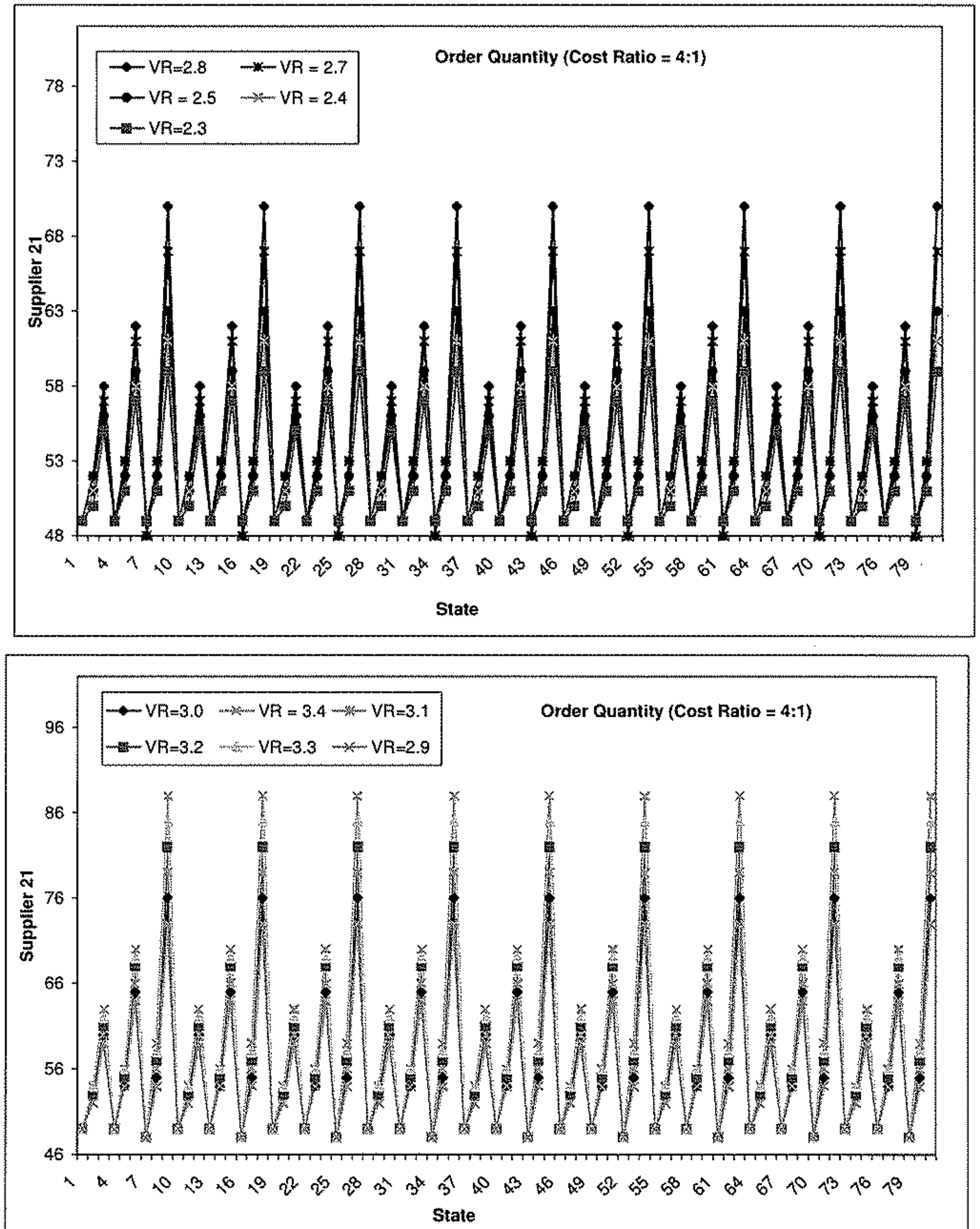


Figure 22: Ordering Quantities from Supplier 2_1 (VR=2.3 though 3.4; Cost Ratio=2:1;Normal Dist)

