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Market Entry Analysis when Releasing Distinctive Products in Independent Markets

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1 Introduction

The purpose of this research is to establish theoretical foundations and provide managerial insight for the market entry timing problem of contract manufacturers, who have incentives to develop their own market presence. A contract manufacturer (CM) often faces the strategic options of devoting its manufacturing capacity entirely to a brand-carrying customer (primary market), or leveraging a portion of its manufacturing capabilities to develop its own market presence (secondary market). In our research, we aim to analyze these conflicting incentives of a CM in a wide variety of settings, and establish theoretical conclusions on the existing empirical market entry timing research.

As a result of major global corporations' shift from vertical integration to outsourcing formerly concentrated production and manufacturing facilities to cheaper locations, contract manufacturing has become popular in high tech industries. Contract manufacturing in electronic industry has grown from a few billion dollar industry in the early 1990's to over $300 billion in 2008. As contract manufacturing became popular in high-tech industries and low cost manufacturing became widely available, the competition among the contract manufacturers has become more intense. In order to obtain the branded customers' businesses, the contract manufacturers started to offer more *value added* services, which in turn led to the emergence of contract manufacturers with design capabilities. (Original Design Manufacturing ODM). ODM firms provide *ready to go* products for their brand carrying customers. For example, many Japanese firms, like Sharp, Hitachi, Canon and NEC, and U.S. firms Xerox, Compaq and Apple use ODM firms for their design and production activities. (WTEC, 1997) [3].

The most advanced ODM producers get early signals about market shifts and technology development, and instantly integrate themselves into advanced products. By the time brand carriers realize the need for new product ideas, ODM firms already have products ready to market. ODM's only need to add their customer's brand name and then proceed to manufacture in volume. For example a Taiwan based ODM producer, Inventec, makes Apple's Newton PDA's and provides Compaq with its high end notebook computers, both as ready to go products.

An ODM producer possesses almost every key element to design, develop and manufacture an advanced product. However, the ability to access correct sales and marketing channels is the key in both high volume sales and high profit margins. While ODM firms are the ones that really design and produce the product, they lack marketing and sales capabilities.

An ODM firm has clear motivations to develop its own brand. Branded products are more profitable. On average gross margin for non-branded products is 19%, while the margin for branded products changes between 40%-100%. (Asiapreneur 2004) [20]. According to BusinessWeek (2003), top 100 contract manufacturers in Asia-Pacific region totaled $85 billion of sales with $4 billion of profits, while world's top 100 consumer goods and retail companies, which rely on oversees production, reported sales of $3,578 billion and profits of $228 billion. [18].

In a 2003 survey by Hong Kong Trade Development Council (HKTDC) more Hong Kong companies have started to develop their own brands, and by releasing their own branded products to versatile markets they diversify their product lines. They consider the Chinese mainland and other emerging markets as a good testing for developing their own brands [4].

In our research, we will analyze incentives of a contract manufacturer (CM), and entry timing decisions considering demand dynamics of technological products. The life span of a high-tech product is mostly determined with the pace of technological innovation. As the extent of technological innovation accelerates, more products are driven to market in shorter periods of time. Due to this fast paced environment, products become obsolete even before the technology used to develop these products mature. The demand dynamics play important role in the analysis of CM's strategic market entry decision. We establish theoretical solutions for the optimal entry policy and generate insights for a wide range of demand-capacity setting. Most of literature on entry timing focus on the problems from the point of view of the brand carrying corporations. In contrast, our research will consider the issues faced by the contract manufacturers. This research is expected to have long-term impact to industries, where contract manufacturing is prevalent, including electronics and computers, semiconductors, communications, automotive, and medical products.

The remainder of the chapter is organized as follows. We summarize the literature as it relates to demand dynamics and market entry timing in high-tech industries. Then, in Section 4, we model and solve CM's entry problem for substitute products. In Section 5, we extend these results for differentiated products and in Section 6, we address the CM's entry problem from game theoretical perspective. Within these sections we provide insights to the problem and demonstrate our results with numerical examples. The chapter finishes with conclusions.

2 Literature Review

There is a rich literature on modeling and describing demand for high-tech products. Earliest *first purchase* models of innovative process were introduced by Bass (1969) [1], Mansfield (1961) [12] and Fourt and Woodlock (1960) [5]. In addition to these, several distributions such as logistic, Weibull, and negative exponential are used to model the first purchase demand of a new product/technology. The most widely used model among the life cycle models is the famous Bass model. Bass (1969) [1] describes the demand for a new product by the theory of adoption and diffusion, and the model is very powerful in estimating the magnitude and timing of the peak sales when the parameters are appropriately estimated. Meade and Islam (1998) [13] document 29 different growth curves that were successfully used in technological forecasting history. These demand curves are categorized into 3 groups; *symmetric*, *nonsymmetric* and *flexible*, according to timing of the point of inflection. [1] Bass (1969) describes the first purchase demand for an innovative product by parameterizing total market size, mass-media influences and word of mouth effect of previous purchasers. The Bass model is distinguished with its ability to describe timing and magnitude of peak demand with these intuitive parameters.

After the introduction of the Bass model, a large body of literature revisiting the structural and conceptual assumptions together with the research on estimation issues has been formed. Mahajan et al. (1990) [10] provide an excellent survey, and categorize these developments until 1990 in five categories: (1) basic diffusion models, (2) parameter estimation considerations, (3) flexible diffusion models, (4) refinement and extensions, and (5) use of diffusion models. In their more recent work, Mahajan et al. (2000) [11] address both the theoretical development and the practice of innovational diffusion models, and provide future research direction in new product acceptance demand modeling.

The earliest technological *substitution* model was introduced by Fisher and Pry (1971) [6]. Substitution process is usually triggered when the introduction of the new product is a line extension of an older generation or an existing model i.e. introduction of a smart phone with Global Positioning System (GPS) functionality added to the previous model. In general substitution process cannibalizes the previous products' or models' demand. Norton and Bass (1987) [15] develop the first model that includes both diffusion and substitution, where successive generation of the product competes with the previous generations. Wilson and Norton (1989) [19] in similar, setting analyze the optimal entry time for the introduction of the second generation product over the first generation. By the imposing simplifying assumptions, they show the conditions when the introduction is optimal at the beginning of the planning horizon and the conditions when it is never optimal to introduce the new generation. Mahajan and Muller (1996) [9] extend Wilson and Norton (1989) [19] and the optimal entry decision is either now or at the maturity of the first generation of product. They empirically demonstrate shape of the profit function in entry time for different demand parameter settings. In Section 4, we extend their work by exactly showing the exact entry time at maturity. In addition to this, we list all possible shapes of the profit function under mutually exclusive and exhaustive demand and model parameter conditions, for substitute products. In Section 5, we extend our results for the products that are highly differentiated. Krankel et al. (2006) [8] analyze the introduction timing decisions of a firm for successive product generations. With the introduction of a new generation the firm incurs a fixed cost. The timing decisions are made considering the available technological level, which stochastically improves over time. They prove the optimality of state dependent threshold policy for the firm's new generation introduction decisions.

The incentives for a firm to outsource a portion of their production or service has been studied by many researchers. Quinn and Hilmer (1994) [16] discuss ways to determine a company's core competencies and which activities are better performed externally. Benson and Ieronimo (1996) [2] discuss the impacts of outsourcing maintenance work on firm performance by comparing Australian firms with Japanese firms operating in Australia. Kamien and Li (1990) [7] formulate a production planning model that explicitly considers subcontracting as a planning tool. They also discuss different subcontracting mechanisms and their costs, concluding a class of subcontracting mechanisms Pareto-dominate other subcontracting mechanisms. Van Mieghem (1999) [14] analyze a competitive two stage stochastic investment game between a manufacturer and a supplier. They discuss the outsourcing conditions for three different contract types. For more recent discussion and survey on subcontracting and outsourcing see Simchi-Levi et al. (2004) [17]. In Section 6, we analyze the CM's entry timing problem from a game theoretical perspective, in which the CM determines entry timing while the brand carrying customer decides the reserve levels on the CM's manufacturing capacity.

3 General Setting and Assumptions

In this section we will describe the general setting of models in Section 4 through Section 6 and introduce the notation that is common to each setting. Later when we introduce our models, we will describe the setting and assumptions relevant to that problem.

In all the subsequent problems, we will analyze a high tech contract manufacturer (CM), with a fixed finite capacity rate, c, who considers a planning horizon of length T. Prior the beginning of the planning horizon, the CM has made commitment to a brand carrying customer (primary market) to devote its entire manufacturing capacity for the brand carrying customer's product (primary product). The expected level of primary product demand, $d^p(t)$ follows life cycle dynamics and the CM has full information on the demand distribution over the planning horizon via brand-carrying customer's (BC) information technology system, which enables information sharing within entire supply chain system. For example, Cisco's eHub network is widely used within major corporations' extended supply chains as a central point for planning at various levels.

The CM has the technological maturity and marketing channels to build and market its own brand, and has an incentive to enter a secondary market with his own product (secondary product), whose demand also follows life cycle dynamics. The CM has the expected distribution of the demand in secondary market, $d^s(t)$ prior the beginning of planning horizon. The cumulative demands for primary and secondary products are represented with $D^p(t)$ and $D^s(t)$ and follow *S-shape* pattern over the planning horizon. The per unit selling prices for the primary and the secondary products are fixed and represented with, π^p and π^s respectively. In the light of above information, the CM would like to determine the best policy for the entry to the secondary market to maximize her profits. The entry may or may not be optimal. In case the CM enters the market, she would like to determine the best entry time, t^e, to maximize her profits over the whole horizon. There is also a fixed cost of K to the market entry, which includes channel and/or advertising costs for the secondary market.

For simplicity, we assume the CM consumes one unit of capacity per unit of primary and secondary product, and the inventory buildup is not proffered due to short product life cycles. Although the CM's production capacity is fixed if the demand from primary or secondary market overshoots the level of capacity, the CM has the option to obtain extra production capacity instantaneously from the spot market at price p. Raising production capacity instantaneously can be easy in environments where used technology is matured and there are high number of manufacturers available using similar technology.

In the next section, we will analyze the CM's entry problem where the primary and secondary products are similar to each other and the same production capacity could be used interchangeably for these products.

4 Entry Time Analysis for Similar Products

The contract manufacturer needs to find an optimal entry policy given the information of expected primary and secondary demand distributions over the planning horizon. The primary and secondary product or product groups share a high level similarities and the production capacity can be used for them interchangeably.

The relationship between the primary and secondary market demand is described as; $d^s(t) = d^p(t-\delta)$ where, δ is a fixed known lag parameter, and the primary market demand is a leading indicator on the secondary market demand. We will assume that the total demand curve $d^p(t)+d^s(t)$ has a unique maximum over the planning horizon. This assumption implies an upper bound for the lag parameter, δ. This assumption for δ reduces to $\delta \leq \frac{ln(7+4\sqrt{3})}{b}$ when logistic demand functions are used, and reduces to $\delta \leq \frac{ln(7+4\sqrt{3})}{p+q}$ when Bass demand functions are used for both product types. (See derivation in the Appendix B) We also assume that the production capacity is less than the expected total demand curve $c < d^p(t) + d^s(t)$ for some period of time. If logistic demand curves are use for the primary and secondary demand with parameters a and b, the bound on c reduces to:

$$c \leq \frac{2b(1+a)}{a(2+e^{-\frac{1}{2}b\delta}+e^{\frac{1}{2}b\delta})} \qquad \text{(See Appendix B)}$$

We make this assumption because, if the production capacity were enough to cover the expected total demand curve then the CM would enter the secondary market as earliest as possible.

In order to formally define the duration when the production capacity is binding for the expected total demand curve, let t_{c1} and t_{c2} be the small and large root of $d^p(t) + d^s(t) = c$. Closed form solution for t_{c1} and t_{c2} cannot be obtained with algebra due to transcendental structure of the demand curves. Sophisticated transformation methods could be used to obtain these roots, however it is outside the scope of this research, moreover the roots can be solved for easily by inserting values

Table 1: Summary of notation for similar products entry time model

Notation	Description
$d^p(t)$	Instantaneous primary market demand at time t
$d^s(t)$	Instantaneous secondary market demand at time t
δ	Time lag between the primary and the secondary demand
m	Scale factor representing the market potential for diffusion models
c	Contract manufacturers maximum production capacity rate
t_{c1}, t_{c2}	The small and the large real roots satisfying, $d^p(t) + d^s(t) = c$
K	Fixed cost of entering the secondary market
p	Per unit penalty cost for each unit of demand overshooting the capacity
π^p	Profit margin of the primary product
π^s	Profit margin of the secondary product
α	$\frac{p-\pi^s}{p}$
$z(t)$	Binary decision variable indicating whether there is an entry or not at t

of the demand parameters into above function. These roots are important to distinguish the periods when capacity is ample and when scarce. According to this, during $[0, t_{c1}]$ capacity is enough to meet total demand, during $[t_{c1}, t_{c2}]$ capacity is scarce and finally during $[t_{c2}, T]$ capacity is ample. Figure 1 illustrates the capacity and the demand setting for the contract manufacturer and Table 1 summarizes the notation for the problem.

The contract manufacturer currently serves the primary demand using her entire production capacity. The capacity is enough to cover the primary market demand at any time during the planning horizon. Having the market information on the secondary product the CM's problems is to decide on entry time t^e to maximize the overall profits during the planning horizon. The problem can be formally stated as:

$$\max_t \pi^p D^p(T) - Kz(t) + \pi^s \int_t^T d^s(u)\mathrm{d}u - p \int_t^T (d^p(u) + d^s(u) - c)^+ \mathrm{d}u \tag{1}$$

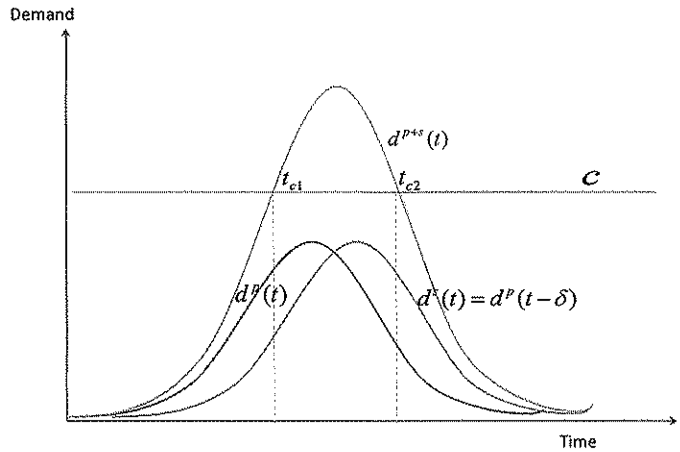
subject to

$$0 \leq t \leq T$$

$$z(t) \in \{0, 1\}$$

The first term represents the profits from the primary market. The second term is the fixed entry cost incurred if the CM decides to enter the market before the end of the planning horizon. $z(t)$ is a binary decision variable representing the market entry decision. $z(t) = 1$ if the CM decides to enter the market at time t and $z(t) = 0$ if the CM does not enter the market. The third term

Figure 1: Problem setting for the similar products entry time model



is the profits obtained from the secondary market, and finally the last term represents the penalty cost of overshooting the production capacity.

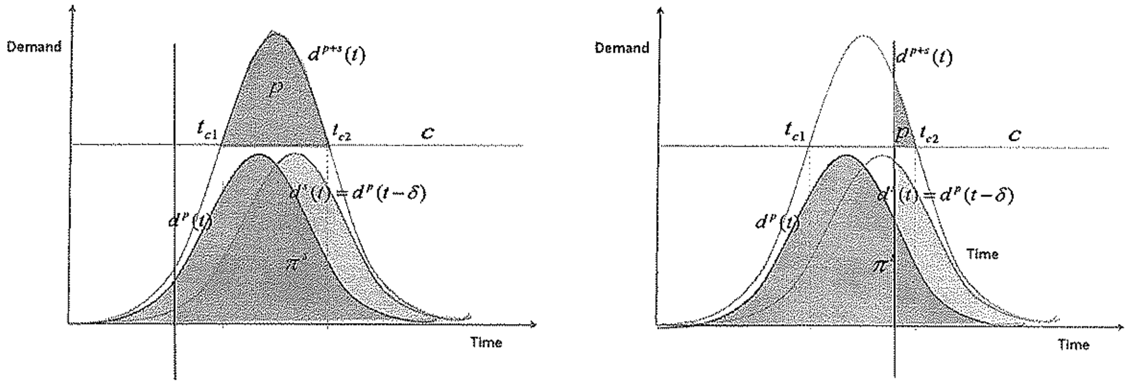
We will analyze Problem 1 considering the demand life cycle dynamics and its relation to the level of production capacity. If the CM enters the market early, she will utilize from high market potential but she has to pay penalty cost for higher number of products. If she enters the market later, the penalty cost for overshooting demand will be less, but the profits from the secondary market may not be enough to cover the fixed market entry cost, K. The trade off between the early and the late entry is represented in Figure 2(a) and Figure 2(b) respectively.

Next, we analyze the CM's entry timing problem and construct the characteristics of the optimal solution.

4.1 Model Analysis

We will use Lemma 4.1 through Lemma 4.4 to determine the characteristics of the CM's profit function in entry time, $\prod^s(t)$. We, then in Proposition 4.1, state the conditions in which the $\prod^s(t)$ displays a different pattern over the planning horizon. After that utilizing the results of 4.5 and 4.6, we summarize the CM's optimal entry policy in Theorem 4.1.

Lemma 4.1. *There are at most two maxima of $\prod^s(t)$. First maximum is always at $t = 0$, and second one (if exists) is at the large root of $\frac{p-\pi^s}{p} d^s(t) + d^p(t) - c = 0$*



(a) Early entry

(b) Late entry

Figure 2: The tradeoff between early and late entry

Proof. We will analyze the profit function in entry time when the entry is realized in one the following three ranges:

- $t < t_{c1}$. The CM's profit function within this range is:

$$\pi^p D^p(T) - K + \pi^s(D^s(T) - D^s(t)) - p(D^{p+s}(t_{c2}) - D^{p+s}(t_{c1}) - c(t_{c2} - t_{c1}))$$

  and the first order derivative of the profit function with respect to t is, $-\pi^s d^s(t)$. The profit function is strictly decreasing in this range.

