Bullwhip and Reverse Bullwhip Effects Under the Rationing Game

Ying Rong
Antai College of Economics and Management, Shanghai Jiao Tong University, Shanghai, China 200052, yrong@sjtu.edu.cn

Lawrence V. Snyder
Department of Industrial and Systems Engineering, Lehigh University, Bethlehem, PA 18015, larry.snyder@lehigh.edu

Zuo-Jun Max Shen
Department of Industrial Engineering and Operations Research, Berkeley, CA 94720, shen@ieor.berkeley.edu

When an unreliable supplier serves multiple retailers, the retailers may compete with each other by inflating their order quantities in order to obtain their desired allocation from the supplier, a behavior known as the rationing game. In this paper, we provide the formal condition of the existence of the bullwhip effect (BWE) under the rationing game when the mean demand changes over time. Moreover, when the capacity information is shared and the capacity reservation mechanism is applied, we provide the condition when the *reverse bullwhip effect* (RBWE) occurs upstream, a consequence of the disruption caused by the supplier. In addition, we show that the smaller unit reservation payment leads to the severe [R]BWE. Finally, we find that capacity information sharing does not necessarily mitigate the [R]BWE and that it may reduce the profitability of the supply chain as a whole.

Key words: rationing game, bullwhip effect, reverse bullwhip effect, supply uncertainty, order variance

1. Introduction

U.S. firms have reduced their supplier base significantly since the late 1980s (Trent and Monczka 1998). In many industries, a majority of the total volume of raw materials is sourced by only a few suppliers (Carbone 1999). Although this consolidation may be beneficial in a stable business environment, the associated supply uncertainty, especially when coupled with demand uncertainty, produces increasingly significant challenges as the importance of each supplier increases (Tang 2006). If a supplier serves several retailers (or other clients), a capacity disturbance at the supplier affects all its clients. Sheffi (2005) gives several examples of this, including the Taiwan Semiconductor Manufacturing Company (TSMC), which was affected by an earthquake in 1999 and which provides chips for numerous leading computer companies, and Philips, whose plant fire in 2000 affected supplies for both Nokia and Ericsson. Another example is that Hurricane Katrina affected many chemical firms, including DuPont and Chevron, whose products are necessities for many of their customers (Storck 2005). Moreover, the retailers must simultaneously cope with the demand uncertainty produced by their own customers.

When the supplier's capacity is insufficient to meet the total demand, a natural response from the retailers is to inflate their order quantities to try to obtain a larger piece of the pie. In this paper, we prove that, under this so-called "rationing game", the supply end of the system experiences the *reverse bullwhip effect* (RBWE), in which order variance increases as one moves downstream in the supply chain, and that the demand end of the system experiences the classical *bullwhip effect* (BWE), in which order variance increases in the opposite direction. When the BWE and RBWE both occur, the middle stages of the supply chain suffer the most from the two types of uncertainty. Our research provides a theoretical explanation to the observation by Cachon et al. (2007) from raw industry-level data that the middle echelon of many supply chains has the greatest order volatility, contradicting the conventional wisdom that the BWE should cause the volatility to be greatest at the upstream end. Moreover, our research highlights the importance of treating the supply chain as an integrated system rather than as a collection of isolated players, since any type of uncertainty-supply or demand, and anywhere it occurs in the supply chain, can be magnified for the remaining parties.

When supply uncertainty exists, the presence or absence of capacity information is often a key determinant of retailers' order-quantity decisions. Under prevailing information-sharing programs such as collaborative planning, forecasting, and replenishment (CPFR), supply and demand information flows in both directions in many supply chains. Moreover, real-time supply information sharing may be particularly important when capacity is a major limiting factor. Sheffi (2005, page 7) provides a vivid example of Nokia demanding to be informed of Philips's supply status on a timely basis after its plant fire in 2000.

Furthermore, capacity reservation is another common practice when capacity is a bottleneck (Wu et al. 2005). A typical example is that Apple offered its suppliers upfront cash payments in an effort to secure its order allocation after the Japanese earthquake in 2011 (AppleInsider 2011). In addition, capacity reservation can serve as a compensation to the supplier for sharing its capacity information.

In model-R (for "reservation"), we introduce capacity information sharing coupled with a reservation payment in order to reduce over-ordering under tight capacity. We show that there always exists a NE, regardless of the size of the reservation payment (provided that it is strictly positive). We characterize the conditions under which the BWE and RBWE do occur, and prove that the smaller the reservation payment is, the more frequently and severely the BWE and RBWE occur.

We also consider a benchmark model, which we call model-L (for Lee et al. (1997), and hereinafter referred to as "LPW"), to evaluate the benefit of capacity information sharing and capacity reservation. This model is equivalent to the seminal rationing game model studied by LPW, in which the retailers place their orders before knowing the realized capacity. LPW use this model to show that the retailers' orders are inflated and to argue for the existence of the BWE when the demand mean changes over time. They also point out, among other advocates(e.g. Chen et al. 2000), that capacity information sharing and capacity reservation are among remedies to mitigate the BWE.

However, by comparing model-L and model-R, we demonstrate that these two treatments may trigger even greater retailer order variability and reduce the profit of the whole supply chain under certain circumstances. Our work also suggests that the insights derived under demand uncertainty may not always apply under supply uncertainty, which is similar in spirit to the inventory placement problem in a network with disrupted supply studied by Snyder and Shen (2006).

The remainder of this paper is organized as follows. In Section 2, we relate our paper to the existing literature on the subject. In Section 3, we outline the basic assumptions for our two analytical models. Sections 4 and 5 provide analyses of the order variability under model-L and model-R, respectively. We illustrate the impact of information sharing and capacity reservation in Section 6. Section 7 concludes with some final remarks on the BWE and RBWE.

2. Literature Review

The BWE was first described by Forrester (1958), though the term "bullwhip effect" was introduced to the literature by LPW. LPW's paper was the first to provide a theoretical understanding of the BWE. They suggest four mechanisms that can trigger the BWE: demand forecasting, rationing game, order batching, and price fluctuations. They introduce analytical models to study demand volatility propagation under all four settings. The subsequent theoretical research on the BWE generally concentrates on exploring additional causes of the BWE, measuring the severity of the BWE, and analyzing mitigation strategies. The reader is referred to Lee et al. (2004) and Geary et al. (2006) for more thorough reviews of the literature on the BWE.

Most of the research on the BWE considers demand uncertainty and ignores supply uncertainty. Of course, both supply and demand uncertainty are present in most supply chains, and Snyder and Shen (2006) show that the insights gained from the study of one type of uncertainty often do not apply to the other. Therefore, when we examine the order variance at one stage of the supply chain, we need to consider the effect of both types of uncertainty. Although empirical studies conducted by Baganha and Cohen (1998) and Cachon et al. (2007) indicate that the BWE does not dominate upstream[1], there is still very little theoretical analysis of the impact of supply uncertainty in the context of the BWE, especially at the stage closest to the source of the supply uncertainty. Our paper explicitly considers the interaction between the two types of uncertainty in creating the BWE and RBWE. Moreover, we demonstrate that capacity information sharing does not necessarily mitigate the [R]BWE, although demand information sharing is a common practice to mitigate the BWE.

[1] Cachon et al. (2007) explain the non-dominance of the BWE by the seasonality of the demand. By removing the seasonality, their data show that there is still a significant portion of industries in the upstream part of the supply chain not exhibiting the BWE. Chen and Lee (2009) argue that the aggregate data at a macro-level may underestimate the magnitude of the BWE when it exists but that using industry-level data does not affect the estimation of the existence of the BWE.

The present paper and the papers by Rong et al. (2009, 2011) are the first to investigate the RBWE. Rong et al. (2009) present a behavioral study that uses a variant of the beer game in which the manufacturer faces supply uncertainty, in contrast to the ample-supply assumption in the traditional version of the game (e.g., Sterman 1989). Rong et al. (2011) investigate how price fluctuations that result from supply uncertainty trigger the RBWE when the firm does not possess a correct model of customer behavior. The present paper demonstrates that competition for scarce resources is an operational cause of the RBWE.

The roots of our models can be found in the literature on the rationing game. Our setting is closest to the rationing game model in Lee et al. (1997) (LPW), who also study multiple retailers sharing a common unreliable supplier. Moreover, we show that under LPW's original model (called "model-L" in our paper), the BWE may not occur. In addition, we provide the formal conditions of the existence of the BWE under both model-L and model-R as well as the existence of the RBWE under model-R.

