Pricing During Disruptions: A Cause of the Reverse Bullwhip Effect

Ying Rong
Department of Industrial Engineering and Operations Research, University of California, Berkeley, CA 94720, rong@berkeley.edu

Zuo-Jun Max Shen
Department of Industrial Engineering and Operations Research, University of California, Berkeley, CA 94720, shen@ieor.berkeley.edu

Lawrence V. Snyder
Department of Industrial and Systems Engineering, Lehigh University, Bethlehem, PA 18015, larry.snyder@lehigh.edu

When supply disruptions occur, firms want to employ an effective pricing strategy to reduce losses. However, firms typically don't know precisely how customers will react to price changes in the short term, during a disruption. In this paper, we investigate three different pricing strategies, which we call naive, one-period correction (1PC), and regression, each of which makes progressively more sophisticated assumptions about customer behavior. We prove that strategies that appear to be more sophisticated may in fact lead to reduced profits and greater volatility. In particular, the 1PC pricing strategy produces a more volatile demand process and smaller revenue than the naive one does. Moreover, when customer behavior is sufficiently strategic, the customer order process under the 1PC pricing strategy is more volatile than the capacity process, a phenomenon known as the reverse bullwhip effect (RBWE). As supply disruptions become longer or more severe, the magnitude of the variability difference between the customer's orders and the capacity increases under the 1PC strategy but decreases under the naive one, while the firm's revenue decreases under both strategies. Furthermore, although the regression pricing strategy is a more advanced approach, it leads to smaller profit and greater customer's order variability than the naive pricing strategy (but the opposite when compared to the 1PC strategy). We conclude that the naive pricing strategy is superior to either of the more sophisticated ones, both in terms of the firm's profit and the magnitude of the customer's order variability in the supply chain as a whole.

Key words: bullwhip effect, reverse bullwhip effect, supply uncertainty, modeling error

1. Introduction

In August 2005, Hurricane Katrina shut down most of the Gulf-region drilling facilities, which produce approximately 7% of America's oil consumption and 16% of its natural gas consumption, and idled approximately 10% of U.S. refining capacity (Mouawad 2005a). Oil producers and refiners worked at nearly their full (newly reduced) capacity, implying very little variability in the production rates of these supply chain stages. Meanwhile, nationwide prices for gasoline rose well over $3 per gallon across the U.S., up from an average of $2.60 the week before (Mouawad and Romero 2005). A hoarding mentality among consumers made gasoline buying patterns chaotic (Gold and Herrick 2005).

In March 2008, Vietnam announced that it was cutting rice exports by 22% due to an rare plant virus (Bradsher 2008). India, Egypt and Cambodia quickly followed suit and curbed overseas sales. In Thailand, the world's top rice exporter, prices topped $1,000 a ton for the first time ever. However, there is substantial evidence that the supply of rice was ample during this time (Balfour 2008, Gogoi 2008). The price spikes were due to uneasy customers' speculative purchasing behavior, buying more rice than their immediate need because they anticipate rising prices in the future.

In both the gasoline and rice cases, supply glitches created panic on the part of customers, resulting in skewed buying patterns. Since the volatility of the supply process was small during the disruptions, these examples suggest the existence of a *reverse bullwhip effect* (RBWE), in which demand variability increases as one moves *downstream* through the supply chain. (In contrast, the classical bullwhip effect (BWE) refers to the amplification of variability as one moves upstream.)

The RBWE is a phenomenon with which supply chain professionals are acquainted. An operations executive at the semiconductor firm LSI (formerly Agere Systems) summarized the phenomenon as follows: 1em

> Whenever there is a perceived shortage of supply, it amplifies as it propagates down the supply chain. In fact, forward and reverse bullwhip effects often act as a system. If you start with a sudden upturn in demand, it gets amplified as it goes upstream, which creates a perceived shortage that amplifies as it propagates downstream. This creates a panic and downstream consumers overstate their demand, which amplifies again as it goes upstream. In turn, a greater scarcity is felt, and so on. Each of these effects feeds the other. (Armbruster 2006)

The same executive argues that the rapidly destabilizing effect of this vicious cycle is at least partly responsible for both the dot-com boom in the late 1990s and the subsequent bust in the 2000s, the effects of which many semiconductor firms and their supply chain partners are still trying to come to terms with.

Lee et al. (1997) show that price fluctuations can cause the BWE if the buyer faces iid demand, if the buyer has complete information about the pricing pattern set by its supplier, and if the price set by the supplier is iid. In contrast, we show that pricing decisions can trigger the RBWE when the price and the buyer's order quantity are interdependent. Unlike Lee et al. (1997), we assume that customers respond not only to the price in the current period but also to the trend in price changes over the periods since the price is no longer iid during a supply disruption. The firm possesses knowledge of the underlying long-run customer demand when the price is stable but does not know how customers will behave in face of price changes during disruptions.