- $t_{c1} \leq t \leq t_{c2}$. The CM's profit function is:

$$\pi^p D^p(T) - K + \pi^s(D^s(T) - D^s(t)) - p(D^{p+s}(t_{c2}) - D^{p+s}(t) - c(t_{c2} - t))$$

  and the first order derivative of the profit function with respect to t is, $(p-\pi^s)d^s(t)+p(d^p(t)-c)$. If we divide each term with p then we obtain $\frac{p-\pi^s}{p}d^s(t) + d^p(t) - c$. The ratio $\frac{p-\pi^s}{p}$ cannot be grater than 1. Hence the solution to the first order condition has at most two real roots within the range $[t_{c1}, t_{c2}]$. If $p < \pi^s$ there is no real root for the first order condition, and the first order derivative of the profit function is strictly decreasing. If the first order condition has a unique root, then the profit function is monotonically decreasing and if the first order condition has two real roots then the profit function is decreasing until the first root, then increasing until the second root and decreasing until t_{c2}. Hence the second maximum exists only if the first order condition has two real roots, the larger root being the maximum point.

- $t_{c2} < t \leq T$. The CM's profit function in this range is $\pi^p D^p(T) - K + \pi^s(D^s(T) - D^s(t))$ and the first order derivative of the profit function with respect to t is $-\pi^s d^s(t)$. The profit function is strictly decreasing in this range.

In summary, the profit function is strictly decreasing during $[0, t_{c1})$ and $(t_{c2}, T]$. Depending on the solution to the first order condition in range $[t_{c1}, t_{c2}]$ the profit function is either decreasing or has an interior maximum at the large root of $\frac{p-\pi^s}{p}d^s(t) + d^p(t) - c$. In either case, $t = 0$ is the first maximum point, and if exists, the large root of $\frac{p-\pi^s}{p}d^s(t) + d^p(t) - c$ is the second maximum point. $\square$

Among the problem parameters π^s and its relation p has the most effect in the CM's profit function's behavior. In order to specify the full characterization of the profit function and the optimal entry policy we will introduce a set of lemmas describing the profit function behavior in π^s and then we will utilize this results to derive Proposition 4.1 for the normalized relationship between π^s and p. Let α be defined as $\frac{p-\pi^s}{p}$ ($\alpha \leq 1$). We will also use t_α (if exists) as the large root of $\alpha d^s(t) + d^p(t) - c = 0$. If π^s is 0 then t_α becomes t_{c1}.

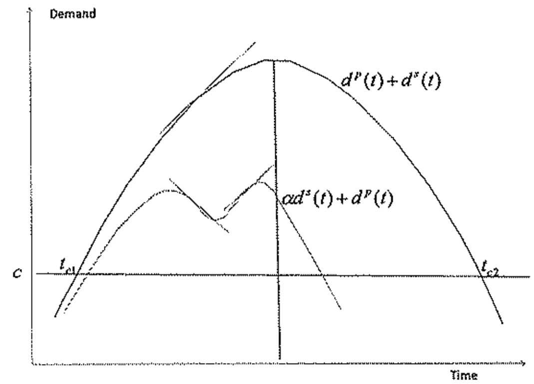
Lemma 4.2. • *There exists a unique level of secondary market profit margin, $\bar{\pi}^s$, such that $\frac{p-\bar{\pi}^s}{p}d^s(t) + d^p(t) - c$ has a unique real root in domain $[0, T]$.*

- *For $0 \leq \pi^s < \bar{\pi}^s$, $\frac{p-\pi^s}{p}d^s(t) + d^p(t) - c$ has two real roots.*

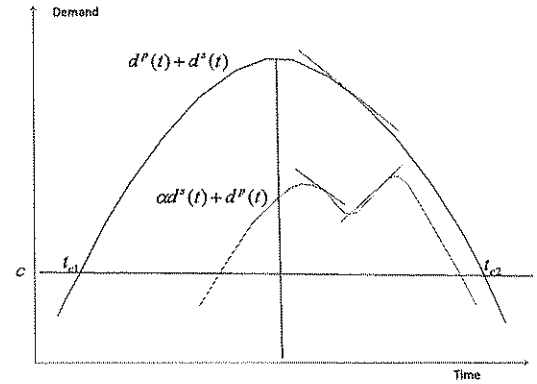
Proof.

- $\frac{p-\pi^s}{p}d^s(t) + d^p(t) - c$ is a continuous function of π^s. We need to show that for any π^s, $\frac{p-\pi^s}{p}d^s(t) + d^p(t)$ has a unique maximum. For $\pi^s = 0$ we know that $\frac{p-\pi^s}{p}d^s(t) + d^p(t)$ has a unique maximum (imposed by the assumption on δ). $\bar{\pi}^s$ has to be less than p because by the our model assumption c is greater than the maximum demand rate of the primary market. Now consider the case where $0 < \pi^s < p$. We will prove our claim by contradiction.

  Suppose that there is more than one maximum of $\alpha d^s(t) + d^p(t)$, where both maxima are before the maximum demand point. (Figure 3 (a)). Consider the slope represented by green line. The total demand must be decreasing around the green line. This cannot be the case when both demands are increasing, so the negative slope should occur after the maximum demand rate of the primary market. i.e. the primary market demand is decreasing and the



(a) Before the maximum demand rate



(b) After the maximum demand rate

Figure 3: Multiple maxima of the total demand curve

secondary market demand is increasing. For $\pi^s = 0$ ($\alpha = 1$) the value of $\alpha d^s(t)$ is maximum. As π^s is increasing (α is decreasing), both the slope and the value of $\alpha d^s(t)$ decreases for a given time. Hence, one the slope of $\alpha d^s(t) + d^p(t)$ becomes negative it cannot go positive in this range.

Now suppose that both maxima are after the maximum demand point. (Figure 3 (b)). Consider the slope represented by green line. $\alpha d^s(t) + d^p(t)$ must be increasing around the green line. This cannot be the case when both demands are decreasing, so the positive slope should occur before the maximum demand rate of the secondary market, i.e. primary market is decreasing and the secondary market is increasing. If $\alpha d^s(t) + d^p(t)$ has a negative slope before the maximum demand rate this means the reduction in the primary market demand dominates the increment in the secondary market demand. In this case, the slope has to stay negative, because the $d^p(t) + d^s(t)$ is also decreasing in this range. Hence, the secondary maximum is not possible.

We showed that for any level of π^s, $\frac{p-\pi^s}{p} d^s(t) + d^p(t) - c$ has a unique maximum and since $\frac{p-\pi^s}{p} d^s(t) + d^p(t) - c$ is continuous in π^s, we can find a value of π^s, where the capacity level c becomes the tangent of the unique maximum of $\frac{p-\pi^s}{p} d^s(t) + d^p(t)$. We call this π^s level as $\bar{\pi}^s$ and $\frac{p-\bar{\pi}^s}{p} = \bar{\alpha}$

- We know that c is tangent to $\bar{\alpha} d^s(t) + d^p(t)$ at its maximum point. For all $0 \leq \pi^s < \bar{\pi}^s$ ($\bar{\alpha} < \alpha \leq 1$), $\alpha d^s(t) + d^p(t)$ will cross capacity level twice, one being before the maximum point and being after the maximum point.

□

Lemma 4.3. *$\prod^s(t_\alpha)$ strictly increases as π^s increases in the domain $0 \leq \pi^s < \bar{\pi}^s$.*

Proof. We will prove our claim by showing that the first order derivative of $\prod^s(t_\alpha)$ with respect to π^s is positive.

$$\Pi^s(t_\alpha) = \pi^p D^p(T) - K + \pi^s(D^s(T) - D^s(t_\alpha)) - p(D^{p+s}(t_{c2}) - D^{p+s}(t_\alpha) - c(t_{c2} - t_\alpha))$$

The first order derivative of $\prod^s(t_\alpha)$ with respect to π^s is;

$$\begin{aligned}\frac{\partial \Pi^s(t_\alpha)}{\partial \pi^s} &= D^s(T) - D^s(t_\alpha) - \pi^s d^s(t_\alpha)\frac{\partial t_\alpha}{\partial \pi^s} + p(d^{p+s}(t_\alpha) - c)\frac{\partial t_\alpha}{\partial \pi^s} \\ &= D^s(T) - D^s(t_\alpha) > 0\end{aligned}$$

□

Lemma 4.4. *There exists a unique level of secondary market profit margin, $\hat{\pi}^s$, ($\hat{\pi}^s < \bar{\pi}^s$) such that $\prod^s(0) = \prod^s(t_\alpha)$*

Proof. The CM's profit function is continuous in π^s. In Lemma 4.3 we showed that $\prod^s(t_\alpha)$ is increasing in π^s. $\prod^s(0)$ is also increasing in π^s. For $\pi^s = 0$, $t_\alpha = t_{c2}$ and,

$$\begin{aligned}\Pi^s(0) &= \pi^p D^p(T) - K - p(D^{p+s}(t_{c2}) - D^{p+s}(t_{c1}) - c(t_{c2} - t_{c1})) \\ \Pi^s(t_\alpha) &= \pi^p D^p(T) - K \\ \Rightarrow \Pi^s(t_\alpha) &> \Pi^s(0)\end{aligned}$$

For $\pi^s = \bar{\pi}^s$, $t = 0$ is the unique maximum of the profit function, thus $\prod^s(0)$ must have exceeded $\prod^s(t_\alpha)$ at a certain level of the secondary market profit margin, $\hat{\pi}^s$. This level is unique since both $\prod^s(0)$ and $\prod^s(t_\alpha)$ is strictly increasing in π^s. □

Now we are ready to state the conditions that describe the every possible shape of $\prod^s(t)$ for varying level of the relationship of π^s and p. We will use the normalized version of the relation in terms of α. Below, $\bar{\alpha} = \frac{p-\bar{\pi}^s}{p}$ and $\hat{\alpha} = \frac{p-\hat{\pi}^s}{p}$.

Proposition 4.1. *Profit function can be in only one and only one of the following four forms depending on the level of α:*

i $\alpha = 1$ *Profit function is represented by Figure I*

ii $\hat{\alpha} < \alpha < 1$ *Profit function is represented by Figure II*

iii $\bar{\alpha} < \alpha < \hat{\alpha}$ *Profit function is represented by Figure III*

iv $\alpha < \bar{\alpha}$ *Profit function is represented by Figure IV*

Proof.

i For $\alpha = 1$ ($\pi^s = 0$), the profit function is monotonically increasing in entry time t. Since $\pi^s = 0$, there is no benefit of entering the secondary market early, however there is a cost benefit of entering late due to the decreased penalty cost.

ii For $\hat{\alpha} < \alpha < 1$ the profit function has two maxima (Lemma 4.1) and the profits are higher at $t = t_\alpha$ than the profits at $t = 0$. (Lemma 4.4)

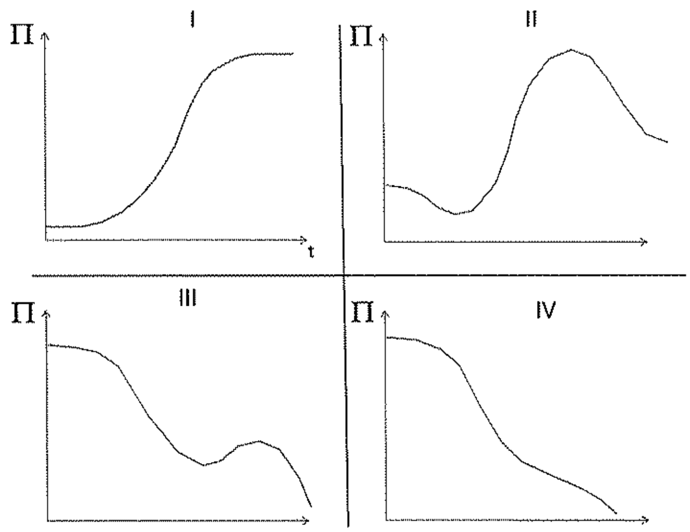
iii For $\bar{\alpha} < \alpha < \hat{\alpha}$ the profit function has two maxima (Lemma 4.1) and the profits are higher at $t = 0$ than the profits at $t = t_\alpha$. (Lemma 4.4)

iv For $\alpha < \bar{\alpha}$ the profit function has a unique maximum at $t = 0$ and the profit function is strictly decreasing in t.

□

Figure 4 displays all possible pattern that $\prod^s(t)$ can display over the planning horizon. We identify 4 distinct patterns in varying π^s levels and establish the bounds on π^s where the pattern switch is observed. With Proposition 4.1, we establish theoretical foundations on the empirical analysis that Mahajan and Muller (1996) [9] provide. In their research, they empirically observe three distinct patterns of the profit function in entry time (Pattern II,III and IV in Figure 4). We, not only formally prove the existence of one more pattern of $\prod^s(t)$, we also show the conditions when the switch in these patterns is realized. Their findings apply to the entry problem with product cannibalization. They find that entry is now or at time where primary product demand matures in a noncapacitated setting. We also show that entry at *now* or at t_α maximizes the profit function in a capacitated setting where products do not cannibalize each other. One major insight of our results is that we show that capacity imitates the cannibalization effect of the generations of technological

Figure 4: All possible realizations of profit function, $\prod^s(t)$



products. It is also intuitive to think that the competition of two products for the limited capacity hampers the total demand satisfied, similar to the parameters in substitution models hampering the previous generation's demand. Hence our model also addresses the entry timing problem among the technological generations of products by offering simpler models yet powerful insights.

Now the following lemmas will be useful in developing optimal entry policy.

Lemma 4.5. *If* $\pi^s \geq \frac{K}{D^s(T)} + p\tilde{\beta}$, *then* $\prod^s(0) \geq \prod^s(T)$, *where* $\tilde{\beta} = \frac{\int_{t_{c1}}^{t_{c2}} (d^{p+s}(u) - c)du}{D^s(T)}$

Proof. We want,

$$\begin{aligned}
\Pi^s(T) \leq& \Pi^s(0) \\
\pi^p D^p(T) \leq& \pi^p D^p(T) + \pi^s D^s(T) - K \\
& - p(D^{p+s}(t_{c2}) - D^{p+s}(t_{c1}) - c(t_{c2} - t_{c1})) \\
\frac{K + p(D^{p+s}(t_{c2}) - D^{p+s}(t_{c1}) - c(t_{c2} - t_{c1}))}{D^s(T)} \leq& \pi^s
\end{aligned}$$

□

Lemma 4.6. *If* $\pi^s \geq \frac{K}{D^s(T) - D^s(t_\alpha)} + p\dot{\beta}$ *then* $\prod^s(t_\alpha) \geq \prod^s(T)$, *where* $\dot{\beta} = \frac{\int_{t_\alpha}^{t_{c2}} (d^{p+s}(u) - c)du}{D^s(T) - D^s(t_\alpha)}$

Proof. We want,

$$\begin{aligned}
\Pi^s(T) \leq& \Pi^s(t_\alpha) \\
\pi^p D^p(T) \leq& \pi^p D^p(T) + \pi^s(D^s(T) - D^s(t_\alpha)) - K \\
& - p(D^{p+s}(t_{c2}) - D^{p+s}(t_\alpha) - c(t_{c2} - t_\alpha)) \\
\frac{K + p(D^{p+s}(t_{c2}) - D^{p+s}(t_\alpha) - c(t_{c2} - t_\alpha))}{D^s(T) - D^s(t_\alpha)} & \leq \pi^s
\end{aligned}$$

□

Theorem 4.1 states the optimal entry policy for every possible realization of profit function, $\prod^s(t)$, where $\dot{\alpha}$ is calculated with smallest value of π^s satisfying the condition in Lemma 4.6 and $\tilde{\alpha}$ is calculated with the smallest value of π^s satisfying Lemma 4.5.

Theorem 4.1. *Optimal entry policy*

i $\alpha = 1$ $\Rightarrow$ *Don't enter*

ii $\hat{\alpha} < \alpha < 1$

- ◇ $\alpha > \dot{\alpha}$ $\Rightarrow$ *Don't enter*
- ◇ $\alpha \leq \dot{\alpha}$ $\Rightarrow$ *Enter at* t_α

iii $\alpha < \hat{\alpha}$

- ◇ $\alpha > \tilde{\alpha}$ $\Rightarrow$ *Don't enter*
- ◇ $\alpha \leq \tilde{\alpha}$ $\Rightarrow$ *Enter at 0*

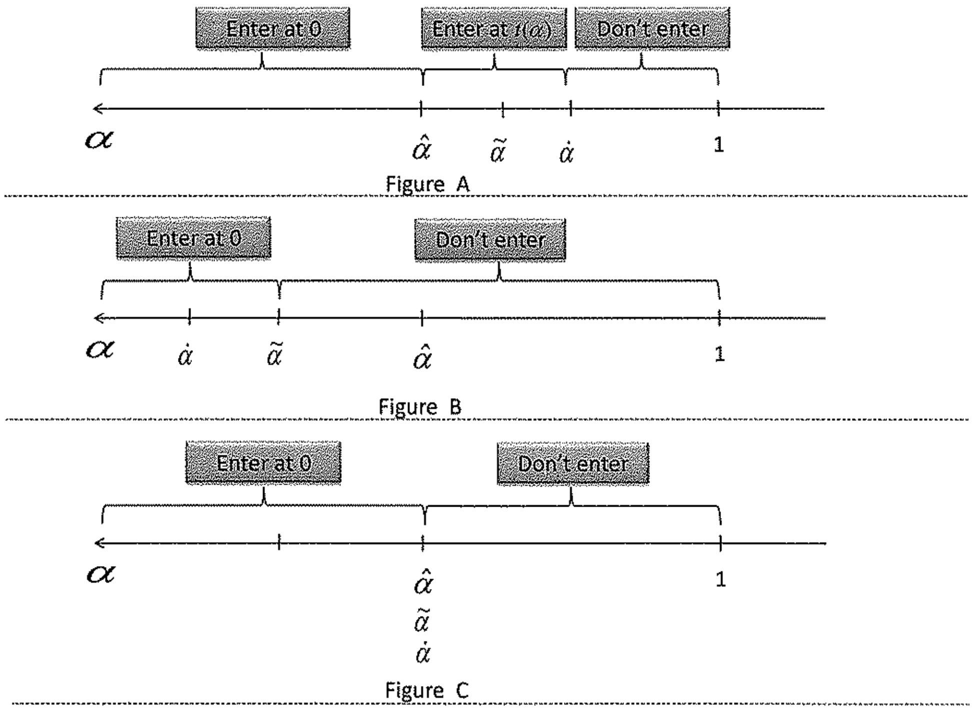
Proof. Profits obtained by CM when there is no entry to the secondary market is $\prod^s(T) = \pi^p D^p(T)$

i For $\alpha = 1$, the global maximum of the profit function is at $t = t_{c2}$, and $\prod^s(t_{c2}) = \pi^p D^p(T) - K$. Since $\prod^s(T) > \prod^s(t_{c2})$ no entry is optimal.

ii For $\hat{\alpha} < \alpha < 1$ the global maximum of the profit function is at $t = t_\alpha$. If $\alpha > \dot{\alpha}$ by Lemma 4.6 $\prod^s(T) > \prod^s(t_\alpha)$ hence no entry is optimal. On the other hand if $\alpha \leq \dot{\alpha}$, $\prod^s(T) \leq \prod^s(t_\alpha)$ and entry at $t = t_\alpha$ is optimal.

iii For $\alpha < \hat{\alpha}$ the global maximum of the profit function is at $t = 0$. If $\alpha > \tilde{\alpha}$, by Lemma 4.5 $\prod^s(T) > \prod^s(0)$ hence no entry is optimal. On the other hand, if $\alpha \leq \tilde{\alpha}$, $\prod^s(T) \leq \prod^s(0)$ and entry at $t = 0$ is optimal.