Cachon and Lariviere (1999a,c) model a rationing game with two retailers, deterministic capacity at the common supplier and a linear demand function in the sales price with two demand states. Cachon and Lariviere (1999c) compare the linear, proportional, and uniform allocation rules. They find that an NE may not exist under the linear and proportional rules, while it always does under the uniform rule. Cachon and Lariviere (1999a) extend the rationing game into two periods and study an allocation rule based on past sales ("turn-and-earn"), as opposed to a fixed allocation. They demonstrate that the supplier always benefits from turn-and-earn since the retailers increase their order quantities. Lu and Lariviere (2011) extend the work by Cachon and Lariviere (1999a) to an infinite-horizon setting and multiple demand states. Their numerical study shows that turn-and-earn may induce the retailers to absorb their local demand variability.

Cachon and Lariviere (1999b) consider two broad classes of allocation mechanisms under a more general form of the retailers' profit function with multiple retailers: those that induce the retailers to increase their orders or those that induce them to tell the truth. They provide conditions under which there exists an NE for the relaxed linear allocation rule. Moreover, they show that truth-inducing mechanisms do not maximize total retailer profit and therefore may not be appealing. To this end, Cachon and Lariviere (1999c) show that, if an NE exists under the linear and proportional allocation rules, the total supply chain is better off on average compared to the uniform allocation rule (a truth-inducing mechanism). Moreover, Rong (2008) shows that the proportional allocation rule induces lower supply chain cost compared to the uniform allocation rule and the sales based allocation rule under a multi-period simulation study. It is because the proportional allocation rule allows the supplier to deliver goods based on retailers' true needs (though it is inflated).

However, the above papers either assume deterministic capacity or stochastic capacity with no information sharing. We investigate the impact of capacity information sharing on both the [R]BWE and supply chain profit when the capacity is random.

Information sharing is regarded as an effective way to improve the performance of the supply chain. Most studies concentrate on demand information sharing (Chen 2003). Chen and Yu (2005) consider upstream leadtime information sharing. Li and Gao (2008) study the benefit of sharing new product development progress. Jain and Moinzadeh (2005) investigate how information about upstream inventory affects the retailer's order decision. They allow the retailer to change from a one-level base stock policy to a two-level base stock policy to take advantage of the shared information. They find that upstream inventory information sharing induces the BWE under stationary Poisson demand. In these three papers, the one-supplier, one-retailer setting enables the retailer to extract a benefit from upstream information sharing. In contrast, we study the behavior of multiple retailers in a competitive environment and find that upstream capacity information sharing may induce higher retailer order variability as well if the reservation payment is small enough. Moreover, the profit of the whole supply chain may be smaller when capacity information is shared.

Most literature on unreliable suppliers assumes a single retailer with one or more unreliable suppliers (see Parlar and Perry (1996), Swaminathan and Shanthikumar (1999), Tomlin (2006), Babich et al. (2007), Wang et al. (2009), Yang et al. (2009), Feng (2010) etc.). There are relatively few papers considering supply uncertainty in distribution networks. Tomlin and Wang (2005) analyze the value of flexibility when multiple products share a single or dual unreliable resources. Snyder and Shen (2006) and Schmitt et al. (2008) study the best location to hold inventory in a one-warehouse, multiple-retailer setting with an unreliable warehouse. Their setting differs from ours in that it assumes a centralized system, and also because they consider a Bernoulli-type supply uncertainty for which the allocation rule does not matter, while the allocation policy plays an important rule in our paper because of the more general capacity process.

3. Model Assumptions

We consider a system with N identical retailers who face independent random demands from customers and replenish their inventory from a common supplier whose capacity is also random. In addition, the retailers observe the shift of the demand distribution (see below) before their order decision.[2] Each retailer makes ordering decisions to maximize its own profit, that is, the system is managed in a decentralized manner.

To be consistent with LPW's model, we consider a single-period setting in our analytical models. The [R]BWE is defined based on the variability of retailers' orders when the game is repeated with a different realization of the random capacity and demand shift.

Before retailers place their orders, they observe a public signal which is translated as a shift in the demand distribution. For example, a sudden stock market crash affects people's willingness to consume. Or, under unusual high temperatures, the sales of icecream would go up. Let X be the *total* demand shift (across all retailers), which is a random variable. We assume that the total demand shift is allocated evenly among the retailers. We assume that the retailers observe X before placing orders. After observing $X = x$, the overall demand for retailer i is $D_i + \frac{x}{N}$, where D_i represents the remaining randomness of demand with pdf $g(d)$ and cdf $G(d)$. We assume that $G(d) = 0$ when $d < 0$ and that $G(d)$ is strictly increasing in this domain. That is, if $0 < G(d) < 1$, then $g(d) > 0$.

[2] LPW also suggests that the reaction to the shift of the demand distribution can cause the BWE.

The capacity at the supplier is random, represented by the random variable V with pdf $f(v)$ and cdf $F(v)$. We assume that X, D_i and V are pairwise independent. D_i is identically distributed for all the retailers. And we assume that the overall demand for retailer i is nonnegative. That is, $D_i + \frac{X}{N} \geq 0$.

For a given observed value x of the demand shift X, retailer i's demand, $D_i + \frac{x}{N}$, has pdf $g_{\frac{x}{N}}(d)$ and cdf $G_{\frac{x}{N}}(d)$, which follow the equations below:

$$\begin{aligned} g_{\frac{x}{N}}(d) &= g\left(d - \frac{x}{N}\right) \\ G_{\frac{x}{N}}(d) &= G\left(d - \frac{x}{N}\right) \\ G^{-1}_{\frac{x}{N}}(\alpha) &= G^{-1}(\alpha) + \frac{x}{N} \end{aligned} \tag{1}$$

In the analysis that follows, if we drop the subscript $\frac{x}{N}$ from a quantity that normally has such a subscript, it means $x = 0$.

We introduce ζ, the wholesale price received by the supplier from the retailers, and ϑ, a unit variable production cost incurred the supplier. The unit sales price for each retailer is ϱ and the unit salvage value is κ. Thus, each retailer faces an underage cost of $p = \varrho - \zeta$ for each unit of unmet demand and an overage cost of $h = \zeta - \kappa$ for each unit in inventory at the end of the period. We assume $h, p > 0$. If there were no capacity constraint and the total demand shift were x, then each retailer would order the newsboy quantity for the distribution $G_{\frac{x}{N}}$, denoted $S_{\frac{x}{N}}$:

$$S_{\frac{x}{N}} = G^{-1}_{\frac{x}{N}}\left(\frac{p}{p+h}\right) = G^{-1}\left(\frac{p}{p+h}\right) + \frac{x}{N} = S + \frac{x}{N}. \tag{2}$$

Note that we use S to denote newsboy quantity when $x = 0$.

In this paper, we propose two models. In model-L, we assume that the supplier does not disclose the capacity status to the retailers before their order decisions. In model-R, we assume that the supplier shares the real-time capacity information with the retailers. In return, the retailers incur a reservation payment r ($r \leq \zeta$) for each unit ordered, whether or not the unit is actually delivered. In addition, for each unit the supplier delivers, the retailers also pay an additional purchase cost

$(\zeta - r)$.[3] The details of each model's setting will be described in Sections 4 and 5, when it is analyzed.

3.1. Definition of BWE and RBWE

Let z_i be the order size of retailer i. Note that z_i and $S_{\frac{x}{N}}$ are not the same: $S_{\frac{x}{N}}$ is the newsboy order quantity, which the retailer would order if there were no capacity constraints, whereas z_i is the retailer's order quantity taking into account both the potential capacity shortage and the other retailers' actions.

Let $y = \sum_i z_i$ be the total order size of all the retailers, and let $\mathbf{z}_{-i}$ be the vector of the other retailers' order quantities. With a slight abuse of notation, we let $\sum_{-i} \mathbf{z}_{-i}$ represent the total order quantity for retailers other than retailer i. Let ω be the index of the model. That is, $\omega = L$ (R) under model-L (model-R).

Let $\pi_i^\omega(z_i|\mathbf{z}_{-i})$ be the expected profit for retailer i when it orders z_i and the other retailers order $\mathbf{z}_{-i}$ under model-ω. Let $z_i^*(\mathbf{z}_{-i})$ be the best response mapping for retailer i when the others order $\mathbf{z}_{-i}$.

Let $z_i^\omega(x, v)$ be the NE of order quantity chosen by retailer i given the realized demand shift x and capacity v. Let

$$y^\omega(x, v) = \sum_i z_i^\omega(x, v)$$

be the total order quantity.