We study three pricing strategies under this setting. The first one is called the "naive" pricing strategy. Under this pricing strategy, the firm ignores any response by customers to price changes and chooses the price solely based on the realized capacity and the underlying long-run customer demand-price curve. We show that the naive pricing strategy always generates a customer ordering process that is more stable than the capacity process, and therefore the BWE occurs.

The second strategy is called the "one-period correction" (1PC) pricing strategy. In this strategy, the firm takes into account the deviation of the customer orders from the long-run demand-price curve in the previous period when it makes its pricing decision. We find that the variability of (an approximation of) the customer order quantity under the 1PC pricing strategy can be higher than that of the capacity process, and therefore that the RBWE occurs. Moreover, we find that the firm's revenue under the 1PC pricing strategy is lower than that under the naive pricing strategy. Thus the 1PC pricing strategy not only creates a more volatile customer ordering pattern, but also matches supply and demand poorly.

The last strategy is called the "regression" pricing strategy and is based on a least-squares fit of historical data on price and customer order pairs. Through numerical analysis, we find that no matter how much history is utilized, the performance of the regression pricing strategy, in terms of customer order variability and the firm's revenue, always lies between that of the naive and 1PC pricing strategies.

Our paper is similar in spirit to that of Cooper et al. (2006), who show that when decision makers do not understand customers' behavior completely, "optimization" may produce increasingly worse flight seat allocation over time for airlines. Our paper also addresses how a modeling error undermines the benefit of a seemingly optimized strategy. Under the setting of supply disruptions, we find that the 1PC and regression pricing strategies, which appear to be the better-optimized strategies since they account for historical data and forecasting in the decision model, produce worse results than the naive strategy does in terms of both the firm's revenue and the stability level of customer's order process. It might be possible to fix the modeling error under normal operating conditions, but it is hard to avoid under supply disruptions due to their infrequent occurrence. As the managers do not have enough experience to build correct intuition, our comparison of different pricing strategies helps them set the price in the face of the disruptions.

The remainder of the paper is organized as follows. In Section 2, we review the literature related to our model. In Section 3, we introduce the settings that are common to all the analysis in this paper. We analyze the naive and 1PC pricing strategies in Sections 4 and 5, respectively. We compare these two strategies in Section 6. In Section 7, we compare the regression pricing strategy with the naive or 1PC pricing strategies through numerical analysis. Finally, we provide concluding remarks and future research directions in Section 8.

2. Literature Review

The BWE is one of the most famous and well studied phenomena in supply chain management. Researchers have performed extensive theoretical analyses and have documented its occurrence in classroom settings using the beer game (Lee et al. 2004). Case studies have noted its presence in a wide range of products and industries. On the other hand, recent evidence suggests that the BWE may not be as prevalent as previously thought. For example, the results of the extensive macroeconomic study by Cachon et al. (2007) suggest that the BWE may be more of a special case than a general rule, and some empirical studies using the beer game have found that the BWE is often absent in upstream supply chain stages (Croson and Donohue 2003, Kaminsky and Simchi-Levi 2000, Rong et al. 2007).

One of the explanations of the absence of the BWE is that supply uncertainty, not just demand uncertainty, brings volatility into the system. Rong et al. (2007) introduce supply disruptions into the beer game and find that players tend to put more emphasis on the status of the supply line when disruptions are present, which tends to be underestimated in the traditional beer game. This change in players' behavior, coupled with other factors, accounts for the RBWE. Rong et al. (2008) use the so-called rationing game to demonstrate that the competition for scarce resources, which can cause the BWE in the downstream portion of the supply chain (Lee et al. 1997), can trigger the RBWE in the upstream portion, as well. (Ozelkan and Cakanyildirim (2009) and Ozelkan and Lim (2009) also use the term "reverse bullwhip effect," but they use it to refer to the amplification of downstream price variability.)

Our results rely on an analysis of customer behavior during disruptions. The study of customer behavior under normal circumstances is an emerging area in the operations management literature (Elmaghraby and Keskinocak 2003, Shen and Su 2007). There are two classes of customers studied in the literature. Myopic customers make purchasing decisions only based on the price in the current period, whereas the decisions made by strategic customers take into account the future path of prices. Anderson and Wilson (2003) and Zhou et al. (2005) analyze optimal purchasing decisions when certain types of pricing strategies implemented by the firms are known to the customers. The work by Su (2007), Su and Zhang (2008), Elmaghraby et al. (2008), Aviv and Pazgal (2008), and others provides an analysis of the optimal pricing decision in the presence of strategic customers.

Unlike the work on dynamic pricing cited above, in which customers optimize their purchasing decisions based on their beliefs about prices in the future, Ahn et al. (2007) assume that the demand is a function of the price over multiple periods in the past. Their assumption is that customers stay in the system for a certain number of periods and will purchase as long as the price is lower than their valuation. Our model of customer behavior is similar to that of Ahn et al. (2007), as we also assume that the customers' order is a function of the price in the current and previous periods. However, our assumption is that customers adjust their orders based the anticipated trend in future price changes inferred from the current trend. Thus, our assumed customer behavior is also in line with the spirit of the strategic customer. However, the major focus of this paper is to study how pricing strategies affect the variability of customers' orders.