Figure 5: Entry policy for varying α levels, similar products case



□

Figure 5 demonstrates the optimal entry policy in various conditions, depending on the relationship between $\hat{\alpha}$, $\dot{\alpha}$ and $\tilde{\alpha}$. In subfigure A, the optimal entry time switches from *never* to t_α and then to *now* as π^s increases. In subfigure B, the optimal entry time switches from *never* to *now* only as the π^s level is increased.

In Proposition 4.2 we specify the minimum level of capacity at which the CM starts getting profitable by entering at the beginning of planning horizon. The bound on the capacity level is useful for contract manufacturers especially when they have the fixed start date and would like to measure the pay off point of their capacity investment.

Proposition 4.2. *The capacity level that is just enough to cover the all the cost of entry at the beginning of the horizon satisfies* $\frac{(\pi^p+\pi^s)D^s(T)-K}{p} = \int_{t_{c1}}^{t_{c2}} d^{p+s}(d(u)-c)du$

Proof. The profit function at $t = 0$ is $(\pi^p + \pi^s)m - K - p\int_{t_{c1}}^{t_{c2}} d^{p+s}(d(u) - c)du$. Only cost component that depends on capacity is the last term, and the last term is decreasing as capacity level is increased. If we set this profit function to 0, and arrange the terms as above, we know that the left hand side is positive because $K < (\pi^s + \pi^p)m$ by assumption. If $c = 0$, then RHS is $2m$, that is greater than the LHS. If $c = d^{p+s}(t^*)$ then RHS is 0 and that is less than the LHS. Since the RHS is decreasing monotonically, there is a unique c level that satisfies the condition. Above that capacity level entry is profitable at 0, below that capacity level entry at 0 is not profitable. □

Next, we demonstrate our results in the following numerical example.

4.2 Numerical Example

Consider that both the primary market and the secondary market demand curves are modeled by logistics curves with parameters; $a = 200$, $b = 1$, $\delta = 2$ and $m = 1000$. The maximum capacity level is, $c = 300$. The fixed entry entry cost is $K = \$500$. The primary market selling price is $\pi^p = \$1$ per unit and the penalty cost per overage demand is $p = \$8$.

(a) What would be the CM's optimal entry decision if the secondary market profit margin were $\pi^s = \$0$, $\pi^s = \$1$, $\pi^s = \$2$, $\pi^s = \$3$, $\pi^s = \$5$?

(b) What is the lowest capacity level that favors entry at $t = 0$, if $\pi^s = \$2$

Solution:

(a) We need to find $\hat{\pi^s}$, $\bar{\pi^s}$, $\dot{\pi^s}$ and $\tilde{\pi^s}$ levels to be able to use the results of Theorem 4.1.

By Lemma 4.2, if we solve $\frac{p-\pi^s}{p}d^s(t) + d^p(t) = c$ for π^s where the equation has single real root in $[0, T]$ we obtain $\bar{\pi^s} = \$4.7$, and $\bar{\alpha} = 0.414$.

By Lemma 4.4, if we solve $\prod^s(0) = \prod^s(t_\alpha)$ for π^s, we obtain the unique solution as $\hat{\pi^s} = \$3.08$, and $\hat{\alpha} = 0.615$

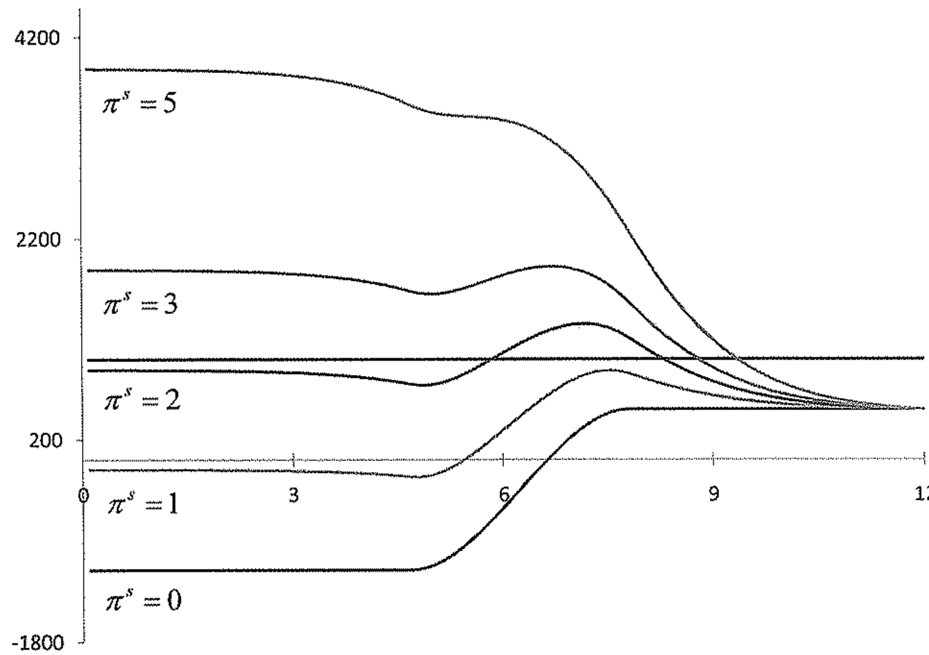
To calculate $\tilde{\pi}^s$ and $\tilde{\alpha}$ we need to use Lemma 4.5. If we solve $d^p(t) + d^s(t) = c$ for t, we obtain, $t_{c1} = 4.74$ and $t_{c2} = 7.85$. Then $\tilde{\beta} = \frac{D^{p+s}(t_{c2}) - D^{p+s}(t_{c1}) - c(t_{c2} - t_{c1})}{D^s(T)} = 0.2$ and $\tilde{\pi}^s = \$2.1$ with $\tilde{\alpha} = 0.74$

Now we should compare $\alpha = \frac{p-\pi^s}{p}$ with the above boundaries and determine the entry time. Table 2 summarizes the optimal entry decision for each secondary market profit margin. Figure 6 shows the plot of CM's profit function $\prod^s(t)$ for different levels of secondary market profit level π^s. According to this for $\pi^s = \$0$, $\pi^s = \$1$, $\pi^s = \$2$, and $\pi^s = \$3$, entry at t_α yields

Table 2: Optimal entry decision for the non allocated capacity numerical example

π^s	α	t_α	Condition	Optimal Decision	Profit Level
\$0	1	7.85	$\alpha = 1$	Don't enter	\$998.7
\$1	0.875	7.58	$\alpha > \dot{\alpha} = 0.832$	Don't enter	\$998.7
\$2	0.75	7.2	$\alpha < \dot{\alpha} = 0.822$	Enter at $t_\alpha = 7.2$	\$1353.5
\$3	0.625	6.75	$\alpha < \dot{\alpha} = 0.8$	Enter at $t_\alpha = 6.75$	\$1922
\$5	0.375	N/A	$\alpha < \bar{\alpha} = 0.414$	Enter at $t = 0$	\$3883.7

Figure 6: CM's profit function for varying π^s levels



higher profits than entering now, however for $\pi^s = \$5$, entering now yields higher profits that entering at t_α.

(b) The capacity level that satisfies the equation in Proposition 4.2 is the lowest capacity level that favors entry at the beginning of the planning horizon. If we insert the value of the parameters into the equation, we have; $\int_{t_{c1}}^{t_{c2}} (d^{p+s}(u) - c)du = 308.7$. If we solve for c, we obtain $c = 267$.

4.3 Key Results

In this section, we formulated and solved the contract manufacturer's strategic market entry decision for similar products. We established the theoretical foundations on the existing empirical market entry literature. Specifically our contributions are;

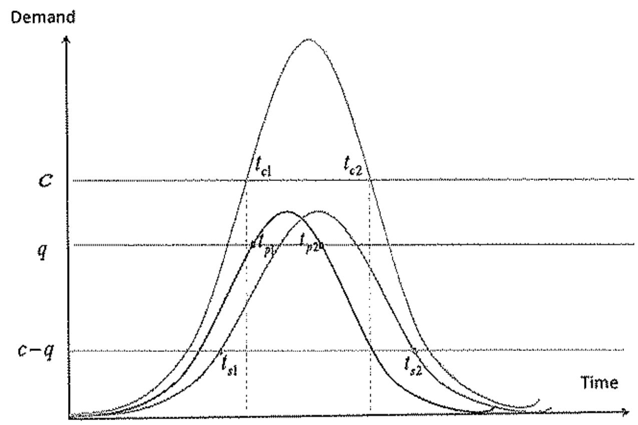
- We extended the empirical profit function characterization of Mahajan and Muller (1996) [9] and theoretically proved that there can be only four distinct patterns of the profit function. Further, we showed bounds on the model parameters where the switch in the pattern of the profit function was observed.
- We showed that capacity imitates the cannibalization effect of technological generations of products. We showed that technological substitution of products can be modeled with more parsimonious models and reduced reliance on estimating the complex demand parameters in classical substitution models.
- We generated theoretical solutions on the optimal entry time, extended now and never results found in Wilson and Norton (1989) [19].
- We showed entry time is sensitive to the small changes in model parameters, i.e. incremental increase in secondary market price may shift the optimal entry from *never* to *now*.
- We demonstrated our findings with numerical examples.

Next we formulate and show the insights on entry timing problem for distinct products

5 Entry Time Analysis for Distinct Products

When there is a high level of product differentiation, or production processes are inflexible, the same production capacity cannot be used for different product groups. Also, there might be limitations that prevent the use of same production capacity for different product groups due to intellectual property rights. In this section, we will analyze the CM's entry problem for where there is a given physical boundary on the capacity level that the CM can use for distinct product groups. Specifically, the CM needs to decide the optimal entry time to a secondary market, before which the CM can use her entire production capacity for the primary market and after the entry time she

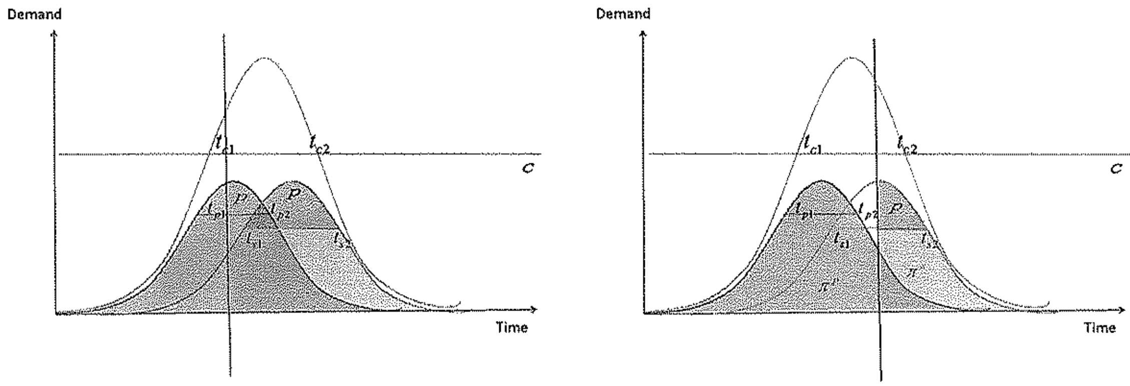
Figure 7: Problem setting for the CM's market entry problem for distinct products



needs to split her capacity among the two products and satisfy the primary and secondary market demands using only the allocated capacity level. The capacity split, q is given and known by the CM. Similar to the model in Section 4 the expected demand distributions for the primary and the secondary market follow the life cycle dynamics and represented with, $d^p(t)$ and $d^s(t)$, respectively with similar relationship between the primary and the secondary demand, $d^s(t) = d^p(t-\delta)$.

Before the entry, the CM uses the entire capacity to satisfy the primary market demand. After the entry, the split level q, $(q \leq c)$, is used to satisfy instantaneous primary market demand and the remaining portion $c - q$ is used to satisfy secondary market demand. Again similar to the setting in Section 4, if the demand from the two markets overshoot the allocated capacity, per unit penalty cost, p is paid for the excessive demand. Since the values of the products are close to each other we will assume per unit penalty cost, p is same for the two markets. We also assume that the production capacity is less than the expected total demand curve $c < d^p(t) + d^s(t)$. Let, t_{c1} and t_{c2} be the small and large roots of $d^p(t) + d^s(t) = c$. Similarly, let t_{p1} and t_{p2} are the small and large roots of $d^p(t) = q, \quad 0 \leq q \leq d^p(t^*)$, and t_{s1} and t_{s2} are the small and large roots of $d^s(t) = c - q$. Had the CM entered the primary market, the CM would incur penalty costs for primary market during $[t_{p1}, t_{p2}]$, and would incur penalty costs during $[t_{s1}, t_{s2}]$ for the secondary market.

The tradeoff between the CM's early entry and late entry completely depends on the capacity split, q for the product groups. When capacity split, q, is high, late entry may be profitable for the CM due to high penalty costs from the secondary market. If capacity split is low, the early entry



(a) Early entry

(b) Late entry

Figure 8: The trade off between early and late entry under for distinct products

Table 3: Summary of notation for distinct products entry time model

Notation	Description
$d^p(t)$	Instantaneous primary market demand at time t
$d^s(t)$	Instantaneous secondary market demand at time t
δ	Time lag between the primary and the secondary demand
q	Capacity level allocated to satisfy primary market demand $q \leq c$
$c - q$	Capacity level left to satisfy secondary market demand
t_{p1}, t_{p2}	The small and the large roots satisfying, $d^p(t) = q$
t_{s1}, t_{s2}	The small and the large roots satisfying, $d^s(t) = c - q$
t_{c1}, t_{c2}	The small and the large real roots satisfying, $d^p(t) + d^s(t) = c$
$d^p(t^*)$	Maximum primary market demand rate
p	Per unit penalty cost for each unit of demand overshooting the capacity
π^s	Profit margin of the secondary market
π^p	Profit margin of the primary market

may be beneficial. Figure 8 demonstrates the tradeoff between the early entry Figure 8(a) and the late entry Figure 8(b) for a given level of capacity split, q. Table 3 summarizes the notation we will be using throughout this section.

The CM's profit, $\prod^d(t)$ maximizing problem can be stated as follows:

$$\begin{aligned}\Pi^d(q) = \max_t \pi^p D^p(T) - Kz(t) + \pi^s \int_t^T d^s(u)\mathrm{d}u \\ - p\int_t^T ((d^p(u) - q)^+ + (d^s(u) - (c-q))^+)\mathrm{d}u\end{aligned} \tag{2}$$

subject to

$$0 \leq t \leq T$$

$$z(t) \in \{0, 1\}$$

First and third terms are expected profits from the primary and secondary markets, second term is the fixed entry costs incurred if entered, and the last term represents the total penalty costs after entry for overshooting the allocated capacity for each type of demands. In the next section, we will formally analyze the CM's Problem 2.

5.1 Model Analysis

Before the analysis of Problem 2, we need to identify all possible allocation structures for the split parameter, q. We do this in Lemma 5.1. We then analyze the CM's problem for each allocation type and compare the results with the previous section.