We are interested in comparing the variance of the total retailer orders, y^ω, with that of the demand and supply processes. To measure the variability of demand process, we use $var(X)$, the variance of the demand shift, not $var(\sum_i D_i + X)$, the variance of the actual demand. This is because in both models we assume the retailers place orders before they know D_i. Thus the realization of D_i does not have an impact on y^ω. This can also be seen in LPW's analysis since

[3] We assume that the supplier keeps the reservation payment even for undelivered units is not strictly required; alternately, one can simply think that the reservation payment returns to the retailers but the opportunity cost associated with the reservation payment incurs during the time between when the retailers order and when the supplier fulfills the orders. These two assumptions are equivalent. All the analysis for the rationing game in this paper is applicable to both of them.

the realization of demand does not affect y^ω no matter how large the variance of D_i is. In other words, y^ω is fixed if X is fixed. That is why we focus on X, the demand shift, which is the actual trigger for the BWE. This is also reflected in the argument by LPW that the BWE occurs when the mean demand changes over time.

To measure the the variability of supply process, we use $var(V)$, the variance of available capacity. Ultimately, we are interested in evaluating the following ratios:

$$\begin{aligned}\theta_X &= \frac{var(y^\omega(X,V))}{var(X)} \\ \theta_V &= \frac{var(y^\omega(X,V))}{var(V)}\end{aligned} \tag{3}$$

If $\theta_X > 1$, we say that the BWE occurs between the retailers and the customers. If $\theta_V > 1$, we say that the RBWE occurs between the supplier and the retailers. If $\theta_X < 1$, rather than saying that the RBWE occurs, we say that the BWE does not occur between the retailers and the customers. Similarly, if $\theta_V < 1$, we do not say the BWE occurs between the supplier and the retailers. This distinction is motivated by our claim that the BWE and the RBWE are triggered by demand uncertainty and supply uncertainty, respectively. The retailers react to supply uncertainty but their customers do not, and they also react to demand uncertainty but the supplier's capacity is independent of the retailers' action. Therefore, if $\theta_X < 1$, it is because the BWE is not occurring, rather than because the RBWE is occurring between the retailers and the customers.

Finally, we define the conditional BWE and RBWE by the following ratios. Let $A \subseteq \Omega$, where Ω is the sample space of the joint random variable (X,V). Then

$$\begin{aligned}\theta_{X|A} &= \frac{var(y^\omega(X,V)|(X,V)\in A)}{var(X|(X,V)\in A)} \\ \theta_{V|A} &= \frac{var(y^\omega(X,V)|(X,V)\in A)}{var(V|(X,V)\in A)}\end{aligned} \tag{4}$$

These two ratios measure the BWE and the RBWE within a sub-region A of the sample space.

4. Order Variability Under Model-L

We define the sequence of events in model-L:

1. The demand shift X is realized by the retailers and the capacity V is realized by the supplier. Thus, $X = x$ and $V = v$. The supplier does **NOT** share the value of the realized capacity v with the retailers.
2. Retailer i places its order z_i, for $i = 1, ..., N$.
3. The supplier produces up to $\sum_{i=1}^{N} z_i$ subject to capacity constraint. The produced products are distributed among the retailers using the proportional allocation rule (see below).
4. The demand at retailer i, D_i, is realized for $i = 1, ..., N$.
5. Customer demands are satisfied to the extent possible, excess demands are lost, and overage and underage costs are incurred.

In step 3, the proportional allocation rule means that if the total retailer order (y) exceeds the realized capacity v, then retailer i receives $\frac{v}{y} z_i$. Otherwise, it receives z_i.

In fact, model-L is the same as the rationing game from LPW except that we explicitly model the demand shift, which is a key to argue the existence of the BWE by LPW. In model-L, there is no capacity information sharing between the supplier and retailers.

For given demand shift x under Model-L, the expected cost function for retailer i when it orders z_i and the other retailers order $\mathbf{z}_{-i}$ is given by the following equation.

$$\begin{aligned}\pi_i^L(z_i|\mathbf{z}_{-i}) = & (\varrho - \zeta)E[D_i] \\ & - \int_{v=0}^{y} \left[p \int_{\frac{v}{y} z_i}^{\infty} \left(d - \frac{v}{y} z_i \right) dG_{\frac{x}{N}}(d) + h \int_{0}^{\frac{v}{y} z_i} \left(\frac{v}{y} z_i - d \right) dG_{\frac{x}{N}}(d) \right] dF(v) \\ & - (1 - F(y)) \left[p \int_{v=z_i}^{\infty} (d - z_i) dG_{\frac{x}{N}}(d) + h \int_{v=0}^{z_i} (z_i - d) dG_{\frac{x}{N}}(d) \right]\end{aligned} \tag{5}$$

Based on (5), LPW gives the following theorem (We customize it for given demand shift x)

THEOREM 1. *The symmetric NE ($z^L(x)$) for given demand shift x, if exist, must satisfy*

$$\begin{aligned}\int_0^{Nz^L(x)} \left[-p + (p+h) G_{\frac{x}{N}} \left(\frac{v}{N} \right) \right] v \left(\frac{1}{Nz^L(x)} - \frac{1}{N^2 (z^L(x))^2} \right) dF(v) \\ + (1 - F(Nz^L(x)) \left[-p + (p+h) G_{\frac{x}{N}} \left(z^L(x) \right) \right] = 0\end{aligned} \tag{6}$$

Moreover, $z^L(x) \geq S_{\frac{x}{N}}$. Further, if $F(\cdot)$ and $G_{\frac{x}{N}}(\cdot)$ are strictly increasing, the inequality strictly holds.

Now, we start to analyze the retailers' order variability under model-L. Since the real-time capacity information is not shared with the retailers under model-L, the retailers' orders are not dependent on the realized capacity. That is, the retailers cannot chase the realized capacity by adjusting their orders dynamically. Therefore, we only analyze the BWE under model-L. In this section, we provide the condition to validate the argument by LPW that the BWE exist "when the mean demand changes over time."

We first provide the following lemma to facilitate the BWE analysis.

LEMMA 1. *Let (U,Ω_U) be a random variable, where Ω_U is contained in the interval $[a,b]$ and U has a strictly positive variance. Let f and g be bounded functions in Ω_U.*

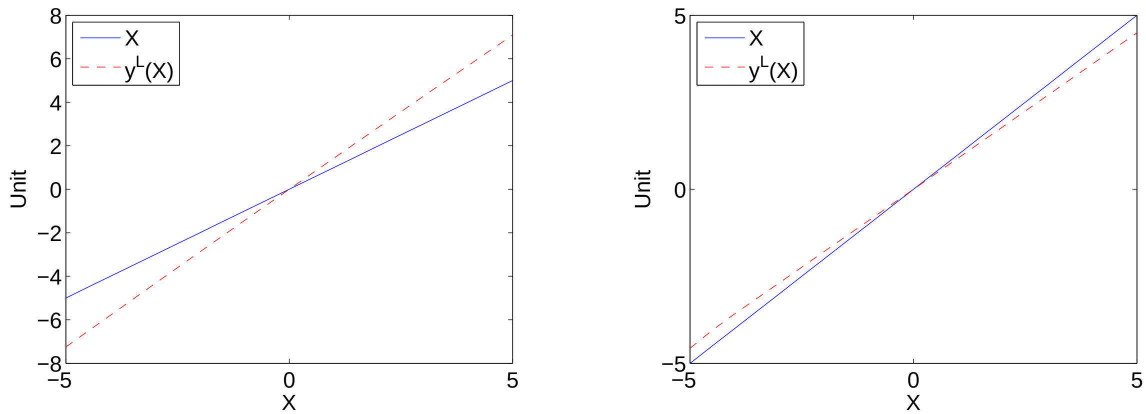
1. *$\forall u\in[a,b]$, if $|g'(u)|>1$ and $g(u)$ is monotone, then $Var(U)<Var(g(U))$.*
2. *$\forall u\in[a,b]$, if $f'(u)>g'(u)>0$, then $Var(f(U))>Var(g(U))$.*

Lemma 1 provides the basis to compare the variance of X and $var(y^L(X))$ based on the the functional relationship between $y^L(x)$ and realization of X. By Lemma 1, we can get the sufficient condition on the existence of the BWE.