When the product is durable, customers need to decide not only when to purchase in order to satisfy their immediate demand but also how much to store for future needs. Empirical studies by Gonul and Srinivasan (1996), Erdem et al. (2003) and Hendel and Nevo (2006) investigate the impact of the customer's anticipation of future prices on purchase decisions for durable goods through scanner data of disposable diapers, ketchup, and laundry detergent, respectively. Su (2008) uses rational expectations to study the equilibrium of the firm's pricing decision and customers' purchasing decision in the presence of stockpiling. Our paper does not model customer stockpiling directly. However, we capture this behavior implicitly through the difference between the order quantity and the underlying demand, which is positively correlated with the pricing change the customer anticipates in the future.

Most pricing literature assumes that the firm has full (or at least partial) knowledge of its customer's demand process. However, customer behavior during disruptions is less predictable, since disruptions occur infrequently and firms have less historical data. Similarly to the work by Cooper et al. (2006), we find that more advanced pricing strategies turn out to produce worse outcomes than a simple one.

The research on supply disruptions provides different strategies to mitigate their impact. Parlar and Berkin (1991), Berk and Arreola-Risa (1994), and many others focus on how to use inventory to hedge against supply disruptions. Tomlin (2006) and Babich et al. (2007b) study the benefits of supplier redundancy. Babich (2007) and Babich et al. (2007a) consider the interaction between finance decisions and operational decisions under supply disruptions.

Swinney and Netessine (2008), Li and Gao (2008) and Shou et al. (2009) use contracting to coordinate a decentralized supply chain subject to disruptions. Feng (2008) integrates pricing and order decisions when the demand model is known under disruptions. Our paper contributes to this stream of research by investigating how the different pricing strategies can mitigate or magnify losses triggered by disruptions when the form of the customer behavior is unknown.

3. Common Settings

We consider a monopolist firm operating in a periodic-review system with a single capacity disruption occurring at the beginning of the horizon. The firm sells a product to multiple customers, whose demand we aggregate into a single customer for modeling purposes. In each period, the firm first realizes its capacity level, then it makes its pricing decision, and then customers reveal their orders based on certain assumed behavior. In the subsections that follow, we describe the settings for the capacity process, our assumptions about customer behavior, and the workings of the three pricing strategies. We also give a formal definition of the bullwhip and reverse bullwhip effects under our setting.

3.1. Capacity Process

We assume that in period 1, the capacity experiences a sudden drop to level A and recovers to the original capacity level, B, linearly over the course of T periods. That is, c_t, the capacity in period t, is given by

$$c_t = \begin{cases} A + \Delta c(t-1), & 1 \le t \le T \\ B, & \text{otherwise} \end{cases} \tag{1}$$

where $\Delta c = \frac{B-A}{T-1}$. Δc represents the rate of capacity recovery. We require $T \geq 3$ for the sake of mathematical deduction. In Appendix 9.1, we relax the assumption that the capacity is deterministic and instead consider random capacity whose mean follows a linear recovery path.

3.2. Assumptions on Customer Behavior

As stated above, our model considers a single customer, though this may represent an aggregate of many customers in the market. We use the term *underlying customer demand*, or simply *demand*, to refer to the customer's underlying need given that the price is fixed over the horizon, and the term *order* to refer to the customer's purchasing decision in a certain period given the price history.

We assume that the underlying customer demand is linear in the price, i.e. $p = mQ + b$, where p and Q are the price and underlying customer demand, respectively. We require $m < 0$ and $b > 0$; that is, the demand cannot be perfectly elastic. We assume that $c_t = B$ and $p_t = P$ for $t \leq 0$. We call B and P the long-run equilibrium capacity and price, respectively. They satisfy $P = mB + b$.

During the supply disruption and the subsequent recovery, the firm may adjust the price in each period, attempting to match demand and supply. The customer observes not only the price in the current period but also the history of the price. We assume that the customer responds to the price change by setting its order quantity in period t to

$$Q_t = \frac{b - p_t}{-m} + r\frac{p_t - p_{t-1}}{-m}, \ r \in [0,1) \tag{2}$$

We call (2) the *short-run demand curve in period t*. The first term represents the customer's underlying demand while the second represents the customer's response to the price change. If the current price p_t is higher than the previous price p_{t-1}, we assume that the customer thinks that the price will be even higher in the future, and as a result, it increases its order in the current period. Conversely, when the price drops in the current period, the customer responds in the opposite manner—that is, its order is smaller than the underlying demand. Based on these two assumptions, we require $r \geq 0$. Moreover, $r < 1$ implies that the customer's order quantity is still negatively correlated with the price in the current period. If $r = 0$, the customer is myopic and ignores the changes in price. In contrast, $r > 0$ implies that the customer is strategic. We say that the larger r is, the more strategic the customer is.