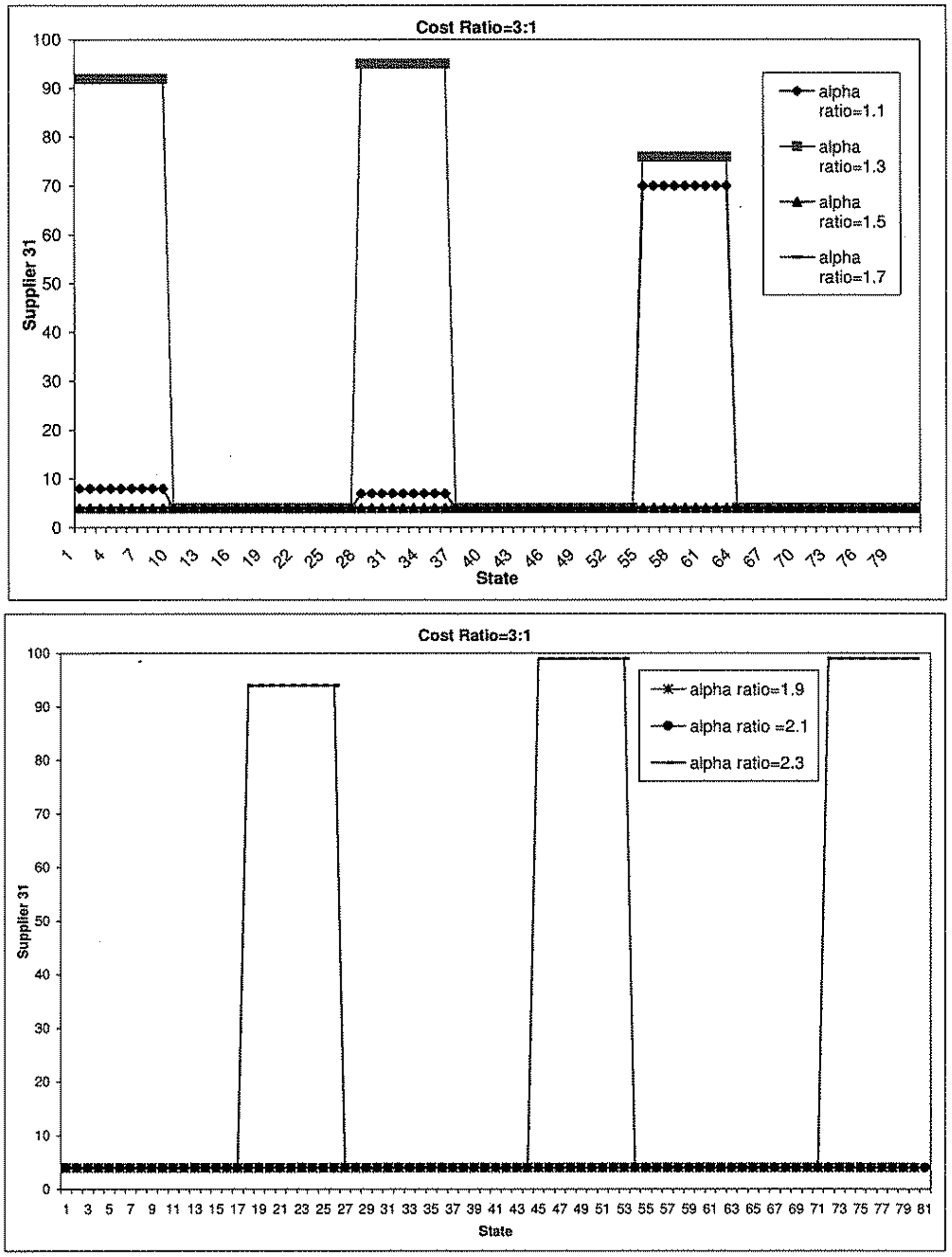
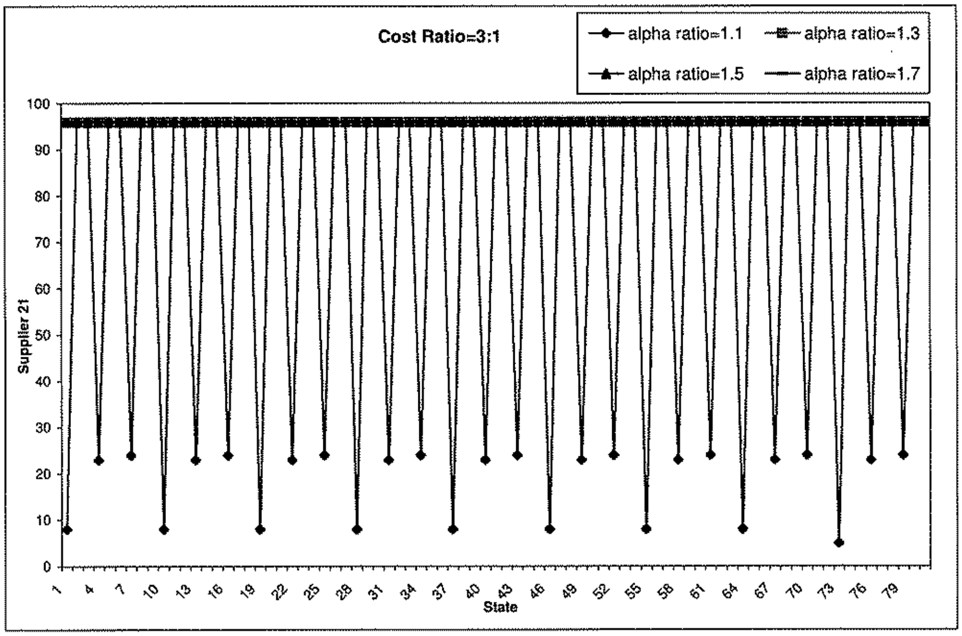