Lemma 5.1. *Allocation policy can be described in one and only one of the following forms depending on the level of q:*

i $d^p(t_{c1}) < q \leq d^p(t^*) \quad \Rightarrow t_{s1} < t_{c1} < t_{p1} < t_{p2} < t_{c2} < t_{s2}$

ii $d^s(t_{c1}) < q \leq d^p(t_{c1}) \quad \Rightarrow t_{p1} < t_{c1} < t_{s1} < t_{p2} < t_{c2} < t_{s2}$

iii $0 \leq q < d^s(t_{c1}) \quad \Rightarrow t_{p1} < t_{c1} < t_{s1} < t_{s2} < t_{c2} < t_{p2}$

Proof. $0 \leq q \leq d^p(t^*)$ by definition. We will divide this range into three regions and analyze the capacity demand curves in these regions:

1. $d^p(t_{c1}) < q \leq d^p(t^*)$ At time t_{c1} and t_{c2}, $d^p(t) + d^s(t) = c$. So if, $q = d^p(t_{c1})$ then $c - q = d^s(t_{c1})$ and $t_{p1} = t_{s1} = t_{c1}$. Similarly, $d^s(t_{c2}) = q$ and $d^p(t_{c2}) = c - q$, hence $t_{p2} = t_{c2} - \delta$ and $t_{s2} = t_{c2} + \delta$. Consider an infinitesimal amount of increase (Δq) on the level q. Then the small root of $d^p(t) = q + \Delta q$ (t_{p1}) and large root of $d^s(t) = c - q - \Delta q$ (t_{s2}) will increase by an infinitesimal amount leading to $t_{p1} = t_{c1}^+$ and $t_{s2} = t_{c2}^+ + \delta$, while the small root of $d^s(t) = c - q - \Delta q$ (t_{s1}) and the large root of $d^p(t) = q + \Delta q$ (t_{p2}) will increase by an infinitesimal amount, resulting in $t_{s1} = t_{c1}^-$ and $d_{p2} = t_{c2}^- - \delta$. q can be as high as $d^p(t^*)$ Putting these together for $d^p(t_{c1}) \leq q \leq d^p(t^*)$ we have $t_{s1} < t_{c1} < t_{p1} < t_{p2} < t_{c2} < t_{s2}$. (See Figure 9 (a))

2. $d^s(t_{c1}) < q \leq d^p(t_{c1})$ With similar argument if $q = d^p(t_{c1})$ then $t_{p1} = t_{s1} = t_{c1}$, $t_{p2} = t_{c2} - \delta$ and $t_{s2} = t_{c2} + \delta$. An infinitesimal amount of decrease Δq in q would increase the large root of $d^p(t) = q - \Delta q$ (t_{p2}) and the small root of $d^s(t) = c - q + \Delta q$ (t_{s1}). Similarly, the small root of $d^p(t) = q - \Delta$ (t_{p1}) and the large root of $d^s(t) = c - q + \Delta$ (t_{s2}) would increase by an infinitesimal amount leading to $t_{p1} = t_{c1}^-$, $t_{s1} = t_{c1}^+$, $t_{p2} = t_{c2}^+ - \delta$ and $t_{s2} = t_{c2}^- + \delta$. At $q = d^s(t_{c1})$, $t_{p1} = t_{c1} - \delta$, $t_{s1} = t_{c1} + \delta$ and $t_{c2} = t_{p2} = t_{s2}$. An infinitesimal amount of increase in q would increase the small root of $d^p(t) = q + \Delta q$ (t_{p1}) and the large root of $d^s(t) = c - q - \Delta q$ (t_{s2}). Similarly the small root of $d^s(t) = c - q - \Delta q$ (t_{s1}) and the large root of $d^p(t) = q + \Delta q$ (t_{p2}) would decrease by an infinitesimal amount leading to, $t_{p1} = t_{c1}^+ - \delta$, $t_{s1} = t_{c1}^- + \delta$, $t_{p2} = t_{c2}^-$ and t_{c2}^+. Putting everything together for $d^s(t_{c1}) < q \leq d^p(t_{c1}$, we have $t_{p1} < t_{c1} < t_{s1} < t_{p2} < t_{c2} < t_{s2}$. (See Figure 9 (b) and (c))

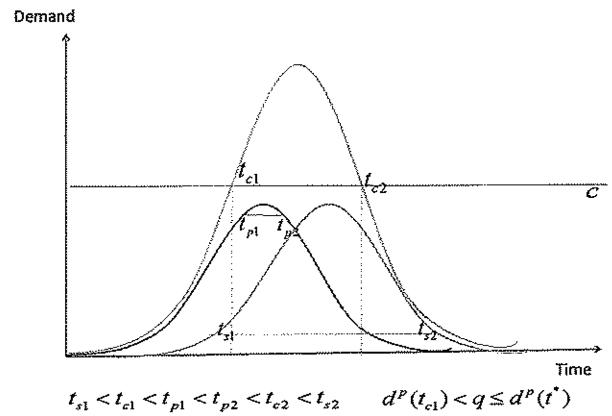
3. $0 \leq q \leq d^s(t_{c1})$ If $q = d^s(t_{c1})$ $t_{p1} = t_{c1} - \delta$, $t_{s1} = t_{c1} + \delta$ and $t_{c2} = t_{p2} = t_{s2}$ An infinitesimal amount of decrease in q would decrease the small root of $d^p(t) = q - \Delta q$ (t_{p1}) and the large root of $d^s(t) = c - q + \Delta q$ (t_{s2}). Similarly the small root of $d^s(t) = c - q + \Delta q$ (t_{s1}) and the large root of $d^p(t) = q - \Delta q$ (t_{p2}) would increase by an infinitesimal amount leading to, $t_{p1} = t_{c1}^- - \delta$, $t_{s1} = t_{c1}^+ + \delta$, $t_{p2} = t_{c2}^+$ and t_{c2}^-. Hence for $q < d^s(t_{c1}$ we have $t_{p1} < t_{c1} < t_{s1} < t_{s2} < t_{c2} < t_{p2}$.

□

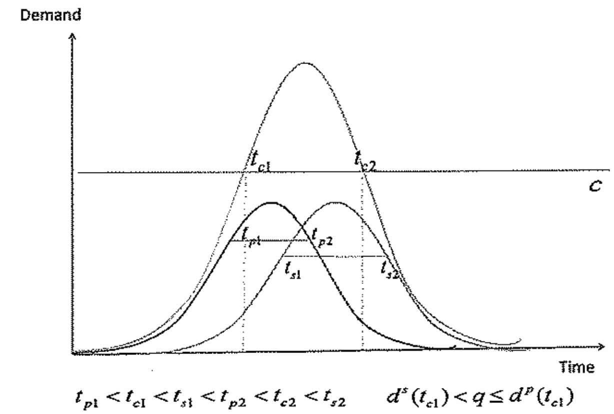
In Lemma 5.2 similar to profit function for similar products, we show that the profit function in distinct products case $\prod^d(t)$ has at most two maxima points.

Lemma 5.2. *For any allocation scheme there are at most two maxima of the profit function. First maximum is always at $t = 0$ and the second one, if exists, is always larger than t_α.*

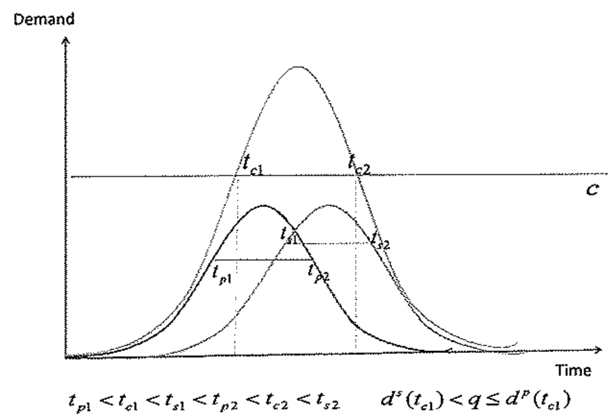
Proof. In the first part of the proof, we will show that for each allocation scheme the profit function is either decreasing always or, first decreasing, then increasing and finally decreasing again. In the former case, the only maximum point is $t = 0$, and in the latter case in addition to $t = 0$, second maximum is the time point where the profit function switches from increasing scheme to decreasing scheme. In the second part of the proof, we will show that the interior maximum, if exists, is always larger than t_α that was defined in Section 4.1. Tables 4-6 show the first order derivatives of the profit function with respect to entry time t for each allocation schemes, $(i), (ii), (iii)$ in Lemma 5.1.



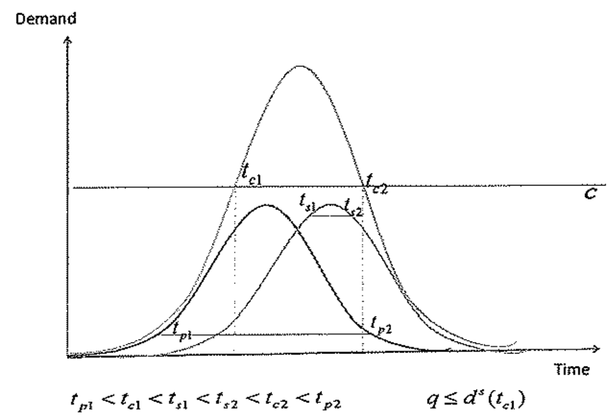
(a) Lemma 5.1 part *i*



(b) Lemma 5.1 part *ii*



(c) Lemma 5.1 part *ii*



(d) Lemma 5.1 part *iii*

Figure 9: All possible capacity allocation structures

(i) $t_{s1} < t_{c1} < t_{p1} < t_{p2} < t_{c2} < t_{s2}$ We will use Table 4; Profit function is decreasing for $0 \leq t \leq t_{s1}$, because the first order derivative is negative in this range. During $t_{s1} < t \leq t_{p1}$ the first order derivative is $-\pi^s d^s(t)$ plus a positive term $p(d^s(t) - c + q)$. Depending on the level of p the first order derivative might become positive, because $d^s(t)$ is increasing in this range. (See figure 9 (a)). If the first derivative does not become positive then the profit function is decreasing in this range too. Now consider the range $t_{p1} < t \leq t_{p2}$. During this range the first order derivative is $(p - \pi^s d^s(t)) - p(c - q)$ (the first order derivative in the previous range plus a positive term $p(d^p(t) - q)$). If the profit function switched from the decreasing scheme to increasing scheme in the previous range, then it will continue to increase during this range too. However, if the profit function were decreasing in the previous range, then the profit function might switch to increasing scheme due to the positive term. Note that the positive term, $p(d^p(t) - q)$ converges to 0 at the end of this range, because $d^p(t_{p2}) = q$. Now consider the range $t_{p2} < t \leq t_{s2}$; during this range the secondary market demand reaches its peak demand rate. If the profit function were decreasing in the previous range, then it might switch to the increasing scheme before the peak demand point. If it doesn't, then the profit function will continue to decrease. If the profit function were increasing in the previous range, then it will continue to increase until the first order derivative becomes 0. (Eventually it will hit 0, because $d^s(t)$ will be decreasing after the peak demand point). During the last range, the profit function is decreasing always. Combining all the information above, we reach the claim in the lemma, that is, either profit function is decreasing always, or has decreasing-increasing-decreasing pattern. Now for this allocation scheme we will show that the inner maximum, if exists is larger than t_α. By definition t_α is the large root of $(p - \pi^s)d^s(t) + d^p(t) - c$, and this large root lies between the peak point of total demand curve and t_{c2}. In other words, t_α lies in range (t_{p2}, t_{s2}) and the first order derivative in this range is:

$$
\begin{aligned}
&(p - \pi^s)d^s(t) - p(c - q) \\
=&(p - \pi^s)d^s(t) + pd^p(t) - pc + p(q - d^p(t))
\end{aligned}
$$

At t_α first three terms sums to 0, and the last term is positive. So, that means the profit function reaches its inner maximum at a point that is greater than t_α.

(ii) $t_{p1} < t_{c1} < t_{s1} < t_{p2} < t_{c2} < t_{s2}$ We will use Table 5. Profit function is decreasing for $0 \leq t \leq t_{p1}$, because the first order derivative is negative in this range. During $t_{p1} < t \leq t_{s1}$

Table 4: First Order Derivative of Profit Function, $\prod^d(q,t)$, under allocation scheme $t_{s1} < t_{c1} < t_{p1} < t_{p2} < t_{c2} < t_{s2}$

Entry Time	First Order Derivative of Profit Function w.r.t t, $\frac{\partial \prod^d(q,t)}{\partial t}$
$0 \leq t \leq t_{s1}$	$-\pi^s d^s(t)$
$t_{s1} < t \leq t_{p1}$	$(p - \pi^s)d^s(t) - p(c - q)$
$t_{p1} < t \leq t_{p2}$	$(p - \pi^s)d^s(t) + pd^p(t) - pc$
$t_{p2} < t \leq t_{s2}$	$(p - \pi^s)d^s(t) - p(c - q)$
$t_{s2} < t \leq T$	$-\pi^s d^s(t)$

the first order derivative is $-\pi^s d^s(t)$ plus a positive term $p(d^p(t) - q)$. Depending on the level of p the first order derivative might become positive. If the first derivative does not become positive then the profit function is decreasing in this range too. Now consider the range $t_{s1} < t \leq t_{p2}$. During this range the first order derivative is $-\pi^s d^s(t) + p(d^p(t) - q)$ (the first order derivative in the previous range) plus a positive term $p(d^s(t) - c + q)$. If the profit function switched from the decreasing scheme to increasing scheme in the previous range, then it will continue to increase during this range too. However, if the profit function were decreasing in the previous range, then the profit function might switch to increasing scheme due to the positive term. Now consider the range $t_{p2} < t \leq t_{s2}$; during this range the secondary market demand reaches its peak demand rate. If the profit function were decreasing in the previous range, then it might switch to the increasing scheme before the peak demand point. If it doesn't, then the profit function will continue to decrease. If the profit function were increasing in the previous range, then it will continue to increase until the first order derivative becomes 0. (Eventually it will hit 0, because $d^s(t)$ will be decreasing after the peak demand point). During the last range, $t_{s2} < t \leq T$, the profit function is decreasing always. Combining all the information above, we reach the claim in the lemma, that is, either profit function is decreasing always, or has decreasing-increasing-decreasing pattern. Now for this allocation scheme we will show that the inner maximum, if exists is larger than t_α. By definition t_α is the large root of $(p - \pi^s)d^s(t) + d^p(t) - c$, and this large root lies between the peak point of total demand curve and t_{c2}. In other words, t_α lies in range (t_{p2}, t_{s2}) and the first order derivative in this range is:

Table 5: First Order Derivative of Profit Function, $\prod^d(q,t)$, under allocation scheme $t_{p1} < t_{c1} < t_{s1} < t_{p2} < t_{c2} < t_{s2}$

Entry Time	First Order Derivative of Profit Function w.r.t t, $\frac{\partial \prod^d(q,t)}{\partial t}$
$0 \leq t \leq t_{p1}$	$-\pi^s d^s(t)$
$t_{p1} < t \leq t_{s1}$	$-\pi^s d^s(t) + pd^p(t) - pq$
$t_{s1} < t \leq t_{p2}$	$(p - \pi^s)d^s(t) + pd^p(t) - pc$
$t_{p2} < t \leq t_{s2}$	$(p - \pi^s)d^s(t) - p(c - q)$
$t_{s2} < t \leq T$	$-\pi^s d^s(t)$

$$
\begin{aligned}
&(p - \pi^s)d^s(t) - p(c - q) \\
=&(p - \pi^s)d^s(t) + pd^p(t) - pc + p(q - d^p(t))
\end{aligned}
$$

At t_α first three terms sums to 0, and the last term is positive. So that means the profit function reaches its inner maximum at a point that is greater than t_α.

(iii) $t_{p1} < t_{c1} < t_{s1} < t_{s2} < t_{c2} < t_{p2}$ We will use Table 6. Profit function is decreasing for $0 \leq t \leq t_{p1}$, because the first order derivative is negative in this range. During $t_{p1} < t \leq t_{s1}$ the first order derivative is $-\pi^s d^s(t)$ plus a positive term $p(d^p(t) - q)$. Depending on the level of p the first order derivative might become positive. If the first derivative does not become positive then the profit function is decreasing in this range too. Now consider the range $t_{s1} < t \leq t_{s2}$. During this range the first order derivative is $-\pi^s d^s(t) + p(d^p(t) - q)$ (the first order derivative in the previous range) plus a positive term $p(d^s(t) - c + q)$. If the profit function switched from the decreasing scheme to increasing scheme in the previous range, then it will continue to increase during this range too. However, if the profit function were decreasing in the previous range, then the profit function might switch to increasing scheme due to the positive term. Now consider the range $t_{s2} < t \leq t_{p2}$; during this range the secondary market demand reaches its peak demand rate. If the profit function were decreasing in the previous range, then it might switch to the increasing scheme before the peak demand point. If it doesn't, then the profit function will continue to decrease. If the profit function were increasing in the previous range, then it will continue to increase until the first order derivative becomes 0. (Eventually it will hit 0, because $d^s(t)$ will be decreasing after the peak demand point). During the last range, $t_{s2} < t \leq T$, the profit function is decreasing always. Combining all the information above, we reach the claim in the lemma, that is, either profit function is decreasing always, or has decreasing-increasing-decreasing pattern. Now for this allocation scheme we will show that the inner maximum, if exists is larger than t_α. By definition t_α is the large root of $(p - \pi^s)d^s(t) + d^p(t) - c$, and this large root lies between the peak point of total demand curve and t_{c2}. In other words, t_α lies in range (t_{p2}, t_{s2}) and the first order derivative in this range is:

Table 6: First Order Derivative of Profit Function, $\prod^d(q,t)$, under allocation scheme $t_{p1} < t_{c1} < t_{s1} < t_{s2} < t_{c2} < t_{p2}$

Entry Time	First Order Derivative of Profit Function w.r.t t, $\frac{\partial \prod^d(q,t)}{\partial t}$
$0 \le t \le t_{p1}$	$-\pi^s d^s(t)$
$t_{p1} < t \le t_{s1}$	$-\pi^s d^s(t) + pd^p(t) - pq$
$t_{s1} < t \le t_{s2}$	$(p - \pi^s)d^s(t) + pd^p(t) - pc$
$t_{s2} < t \le t_{p2}$	$-\pi^s d^s(t) + pd^p(t) - pq$
$t_{p2} < t \le T$	$-\pi^s d^s(t)$

$$
\begin{aligned}
&(p - \pi^s)d^s(t) - p(c - q) \\
=&(p - \pi^s)d^s(t) + pd^p(t) - pc + p(q - d^p(t))
\end{aligned}
$$

At t_α first three terms sums to 0, and the last term is positive. So that means the profit function reaches its inner maximum at a point that is greater than t_α. □

Theorem 5.1. *For any entry time, profits in similar products case always dominates the profit function for distinct products. i.e. For any t and q, $\prod^s(t) \ge \prod^d(t,q)$*

Proof. The proof is done by showing that at any given entry time, t the total unsatisfied demand in non allocated capacity case is always less than or equal to the total unsatisfied demand in the allocated capacity case. There are three allocation schemes as shown in Lemma 5.1, and we need to prove our claim for every capacity allocation schemes. The methodology to prove the claim is to compare the unsatisfied demand rate in two setting by going backwards in time, since total unsatisfied demand at a future entry time is a subset of total unsatisfied demand at an earlier entry time. Below tables summarizes the unsatisfied demand accumulation rate for the allocated and non allocated capacity models for each allocation schemes. The last column at each table shows the difference between the unsatisfied demand rate.