3.3. Three Pricing Strategies

We assume that the firm only has information about the customer's underlying demand-price curve, but not about the customer behavior mechanism during the disruption. However, the firm can use the customer's past orders to infer information about this mechanism when choosing its price.

We assume that in every period t, the firm produces its capacity c_t, since c_t is less than the long-run capacity equilibrium. (This assumption is often reasonable in the real world. For example, the U.S. oil refining industry currently operates with nearly no excess capacity (Mouawad 2005b).) The firm's only decision is therefore the price. However, due to its limited information about customer behavior, the pricing decision may result in an order quantity Q_t that is not equal to c_t. In general, the quantity sold equals $\min\{Q_t, c_t\}$. We do not consider inventory carry-over at the firm. We assume that if $Q_t > c_t$, the excess demand is lost, and if $Q_t < c_t$, the surplus inventory is scrapped.

During the supply disruption and recovery, the firm employs one of three pricing strategies to account for its lack of knowledge about the short-run demand curve. We call these the *naive*, *one-period correction (1PC)*, and *regression pricing strategies*. Let p_t^n [p_t^c, p_t^r] and Q_t^n [Q_t^c, Q_t^r] represent the price and customer order in period t under the naive [1PC, regression] pricing strategy.

Under the naive pricing strategy, the firm ignores any strategic part of the customer and sets the price according to the realized capacity and the underlying long-run demand curve:

$$p_t^n = mc_t + b. \tag{3}$$

The customer's order under such a pricing strategy is given by

$$Q_t^n = \frac{b - p_t^n}{-m} + r\frac{p_t^n - p_{t-1}^n}{-m} = c_t - r(c_t - c_{t-1}). \tag{4}$$

Clearly, the customer's order quantity does not equal the capacity during the supply disruption and recovery if $r \neq 0$. However, under this pricing strategy, instead of reacting to the difference between Q_t and c_t, the firm simply treats the underlying demand curve as the short-run demand curve.

Under the 1PC pricing strategy, the firm recognizes the discrepancy between the revealed order and the underlying demand. We assume that the firm believes that the short-run demand curve is a translation of the long-run one; that is, that the slope of the short-run demand curve stays the same but the intercept changes. Accordingly, the firm estimates the intercept of the short-run demand curve in period t based on its observation of the discrepancy in the previous period, as captured by the formula $b_t^c = p_{t-1}^c - mQ_{t-1}^c$. Then the firm sets the price in period t according to $p_t^c = mc_t + b_t^c$. The decision dynamic under the 1PC pricing strategy is equivalent to

$$p_t^c = mc_t + r(p_{t-1}^c - p_{t-2}^c) + b \tag{5}$$

$$Q_t^c = \frac{(1-r)p_t^c + rp_{t-1}^c - b}{m} \tag{6}$$

Under the regression pricing strategy, the firm applies the least-squares fit to N pairs of prices and customer orders, from periods $t-N$ through $t-1$, to estimate the short-run demand curve in period t. For period $i < N$, the firm does not have N periods' worth of history. In this case, the firm sets $p_i^r \sim U(0,b)$ and $Q_i^r = \frac{p_i^r - b}{m}$ for $-N \leq i \leq -1$; that is, it uses N randomly selected points along the underlying demand curve in place of the missing historical data. Under these assumptions, for $t \geq 0$, the estimated short-run demand slope and intercept are

$$b_t^r = \frac{\sum_{i=t-N}^{t-1} p_i^r \sum_{i=t-N}^{t-1} (Q_i^r)^2 - \sum_{i=t-N}^{t-1} Q_i^r \sum_{i=t-N}^{t-1} p_i^r Q_i^r}{n \sum_{i=t-N}^{t-1} (Q_i^r)^2 - (\sum_{i=t-N}^{t-1} Q_i^r)^2}$$

$$m_t^r = \frac{n \sum_{i=t-N}^{t-1} p_i^r Q_i^r - \sum_{i=t-N}^{t-1} Q_i^r \sum_{i=t-N}^{t-1} p_i^r}{n \sum_{i=t-N}^{t-1} (Q_i^r)^2 - (\sum_{i=t-N}^{t-1} Q_i^r)^2}$$