Figure 23: Ordering Quantities from Supplier 3_1 (Cost Ratio 4:1; Beta Dist)



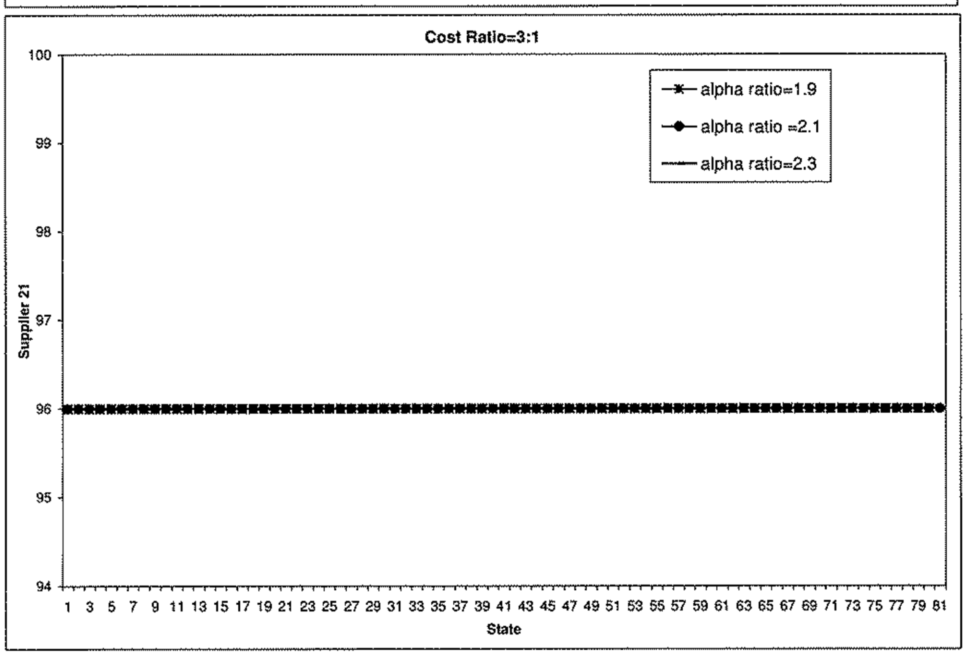


Figure 24: Ordering Quantities from Supplier 2_1 (Cost Ratio 3:1; Beta Dist)