THEOREM 2. *Suppose that (6) has a solution for $x\in[a,b]$. If $\frac{dy^L(x)}{dx}>1$ for all $x\in[a,b]$, then $\theta_X>1$ for all possible distributions of X within $[a,b]$. That is, the BWE always exists.*

We utilize two examples to demonstrate why we need such condition to ensure the existence of the BWE for all possible distributions of $X\in[a,b]$. Figure 1 contains two plots, in which we set $N=5, h=1, p=1, D_i\sim U[10,50]$. In Figure 1.a, we assume that, with probability 0.5, $V\sim U[120,121]$, and with probability 0.5, $V\sim U[180,181]$. In Figure 1.b, we assume that, with probability 0.5, $V\sim U[45,46]$ and with probability 0.5, $V\sim U[180,181]$. We plot X and $y^L(X)-y^L(0)$ in both plots.

We can see that the slope of the NE is larger than that of X in Figure 1.a. The existence of the BWE is independent on the distribution of $X\in[-5,5]$ because the disturbance brought by X triggers an even larger change in $y^L(X)$. In contrast, the slope of NE is smaller than that of X in Figure 1.b, which indicates the non-existence of the BWE when X varies in the same area.



(a: BWE) (b: No BWE)

Figure 1 BWE Results in Model-L

Thus, Theorem 2 indicates that the BWE does not always occur under the rationing game when the mean demand changes overtime. This also provides an explanation of why the BWE is not ubiquitous in real-world supply chains (Cachon et al. 2007).

5. Order Variability under Model-R

The remedies to reduce the BWE given by LPW (Table 1 in their paper) are to 1) share the capacity information between the retailers and the supplier and 2) adopt the capacity reservation mechanism. In order to examine the effectiveness of these remedies, we provide model-R. We define the sequence of events in model-R:

1. The demand shift X is realized by the retailers and the capacity V is realized by the supplier. Thus, $X = x$ and $V = v$. The supplier shares the value of the realized capacity v with the retailers.

2. Retailer i places its order z_i and incur the reservation payment rz_i, for $i = 1, ...N$.

3. The supplier produces up to $\sum_{i=1}^{N} z_i$ subject to capacity constraint. The produced products are distributed among the retailers using the proportional allocation rule. The remaining purchase cost $(\zeta - r)\left(\frac{v}{\max(y,v)} z_i\right)$ incurs for the allocated order quantity to retailer i.

4. The demand at retailer i, $D_i + \frac{x}{N}$, is realized for $i = 1, ...N$.

5. Customer demands are satisfied to the extent possible, excess demands are lost, and overage

and underage costs are incurred.

We first characterize the NE of retailers' order quantity. Then, we provide the condition for the existence of the BWE and RBWE.

5.1. Existence of NE Under Model-R

When $v \geq NS_{\frac{x}{N}}$, it is obvious that $z_i^R(x,v) = S_{\frac{x}{N}}$. Let's consider how to derive the NE of retailers' order quantity when $v \leq NS_{\frac{x}{N}}$. If an NE z_i^R, $i = 1, ..., N$, exists under model-R, then the following lemma states that the NE of the retailers' total order size cannot be smaller than the available capacity.

Lemma 2. *When $v \leq NS_{\frac{x}{N}}$, we have $\sum_i z_i^R(x,v) \geq v$.*

Now we only need to consider candidates for the NE that satisfy Lemma 2, i.e., $y = \sum_i z_i \geq v$. Then the proportional allocation rule will always be in force. For each of the $z_i - \frac{v}{y} z_i \geq 0$ units that retailer i orders beyond its allocation, it incurs the extra unit reservation payment r under model-R. Therefore, when $\sum_i z_i \geq v$, the expected cost function for retailer i under model-R is

$$\pi_i^R(z_i|\mathbf{z}_{-i}) = (\varrho - \zeta)E[D_i] - \left[p \int_{\frac{v}{y}z_i}^{\infty} \left(d - \frac{v}{y} z_i \right) dG_{\frac{x}{N}}(d) + h \int_0^{\frac{v}{y}z_i} \left(\frac{v}{y} z_i - d \right) dG_{\frac{x}{N}}(d) + r \left(z_i - \frac{v}{y} z_i \right) \right] \tag{7}$$

Based on Lemma 3, we obtain the following theorem, which establishes the existence of a unique symmetric NE of order quantities and characterizes its magnitude.

Theorem 3. *Suppose that $v \leq NS_{\frac{x}{N}}$. Let*

$$v^0(x) = NG_{\frac{x}{N}}^{-1} \left(\max \left\{ 0, \frac{p}{p+h} - \frac{r}{p+h} \frac{1}{N-1} \right\} \right) \tag{8}$$

Then a unique symmetric NE exists, and it satisfies

$$y^R(x,v) = Nz^R(x,v) = \max \left\{ v, \frac{v}{r} \left(1 - \frac{1}{N} \right) \left[p + r - (p+h) G_{\frac{x}{N}} \left(\frac{v}{N} \right) \right] \right\} \tag{9}$$

Moreover, the former case in the maximum prevails iff $v \geq v^0(x)$.

Note that y^R is a function of x, v and r, but for the sake of simplicity, we include arguments for y^R only as necessary.

Since $v^0(x)$ decreases with r, for large enough r, $v^0(x) \le v < NS_{\frac{x}{N}}$. In this case, we can conclude, by Theorem 3, that the NE of the total order quantity, y^R, can be smaller than the sum of the newsboy quantities, $NS_{\frac{x}{N}}$. If that is the case, the reservation payment prevents over-ordering completely.

COROLLARY 1. *Suppose that* $v \le NS_{\frac{x}{N}}$. *Then we have* $\lim_{r\to 0} y^R(r) = \infty$

Corollary 1 shows that the supplier needs to implement the capacity reservation mechanism when the real-time capacity information is shared. Otherwise, there is no NE when the capacity shortage incurs unless there is a limit on the order quantity.

5.2. BWE & RBWE Analysis Under Model-R

In this section, we characterize the conditions under which the BWE and the RBWE exist by studying the partial derivatives of $y^R(x,v)$. In addition, we analyze the impact of the reservation payment r on the conditional BWE and RBWE.

5.2.1. BWE Analysis Under Model-R

Let $\Omega_{v^0} = \{(x,v) \in \Omega | v < v^0(x)\}$; Ω_{v^0} is the set of all capacities and demand shifts for which the capacity is strictly less than the total order quantity (by Theorem 3). We focus our analysis on Ω_{v^0} since if $(x,v) \notin \Omega_{v^0}$, then $y^R(x,v) = v$ for $v \le NS_{\frac{x}{N}}$ and $y^R(x,v) = NS_{\frac{x}{N}}$ for for $v > NS_{\frac{x}{N}}$. Both cases are trivial.

By (1) and (9), the equilibrium total order quantity satisfies

$$y^R(x,v) = \frac{v}{r}\left(1-\frac{1}{N}\right)\left[p+r-(p+h)G\left(\frac{v}{N}-\frac{x}{N}\right)\right]$$

Taking a partial derivative with respect to x, we get

$$\frac{\partial}{\partial x} y^R(x,v) = \frac{v}{r}\frac{1}{N}\left(1-\frac{1}{N}\right)(p+h)g\left(\frac{v}{N}-\frac{x}{N}\right) > 0. \tag{10}$$

(10) indicates that the retailers' order quantity increases in the demand shift x. But this does not guarantee that the change in the retailers' order quantity is always greater than that in the

demand shift. To determine when the BWE exists, by Lemma 1, we need to explore the region in which $\frac{\partial}{\partial x} y^R(x,v) > 1$, i.e., when

$$vg\left(\frac{v}{N} - \frac{x}{N}\right) > \frac{N^2}{N-1}\frac{r}{p+h}. \tag{11}$$

In the hope of extracting more insights, we restrict our attention to demand distributions (D_i) that are unimodal. This is, in fact, not an overly restrictive assumption because a great many distribution families, including normal, uniform, and gamma, fall into this category. This assumption implies that $vg(\frac{v}{N} - \frac{x}{N})$ is unimodal in x. Let $g(d^*)$ be the maximal value of $g(\cdot)$. If $vg(d^*) > \frac{N^2}{N-1}\frac{r}{p+h}$, then there exist $\gamma^0(v)$ and $\gamma^1(v)$ such that $\gamma^0(v) < \gamma^1(v)$ and

$$vg\left(\frac{v}{N} - \frac{\gamma^0(v)}{N}\right) = vg\left(\frac{v}{N} - \frac{\gamma^1(v)}{N}\right) = \frac{N^2}{N-1}\frac{r}{p+h}$$

and (11) holds when $\gamma^0(v) < x < \gamma^1(v)$.

Let $\Omega_{\gamma(v)} = \{x|(x,v) \in \Omega \text{ and } \gamma^0(v) < x < \gamma^1(v)\}$. $\Omega_{\gamma(v)}$ is the set of all demand shifts for fixed v for which (11) holds, and its closure. If $\gamma^0(v)$ and $\gamma^1(v)$ do not exist, then $\Omega_{\gamma(v)} = \emptyset$.