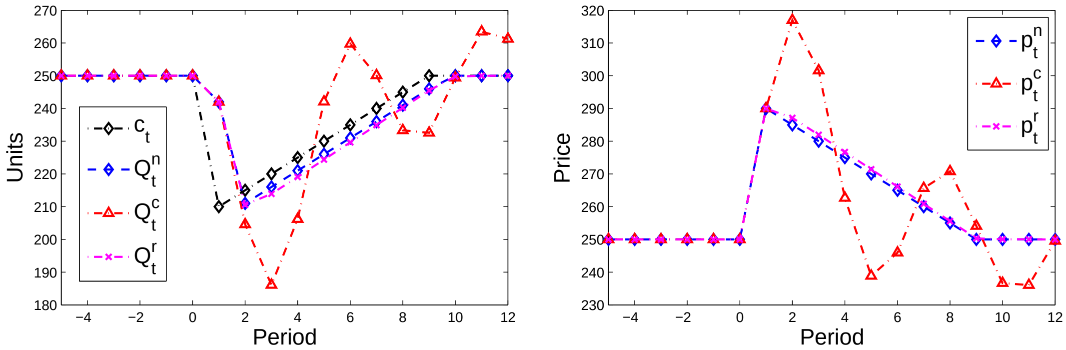
Notice that b_t^r and m_t^r are random due to the uniform draws representing the initial conditions. The decision dynamic under the regression pricing strategy is equivalent to

$$p_t^r = m_t^r c_t + b_t^r \tag{7}$$

$$Q_t^r = \frac{(1-r)p_t^r + rp_{t-1}^r - b}{m} \tag{8}$$

In Figure 1, we plot the customer's orders and the firm's prices under the three different pricing strategies. The capacity follows the linear recovery described by (1). Note that Q_t^c appears to be more volatile than c_t. On the other hand, the volatility of Q_t^n and Q_t^r are similar to that of c_t. Moreover, the naive and regression prices follow a downward trend during the recovery process, while the 1PC prices follow a cyclic pattern, with diminishing amplitude, around the the naive and regression prices. (A cyclic price pattern is also identified by Su (2008) under different settings.) We provide a more detailed investigation of these results in Sections 4–7.

Below, we omit the superscript on p^n, p^c, p^r, Q^n, Q^c, and Q^r when the description is applicable to all three pricing startegies.



a: Customer Orders b: Firm's Prices

Figure 1 **Difference among Three Different Pricing Strategies**

3.4. Definition of BWE and RBWE

Although the production c_t and demand Q_t are both deterministic, neither is constant. We define the average value and variability for Q_t from period 1 to period t as

$$avg(Q,1,t) = \frac{1}{t}\sum_{1}^{t} Q_i \tag{9}$$
$$var(Q,1,t) = \frac{1}{t}\sum_{1}^{t}(Q_i - avg(Q,1,t))^2$$

(The terms "mean" and "variance" are not appropriate here since the quantities are deterministic.) The average value and variability for c_t are defined similarly, with Q replaced by c in (9).

Similar to the well known identity relating variance and mean, we have

$$var(Q,1,t) = \frac{1}{t}\sum_{1}^{t} Q_i^2 - (avg(Q,1,t))^2 \tag{10}$$
$$var(c,1,t) = \frac{1}{t}\sum_{1}^{t} c_i^2 - (avg(c,1,t))^2$$

We say that the bullwhip effect (BWE) occurs if $var(c,1,t) > var(Q,1,t)$, and the reverse bullwhip effect (RBWE) occurs if $var(c,1,t) < var(Q,1,t)$.

4. Naive Pricing Strategy

In this section, we show that the customer's order process is less variable than the capacity process under the naive pricing strategy. Under the linear capacity recovery process, the customer's order quantity in (4) can be transformed to

$$Q_t^n = \begin{cases} A + r(T-1)\Delta c & t = 1 \\ A + (t-1-r)\Delta c & 2 \le t \le T \\ c_t & \text{otherwise} \end{cases} \tag{11}$$

Note that $Q_t^n = c_t$ for $t \geq T+1$. Thus we only need to compare the variability of demand and capacity from period 1 through period $T+1$, that is, from the start of the disruption through the start of the normal state. By comparing Q_t^n and c_t, one can show that $Q_1^n \geq c_1$ and for $t > 1$, $Q_t^n \leq c_t$, which is also evident from Figure 1. This is because, under the naive pricing strategy, the price increases in the first period as the capacity drops. Then the price decreases as the capacity recovers over the remaining periods.

Using (11), we can obtain the following lemma.

Lemma 1. 1.

$$avg(Q^n, 1, T+1) = avg(c, 1, T+1)$$

2.

$$var(Q^n, 1, T+1) - var(c, 1, T+1) = r(r-1)\frac{T(T-1)}{(T+1)}\Delta^2 c \tag{12}$$

Proof Part 1: Follows from (11).

Part 2: By (10), we only need to compare $\sum_1^t (Q_i^n)^2$ and $\sum_1^t c_i^2$. From (11), we have:

$$\begin{aligned} & var(Q^n, 1, T+1) - var(c, 1, T+1) \\ = & \frac{1}{T+1}\left(\sum_1^{T+1}(Q_i^n)^2 - \sum_1^{T+1} c_i^2\right) \\ = & \frac{\Delta c^2}{T+1}\left(r^2(T-1)^2 + (1-r)^2 +, ... + (T-1-r)^2 - (1^2 + 2^2 + ... + (T-1)^2)\right) \\ = & \frac{\Delta c^2}{T+1} T(T-1)r(r-1) \end{aligned}$$

□

Using Lemma 1, it is straightforward to show that the BWE exists between the firm and the customer.