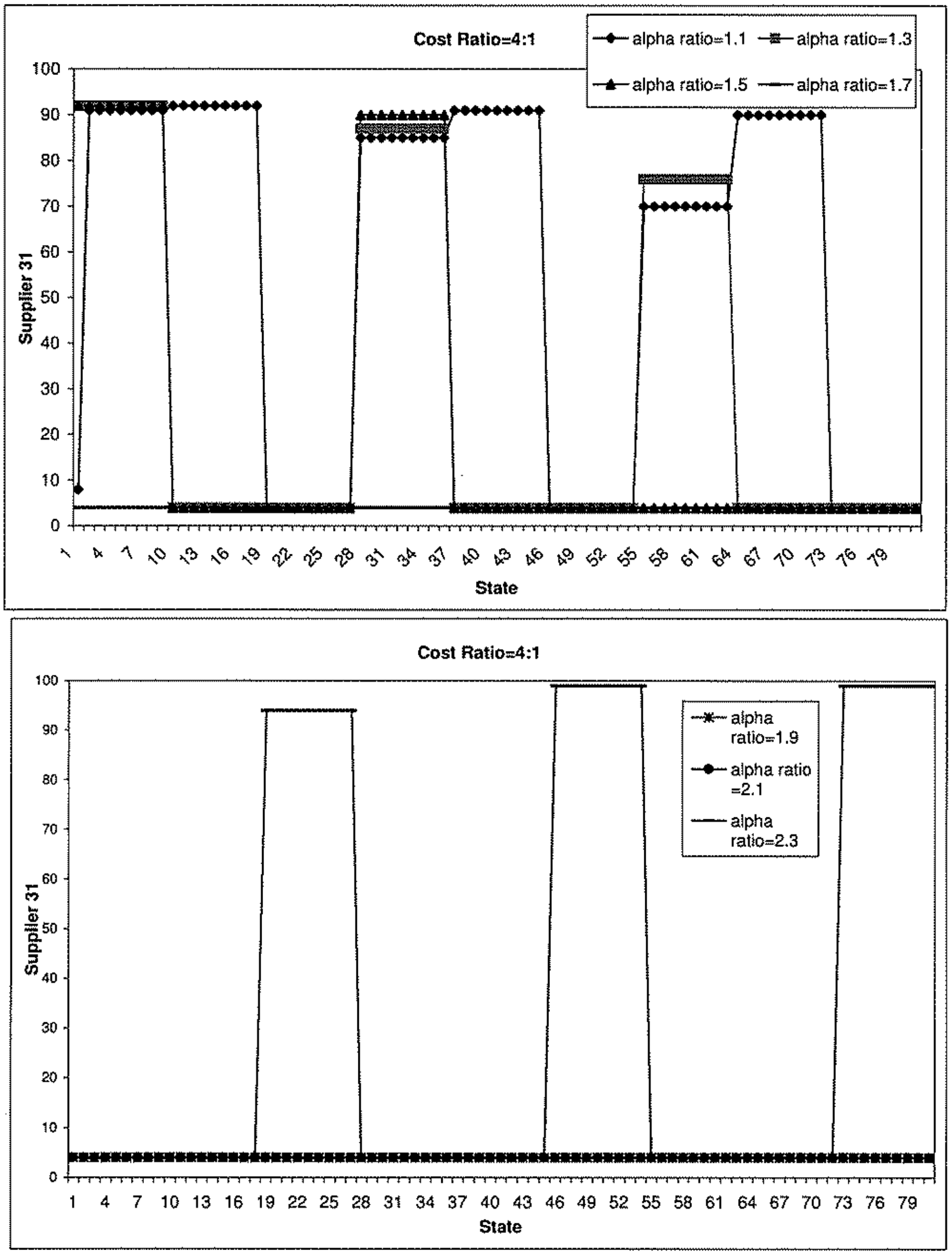


Figure 25: Ordering Quantities from Supplier 3_1 (Cost Ratio 4:1; Beta Dist)