Table 7: Allocation scheme $t_{s1} < t_{c1} < t_{p1} < t_{p2} < t_{c2} < t_{s2}$

Entry Time	Unsatisfied Demand Accumulation Rate		Difference
	Complete Capacity	Split Capacity	
$0 \leq t \leq t_{s1}$	0	0	0
$t_{s1} < t \leq t_{c1}$	0	$d^s(t) - (c-q)$	$d^s(t) - (c-q)$
$t_{c1} < t \leq t_{p1}$	$d^s(t) + d^p(t) - c$	$d^s(t) - (c-q)$	$q - d^p(t)$
$t_{p1} < t \leq t_{p2}$	$d^s(t) + d^p(t) - c$	$d^s(t) + d^p(t) - c$	0
$t_{p2} < t \leq t_{c2}$	$d^s(t) + d^p(t) - c$	$d^s(t) - (c-q)$	$q - d^p(t)$
$t_{c2} < t \leq t_{s2}$	0	$d^s(t) - (c-q)$	$d^s(t) - (c-q)$
$t_{s2} < t \leq T$	0	0	0

Table 8: Allocation scheme $t_{p1} < t_{c1} < t_{s1} < t_{p2} < t_{c2} < t_{s2}$

Entry Time	Unsatisfied Demand Accumulation Rate		Difference
	Complete Capacity	Split Capacity	
$0 \leq t \leq t_{p1}$	0	0	0
$t_{p1} < t \leq t_{c1}$	0	$d^p(t) - q$	$d^p(t) - q$
$t_{c1} < t \leq t_{s1}$	$d^s(t) + d^p(t) - c$	$d^p(t) - q$	$c - q - d^s(t)$
$t_{s1} < t \leq t_{p2}$	$d^s(t) + d^p(t) - c$	$d^s(t) + d^p(t) - c$	0
$t_{p2} < t \leq t_{c2}$	$d^s(t) + d^p(t) - c$	$d^s(t) - (c-q)$	$q - d^p(t)$
$t_{c2} < t \leq t_{s2}$	0	$d^s(t) - (c-q)$	$d^s(t) - (c-q)$
$t_{s2} < t \leq T$	0	0	0

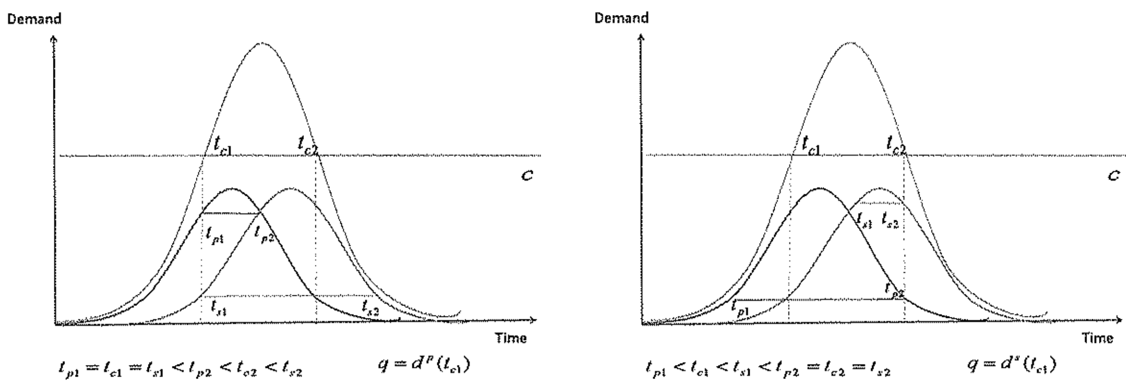
The last column in Tables 7-9 are nonnegative for every entry time specified in the first columns. This suggests that accumulation of unsatisfied demand in the split capacity model is higher than the nonallocated capacity model, For any entry time less penalty cost is paid in the nonallocated capacity case, resulting higher profits as suggested in our claim. □

5.1.1 Special Cases

In this section, we will compare the profit functions for the distinct products with the profit function for similar products for two special cases and generate insights on the complete characterization of the profit functions. These special cases are 1. $q = d^p(t_{c1})$ and 2. $q = d^s(t_{c1})$. Figure 10(a) demonstrates special case $q = d^p(t_{c1})$ and Figure 10(b) demonstrates special case $q = d^s(t_{c1})$. Next, we analyze the special case $q = d^p(t_{c1})$

Table 9: Allocation scheme $t_{p1} < t_{c1} < t_{s1} < t_{s2} < t_{c2} < t_{p2}$

Entry Time	Unsatisfied Demand Accumulation Rate		Difference
	Complete Capacity	Split Capacity	
$0 \leq t \leq t_{p1}$	0	0	0
$t_{p1} < t \leq t_{c1}$	0	$d^p(t) - q$	$d^p(t) - q$
$t_{c1} < t \leq t_{s1}$	$d^s(t) + d^p(t) - c$	$d^p(t) - q$	$c - q - d^s(t)$
$t_{s1} < t \leq t_{s2}$	$d^s(t) + d^p(t) - c$	$d^s(t) + d^p(t) - c$	0
$t_{s2} < t \leq t_{c2}$	$d^s(t) + d^p(t) - c$	$d^p(t) - q$	$c - q - d^s(t)$
$t_{c2} < t \leq t_{p2}$	0	$d^p(t) - q$	$d^p(t) - q$
$t_{p2} < t \leq T$	0	0	0



(a) Special case $q = d^p(t_{c1})$ (b) Special case $q = d^s(t_{c1})$

Figure 10: Special Cases of allocation schemes: $q = d^p(t_{c1})$ and $q = d^s(t_{c1})$

1. $\mathbf{q = d^p(t_{c1})}$

If the split level is set as, $q = d^p(t_{c1})$, then we have $t_{p1} = t_{c1} = t_{s1}$, and overall we have $t_{c1} < t_{p2} < t_{c2} < t_{s2}$. Table 10 displays the difference between $\prod^s(t)$ and $\prod^d(d^p(t_{c1}), t)$ for a entry time t. According to this, after the time point t_{s2} the two functions become equal. Figure demonstrates the comparison of $\prod^s(t)$ and $\prod^d(d^p(t_{c1}, t))$ under the all possible parametric situation we constructed in Section 4. As shown in all subfigures, the profit function in this special case is dominated until the time point t_{s2}, later they become equal.

2. $\mathbf{q = d^s(t_{c1})}$

If we set the primary market capacity level as, $q = d^s(t_{c1})$, then we have $t_{c2} = t_{p2} = t_{s2}$, and overall we have $t_{p1} < t_{c1} < t_{s1} < t_{c2}$. Table 11 displays the difference between the $\prod^s(t)$ and $\prod^d(d^s(t_{c1}), t)$ during the planning horizon. According to this, after the time point t_{s1} the two

Table 10: Comparison of profit functions, $\prod^s(t)$ and $\prod^d(d^p(t_{c1}), t)$ in entry time

Entry Time	$\prod^s(t) - \prod^d(d^p(t_{c1}), t)$
$0 \leq t \leq t_{c1}$	$p\Bigg((2d^p(t_{c1} - c)\delta + D^s(t_{s2}) + D^p(t_{p2}) - D^{p+s}(t_{c2})\Bigg)$
$t_{c1} < t \leq t_{p2}$	$p\Bigg((2d^p(t_{c1} - c)\delta + D^s(t_{s2}) + D^p(t_{p2}) - D^{p+s}(t_{c2})\Bigg)$
$t_{p2} < t \leq t_{c2}$	$p\Bigg((d^p(t_{c1} - c)\delta + D^s(t_{s2}) + D^p(t) - D^{p+s}(t_{c2}) + d^p(t_{c1}(t_{c2} - t))\Bigg)$
$t_{c2} < t \leq t_{s2}$	$p\Bigg((d^p(t_{c1} - c)(t_{s2} - t) + D^s(t_{s2}) - D^s(t))\Bigg)$
$t_{s2} < t \leq T$	0

Figure 11: Comparison of profit functions, $\prod^s(t)$ and $\prod^d(d^p(t_{c1}, t))$

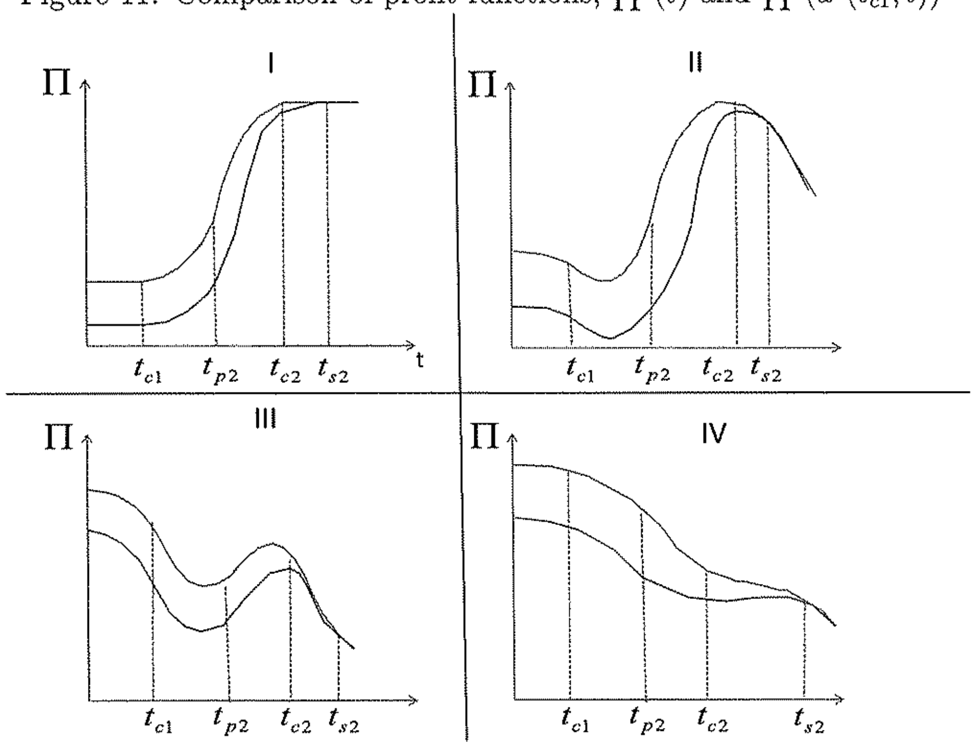
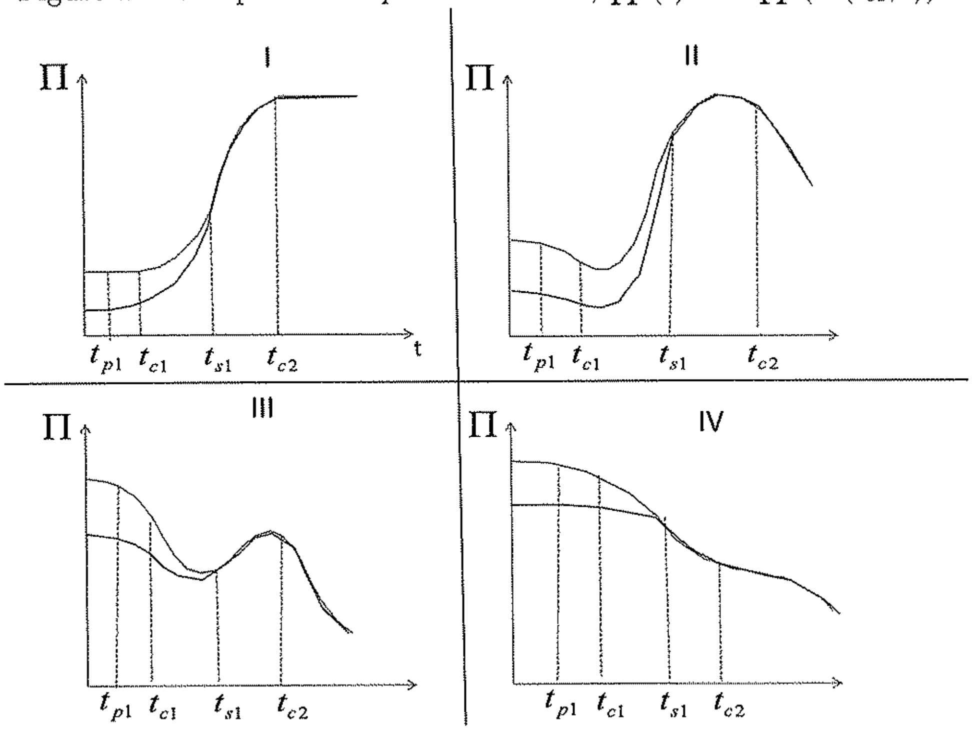


Figure 12: Comparison of profit functions, $\prod^s(t)$ and $\prod^d(d^s(t_{c1},t))$



functions become equal. Figure demonstrates the comparison of $\prod^s(t)$ and $\prod^d(d^s(t_{c1},t))$ under the all possible parametric situation we constructed in Section 4. As shown in all subfigures, the profit function in this special case is dominated until the time point t_{s1}, later they become equal.

Below, we show that the profits in split level $q = d^s(t_{c1})$ always dominates the profits in split level $q = d^p(t_{c1})$.

Proposition 5.1. $\prod^d(d^s(t_{c1}),t) \geq \prod^d(d^p(t_{c1}),t)$

Table 11: Comparison of profit functions, $\prod^s(t)$ and $\prod^d(d^s(t_{c1}),t)$ in entry time

Entry Time	$\prod^s(t) - \prod^d(d^s(t_{c1}),t)$
$0 \leq t \leq t_{p1}$	$p\Big((c-2d^s(t_{c1}))\delta + D^{p+s}(t_{c1}) - D^p(t_{p1}) - D^s(t_{s1})\Big)$
$t_{p1} < t \leq t_{c1}$	$p\Big((c-d^s(t_{c1}))\delta + D^{p+s}(t_{c1}) - D^p(t) - D^s(t_{s1}) - d^s(t_{c1})(t_{c1}-t))\Big)$
$t_{c1} < t \leq t_{s1}$	$p\Big((c-d^s(t_{c1}))\delta + D^s(t_{s1}) - D^s(t)\Big)$
$t_{s1} < t \leq t_{c2}$	0
$t_{c2} < t \leq T$	0

Proof. We need to show that for any entry time the profit function of the special case $q = d^s(t_{c1})$ is always higher than or equal to the profit function in special case, $q = d^p(t)$. For this we will utilize from Tables 10 and 11. Let the roots of $d^p(t) = d^s(t_{c1})$ be identified as t'_{p1} and t'_{p2} and the roots of $d^s(t) = c - d^s(t_{c1})$ be identified as t'_{s1} and t'_{s2}. These time points represent the allocation scheme $q = d^s(t_{c1})$. Now let the roots of $d^p(t) = d^p(t_{c1})$ be identified as t''_{p1} and t''_{p2} and the roots of $d^s(t) = c - d^p(t_{c1})$ be identified as t''_{s1} and t''_{s2}. If we subtract the second column of Table 11 from Table 10 we obtain $\prod^d(d^s(t_{c1}), t) - \prod^d(d^p(t_{c1}), t)$. For each row we have; (see Figure 10)

$$\begin{aligned}
\Pi^d(d^s(t_{c1}), t) - \prod^d(d^p(t_{c1}), t) &= p\Big((2d^p(t_{c1} - c)\delta + D^s(t''_{s2}) + D^p(t''_{p2}) - D^{p+s}(t_{c2}))\\
&\quad - ((c - 2d^s(t_{c1}))\delta + D^{p+s}(t_{c1}) - D^p(t'_{p1}) - D^s(t'_{s1}))\Big)\\
&= 0 \qquad \forall t \in [0, t'_{p1}]
\end{aligned}$$

$$\begin{aligned}
\Pi^d(d^s(t_{c1}), t) - \prod^d(d^p(t_{c1}), t) &= p\Big((2d^p(t_{c1} - c)\delta + D^s(t''_{s2}) + D^p(t''_{p2}) - D^{p+s}(t_{c2}))\\
&\quad - ((c - d^s(t_{c1}))\delta + D^{p+s}(t_{c1}) - D^p(t) - D^s(t'_{s1})\\
&\quad - d^s(t_{c1})(t_{c1} - t))\Big) > 0 \quad \forall t \in (t'_{p1}, t_{c1}]
\end{aligned}$$

$$\begin{aligned}
\Pi^d(d^s(t_{c1}), t) - \prod^d(d^p(t_{c1}), t) &= p\Big((2d^p(t_{c1} - c)\delta + D^s(t''_{s2}) + D^p(t''_{p2}) - D^{p+s}(t_{c2}))\\
&\quad - ((c - d^s(t_{c1}))\delta + D^s(t'_{s1}) - D^s(t))\Big) > 0 \qquad \forall t \in (t_{c1}, t'_{s1}]
\end{aligned}$$

$$\begin{aligned}
\prod^d(d^s(t_{c1}), t) - \prod^d(d^p(t_{c1}), t) &= p\Big(2d^p(t_{c1} - c)\delta + D^s(t''_{s2}) + D^p(t''_{p2}) - D^{p+s}(t_{c2})\Big)\\
&> 0 \qquad \forall t \in (t'_{s1}, t''_{p2}]
\end{aligned}$$

$$\begin{aligned}
\Pi^d(d^s(t_{c1}), t) - \prod^d(d^p(t_{c1}), t) &= p\Big((d^p(t_{c1} - c)\delta + D^s(t''_{s2}) + D^p(t) - D^{p+s}(t_{c2})\\
&\quad + d^p(t_{c1})(t_{c2} - t)\Big) > 0 \qquad \forall t \in (t''_{p2}, t_{c2}]
\end{aligned}$$

$$\begin{aligned}
\Pi^d(d^s(t_{c1}), t) - \Pi^d(d^p(t_{c1}), t) &= p\Big((d^p(t_{c1}) - c)(t_{s2''} - t) + D^s(t_{s2''}) - D^s(t))\Big)\\
&> 0 \forall t \in (t_{c2}, t''_{s2}]
\end{aligned}$$

$$\Pi^d(d^s(t_{c1}), t) - \prod^d(d^p(t_{c1}), t) = 0 \qquad \forall t \in (t''_{s2}, T]$$

□

Next, we demonstrate our findings with numerical examples.

Table 12: Optimal entry decision for the allocated capacity numerical example

q	$[t_{p1}, t_{p2}]$	$[t_{s1}, t_{s2}]$	Optimal Decision	Profits
40	[2.16,8.44]	N/A	Enter at 7.3	\$1227.1
90	[3.09,7.50]	[6.44,8.16]	Enter at 7.1	\$1292.2
140	[3.69,6.90]	[5.90,8.69]	Don't enter	\$998.7
190	[4.22.6.38]	[5.35,9.24]	Don't enter	\$998.7
240	[4.86,5.72]	[4.61,9.98]	Don't enter	\$998.7

5.2 Numerical Example

Consider that both the primary market and the secondary market demands are modeled by logistics curves with parameters; $a = 200$, $b = 1$, $\delta = 2$ and $m = 1000$. The maximum capacity level is, $c = 300$. The fixed entry entry cost is $K = \$500$. The primary product profit margin is $\pi^p = \$1$ per unit, the secondary product profit margin is $\pi^s = \$2$ per unit, and the penalty cost per overage demand is $p = \$8$ per unit.