The following theorem specifies a subregion of $\Omega_{\gamma(v)}$ in which the conditional BWE occurs and says that it never occurs if $\Omega_{\gamma(v)}$ is empty.

THEOREM 4. *Under model-R, when v is fixed,*

1. *If $\Omega_{\gamma(v)}$ is empty, then $\frac{\partial}{\partial x} y^R(x,v) \le 1$. Thus $Var(X) > Var(y^R(X,v))$. That is, there is no conditional BWE (conditioned on $V = v$) between the retailers and the customers.*

2. *If $\Omega_{\gamma(v)}$ is non-empty, let $A = \Omega_{\gamma(v)} \cap \Omega_{v^0}$. If $(x,v) \in A$, then $\frac{\partial}{\partial x} y^R(x,v) > 1$. Thus $Var(X|A) < Var(y^R(X,V)|A)$. That is, the conditional BWE (conditioned on $V = v$) occurs between the retailers and the customers.*

Next we examine the impact of magnitude of the reservation payment on the BWE. The following proposition states that the BWE becomes more severe between the retailers and the customers as r decreases. First note that $\Omega_{\gamma(v)}$ and Ω_{v^0} both depend on r since $v^0(x)$, $\gamma^0(v)$, and $\gamma^1(v)$ do. Let $A(r_1) = \Omega_{\gamma(v)} \cap \Omega_{v^0}$ when $r = r_1$.

PROPOSITION 1. *If $A(r_1) \neq \emptyset$, then for all $r_2 < r_1$, we have*

$$Var(y^R(X,V)|A(r_1), r = r_1) < Var(y^R(X,V)|A(r_1), r = r_2).$$

5.2.2. RBWE Analysis Under Model-R

Now we consider the impact of the variability of V on $y^R(x,V)$. We have

$$\frac{\partial}{\partial v} y^R(x,v) = -\frac{1}{r}\left(1-\frac{1}{N}\right)\left[-(p+r)+(p+h)\left[G\left(\frac{v}{N}-\frac{x}{N}\right)+\frac{v}{N}g\left(\frac{v}{N}-\frac{x}{N}\right)\right]\right]. \tag{12}$$

The partial derivative $\frac{\partial}{\partial v} y^R(x,v)$ can be either positive or negative. If it is positive, then the increased capacity induces the retailers to increase their order size. It can happen if the risk of paying too much in the reservation payments of undelivered units is outweighed by the benefit attained by ordering more when the capacity is tighter. On the other hand, when $\frac{\partial}{\partial v} y^R(x,v)$ is negative, an increase in capacity causes a decrease in the retailers' order quantity, i.e., the gaming behavior among the retailers is mitigated. Therefore, the sign of $\frac{\partial}{\partial v} y^R(x,v)$ is determined by the tradeoff between the penalty for over-ordering (and incurring the extra reservation payment) and the shortage risk from under-ordering (and receiving too few units because of rationing). However, we will show next that it is the magnitude of $\frac{\partial}{\partial v} y^R(x,v)$, and not its sign, that determines whether the RBWE occurs.

For any fixed x, there is a (possibly empty) collection of non-overlapping intervals in the domain of V such that, on a given interval, $y^R(x,v)$ is monotone in v and $|\frac{\partial}{\partial v} y^R(x,v)| > 1$. Let $I_k(x)$ be the kth such interval.

THEOREM 5. *Let x be fixed and let $A_k = I_k(x) \cap \Omega_{v^0}$. Then $Var(V|A_k) < Var(y^R(X,V)|A_k)$. That is, the conditional RBWE (conditioned on $X = x$) occurs between the supplier and the retailers on the interval A_k, for each k.*

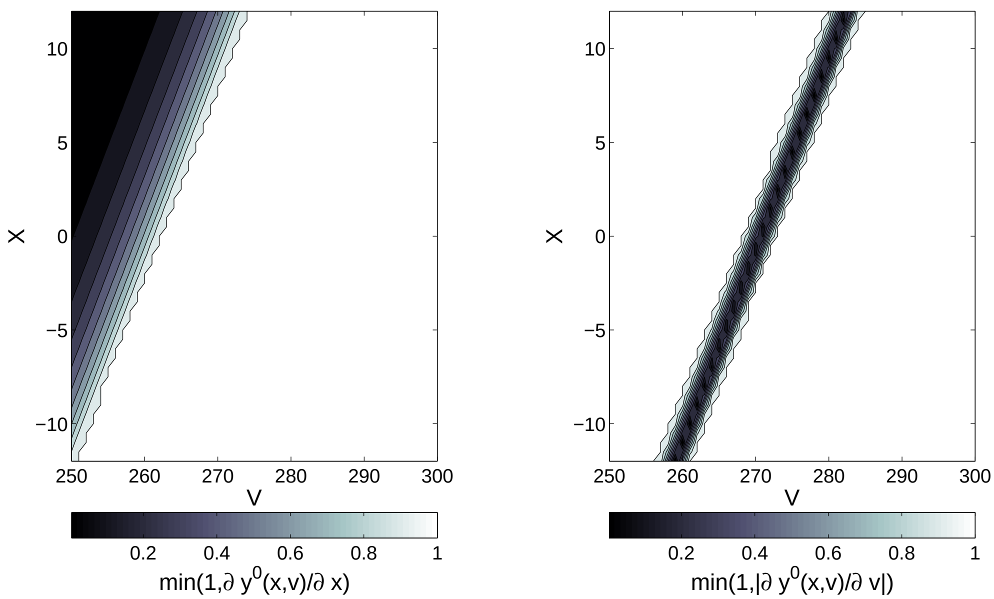
Similar to Proposition 1, we let $B_k(r_1) = I_k(x) \cap \Omega_{v^0}$ when $r = r_1$.

PROPOSITION 2. *If $B_k(r_1) \neq \emptyset$, then for all $r_2 < r_1$, we have*

$$Var(y^R(X,V)|B_k(r_1), r = r_1) < Var(y^R(X,V)|B_k(r_1), r = r_2).$$

Proposition 2 states that a smaller r triggers a larger RBWE. This implies that the supplier's preferred choice of reservation payment aggravates the RBWE, just as it does the BWE.

We provide numerical examples to demonstrate the occurrence of the BWE and RBWE in Figure 2. We set $r=2$. We use $N=3$, $D_i \sim N(100, 5^2)$, $p=10$ and $h=1$. In addition, V varies from 250 to 300 and X varies from -12 to 12.



(a: BWE) (b: RBWE)

Figure 2 **When the BWE and RBWE Occur Under Model-R**

In Figure 2.a, in the white area, the conditional BWE occurs ($\frac{\partial}{\partial x} y^R(x,v) > 1$), while in the black and gray areas, it does not ($\frac{\partial}{\partial x} y^R(x,v) < 1$). If the mismatch between demand and supply is exceptionally severe, the BWE does not occur since the reservation payment tends to reduce the benefit of over-ordering. Therefore, the retailers' gaming behavior is mitigated.

In Figure 2.b, in the white areas, the conditional RBWE occurs ($|\frac{\partial}{\partial v} y^R(x,v)| > 1$), while in black and gray areas, it does not ($|\frac{\partial}{\partial v} y^R(x,v)| < 1$). The two separate white areas represent the two possible signs of $\frac{\partial}{\partial v} y^R(x,v)$, as discussed at the start of Section 5.2.2. The retailers needs to balance between the reservation payment and the competition among the retailers. When the capacity is exceptionally severe, the retailers are more concern about the reservation payment. Therefore, the *decreasing* capacity leads to the lower retailers' order quantity. On the other hand, if the capacity is not tight enough, the retailers' are more concern about the competition among the retailers. Therefore, the *increasing* capacity relieves the competition which leads to the lower retailers' order quantity. When the shortage of capacity is moderate, the two driving forces cancel each other. Therefore, the change of capacity does not trigger even larger change in the retailers' order quantity.

6. Impact of Capacity Information Sharing and Capacity Reservation

In this section, we analyze how capacity information sharing and capacity reservation affect the profits of the retailers, the supplier and the whole supply chain. In addition, we closely examine the relationship between the retailers' order variance and their profit or the supplier's profit.