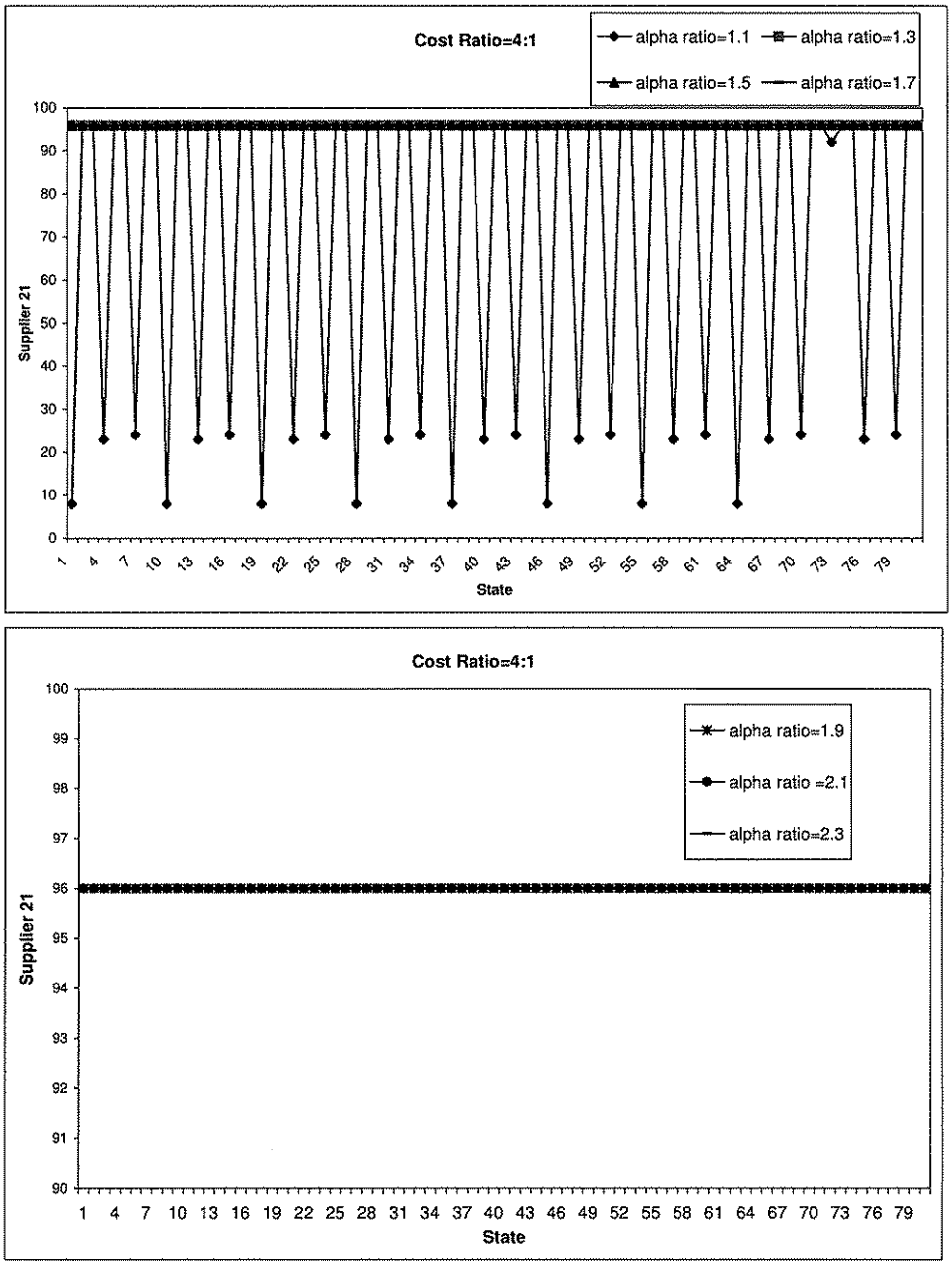


Figure 26: Ordering Quantities from Supplier 2_1 (Cost Ratio 4:1; Beta Dist)