(a) What is the optimal entry decision for CM if the capacity allocated for the primary product demand is $q = 40$, $q = 90$, $q = 140$, $q = 190$, $q = 240$ units?

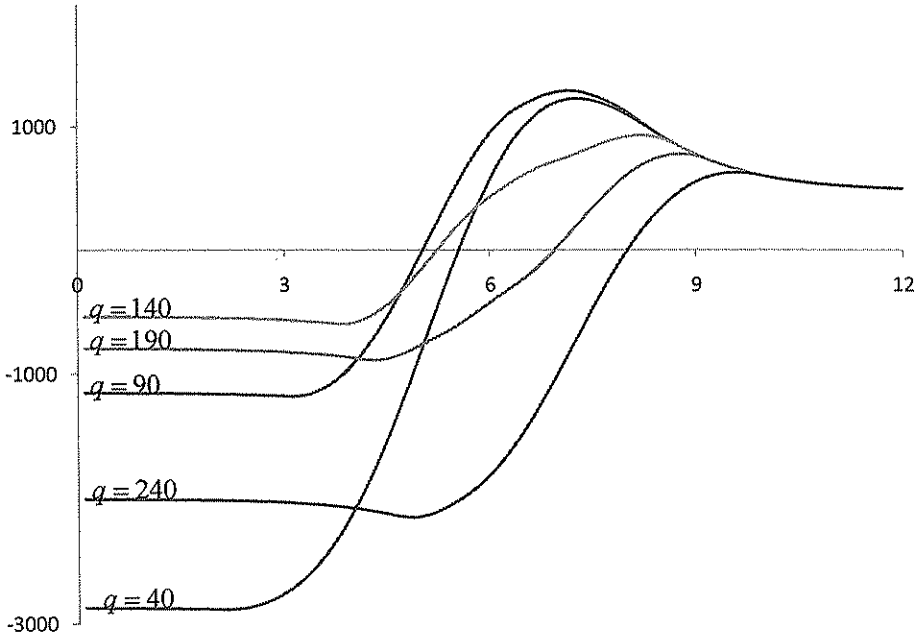
(b) What is the optimal entry decision under special cases $q = d^s(t_{c1})$ and $q = d^p(t_{c1})$?

Solution:

(a) Table 12 summarizes the times at which the capacity allocated for the primary and the secondary products are binding ($[t_{p1}, t_{p2}]$ and $[t_{s1}, t_{s2}]$), optimal entry decision and maximum level of profits for each allocation level q. Figure 5.2 illustrates the CM's profit function in entry time for these allocation quantities. According to this, the highest profits are generated for split level $q = 90$ and lowest profits are generated for split level $q = 240$. As seen from the figure it is hard to generalize the split levels that dominating profit functions. We can say that split level $q = 90$ dominates the split levels 40 and 240. We can also say that split level $q = 140$ dominates the split level $q = 190$; however we cannot say anything specific for other split level comparisons.

(b) The small and the large roots of $d^p(t) + d^s(t) = c$ are $t_{c1} = 4.74$ and $t_{c2} = 7.85$.

Figure 13: CM's profit function for varying q levels



For the special case $q = d^p(t_{c1}) = 233$, we have the small and the large roots of $d^p(t) = 233$ as $t''_{p1} = 4.74$ and $t''_{p2} = 5.85$, and the roots of $d^s(t) = 300 - 233 = 67$ as $t''_{s1} = 4.74$ and $t''_{s2} = 9.85$. As shown in Figure 10 (a), we have $t''_{s1} = t''_{p1} = t_{c1} = 4.74$.

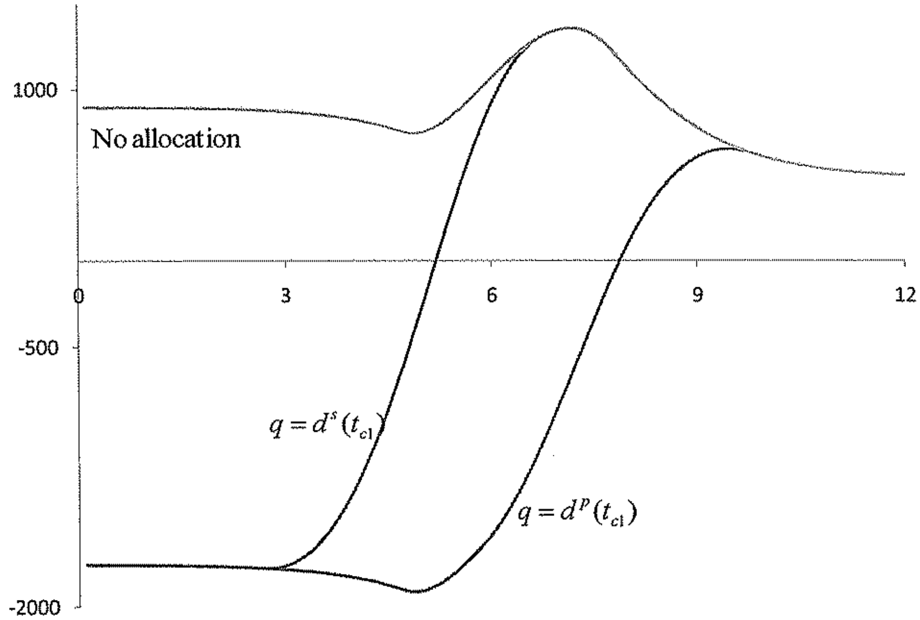
For the special case $q = d^s(t_{c1}) = 67$, we have the small and the large roots of $d^p(t) = 67$ as $t''_{p1} = 2.74$ and $t''_{p2} = 7.85$, and the roots of $d^s(t) = 300 - 67 = 233$ as $t'_{s1} = 6.74$ and $t'_{s2} = 7.85$. As shown in Figure 10 (b), we have $t'_{s2} = t'_{p2} = t_{c2} = 7.85$.

Figure 14 displays the CM's profit function in entry time for the special cases $q = d^p(t_{c1})$ and $d^s(t_{c1})$ and the profit function when capacity is not allocated (model in Section 4.1). As stated in Proposition 5.1, the allocation policy $q = d^s(t_{c1})$ always dominates the allocation policy $q = d^p(t_{c1})$.

5.3 Key Results

In this section, we formulated and solved the contract manufacturer's strategic market entry decision for distinct products. We showed that there are only three possible allocation patterns for a given split level q, and under each allocation type we generalized the characteristics of the profit function.

Figure 14: CM's profit function for special cases $q = d^s(t_{c1})$ and $q = d^p(t_{c1})$ numerical example



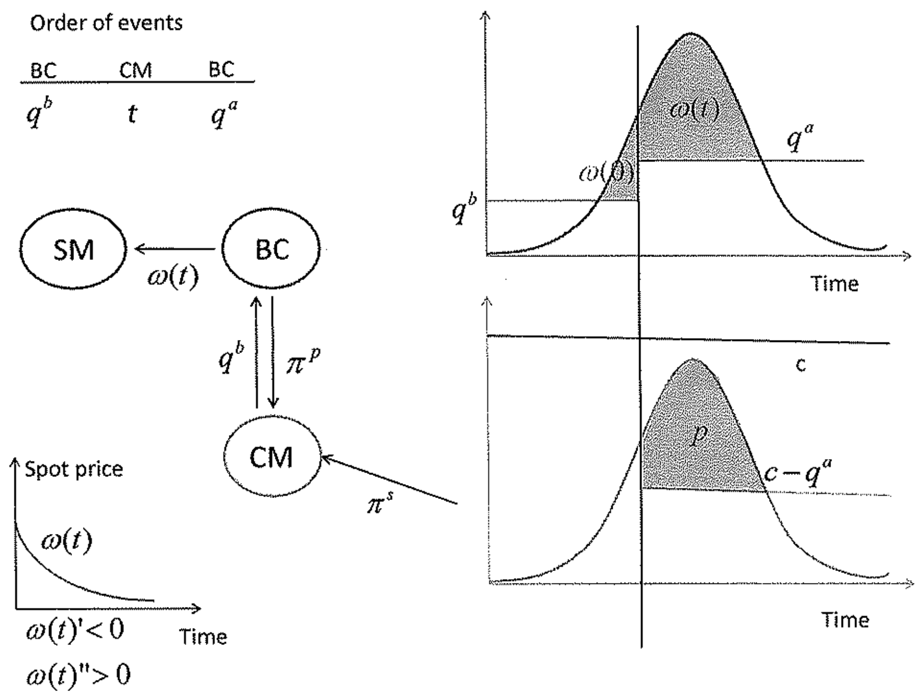
We showed that the profits in similar products setting always dominates the profits in distinct products, regardless of the split level. This is intuitive since, the capacity is less efficiently utilized in the distinct products case than the similar products case. We also demonstrated two special cases defined by values of split levels and showed that the pattern of the profit functions in special cases follow the pattern of the profit function in similar products case. We supported our findings with numerical examples.

In the next section we will analyze the CM's market entry timing problem from game theoretical perspective.

6 Entry Time Analysis with Game Theoretical Models

In real world situations, if a contract manufacturer (CM) while serving a brand-carrying customer (BC), has an incentive to develop its own market presence, then the BC would take precautious actions before the entry is realized or recourse actions after the entry is realized. The intensity of these actions depends on different circumstances. One precautious strategy for a potential entry is

Figure 15: Problem setting of the game theoretic model



to build up inventories, because later when CM enters the market the CM's production capacity may not be available. Another is to reserve the capacity of the CM pay paying a premium upfront. As an example precautions actions, the BC may threaten the CM that he will cut his business with the CM upon her entry to an secondary market. In this portion of our research we will try to answer how the CM's market entry timing decisions are affected when the brand carrying customer is also a decision maker.

The BC has an option to satisfy his demand, $d(t)$ with instantaneous spot market purchase or with a contract manufacturer who offers better prices than the spot market, hence the CM is the preferred supplier of the BC. The price of the component obtained from the spot market is time dependent. Initially, the purchase price of the component is high because there are not many manufacturers that have the technology for this component. However, as time passes the required technology is adopted by more manufacturers hence the price of the component in the spot market decreases as a result of increased competition. For a simplified model, we assume that $\delta = 0$, that is same amount of product is demanded from each market at any time. We use $d(t)$ to describe the instantaneous demand from the BC's market and the secondary market.

We assume that the spot market has unlimited capacity and the units purchased are delivered instantaneously. The purchase price in the spot market is $\omega(t)$, with $\omega(t)' < 0$ and $\omega(t)'' > 0$. BC has the option to reserve CM's production capacity by paying a reserve price, r, per unit capacity per time. Assume $r < w(t), \forall t \in [0, T]$. Then, when the BC faces customer demand, the option is exercised at price π^p. Order of events are as follows; Before any demand is observed, BC determines the capacity reserve level of q^b, then CM responds with the entry time, t and last BC adjusts the after entry reserve level q^a. BC's after entry problem is to minimize ordering costs and this problem is called $P3$:

$$P3 = \min_{q^a} rq^a(T-t) + \omega(t)\int_t^T (d(u) - q^a)^+ du \tag{3}$$

The first term is the total price paid to reserve q^a units of capacity, and the second term is the purchase costs from the spot market. The optimal q^a is a function of entry time. At the second stage knowing how BC would change his allocation level, q^a, CM's determines the optimal entry time for a given before entry reserve level q^b. Let the second stage problem called $P2$.

$$\begin{aligned} P2 = \max_t \ & rq^b t + rq^a(T-t) + \pi^p\left(\int_0^t \min\{q^b, d(u)\}du + \int_t^T \min\{q^a(t), d(u)\}du\right) \\ & + \pi^s \int_t^T d(u)du - p\int_t^T (d(u) - c + q^a(t))^+ du \end{aligned} \tag{4}$$

The first two terms are profits from BC for reserving q^a and q^b units of capacity for him. The third and the fourth terms are the profits from BC and the secondary markets. The last term is the penalty paid for unsatisfied demand for the secondary market. Note that the CM does not pay penalty for the BC demand because, by reserving production capacity of CM, the BC accepts not to demand more than the reserve level. The CM decides on the entry time for a given level of q^b and. Finally in the first stage, knowing the CM's problem hence the entry time BC would decide the before entry reserve level, q^b. This problem is called $P1$.

$$P1 = \min_{q^b} rq^b t + \omega(0)\int_0^t (d(u) - q^b)^+ du + P3 \tag{5}$$

The first term in problem $P1$ is the cost of reserving q^b unit of capacity at a unit cost of r until entry time. The second term is the cost of satisfying the extra demand at initial spot price $\omega(0)$. The third term is the cost of satisfying after entry demand.

Table 6 summarizes the notation of each problem. In the following sections, we will analyze each decision via backward induction.

Table 13: Summary of Notation for the game theoretical model

Notation	Description
$d(t)$	Instantaneous primary and secondary market demand at time t
q^b	Before entry capacity reserve level set by the BC
q^a	After entry capacity reserve level set by the BC
$\omega(t)$	Spot market price of the CM's service, available to the BC at time t
π^s	CM's profit margin from the secondary product
π^p	CM's profit margin from the primary product
c	CM's maximum production capacity rate
r	CM's price to reserve unit of capacity for BC's product
$t_{q_1^a}, t_{q_2^a}$	The small and the large roots of equation $d(t) = q^a$
$t_{q_1^b}, t_{q_2^b}$	The small and the large roots of equation $d(t) = q^b$
$d(t^*)$	Maximum demand rate of the primary and secondary product demand
t_1	The small root of $d(t) = \frac{rq^b}{\pi^s - \pi^p}$
t_2	The large root of $d(t) = \frac{p(c-q^b)}{p-\pi^s}$

6.1 Analysis of After Entry Reserve Level

There is a unique optimal solution to the problem $P3$ and the optimal reserve level $q^a(t)$. Lemma 6.1 through Lemma 6.2 help the analysis of problem P3.

Lemma 6.1. $\frac{r(T-t)}{\omega(t)}$ *is a increasing function of* t.

Proof. We show our claim by taking the first derivative of the function and show that it is always positive. First derivative is:

$$\frac{\partial r(T-t)/\omega(t)}{\partial t} = -\frac{r}{\omega(t)}\left(1 + \frac{\omega'(t)(T-t)}{\omega(t)}\right)$$

The inside of the parenthesis is negative, since the fraction in the parenthesis is less than -1. Because we have $\omega(t)$ is a convex function $\omega'(t)(T-t) < -\omega(t)$. So the first derivative is always positive hence the function is increasing in t. □

Lemma 6.2. *There exists a unique interior time point* τ *such that* $d(\tau) = d(\tau + r(T-\tau)/\omega(\tau))$

Proof. $\tau = T$ satisfies the above statement, however we are in search of an interior time point that makes the above statement true. Since $r(T-\tau)/\omega(\tau)$ is a positive quantity, τ and $\tau + r(T-\tau)/\omega(\tau)$ are the small and the large roots of a statement $d(t) = q$. For symmetric curves the small and large roots are symmetric around the maximum demand point t^*. That is the arithmetic mean of these two roots is t^*; Hence we have:

$$\tau + \frac{r(T-\tau)}{2\omega(\tau)} = t^*$$

Now we will show that there exists a unique τ level that satisfies above statement. Remember that t^* is the time point where maximum demand rate occurs and $t^* = T/2$ by assumption. By Lemma 6.1, $t + \frac{r(T-t)}{2\omega(t)}$ is an increasing function of t. Hence the lowest level of this function is achieved at $t = 0$. At $t = 0$ we have $t + \frac{r(T-t)}{2\omega(t)} = rt^*/\omega(0) < t^*$. Since $r < \omega(t)$ for all $t \in [0, T]$ by assumption. Now evaluate the function at $t = t^*$. We have $t^* + \frac{r(T-t^*)}{2\omega(t^*)} > t^*$ The time point where equality holds is less than t^*, and since the function is always increasing in t the cross over point is unique. □

Proposition 6.1. *There exists a unique optimal reserve level q^a for any entry time, t. Furthermore,*

- *Optimal reserve level:*
  - $q^a = d(t^* + \frac{r(T-t)}{2\omega(t)})$ *for* $t \leq \tau$
  - $q^a = d(t + \frac{r(T-t)}{\omega(t)})$ *for* $t > \tau$
- *Optimal reserve level q^a decreases with in entry time.*

Proof. By Lemma 6.2, $t + \frac{r(T-t)}{2\omega(t)} < t^*$ and $t + \frac{r(T-t)}{2\omega(t)} \geq t^*$ conditions refer to two mutually exclusive ranges of t. By assumption we have $q^a \leq d(t^*)$. For $q^a < d(t^*)$, there are two roots of the equation $d(t) = q^a$. Let the small and the large root of this equation be called $t_{q_1^a}$ and $t_{q_2^a}$. For $q^a = d(t^*)$ small root is equal to the large root and both are equal to t^*. For any entry time $t > \tau$, the objective function is:

$$rq^a(T-t) + \omega(t)(D(t_{q_2^a}) - D(t) - q^a(t_{q_2^a} - t))$$

If we take the derivative of the objective function with respect to q^a and force it to be 0, we have:

$$r(T-t) + \omega(t)\left(d(t_{q_2^a})\frac{\partial t_{q_2^a}}{\partial q^a} - (t_{q_2^q} - t) - q^a\frac{\partial t_{q_2^a}}{\partial q^a}\right) = 0$$

$$r(T-t) - \omega(t)(t_{q_2^q} - t) = 0$$

$$t_{q_2^a} = t + \frac{r(T-t)}{\omega(t)}$$

Note that first term and the third term in parenthesis in the first line above cancel each other out, since $d(t_{q_2^a}) = q^a$. So optimal $q^a = d(t + \frac{r(T-t)}{\omega(t)})$

For $t \leq \tau$ the objective function is:

$$rq^a(T-t) + \omega(t)(D(t_{q_2^a}) - D(t_{q_1^a}) - q^a(t_{q_2^a} - t_{q_1^a}))$$

If we take the derivative of the objective function with respect to q^a and force it to be 0, we have:

$$r(T-t) + \omega(t)\Bigg(d(t_{q_2^a})\frac{\partial t_{q_2^a}}{\partial q^a} - d(t_{q_1^a})\frac{\partial t_{q_1^a}}{\partial q^a} - (t_{q_2^q} - t_{q_1^a}) - q^a(\frac{\partial t_{q_2^a}}{\partial q^a} - \frac{\partial t_{q_1^a}}{\partial q^a})\Bigg) = 0$$

$$r(T-t) - \omega(t)(t_{q_2^q} - t_{q_1^a}) = 0$$

$$t_{q_2^a} - t_{q_1^a} = \frac{r(T-t)}{\omega(t)} \quad \text{and using the fact that } t_{q_1^a} + t_{q_2^a} = 2t^* \text{ we have:}$$

$$\Rightarrow t_{q_2^a} = t^* + \frac{r(T-t)}{2\omega(t)}$$

In the first line above, the first two terms in parenthesis are canceled with the last term in parenthesis, because $d(t_{q_1^a}) = d(t_{q_2^a}) = q^a$. After using some algebra we obtain $q^a = d(t^* + \frac{r(T-t)}{2\omega(t)})$ for $t \leq \tau$.