Under model-L, the profit of retailer i under NE is given by

$$\pi_i^L = E\left[\pi_i^L(z_i^L(X)|\mathbf{z}_{-i}^L(X))\right] \tag{13}$$

The supplier gains $\zeta - \vartheta$ for each delivered product. Therefore, the supplier's profit under NE follows

$$\pi_s^L = (\zeta - \vartheta)E\left[\min(V, y^L(X))\right] \tag{14}$$

And the profit of the whole supply chain under NE follows

$$\pi^L = \pi_s^L + \sum_i^N \pi_i^L \tag{15}$$

Similarly, under model-R, the profit of retailer i, the supplier and the whole supply chain under NE is respectively given by

$$\pi_i^R = E\left[\pi_i^R(z_i^R(X,V)|\mathbf{z}_{-i}^R(X,V))\right] \tag{16}$$

$$\pi_s^R = (\vartheta - \zeta)E[\min(V, y^R(X,V))] + rE[(y^R(X,V) - V)^+] \tag{17}$$

$$\pi^R = \pi_s^R + \sum_{i=1}^N \pi_i^R \tag{18}$$

In (17), the supplier's profit function under NE contains both the gain of each delivered product and the reservation payment it receives.

Note that all the profit components can be affected by parameters such as the unit product cost, the wholesale price and the unit reservation payment. For the sake of simplicity, we include arguments for these profit components only as necessary.

Then the following proposition categorizes the impact of the magnitude of the reservation payment.

Proposition 3. *For $r_1 < r_2$, we have*

1. $\pi^R(r_1) = \pi^R(r_2)$
2. $\pi_s^R(r_1) \geq \pi_s^R(r_2)$.
3. $\pi_i^R(r_1) \leq \pi_i^R(r_2)$.

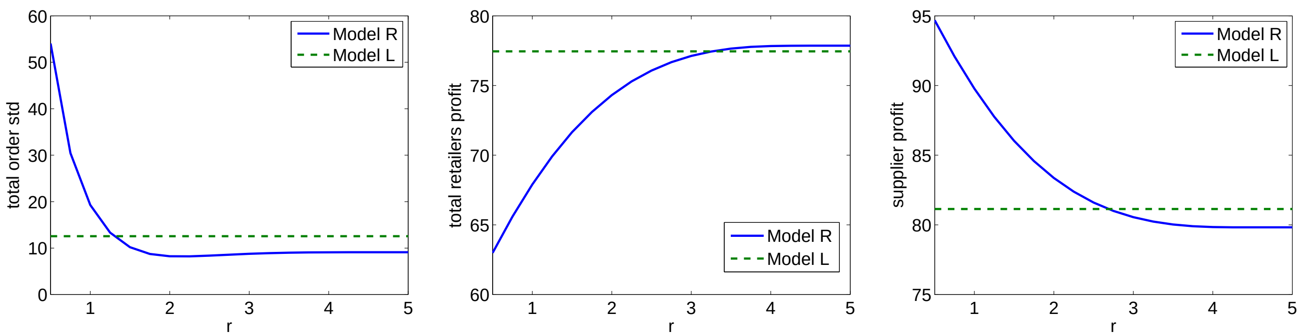
Proposition 3.1 reveals that the whole supply chain profit does not change as the value of r does. But how the supplier and retailers split the profit differs at varied r since the total reservation payment, together with the purchase payment, constituting the transfer payment from the retailers to the supplier, depends on the magnitude of r and the degree of over-ordering, which is also a function of r. Moreover, Propositions 3.2 and 3.3 indicate that the decrease of r is overwhelmed by the increase cost of over-ordering.

Next, we provide the following proposition to examine the benefit of capacity information sharing and capacity reservation.

Proposition 4. *There exists a critical value ϑ^0. When $\vartheta < \vartheta^0$, $\pi^L(\vartheta) \geq \pi^R(\vartheta)$. Otherwise, $\pi^L(\vartheta) \leq \pi^R(\vartheta)$*

Proposition 4 shows that capacity information sharing and capacity reservation do not necessarily lead to a higher supply chain profit. In fact, it can reduce the total profit when the marginal profit is sufficiently high for the supplier (i.e. when the production cost is low enough). In such a situation, the over-ordering behavior among the retailers under model-L can actually lessen the double marginalization since the system-wide optimal order quantity is much larger than the newsvendor order quantity.

Now we investigate the relationship between the retailers' order variability and their profit or the supplier's profit under both model-L and model-R through a simulation test. We assume $\zeta = 5$, $\vartheta = 4.6$, $\varrho = 10$ and $\kappa = 4$. We also set $N = 5$, $D_i \sim U[10, 50]$, $V \sim U[180, 250]$ and $X \sim U[-25, 0]$. In Figure 3, we plot the retailers' order standard deviation, the retailers' profit, and the supplier's profit under both models.



(a: Retailers' Order Std) (b: Retailers' Profit) (c: Supplier's Profit)

Figure 3 Impacts of Capacity Information Sharing and Reservation Payment

Figure 3.a shows that the retailers' order variability increases significantly under model-R compared to that under model-L when r is small enough. However, the result is reversed when r is large. The reason is that in model-R, the retailers adjust their order decision dynamically based on real-time capacity information and demand shift information, and the magnitude of r determines how they react to these information. When r is small, the retailers overreact to the supply shortage, while they just chase the capacity when r is large. As a result, the order variability decreases when r increases under model-R.

One may expect that capacity information sharing will improve the retailers' profit. However, this is not always the case, as shown in Figure 3.b. Excluding the reservation payment, the retailers always obtain no-less-than-newsboy-type profit in model-R since their allocated products never exceed $S_{\frac{x}{N}}$. However, when r is small, the reservation payment that is introduced to compensate the supplier for sharing its capacity information outweighs the newsvendor-type profit gain for the retailers under model-R. In contrast, as shown in Figure 3.c, when r is small, the supplier can collect more money from the reservation payment under model-R than the additional sales profit it earns from the retailers' over-ordering under model-L.

Lastly, we examine the three plots collectively and make a few more observations regarding the relationship between the retailer's order variability and their profit. If we confine our attention only to model-R, then the higher the retailers' order variability, the lower profit they gain. But if we allow the system to shift between model-L and model-R, such relationship may break down. For example, when the system shifts from model-L to model-R for $r \in [2, 3]$, then both the retailers' order variability and profit drop at the same time. In sum, the changing pattern of the order variability can be a good indicator of that of the profit only for the retailers who react to both uncertainties in one particular model. If there is a significant change in the supply chain settings, then the relationship between the retailers' order variability and their profit becomes loose.

7. Conclusions

In this paper, we find that the retailers' order size changes in response to both demand and supply uncertainty. Depending on the settings, the likelihood and severity of the occurrence of the BWE and RBWE vary.

We also document the inability of capacity information sharing to reduce the [R]BWE. Moreover, capacity information sharing may also hurts the retailers as well as the whole supply chain. However, the supplier is willing to disclose its capacity information in order to collect an additional reservation payment from the retailers for sufficiently small unit reservation payment.

When both the BWE and RBWE happen, the whole supply chain exhibits an "umbrella pattern." But this does not necessarily occur in every supply chain. It does in ours because the supply and demand variances are exogenous, whereas the retailers' decisions are endogenous. In other words, the retailers react to both the supply and demand processes, but the upstream echelon does not react to the demand uncertainty created by the retailers, nor does the downstream end react to the supply uncertainty created by the retailers. If, instead, we assumed that the customers, too, game the system as a reaction to supply shortages, then the RBWE would occur between the retailers and the customers. Similarly, if the supplier could control its capacity in response to the retailers' order variability, then the BWE would occur between the supplier and the retailers.

Put another way, in a system with random capacity and demand shifts, the BWE originates from downstream and the RBWE originates from upstream, and both effects propagate through the supply chain until they reach a stage that does not react to the uncertainty that created it.

Technical Appendix

In this appendix, we drop the subscript $\frac{x}{N}$ and superscript L and R to simplify the notation whenever it causes no confusion.

Lemma 1

Part 1:

WLOG, we prove this part for $g'(u) > 1$ for all $u \in \Omega_U$.

First, we show that it holds for the discrete distribution with finite mass points. Then we use the Riemann sum to extend this result to the continuous distribution.