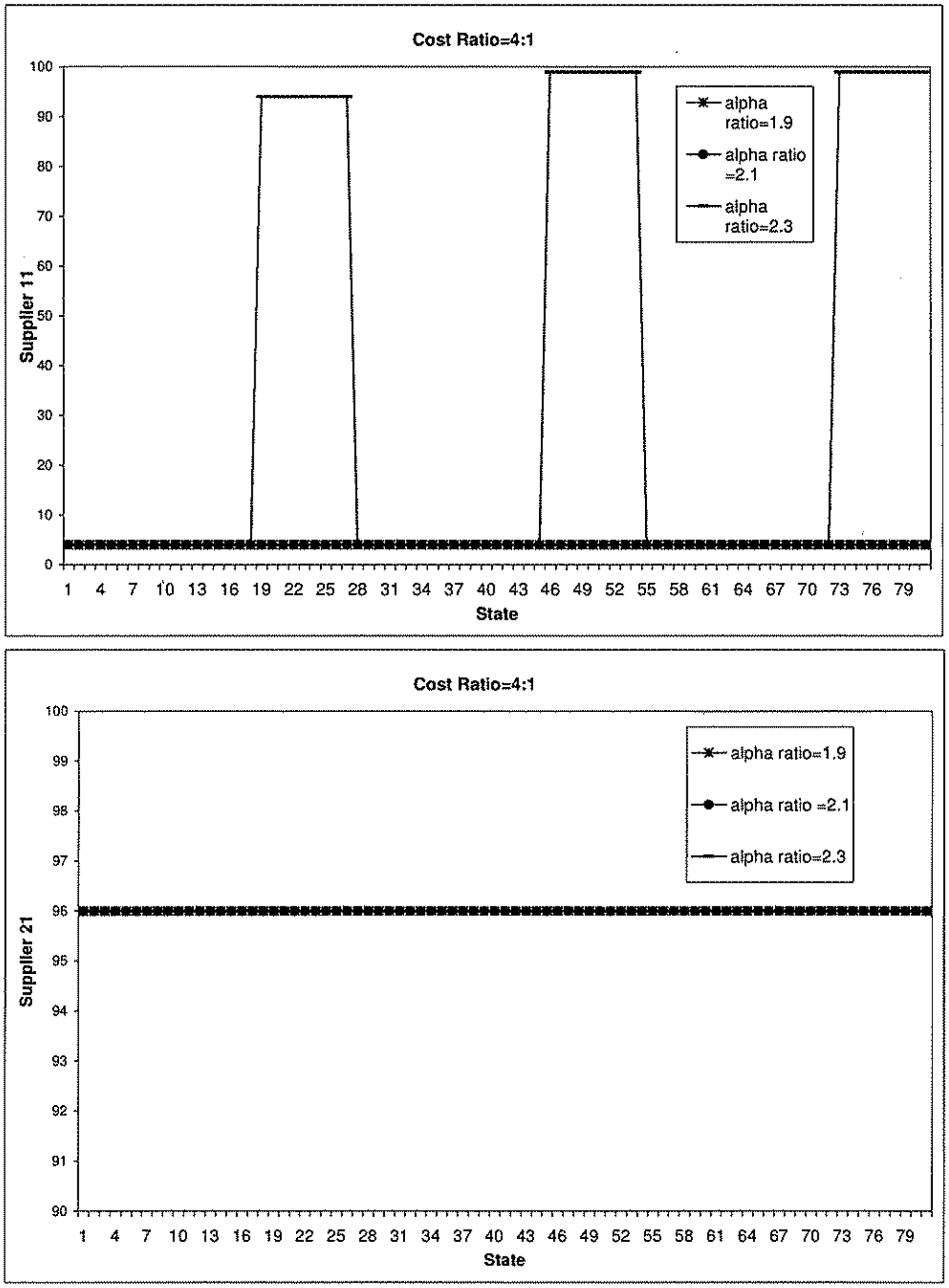


Figure 27: Ordering Quantities - Cost Ratio 4:1 (Beta Dist)

APPENDIX IV
Normal and Beta Distribution Parameters for the UCM

The normal distribution parameters for the uncontrolled situation were determined by generating 300 normal random variables (100 for each of the 3 states: light, normal, congested) for each of the 24 variance ratios (VR) in Table ??. The mean and variance of the 300 random numbers was determined and used as the uncontrolled distribution mean and variance. This was done for each VR.