Theorem 1. 1. *When $r \in (0,1)$, $var(Q^n, 1, T+1) < var(c, 1, T+1)$. That is, the BWE exists between the firm and the customer under the naive pricing strategy.*

2. *When* $r = 0$, $var(Q^n, 1, T+1) = var(c, 1, T+1)$.

Theorem 1 shows that the naive pricing strategy makes the customer's order pattern less volatile than the capacity process. Note that the definition of the variability in (9) is independent of the sequence of Q_t^n. After accounting for the sequence, Q_t^n is still more stable since it takes two periods instead of one period for Q_t^n to drop to its lowest point, and then Q_t^n has the same rate of increase, Δc, as c_t does.

Moreover, we can determine how customer behavior and the capacity process affect the system.

PROPOSITION 1. 1. *When* $r = 0.5$, *the BWE is most severe. The severity of the BWE decreases as* r *deviates from* 0.5.

2. *The severity of the BWE increases when* Δc *increases.*

3. *The severity of the BWE increases when* T *increases.*

Proof Follows from (12). □

Notice that the severity of the BWE is not monotonic in r. We can see this from the extreme cases when $r = 0$ and $r = 1$. When $r = 0$, the customer's order process is exactly the same as the capacity process. When $r = 1$, the customer's order process is the same as the capacity process except that the customer's order process has a one-period lag. Thus, there is no BWE in the two extreme cases.

5. One-Period Correction (1PC) Pricing Strategy

In employing the 1PC pricing strategy, the firm treats the discrepancy between the customer's order and the underlying demand as feedback and revises the price accordingly.

Under the naive pricing strategy, it is easy to see that the system returns to its normal state after period $T+1$. The next lemma demonstrates that the system under the 1PC pricing strategy is stable, i.e. the system converges to the normal state.

LEMMA 2. *Suppose there exists* $N \geq 1$ *such that* $c_t = B$ *for all* $t > N$. *Then if* $0 \leq r < 1$, *no matter what initial values* p_N^c *and* Q_N^c *have,* $\lim_{t\to\infty} p_t^c = P$ *and* $\lim_{t\to\infty} Q_t^c = B$.

Proof The results is trivial if $r = 0$. Therefore, we assume that $0 < r < 1$.

When $t \geq N$, (5) is equal to $p_t^c = mB + b + r(p_{t-1}^c - p_{t-2}^c)$. Since $B = \frac{P-b}{m}$, this can be rewritten as $(p_t^c - P) = r(p_{t-1}^c - P) - r(p_{t-2}^c - P)$. Then we have

$$\begin{pmatrix} p_t^c - P \\ p_{t+1}^c - P \end{pmatrix} = \begin{bmatrix} r & -r \\ r^2 - r & -r^2 \end{bmatrix} \begin{pmatrix} p_{t-1}^c - P \\ p_{t-2}^c - P \end{pmatrix} \tag{13}$$

It can be shown that the eigenvalue of the matrix in (13) is

$$\lambda = \frac{r^2 - 2r}{2} \pm r\sqrt{\frac{4r - r^2}{4}}i$$

The norm of λ is less than 1, and the rows of the matrix are linearly independent. Therefore, by the Spectral Radius Formula (Dunford and Schwartz 1988), we have

$$\lim_{i\to\infty} \begin{pmatrix} p_{N+2i}^c - P \\ p_{N+2i+1}^c - P \end{pmatrix} = \lim_{i\to\infty} \begin{bmatrix} r & -r \\ r^2 - r & -r^2 \end{bmatrix}^{i+1} \begin{pmatrix} p_{N-1}^c - P \\ p_{N-2}^c - P \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}.$$

Hence, $\lim_{t\to\infty} p_t^c = P$. Moreover, $\lim_{t\to\infty}(p_t^c - p_{t-1}^c) = 0$ indicates that Q_t^c converges to B. □

From the proof of Lemma 2, one can see that the rate of convergence depends on the norm of λ, which is equal to r^2. Therefore, the more strategic the customer is, the slower the system converges to the normal state.

From (6), it is clear that Q_t^c depends on p_t^c, and therefore it is important to derive an expression for p_t^c. We first derive an exact expression and later derive an approximate one. Let $\lceil x \rceil$ represent the smallest integer no less than x. We also define $\sum_{j=N}^{M} a_j \equiv 0$ for any a_j if $M < N$.