Now we will prove the second part in Proposition; that is optimal reserve level, q^a decreases with time. Consider the entry times satisfying $t \leq \tau$. If we take derivative of optimal q^a with respect to t, we have:

$$\frac{\partial q^a}{\partial t} = \frac{\partial d(t^* + \frac{r(T-t)}{2\omega(t)})}{\partial t}\Bigg(-\frac{r}{2\omega(t)}\Bigg(1 + \frac{\omega'(t)(T-t)}{\omega(t)}\Bigg)\Bigg)$$

The term in outer parenthesis on right hand side is positive by Lemma 6.1 and the term before the outer parenthesis is negative, because the demand function is decreasing. So, the reserve level decreases as entry time increase. Now consider the entry times satisfying $t > \tau$. If we take the derivative of optimal reserve level with respect to t, we have:

$$\frac{\partial q^a}{\partial t} = \frac{\partial d(t + \frac{r(T-t)}{\omega(t)})}{\partial t}\Bigg(1 - \frac{r}{\omega(t)}\Bigg(1 + \frac{\omega'(t)(T-t)}{\omega(t)}\Bigg)\Bigg)$$

Again, the first term in front of the outer parenthesis is negative and the term inside the parenthesis is positive, yielding a negative first order derivative. Hence, optimal reserve level decreases with entry time. $\square$

Next we analyze CM's problem of entry time.

6.2 Analysis of Entry Time

In the previous problem, we found the BC's optimal capacity reserve level as a recourse upon CM's entry decision. In this section, we will analyze the CM's entry problem with the knowledge of BC's best response function, q^a.

For $t \leq \tau$ BC's best response will be $q^a = d(t^* + \frac{r(T-t)}{2\omega(t)})$ by Proposition 6.1. The objective function for the CM becomes:

$$\begin{aligned} & rq^b t + rd(t^* + \frac{r(T-t)}{2\omega(t)})(T-t) + \pi^p \Bigg(\int_0^t \min\{q^b, d(u)\} du \\ & + \int_t^T \min\{d(t^* + \frac{r(T-t)}{2\omega(t)}), d(u)\} du \Bigg) + \pi^s \int_t^T d(u) du \\ & - p \int_t^T (d(u) - c + d(t^* + \frac{r(T-t)}{2\omega(t)}))^+ du \end{aligned} \tag{6}$$

If we take the derivative of above function with respect to t, and after using some algebra (See Appendix B) we obtain:

$$\begin{aligned} \frac{\partial \prod^G(t)|t \leq \tau}{\partial t} = & rq^b - (\pi^p + \pi^s)d(t) + \pi^p \min\{q^b, d(t)\} - rd(t^* - \frac{r(T-t)}{2\omega(t)}) \\ & + \frac{r(T-t)}{\omega(t)}(\pi^p - r/2) \frac{\partial d(t^* - \frac{r(T-t)}{2\omega(t)})}{\partial t} \left(1 + \frac{\omega'(t)(T-t)}{\omega(t)} \right) \\ & - p(c - d(t^* - \frac{r(T-t)}{2\omega(t)}))(t_{s2} - \max\{t_{s1}, t\}) \end{aligned} \tag{7}$$

For $t > \tau$ again by Proposition 6.1 the BC's best response function is $q^a = d(t + \frac{r(T-t)}{\omega(t)})$. The objective function of the CM becomes:

$$\begin{aligned} & rq^b t + rd(t + \frac{r(T-t)}{\omega(t)})(T-t) + \pi^p \Bigg(\int_0^t \min\{q^b, d(u)\} du \\ & + \int_t^T \min\{d(t + \frac{r(T-t)}{\omega(t)}), d(u)\} du \Bigg) + \pi^s \int_t^T d(u) du \\ & - p \int_t^T (d(u) - c + d(t + \frac{r(T-t)}{\omega(t)}))^+ du \end{aligned} \tag{8}$$

If we take the derivative of above function with respect to t, and after using some algebra (See

Appendix B) we obtain:

$$\begin{aligned}\frac{\partial \prod^G(t)|t>\tau}{\partial t} =& rq^b - (\pi^p+\pi^s)d(t) + \pi^p \min\{q^b, d(t)\} - rd(t+\frac{r(T-t)}{\omega(t)}) \\ &+ \frac{r(T-t)}{\omega(t)}(\pi^p + \frac{\omega(t)-r}{\omega(t)})\frac{\partial d(t+\frac{r(T-t)}{\omega(t)})}{\partial t}\left(1+\frac{\omega'(t)(T-t)}{\omega(t)}\right) \\ &- p(c-d(t^* - \frac{r(T-t)}{2\omega(t)}))(t_{s2} - \max\{t_{s1}, t\})\end{aligned} \tag{9}$$

The derivative of the objective functions in Equation 7 and 9 are extremely hard to evaluate. Hence, in order to provide insights we will analyze special cases and obtain the solution for CM's entry time. We will analyze two special cases; Upon CM's entry decision 1)BC sets his reserve level $q^a = 0$ and 2)BC continues to reserve before entry reserve level, $q^a = q^b$.

6.2.1 $q^a = 0$ Policy

For a given BC's before entry reserve level and $q^a = 0$, the CM's objective function becomes:

$$\max_t rq^b t + \pi^p \int_0^t \min\{q^b, d(u)\}du + \pi^s \int_t^T d(u)du \tag{10}$$

Since, $q^a = 0$ all capacity is used to satisfy secondary market demand, as a result there is no penalty cost incurred for the unsatisfied demand of the secondary market. The entry time depends the before entry reserve level q^b. Proposition 6.2 summarizes the optimal entry time for any level of q^b.

Lemma 6.3.
- *For $q^b \leq d(0)$, unique maximizer of objective function is t=0*
- *For $d(0) < q^b \leq d(t^*)$, there are two maxima of the objective function; first one is the small root of $d(t) = \frac{rq^b}{\pi^s - \pi^p}$ and the second is $t = T$*

Proof. Let, $t_{q_1^b}$ and $t_{q_1^b}$ as the small and the large root of $d(t) = q^b$.

- For $q^b \leq d(0)$ the objective function is $(\pi^p + r)q^b t + \pi^s(D(T) - D(t))$ and the first derivative of the objective function with respect to t is $(\pi^p + r)q^b - \pi^s d(t)$. The derivative of the objective function is negative because $\pi^s > \pi^p + r$ by assumption and $d(t) > d(0), \forall t \in (0, T)$. Hence $t = 0$ is the unique maximizer of the objective function for $q^b \leq d(0)$

- For $d(0) < q^b \leq d(t^*)$ the objective function is

◇ $rq^bt + \pi^p D(t) + \pi^s(D(T) - D(t))$ for $t < t_{q_1^b}$

◇ $rq^bt + \pi^p(D(t_{q_1^b}) + q^b(t_{q_2^b} - t)) + \pi^s(D(T) - D(t))$ for $t_{q_1^b} \leq t < t_{q_2^b}$

◇ $rq^bt + \pi^p(D(t_{q_1^b}) + q^b(t_{q_2^b} - t_{q_1^b}) + D(t) - D(t_{q_2^b})) + \pi^s(D(T) - D(t))$ for $t_{q_2^b}t \leq T$

and the derivative of the objective function with respect to t is:

◇ $rq^b - (\pi^s - \pi^p)d(t)$ for $t < t_{q_1^b}$ and first order condition is: $d(t) = \frac{rq^b}{\pi^s - \pi^p}$. The small root of the equation is the maximizer of the profit function for $t < t_{q_1^b}$.

◇ $-(\pi^p - r)q^b - \pi^s d(t)$ for $t_{q_1^b} \leq t < t_{q_2^b}$. The first derivative is negative, hence $t_{q_1^b}$ is the maximizer of the profit function in this range

◇ $rq^b - (\pi^s - \pi^p)d(t)$ for $t_{q_2^b}t \leq T$ and first order condition is $d(t) = \frac{rq^b}{\pi^s - \pi^p}$. The derivative switches from negative to positive at the large root of the first order condition, hence $t = T$ is the maximizer of this range.

So there are two maximizer of the profit function when $d(0) < q^b \leq d(t^*)$ □

We need to compare the profit function at the two local maxima points and specify conditions which root becomes the global optimal. Let the small root of $\frac{rq^b}{\pi^s - \pi^p} = d(t)$ be called t_1. Lemma 6.4 specifies the conditions when entry time $t = t_1$ is global optimal and when never entry $t = T$ is global optimal.

Lemma 6.4. *There exists a unique before entry reserve level $\hat{q}^b$ such that above which never entry is optimal and below which entry at t_1 is optimal*

Proof. Total profits of the CM if she enters at t_1 is $rq^bt_1 + \pi^p D(t_1) + \pi^s(D(T) - D(t_1))$ where $d(t_1) = \frac{rq^b}{\pi^s - \pi^p}$. Total profits of the CM if she does not enter the secondary market is, $rq^bT + \pi^p(D(t_{q_1^b}) + q^b(t_{q_2^b} - t_{q_1^b}) + D(T) - D(t_{q_2^b}))$. If we subtract the profits at $t = t_1$ from the profits at $t = T$ we have;

$$rq^b(T - t_1) + \pi^p(D(t_{q_1^b}) + q^b(t_{q_2^b} - t_{q_1^b}) + D(T) - D(t_{q_2^b}) - D(t_1)) - \pi^s(D(T) - D(t_1))$$

If we take derivative of the difference of profits at t_1 and $t = T$ with respect to q^b, we obtain:

$$r(T - t_1) - rq^b\frac{\partial t_1}{\partial q^b} + (\pi^s - \pi^p)d(t_1)\frac{\partial t_1}{\partial q^b}$$
$$+ \pi^p(-d(t_{q_2^b})\frac{\partial t_{q_2^b}}{\partial q^b} + d(t_{q_1^b})\frac{\partial t_{q_1^b}}{\partial q^b} + (t_{q_2^b} - t_{q_1^b}) + q^b(\frac{\partial t_{q_2^b}}{\partial q^b} - \frac{\partial t_{q_1^b}}{\partial q^b}))$$

The first and the second terms in the second line are canceled with the last term on the second line because $q^b = d(t_{q_2^b}) = d(t_{q_1^b})$ by definition. Also if we insert $d(t_1) = \frac{rq^b}{\pi^s - \pi^p}$ into the third term in the first line, the second and the third terms in the first line cancel each other. Hence the derivative becomes, $r(T - t_1) + \pi^p(d(t_{q_2^b}) - d(t_{q_1^b})) > 0$, which means that as the before entry capacity reserve level q^b increases, the profits at $t = T$ increases more than the profits at t_1.

If we show two capacity reserve levels such that at one level the profits at t_1 is higher than profits at $t = T$ and at second level (which is greater than the first level) the profits at t_1 is less than the profits at $t = T$, we prove that there is a unique cross over point. Consider a reserve level $q^b = d(t^*)$. Then profits at $t_1 = rd(t^*)t_1 + \pi^p D(t_1) + \pi^s(D(T) - D(t_1))$ and profits at $T = rd(t^*)T + \pi^p D(T)$. If we take the difference of each other:

$$\begin{aligned}\Pi(T) - \Pi(t_1) &= rd(t^*)(T - t_1) - (\pi^s - \pi^p)(D(T) - D(t_1)) \\ &= d(t_1)(T - t_1) - (D(T) - D(t_1)) > 0\end{aligned}$$

Now consider a reserve level $q^b = d(0) + \varepsilon$, where ε is a small value. Then the difference of profits at t_1 and the profits at T are:

$$\begin{aligned}\Pi(T) - \Pi(t_1) = &\, rq^b(T - t_1) - (\pi^s - \pi^p)(D(t_{q_1^b}) + q^b(t_{q_2^b} - t_{q_1^b}) + D(T) - D(t_{q_2^b}) - D(t_1)) \\ &- \pi^s(D(t_{q_1^b}) + q^b(t_{q_2^b} - t_{q_1^b})) \\ &< 0\end{aligned}$$

Because the last term is greater than the first term. Our proof is complete. $\square$

Now we can generalize the optimal entry time as in the following Proposition

Proposition 6.2. *The optimal entry time t for CM under BC's $q^a = 0$ policy is:*

- $q^b \le d(0)$, $t = 0$
- $q^b \le \hat{q}^b$, $t = t_1$,
- $\hat{q}^b < q^b < d(t^*)$, $t = T$

Proof. The statements are direct results of Lemma 6.3 and Lemma 6.4.

Numerical Example

Consider the case where the CM has a capacity level, $c = 300$ units per time. The CM mainly uses her capacity to meet the demand from the BC. The BC has the option to reserve some portion of the CM's capacity by paying a premium upfront. The reserve price per unit capacity the BC pays to the CM is $r = 1$. In exchange for this per unit capacity price, the CM reserves her capacity up to q^b and does not use for any other purposes. In addition to this the BC pays CM $\pi^p = 2$ per unit demanded up to the reserve level. The BC cannot demand more than the reserve level. The demand for the BC's market is represented by the logistics curve with parameters $b = 1$, $a = 200$, $m = 1000$. The length of the planning horizon is $T = 12$ periods. The CM's The CM's capacity is more than enough to serve only one market. She wants to have maximum utilization on her capacity, hence considers using her capacity to meet the demands for a secondary market. The secondary market demand is represented with exactly the same parameters of the BC's own market demand.

The CM knows that at the time she announces her market entry, the BC will not continue to pay for the reserve level from the entry time to the end of the horizon. In other words the CM will loose the BC's business. If she enters the secondary market she will be able to obtain a profit margin of $\pi^s = 3.1$ per unit sold. What should be the CM's best response if the BC reserved, $q^b = 1$, $q^b = 80$, $q^b = 150$, $q^b = 210$, $q^b = 280$ unit capacity per time?

Solution:

Table 14 and Figure 6.2.1 summarize the data and optimal solution for the problem. We will utilize Proposition 6.2 to determine the optimal entry time.

- For $q^b = 1$, we will use the first condition in Proposition 6.2, since $q^b < d(0) = 5$. The entry should be at $t = 0$.

- For $q^b = 80$, the entry is either at $t = t_1$ or $t = T$ whichever is higher. We see in Table 14 that entry at $t_1 = 2.83$ yield higher profits than entry at $t = T$.

- For $q^b = 150$, the entry is either at $t = t_1$ or $t = T$ whichever is higher. We see in Table 14 that entry at $t_1 = 3.65$ yield higher profits than entry at $t = T$.

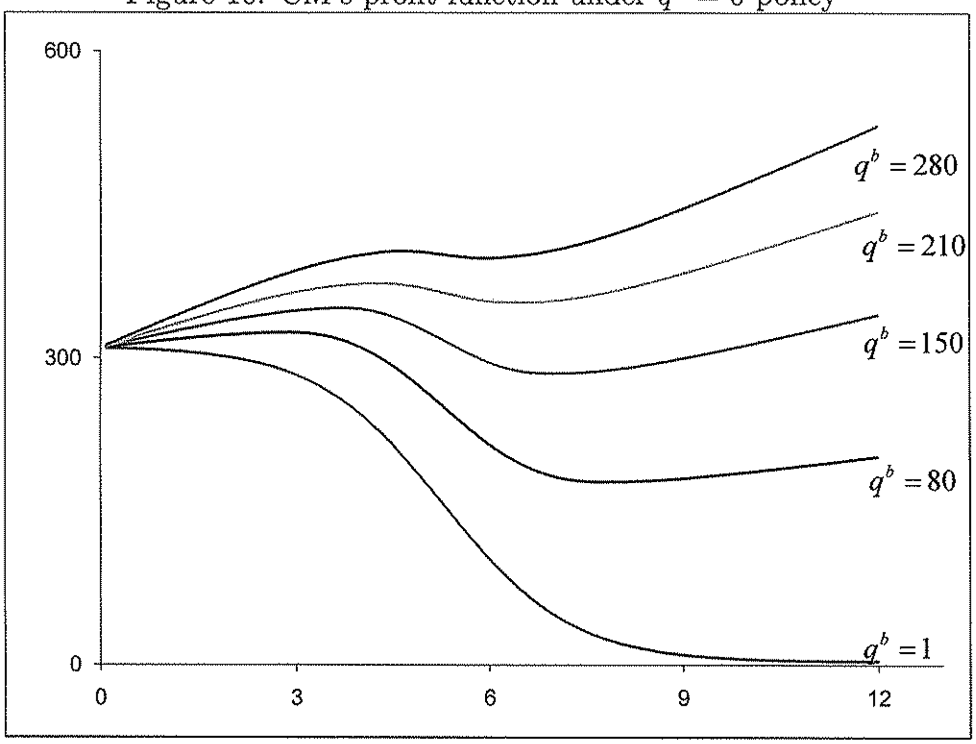
- For $q^b = 210$, the entry is either at $t = t_1$ or $t = T$ whichever is higher. We see in Table 14 that entry at $t = T$ yield higher profits than entry at $t_1 = 4.22$.