Let u_k, $k = 1, ...K$ be the mass points in Ω_U. Let $h(u_k)$ be the pmf at u_k. Then

$$
\begin{aligned}
var(g(U)) &= E[g^2(U)] - E[g(U)]^2 \\
&= \sum_{k=1}^{K} g^2(u_k)h(u_k) - \left[\sum_{k=1}^{K} g(u_k)h(u_k)\right]^2 \\
&= g^2(u_1)h(u_1) + g^2(u_2)h(u_2)+, ..., +g^2(u_K)h(u_K) \\
&\quad - [g(u_1)h(u_1) + g(u_2)h(u_2)+, ..., +g(u_K)h(u_K)]^2 \\
&= g^2(u_1)h(u_1)(1 - h(u_1))+, ..., g^2(u_K)h(u_K)(1 - h(u_K)) \\
&\quad -2\sum_{k<j} g(u_k)g(u_j)h(u_k)h(u_j) \\
&= \sum_{k \neq j} g^2(u_k)h(u_k)h(u_j) - 2\sum_{k<j} g(u_k)g(u_j)h(u_k)h(u_j)
\end{aligned}
$$

$$
\begin{aligned}
&= \sum_{k<j} (g(u_k) - g(u_j))^2 h(u_k) h(u_j) \\
&> \sum_{k<j} (u_k - u_j)^2 h(u_k) h(u_j) \\
&= var(U)
\end{aligned}
$$

The inequality is due to $f'(u) > 1$. Thus $|g(u_k) - g(u_j)| > |u_k - u_j|$ for $u_k \neq u_j$.

Then let U be a continuous variable and $H(u)$ be its cdf. We define a sequence of new random variables U_n which have the mass points $u_n(i) := a + \frac{b-a}{n} i$, for $i = 1, \ldots n$, with pmf $h_n(u_n(i)) = H\left(a + \frac{b-a}{n} i\right) - H\left(a + \frac{b-a}{n}(i-1)\right)$. Since $E[\cdot]$ is continuous and g is bounded, by the Riemann sum, we have

$$
\lim_{n \to \infty} var(U_n) = var(U)
$$

$$
\lim_{n \to \infty} var(g(U_n)) = var(g(U))
$$

Since we have $var(U_n) < var(g(U_n))$, we only need to show that $var(U)$ is strictly less than $var(g(U))$.

Because $\lim_{n \to \infty} var(U_n) = var(U)$, $\exists N$ s.t. $\forall n > N$, $var(U_n) > \frac{var(U)}{2} > 0$. Moreover, $g'(u) > 1$ implies that $\forall \varepsilon$, $\exists e > 1$ such that the set $\{u \in [a,b] : g'(u) < e\}$ has Lebesgue measure less than ε. Then $\forall u, v \in [a,b]$, $|g(u) - g(v)| > e|u - v| - (e-1)\varepsilon$, which helps to establish the following inequality.

$$
\begin{aligned}
& var(g(U_n)) \\
&= \sum_{k<j} (g(u_n(k)) - g(u_n(j)))^2 h_n(u_n(k)) h_n(u_n(j)) \\
&> e^2 \sum_{k<j} (u_n(k) - u_n(j))^2 h_n(u_n(k)) h_n(u_n(j)) \\
&\quad -2e(e-1)\varepsilon \sum_{k<j} |u_n(k) - u_n(j)| h_n(u_n(k)) h_n(u_n(j)) \\
&> e^2 var(U_n) - 2e(e-1)(b-a)\varepsilon
\end{aligned}
$$

The last inequality holds since $|u_n(k) - u_n(j)| < b - a$.

Then, we have

$$
var(g(U_n)) - var(U_n)
$$

$$> (e^2-1)var(U_n) - 2e(e-1)(b-a)\varepsilon$$

$$> (e^2-1)(\frac{var(U)}{2} - 2(b-a)\varepsilon)$$

By choosing $\varepsilon = \frac{var(U)}{8(b-a)}$, we get

$$var(g(U_n)) - var(U_n) > \frac{e^2-1}{4} var(U) > 0.$$

This implies that $var(g(U_n)) - var(U_n)$ is bounded below by a strictly positive number, if $n \geq N$. Then we have $var(U) < var(g(U))$.

Part 2:

Let $y = g(u)$. We want to show that $[f(g^{-1}(y))]' > 1$. Applying the result of part 1, we get $var([f(g^{-1}(Y))]) > var(Y)$, which implies $var[f(U)] > var[g(U)]$. Since

$$[f(g^{-1}(y))]' = \frac{f'(g^{-1}(y))}{g'(g^{-1}(y))} > 1,$$

the proof is complete. □

Theorem 2

It follows directly from Lemma 1. □

Lemma 2

We use contradiction to prove this lemma. Suppose that $\sum_i z_i^R = \tau < v$. Then let $j = \arg\min_i z_i^R$. Observe that $z_j^R < \frac{v}{N}$. If retailer j orders $\widehat{z}_j = \min(z_j^R + v - \tau, \frac{v}{N})$, its allocation is equal to $\widehat{z}_j$ since $\tau - z_j^R + \widehat{z}_j \leq v$. Observe that $z_j^R < \widehat{z}_j < S$. Therefore, retailer j's cost by ordering $\widehat{z}_j$ is strictly lower than that by ordering z_j^R because the newsboy cost function is strictly convex. This contradicts the fact that $\{z_i^R\}_{i=1}^N$ is a NE. Therefore, $\sum_i z_i^R \geq v$. □

Theorem 3

The NE of maximizing the newsvendor profit is equivalent to the NE of minimizing the newsvendor cost. Based on (7), the newsvendor cost function follows

$$C_i^R(z_i|\mathbf{z}_{-i}) = p\int_{\frac{v}{y}z_i}^{\infty} \left(d - \frac{v}{y}z_i\right) dG_{\frac{x}{N}}(d) + h\int_0^{\frac{v}{y}z_i} \left(\frac{v}{y}z_i - d\right) dG_{\frac{x}{N}}(d) + r\left(z_i - \frac{v}{y}z_i\right). \tag{19}$$

Before we prove Theorem 3, we need the following lemma.

Lemma 3. *Suppose that $v \le NS_{\frac{x}{N}}$. Then*

1. *When $\sum_i z_i \ge v$,*

$$\frac{\partial}{\partial z_i} C_i^R(z_i|\boldsymbol{z}_{-i}) = v\left(\frac{1}{y} - \frac{z_i}{y^2}\right)\left[-(p+r)+(p+h)G_{\frac{x}{N}}\left(\frac{v}{y}z_i\right)\right] + r; \tag{20}$$

2. *$C_i^R(z_i|\boldsymbol{z}_{-i})$ is strictly pseudoconvex;*

3. *there is a unique $z_i^*(\boldsymbol{z}_{-i})$ such that either (a) $\frac{\partial}{\partial z_i}C_i^R(z_i^*, \boldsymbol{z}_{-i}) = 0$ or (b) $z_i^* = 0$ and $\frac{\partial}{\partial z_i}C_i^R(z_i^*, \boldsymbol{z}_{-i}) > 0$.*

Proof of Lemma 3

It is straightforward to obtain Part 1 by taking the derivative of (19).

Part 2:

It is easy to see that $C_i^R(z_i|\mathbf{z}_{-i})$ is strictly pseudoconvex if $\frac{\partial}{\partial z_i}C_i^R(z_i|\mathbf{z}_{-i})$ is monotone and has no saddle points.

Then suppose that there exists $\widehat{z}_i$ satisfying $\frac{\partial}{\partial z_i}C_i^R(z_i|\mathbf{z}_{-i})|_{z_i=\widehat{z}_i} = 0$. In order to show that $C_i^R(z_i|\mathbf{z}_{-i})$ is strictly pseudoconvex, we need to show 1) if $z_i > \widehat{z}_i$, $\frac{\partial}{\partial z_i}C_i^R(z_i|\mathbf{z}_{-i}) > 0$, 2) if $z_i < \widehat{z}_i$, $\frac{\partial}{\partial z_i}C_i^R(z_i|\mathbf{z}_{-i}) < 0$. This implies that $\widehat{z}_i$ is unique.