Similarly, for the beta distribution, 300 beta random variables were generated for each of the 7 alpha ratios (AR). In order to determine the parameters at each AR, the *betafit* function in Matlab (Version 7.01) was used with a 99% confidence interval. From this, the revised PERT equations (7 and 8) were used to find the mean and variance parameters of the uncontrolled distribution. Since the same 50 demands were used for each scenario and the uncontrolled simulation uses only one leadtime distribution, the 50 instances of leadtime were found once for each of the 31 scenarios (24 normal and 7 beta) and used for all three cost ratios.

References

[1] T.J. Becker. The weakest link: New study quantifies financial fallout from supply-chain malfunctions. *Georgia Tech Research News*, February 2, 2004, 2004.

[2] J. Blackhurst, C.W. Craighead, D. Elkins, and R.B. Handfield. An empirically derived agenda of critical research issues for managing supply-chain disruptions. *International Journal of Production Research*, 43(19):4067–4081, 2005.

[3] Maqbool Dada, Nicholas Petruzzi, and Leroy B. Schwarz. A newsvendor model with unreliable suppliers. Technical report, Krannert Graduate School of Management, Purdue University, 2003.

[4] Mark S. Daskin, Erdem Eskigun, and Jeff Tew. A genetic algorithm for designing a reliable supply chain. *IIE Annual Conference and Exhibition 2004*, 2004.

[5] Julie Drzymalski. *A Synchronized Supply Chain For A Multi-Echelon, Multi-Stage System*. PhD thesis, Lehigh University, 2008.

[6] N.R. Farnum and L.W. Stanton. Some results concerning the estimation of beta distribution parameters in pert. *Journal of the Operational Research Society*, 38:287–290, 1987.

[7] G. Feeny and C. Sherbrooke. The (s-1,s) inventory policy under compound popisson demand. *Management Science*, 12:391–411, 1966.

[8] D. Golenko-Ginzburg. On the distribution of activity time in pert. *Journal of the Operational Research Society*, 39:767–771, 1988.

[9] Donald L. Keefer and William A¿ Verdini. Better estimation of pert activity time parameters. *Management Science*, 39(9):1086–1091, 1993.

[10] Amy Hing-Ling Lau, Hon-Shiang Lau, and Yue Zhang. A simple and logical alternative for making pert time estimates. *IIE Transactions*, 28:183–192, 1996.

[11] Hon-Shiang Lau and Amy Hing-Ling Lau. An improved pert-type formula for standard deviation. *IIE Transactions*, 30:273–275, 1998.

[12] R.E. Megill. *An Introduction to Risk Analysis*. Petroleum Publishing Company, 1977.

[13] Hokey Min and Gengui Zhou. Supply chain modeling: past, present and future. *Computers and Industrial Engineering*, 43, 2002.

[14] Suleyman Oekici and Mahmut Parlar. Inventory models with unreliable suppliers in a random environment. *Annals of Operations Research*, 91:123–136, 1999.

[15] E.S. Pearson and J.W. Tukey. Approximate means and standard deviations based on distances between percentage points of frequency curves. *Biometrika*, 52:533–546, 1965.

[16] Larry Snyder. Quantitative models for supply chain management. class notes.

[17] Jing-Sheng Song. The effect of leadtime uncertainty in a simple stochastic inventory model. *Management Science*, 40(5):603–613, 1994.

[18] Anthony Svoronos and Paul Zipkin. Evaluation of one-for-one replenishment policies for multiechelon inventory systems. *Management Science*, 37(1):68–83, 1991.

[19] Brian Tomlin. On the value of mitigation and contingency strategies for managing supply chain disruption risks. *Management Science*, 52(5):649–657, 2006.

[20] Brian T. Tomlin and Lawrence V. Snyder. On the value of a threat advisory system for managing supply chain disruptions. Submitted for publication, April 2007.

[21] M.D. Troutt. On the generality of the pert average time formula. *Decision Sciences*, 20:410–412, 1989.

[22] Jin-Hsien Wang. Fuzzy linguistic pert. *IEEE Transactions on Fuzzy Systems*, 15(2):133–144, 2007.