- For $q^b = 280$, the entry is either at $t = T$

Table 14: Numerical Example of entry time under $q^a = 0$ policy

q^b	t_1	Profits at t_1	Profits at T	Optimal Policy
1	N/A	N/A	0	Enter at 0
80	2.83	325	205	Enter at 2.83
150	3.65	348.5	342	Enter at 3.65
210	4.22	372.6	442.5	Don't Enter
280	N/A	N/A	526.5	Don't Enter

Figure 16: CM's profit function under $q^a = 0$ policy



6.2.2 $q^a = q^b$ Policy

Now consider the BC's strategy $q^a = q^b$. Then CM's problem is:

$$\max_{t^e} rq^bT + \pi^p \int_0^T \min\{q^b, d(u)\}du + \pi^s \int_t^T d(u)du - p\int_t^T (d(u) - c + q^b)^+ du \qquad (11).$$

The profits are now independent of BC's price and demand parameters because the CM knows that the BC will commit to the q^b reserve level although she enters to the secondary market. The profits are maximized considering the trade off between incurring the penalty cost and profits obtained from the secondary market. Let $t^s_{q_1}$ and $t^s_{q_2}$ be the small and the large roots of $d(t) = c - q^b$. If $c - q^b > d(t^*)$ then no such roots exist. Proposition 6.3 describes the optimal solution to CM's problem under BC's $q^a = q^b$ policy.

Proposition 6.3. • *For* $d(t^*) \leq c - q^b$, $t = 0$

- *For* $d(0) \leq c - q^b < d(t^*)$, *there exists at most two local maximizers of the profit function, one is at* $t = 0$ *and the other is at the large root of* $d(t) = \frac{p(c-q^b)}{p-\pi^s}$ *if* $\frac{p-\pi^s}{p} > \frac{c-q^b}{d(t^*)}$
- *For* $c - q^b < d(0)$ *the maximizer is at the large root of* $d(t) = \frac{p(c-q^b)}{p-\pi^s}$ *if* $\frac{p-\pi^s}{p} > \frac{c-q^b}{d(t^*)}$. *Otherwise* $t = T$ *is optimal.*

Proof.

- For $d(t^*) \leq c - q^b$, $t = 0$ capacity is ample to meet all the demand from the secondary market, hence the CM best response is to enter the market earliest possible.
- For $d(0) \leq c - q^b < d(t^*)$, the function is decreasing between $[0, t^s_{q_1})$, because the first derivative with respect to t is $-\pi^s d(t) < 0$. For $t \in [t^s_{q_1}, t^s_{q_2}]$ the derivative of the profit function is $(p - \pi^s)d(t) - p(c - q^b)$. At time $t^s_{q_1}$ the derivative is negative. If $\frac{p-\pi^s}{p} < \frac{c-q^b}{d(t^*)}$ the derivative will always stay negative hence the profit function will continue to decrease. On the other hand if $\frac{p-\pi^s}{p} > \frac{c-q^b}{d(t^*)}$, the derivative will hit 0 at the small and large roots of $d(t) = \frac{p(c-q^b)}{p-\pi^s}$. The large root is the maximizer since the derivative becomes negative from positive at the large root.
- For $c - q^b < d(0)$ the derivative of the profit function is $(p - \pi^s)d(t) - p(c - q^b)$. again with the same argument as above if $\frac{p-\pi^s}{p} < \frac{c-q^b}{d(t^*)}$ the derivative will stay positive at all times hence

$t = T$ is optimal. On the other hand if $\frac{p-\pi^s}{p} < \frac{c-q^b}{d(t^*)}$ then the profit function will reach the highest profits at the large root of $d(t) = \frac{p(c-q^b)}{p-\pi^s}$ □

Numerical Example:

Consider the similar example in Section 6.2.1. However, in this section upon the CM's entry, the BC continues to utilize from the CM capacity with the same price and before entry reserve level. Differing from the previous section now CM's has to weigh the trade off between using the capacity for the secondary market or for the BC's market after entry. Since CM has to reserve the same capacity level for the BC after entry, there might not be enough capacity left for the secondary market demand. The CM has to pay penalty cost per unit unsatisfied secondary market demand at a level $p = 9$. What should be the CM's best response if the BC reserved, $q^b = 1$, $q^b = 80$, $q^b = 150$, $q^b = 210$, $q^b = 280$ unit capacity per time?

Solution:

We will utilize from the results of Proposition 6.3. We need to check whether $\frac{p-\pi^s}{p} > \frac{c-q^b}{d(t^*)}$ hold for the reserve level. Table 6.2.2 summarize the data and the optimal policy under different BC's given reserve levels.

- For $q^b = 1$, the condition $\frac{p-\pi^s}{p} > \frac{c-q^b}{d(t^*)}$ does not hold. Hence, according to the Proposition 6.3 the optimal entry time is at $t = 0$. It is also illustrated in Figure 17.

- For $q^b = 80$ again the condition $\frac{p-\pi^s}{p} > \frac{c-q^b}{d(t^*)}$ does not hold. Hence, similar to $q^b = 1$, optimal entry time is at $t = 0$.

- For $q^b = 150$, the condition $\frac{p-\pi^s}{p} > \frac{c-q^b}{d(t^*)}$ holds and according to Proposition 6.3 either entry at $t = t_2$ or $t = 0$ optimal. From Table 6.2.2 we see that profits at $t = 0$ are higher. This is also easily seen in Figure 17

- For $q^b = 210$, the condition $\frac{p-\pi^s}{p} > \frac{c-q^b}{d(t^*)}$ holds and according to Proposition 6.3 either entry at $t_2 = 7$ or $t = 0$ optimal. From Table 6.2.2 we see that profits at $t_2 = 7$ are higher than profits at $t = 0$. This is also easily seen in Figure 17

- For $q^b = 280$, the condition $\frac{p-\pi^s}{p} > \frac{c-q^b}{d(t^*)}$ holds and according to Proposition 6.3 either entry at $t_2 = 8.8$ or $t = 0$ optimal. From Table 6.2.2 we see that profits at $t_2 = 8.8$ are higher than profits at $t = 0$. This is also easily seen in Figure 17

Figure 17: CM's profit function under $q^a = q^b$ policy

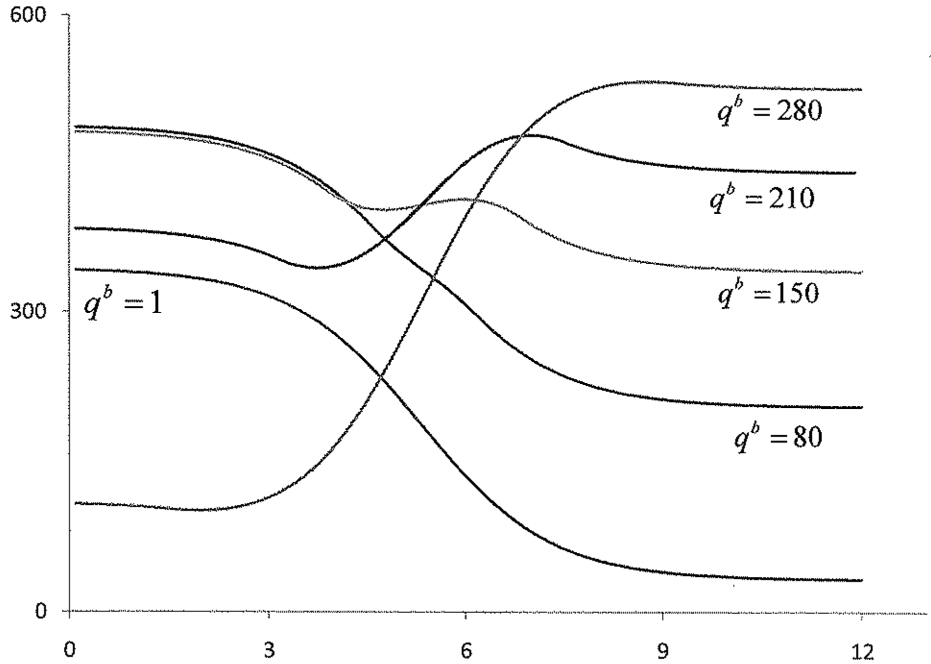


Table 15: Numerical Example of entry time under $q^a = q^b$ policy

q^b	$\frac{p-\pi^s}{p} \overset{?}{>} \frac{c-q^b}{d(t^*)}$	Profits at 0	Profits at t_2	Profits at T	Optimal Policy
1	No	342.6	N/A	33.1	Enter at 0
80	No	487.2	N/A	204.9	Enter at 0
150	Yes	482.1	414.7	342	Enter at 0
210	Yes	384.4	479.4	442.3	Enter at $t_2 = 7$
280	Yes	107.7	533.5	526.5	Enter at $t_2 = 8.8$

We obtained the optimal best response functions for the CM after observing the BC's before entry capacity reserve level q^b. Propositions 6.2 and 6.3 summarizes the best response decisions of the CM under the BC after entry $q^a = 0$ and $q^a = q^b$ policies, respectively. In the next section we will analyze the first stage of the game, namely BC's before entry reserve level, q^b for his after entry $q^a = 0$ and $q^a = q^b$ policies.

6.3 Analysis of Before Entry Reserve Level

Using the results of Proposition 6.2 and 6.3, we will solve for the BC's optimal before entry reserve level, q^b under $q^a = 0$ and $q^a = q^b$ policies.

6.3.1 $q^a = 0$ Policy

Under $q^a = 0$ policy BC's problem 5 becomes:

$$\min_{q^b} rq^b t + \omega(0)\int_0^t (d(u) - q^b)^+ + \omega(t)\int_t^T d(u)du$$

According to Proposition 6.2 for a given q^b level, CM's best response is either $t = 0$, $t = t_1$ or $t = T$. Let the small and large roots of $d(t) = \hat{q}^b$ be called $\hat{t}_{q^1}$ and $\hat{t}_{q^2}$ respectively. The following Lemmas describe BC's best policy.

Proposition 6.4. *If $\hat{t}_{q^2} - \hat{t}_{q^1} < \frac{rT}{\omega(0)}$ then BC is better off by setting $q^b = d(t^* - \frac{rT}{2\omega(0)})$ and force CM never enter the market than setting $q^b < d(0)$ and letting CM enter the market at the beginning of the horizon*

Proof. If BC sets $q^b < d(0)$, the CM's best response would be $t = 0$ and BC's cost would be $\omega(0)D(T)$. $d(t^* - \frac{rT}{2\omega(0)}) > \hat{q}^b$ because $\hat{t}_{q^2} - \hat{t}_{q^1} < \frac{rT}{\omega(0)}$. If BC sets $q^b = d(t^* - \frac{rT}{2\omega(0)})$ the CM's best response is to never enter the market. In this case the BC's cost function becomes $rd(t^* - \frac{rT}{2\omega(0)})T + \omega(0)(D(t_{q_2^b}) - D(t_{q_1^b}) - q^b(t_{q_2^b} - t_{q_1^b})))$. If we insert $t_{q_2^b} - t_{q_1^b} = \frac{rT}{\omega(0)}$ the BC's cost would become $\omega(0)(D(t_{q_2^b} - D(t_{q_1^b}))$ which is less than $\omega(0)D(T)$. Hence we prove our claim. $\square$

If BC sets the before entry reserve level, $d(0) < q^b < \hat{q}^b$ then the CM would enter the market at $t = t_1$ where $d(t_1) = \frac{rq^b}{\pi^s - \pi^p}$. In this case BC's cost would be, $rq^b t_1 + \omega t_1(D(T) - D(t_1))$. We are not able to show analytically which before entry reserve level within the range $(d(0), \hat{q}^b$ minimizes the BC's cost. Also without knowing the spot market price function and the demand parameters it is hard to compare $rq^b t_1 + \omega t_1(D(T) - D(t_1))$ with the never entry cost $\omega(0)(D(t_{q_2^b} - D(t_{q_1^b}))$.

6.3.2 $q^a = q^b$ Policy

Under $q^a = q^b$ policy BC's problem 5 becomes:

$$\min_{q^b} rq^bT + \omega(0)\int_0^t (d(u) - q^b)^+ + \omega(t)\int_t^T (d(u) - q^b)^+ du$$

According to Proposition 6.3 for a given q^b level, CM's best response is either $t = 0$, $t = t_2$ or $t = T$. Remember that t_2 is a local minimizer provided that $\frac{p-\pi^s}{p} > \frac{c-q^b}{d(t^*)}$. If the condition $\frac{p-\pi^s}{p} > \frac{c-q^b}{d(t^*)}$ does not hold the BC would optimize his cost function over the entry times $t = 0$ and $t = T$.

If $\frac{p-\pi^s}{p} < \frac{c-q^b}{d(t^*)}$ the CM will either enter the market at $t = 0$ or never enter the market. In both cases the BC's cost function will be $rq^bT + \omega(0)(D(t_{q_2^b}) - D(t_{q_1^b}) - q^b(t_{q_2^b} - t_{q_1^b}))$. If we take derivative with respect to q^b and set it to 0, we obtain $q^b = d(t^* - \frac{rT}{2\omega(0)})$; and at this reserve level the BC cost function becomes $\omega(0)(D(t_{q_2^b}) - D(t_{q_1^b}))$. This cost is equivalent to the costs under $q^a = 0$ policy and preventing CM enter the market.

We showed that under the special case $q^a = 0$, and $q^a = q^b$ there are only three time points that are candidate to be optimal entry time. Among these entry now or never is the common to the two special cases. Under $q^a = 0$ policy the CM's interior local maximum is earlier than the interior local maximum in $q^a = q^b$ policy. When considered it is also intuitive, the CM would benefit entering the market earlier if she is not getting the BC's business. With $q^a = q^b$ policy the BC has less option to manipulate the CM's entry time, and the minimum level of costs that he can generate can be also generated in $q^a = 0$ policy. $q^a = 0$ policy provides more choices for BC to manipulate the CM's entry time.

Next we demonstrate the BC's cost using the same numerical examples in CM's entry time model.

Numerical Example: Consider the numerical example in Section 6.2.1. Assume that the spot market price is modeled as $\omega(t) = r + \frac{1}{\sqrt{t}}$. What would be the BC's overall cost under:

(a) $q^a = 0$ policy?

(b) $q^a = q^b$ policy?

if before entry reserve levels are $q^b = 1$, $q^b = 80$, $q^b = 150$, $q^b = 210$, $q^b = 280$.

<u>Solution:</u>

Table 16: BC's cost under $q^a = 0$ policy, numerical example

q^b	CM's entry Time	BC's costs
1	0	4158.15
80	2.83	1693.97
150	3.65	1836.46
210	Never	3476.00
280	Never	3360.00

Table 17: BC's cost under $q^a = q^b$ policy, numerical example

q^b	CM's entry Time	BC's costs
1	0	4118.52
80	0	2849.67
150	0	2582.43
210	Never	2712.29
280	Never	3360.00

(a) Table 16 summarizes the CM's entry time and BC's overall cost under $q^a = 0$ policy. If BC set $q^b = 1$, then from the numerical example in previous section, we know that the CM will enter at time $t = 0$ and if we insert $t = 0$ into the BC's problem $\min_{q^b} rq^b t + \omega(0) \int_0^t (d(u) - q^b)^+ + \omega(t) \int_t^T d(u)du$ and plug in the parameters we obtain the BC's cost as 4158.15 as in Table 16. Similarly for other q^b levels, we use the CM's optimal entry decision and calculate the BC's cost accordingly. According to this, among the before entry reserve levels $q^b = 1$, $q^b = 80$, $q^b = 150$, $q^b = 210$, $q^b = 280$, $q^b = 80$ yields the smallest BC's cost.

(b) Table 17 summarizes the CM's entry time and BC's overall cost under $q^a = q^b$ policy. If the BC sets $q^b = 1$, the from the numerical example in previous section, the CM's entry decision is at $t = 0$. If we plug $t = 0$ into the BC's model $\min_{q^b} rq^b T + \omega(0) \int_0^t (d(u) - q^b)^+ + \omega(t) \int_t^T (d(u) - q^b)^+ du$ and use the problem parameters, we obtain the BC's cost as 4118.52. Similarly we obtain the BC's cost for other before entry reserve levels. According to this, among the before entry reserve levels $q^b = 1$, $q^b = 80$, $q^b = 150$, $q^b = 210$, and $q^b = 280$, $q^b = 150$ yields the smallest cost for the BC. However, the costs are higher than the $q^a = 0$ policy.

$q^a = 0$ yields the minimum cost for the BC, although the CM enters the market in the planning horizon. As seen from the numerical example, the CM's early entry might actually be good for the

BC. Overall, BC's best policy in terms of reserve level q^b depends on the demand parameters and spot market prices. Next we summarize the contributions of this section.

6.4 Summary of Results

We can summarize this section's contributions and results as follows:

- We showed that the BC's optimal after entry allocation policy (solution to P3) is decreasing in entry time. This is intuitive because first the after entry reserve level is independent of the before entry reserve level and as the entry time increases, the BC's remaining market potential decreases, hence he requires less reserved capacity to meet the demand.

- Due to inability to solve second stage problem analytically, we solved CM's entry time in special cases $q^a = 0$ and $q^a = q^b$. Under $q^a = 0$ we showed that entry time is now, never or an interior point closer to the beginning of planning horizon. We provided the bounds at which the shift in optimal entry time is realized. Under $q^a = q^b$ policy we showed that entry time is at now, never or at an interior point closer to the end of horizon.

- Under $q^a = q^b$ policy the BC does not have any bargaining power to effect the CM's entry time.

Above results provides insights on the BC's possible capacity reservation policies in the presence of CM's incentive to enter a new market. These results are obtained when the BC has an alternative supplier spot market whose benefits increase over time. More detailed analysis could be done in a future research when the spot market price relations has direct effect on the BC's demand.

7 Conclusions

In this chapter, we thoroughly analyzed a contract manufacturer's incentive to develop her own market presence and her strategic market entry timing problem from a wide variety of settings. We established theoretical foundations on the existing empirical market entry literature. We also extend the theoretical results obtained in the previous research models. In the first part, we approached this problem when the products in the existing market and the potential market are similar. We showed the complete characterization of the profit function and optimal entry decision. As a side benefit in this section we showed that limited capacity imitates the cannibalization parameter in the demand models of technological substitution models. In the second model we show that when the products are different or competing each other the CM's profits are always less than the profits in the first model. This is also intuitive hence, distinct products generally lead to inefficient use of resources and competing products reflects the conflicting goals which yield to lower profit levels. In the final section we modeled the entry timing problem in a game theoretical environment and showed that under special cases, the CM's entry decision is shifted earlier when there is a threat of losing BC's business.

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