Lemma 3. *p_t^c can be written as*

$$p_t^c = m\sum_{i=1}^{t}\left[c_{t-i+1}\sum_{j=\lceil\frac{i+1}{2}\rceil}^{i}(-1)^{i+j}\binom{j-1}{i-j}r^{j-1}\right] \tag{14}$$
$$+b + (b-P)\sum_{j=\lceil\frac{t}{2}\rceil+1}^{t}(-1)^{t+j}\binom{j-2}{t-j}r^{j-1}$$

Proof By (5) and (6), we have $p_1^c = mc_1 + b$ and $p_2^c = m(rc_1 + c_2) + b + (b-P)r$. Therefore, (14) holds for $t = 1$ and $t = 2$. We use mathematical induction to show it is valid for any t. We provide a proof for the coefficient of $(b-P)$ in (14) when t is even. The proof of the coefficient of m when t is even, and of both coefficients when t is odd, follow a similar procedure.

Suppose that (14) holds for all periods through $t-1$. By (5), the coefficient of $b-P$ for p_t^c can be derived from p_{t-1}^c and p_{t-2}^c and is given by

$$\sum_{j=\frac{t}{2}+1}^{t-1}(-1)^{j-1}\binom{j-2}{t-j-1}r^j-\sum_{j=\frac{t}{2}}^{t-2}(-1)^j\binom{j-2}{t-j-2}r^j.$$

Let $k=j+1$; then we have

$$\begin{aligned}
&\sum_{k=\frac{t}{2}+2}^{t}(-1)^k\binom{k-3}{t-k}r^{k-1}+\sum_{k=\frac{t}{2}+1}^{t-1}(-1)^k\binom{k-3}{t-k-1}r^{k-1}\\
=&\sum_{k=\frac{t}{2}+2}^{t-1}(-1)^k\left[\binom{k-3}{t-k}+\binom{k-3}{t-k-1}\right]r^{k-1}+(-1)^t r^{t-1}+(-1)^{\frac{t}{2}+1}r^{\frac{t}{2}}\\
=&\sum_{k=\frac{t}{2}+2}^{t-1}(-1)^k(k-3)!\left[\frac{1}{(t-k)!(2k-t-3)!}+\frac{1}{(t-k-1)!(2k-t-2)!}\right]r^{k-1}\\
&+(-1)^t r^{t-1}+(-1)^{\frac{t}{2}+1}r^{\frac{t}{2}}\\
=&\sum_{k=\frac{t}{2}+1}^{t}(-1)^k\binom{k-2}{t-k}r^{k-1}
\end{aligned}$$

□

Unfortunately, it is difficult to analyze p_t^c directly. Instead, we take a first-order approximation of p_t^c with respect to r. When $t\geq 3$, (14) involves r with powers greater than 2. We only keep the terms with r^1 and r^0. Let $\tilde{p}_t^c$ represent this approximation of p_t^c. Then we have the following:

$$\begin{aligned}
\tilde{p}_1^c &= mc_1+b\\
\tilde{p}_2^c &= m(c_2+rc_1)+b(r+1)-rP\\
\tilde{p}_t^c &= m(c_t+r(c_{t-1}-c_{t-2}))+b \qquad \text{for } t\geq 3
\end{aligned} \tag{15}$$

Note that, by (14), $\tilde{p}_1^c=p_1^c$ and $\tilde{p}_2^c=p_2^c$.

Let $\tilde{Q}_t^c$ be the approximate value of Q_t^c that results from approximating p_1^c with $\tilde{p}_1^c$; that is, $\tilde{Q}_t^c=\frac{(1-r)\tilde{p}_t^c+r\tilde{p}_{t-1}^c-b}{m}$ (from (6)). (We test the effectiveness of this heuristic in Section 6.)

Lemma 4. *For $4\leq t\leq T$ and $t\geq T+3$, $\tilde{Q}_t^c=c_t$. For the remaining values of t, we have*

$$\begin{aligned}
\tilde{Q}_1^c =&\ (1-r)A+rB\\
\tilde{Q}_2^c =&\ (1+r-r^2)A+(1-r)\Delta c+(r^2-r)B\\
\tilde{Q}_3^c =&\ (1+r^2)A+(2-r^2)\Delta c-r^2B
\end{aligned} \tag{16}$$

$$\tilde{Q}^c_{T+1} = B + r\Delta c$$

$$\tilde{Q}^c_{T+2} = B + r^2\Delta c \tag{17}$$

Proof See Appendix 9.2. □

Lemma 4 indicates that $\tilde{Q}^c_t$ returns to its normal level B in period $T+3$. (Recall that, by Lemma 2, this is only true for the exact price in the limit.) Therefore, we focus on the approximate system's behavior during periods 1 to $T+3$. The following lemma shows that the approximate average customer order is equal to the average capacity during the whole supply disruption and recovery process. It also provides a formula for the difference in variability between the customer's order and the capacity.