Let $\widehat{y} = \widehat{z}_i + \sum_{-i} z_{-i}$. Observe that $\frac{1}{y} - \frac{z_i}{y^2} \ge 0$ since $z_i \le y$. Then $\frac{\partial}{\partial z_i}C_i(z_i|\mathbf{z}_{-i})|_{z_i=\widehat{z}_i} = 0$ implies $-(p+r)+(p+h)G(\frac{v}{\widehat{y}}\widehat{z}_i) < 0$. As $G(d)$ is strictly increasing, when $z_i > \widehat{z}_i$,

$$-(p+r)+(p+h)G\left(\frac{v}{y}z_i\right) > -(p+r)+(p+h)G\left(\frac{v}{\widehat{y}}\widehat{z}_i\right)$$

There are two cases. First, $-(p+r)+(p+h)G\left(\frac{v}{y}z_i\right) \ge 0$; then $\frac{\partial}{\partial z_i}C_i(z_i|z_{-i}) > 0$. Second,

$$0 > -(p+r)+(p+h)G\left(\frac{v}{y}z_i\right) > -(p+r)+(p+h)G\left(\frac{v}{\widehat{y}}\widehat{z}_i\right).$$

Since $\frac{1}{y} - \frac{z_i}{y^2}$ is a decreasing function in z_i, $0 < \frac{1}{y} - \frac{z_i}{y^2} < \frac{1}{\widehat{y}} - \frac{\widehat{z}_i}{\widehat{y}^2}$. Thus we have

$$\begin{aligned}
& v\left(\frac{1}{y} - \frac{z_i}{y^2}\right)\left[-(p+r)+(p+h)G\left(\frac{v}{y}z_i\right)\right] + r \\
> \ & v\left(\frac{1}{\widehat{y}} - \frac{\widehat{z}_i}{\widehat{y}^2}\right)\left[-(p+r)+(p+h)G\left(\frac{v}{\widehat{y}}\widehat{z}_i\right)\right] + r \\
= \ & 0
\end{aligned}$$

Similarly, we can prove that if $z_i < \widehat{z}_i$, $\frac{\partial}{\partial z_i} C_i^R(z_i|\mathbf{z}_{-i}) < 0$. Therefore, we have part 2.

Part 3:

Set z_i high enough so that $\frac{v}{y} z_i > S$. By (20),

$$\frac{\partial}{\partial z_i} C_i^R(z_i|z_{-i}) > -v\left(\frac{1}{y} - \frac{z_i}{y^2}\right) r + r > 0.$$

The first inequality is due to the fact that $G(\frac{v}{y} z_i) > \frac{p}{p+h}$ for $\frac{v}{y} z_i > S$. The last inequality is due to the fact that $v \leq y$. Then we can get part 3 directly from part 2 since eventually $\frac{\partial}{\partial z_i} C_i(z_i|z_{-i})$ will have positive values. $\square$

Proof of Theorem 3

If $v < v^0(x)$, then $\frac{\partial}{\partial z_i} C_i(z_i, z_{-i})|_{z_i = z^0, z_{-i} = z^0} = 0$ for $z^0 = \frac{v}{r}\left(1 - \frac{1}{N}\right)\left[p + r - (p+h)G\left(\frac{x}{N}\right)\right]$. Therefore, $z_i^R = z^0$ is a NE due to the pseudoconvexity of $C_i(z_i|z_{-i})$.

If $v \geq v^0(x)$, then $\frac{\partial}{\partial z_i} C_i(z_i, z_{-i})|_{z_i = \frac{v}{N}, z_{-i} = \frac{v}{N}} \geq 0$. $z^R = \frac{x}{N}$ is a NE since no retailer will order higher than $\frac{v}{N}$ and $y^R \geq v$ by Lemma 2.

The uniqueness of y^R relies on the condition that $\frac{\partial}{\partial z_i} C_i(z_i, z_{-i})|_{z_i = z_{-i}}$ is strictly increasing in y. $\square$

Corollary 1

It directly comes from Theorem 3. $\square$

Theorem 4

It directly comes from Lemma 1 and the definition of $\gamma^0(v)$ and $\gamma^1(v)$. $\square$

Proposition 1

When $r_2 < r_1$, we have $\frac{v}{r_2}\frac{1}{N}(1-\frac{1}{N})(p+h)g(\frac{v}{N} - \frac{x}{N}) > \frac{v}{r_1}\frac{1}{N}(1-\frac{1}{N})(p+h)g(\frac{v}{N} - \frac{x}{N}) > 0$. Applying part 2 of Lemma 1, we get the result directly. $\square$

Theorem 5

It directly comes from Lemma 1 and definition of $I_k(x)$. $\square$

Proposition 2

If for any $(x, v) \in B_k(r_1)$,

$$1 < -\frac{1}{r_1}\left(1 - \frac{1}{N}\right)\left[-(p+r_1) + (p+h)\left[G\left(\frac{v^1}{N} - \frac{x}{N}\right) + \frac{v^1}{N} g\left(\frac{v^1}{N} - \frac{x}{N}\right)\right]\right]$$

$$= (1-\frac{1}{N}) - \frac{1}{r_1}(1-\frac{1}{N})\left[-p+(p+h)\left[G\left(\frac{v^1}{N}-\frac{x}{N}\right)+\frac{v^1}{N}g\left(\frac{v^1}{N}-\frac{x}{N}\right)\right]\right],$$

then $\left[-p+(p+h)\left[G\left(\frac{v^1}{N}-\frac{x}{N}\right)+\frac{v^1}{N}g\left(\frac{v^1}{N}-\frac{x}{N}\right)\right]\right]<0$. Then

$$\begin{aligned}
&(1-\frac{1}{N}) - \frac{1}{r_2}(1-\frac{1}{N})\left[-r_2+(p+h)\left[G\left(\frac{v^1}{N}-\frac{x}{N}\right)+\frac{v^1}{N}g\left(\frac{v^1}{N}-\frac{x}{N}\right)\right]\right] \\
&> (1-\frac{1}{N}) - \frac{1}{r_1}(1-\frac{1}{N})\left[-r_1+(p+h)\left[G\left(\frac{v^1}{N}-\frac{x}{N}\right)+\frac{v^1}{N}g\left(\frac{v^1}{N}-\frac{x}{N}\right)\right]\right] \\
&> 0
\end{aligned}$$

Applying part 2 of Lemma 1, we get the result. It is similar to the case where

$$-\frac{1}{r_1}(1-\frac{1}{N})\left[-(p+r_1)+(p+h)\left[G\left(\frac{v^1}{N}-\frac{x}{N}\right)+\frac{v^1}{N}g\left(\frac{v^1}{N}-\frac{x}{N}\right)\right]\right]<-1$$

□

Proposition 3

Part 1:

When $v \ge S_{\frac{x}{N}}$, $y^R(x,v) = S_{\frac{x}{N}}$. When $v \le S_{\frac{x}{N}}$, $y^R(x,v) \ge v$. Since $E[\min(V, y^R(X,V))] = E[S_{\frac{x}{N}}|V \ge S_{\frac{x}{N}}]\Pr(V \ge S_{\frac{x}{N}}) + E[V|V \le S_{\frac{x}{N}}]\Pr(V \le S_{\frac{x}{N}})$, the first part of (17) is independent on the choice of r. Moreover, the allocation to the retailers under NE are independent on the reservation payment and the reservation payment is the internal transfer payment between the retailers and supplier. Thus, we have Part 1.

Part 2:

Since the first part in (17) is independent on the choice of r, we only need to show that $r_1(y^R(x,v|r_1)-v)^+ \ge r_2(y^R(x,v|r_2)-v)^+$. For $v \ge S_{\frac{x}{N}}$, $(y^R(x,v|r)-v)^+ = 0$ for all r. Therefore, we only need to consider $v \le S_{\frac{x}{N}}$. Since $r(y^R(r)-v)^+ = \left[-\frac{v}{N}r + v\left(1-\frac{1}{N}\right)\left[p-(p+h)G_{\frac{x}{N}}\left(\frac{v}{N}\right)\right]\right]^+$ is a decreasing function in r, we have shown Proposition 3.

Part 3:

Part 1 and 2 result in Part 3 directly. □

Proposition 4

Define a newsvendor cost function

$$c(O) = p\int_O^\infty (d-O)\, dG_{\frac{x}{N}}(d) + h\int_0^O (O-d)\, dG_{\frac{x}{N}}(d).$$

Under model-R, the retailer's allocation is $\min(V/N, S_{\frac{x}{N}})$ based on Theorem 3. On the other hand, the retailers' allocation is $\min(V/N, y^L)$. Therefore we have,

$$\begin{aligned}\pi^L - \pi^R = \;& (\zeta-\vartheta)\left(E[\min(V, y^L(X)) - E[\min(V, NS+X)]\right) \\ & -NE\left[(c(\min(V/N, y^L(X))) - \min(V/N, S+X/N)))\right].\end{aligned}$$

Since $y^n < y^L$ by Theorem 1, then $E[\min(V, y^L) - E[\min(V, y^n)] > 0$.

Base on the fact that $S_{\frac{x}{N}}$ is the minimizer of the allocation to the newsvendor cost function, that the newsvendor cost function is a convex function, and that $F(\cdot)$ and $G_{x/N}(\cdot)$ are strictly increasing in their domain, we have $E\left[(c(\min(V/N, y^L(X))) - \min(V/N, S+X/N)))\right] > 0$. Moreover, Theorem 1 leads to $z^L(x) > S + x/N$. Thus, we have Proposition 4. $\square$

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