LEMMA 5. 1. *$avg(c,1,T+3) = avg(\tilde{Q}^c,1,T+3)$.*

2. *Let $d(r,T,\Delta c)$ be the difference in variability between the customer order and the capacity, treated as a function of r, T and Δc. Then*

$$\begin{aligned} d(r,T,\Delta c) &= var(\tilde{Q}^c,1,T+3) - var(c,1,T+3) \\ &= \frac{2\Delta c^2}{T+3}[T^2(r^4 - r^3 + r^2) + T(-r^4 + r^3 - r^2) + r^4 - r^2 - r] \end{aligned} \tag{18}$$

Proof See Appendix 9.2. □

LEMMA 6. 1. *For any recovery time $T \geq 3$, there is a unique $r^*(T) \in (0,1)$ such that $d(r^*(T),T,\Delta c) = 0$.*

2. *$r^*(T)$ is a monotonically decreasing function of T. Also, $r^*(3) = 0.2547$ and $r^*(\infty) = 0$. (Therefore $0 \leq r^*(T) \leq 0.2547$ for all $T \geq 3$.)*

Proof See Appendix 9.2. □

We now apply Lemma 5 and Lemma 6 to obtain results concerning the BWE and RBWE for the 1PC pricing strategy.

THEOREM 2. 1. *When $r = 0$ or $r = r^*(T)$, the approximate customer order variability is equal to the capacity variability.*

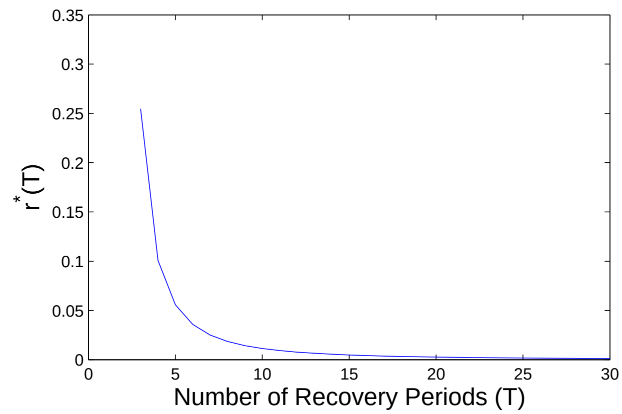


Figure 2 **The decreasing trend of** $r^*(T)$

2. *When* $r \in (0, r^*(T))$*, the approximate customer order variability is less than the capacity variability. That is, the BWE occurs.*

3. *When* $r \in (r^*(T), 1)$*, the approximate customer order variability is greater than the capacity variability. That is, the RBWE occurs.*

Proof Part 1: Follows from Lemma 6.

Parts 2 and 3: Since $d(r, T, \triangle c)$ only has two real roots, $d(r, T, \triangle c)$ is either positive or negative when $r > r^*(T)$. Clearly $d(r, T, \triangle c)$ goes to infinity when r goes to infinity. Therefore $d(r, T, \triangle c) > 0$ when $r > r^*(T)$. Similarly, $d(r, T, \triangle c)$ is either positive or negative when $0 < r < r^*(T)$. It is not hard to see that $d\left(\frac{1}{T^3}, T, \triangle c\right) < 0$. Thus $d(r, T, \triangle c) < 0$ when $0 < r < r^*(T)$. $\square$

This theorem demonstrates that when the customer's reaction to price changes is significant (r is large), that is, when the customer is more strategic, the RBWE can occur as a result. However, when the customer is prone to be myopic, the BWE occurs. Moreover, since $r^*(T)$ is a decreasing function, the longer the recovery lasts, the less strategic the customer needs to be in order to cause the RBWE. Figure 2 demonstrates that $r^*(T)$ quickly approaches 0 as T increases. (Note that $r^*(T)$ depends on T but no other parameters.)

Proposition 2. 1. *The difference in variability between the approximate customer order and the capacity,* $d(r, T, \Delta c)$*, is increasing in* Δc *when* $r \geq r^*(T)$*.*

2. *The difference in variability between the approximate customer order and the capacity, $d(r,T,\Delta c)$, is increasing in T*

Proof See Appendix 9.2. □

We know that $\Delta c(T-1) = B - A$. Proposition 2.1 says that, given the same length of recovery (T), a large disruption with a quick recovery (large Δc) causes more RBWE than a small disruption with a slow recovery (small Δc) does. Proposition 2.2 says that a large disruption causes more RBWE than a small disruption does given the same recovery rate.

6. Comparison between Naive and One-Period Correction (1PC) Pricing Strategies

In this section, we compare the naive and 1PC pricing strategies in terms of their effects on customer order variability and the firm's revenue, and we evaluate the impact of the degree of strategic-ness and the capacity process. We also examine the effectiveness of the approximation for the 1PC pricing strategy given by (15) and Lemma 4.

By comparing Theorems 1 and 2, one can see that when $r > r^*(T)$, the 1PC pricing strategy produces a more volatile order process than the naive one does. The next proposition strengthens this result by showing that it holds for all $r \in (0,1)$.

PROPOSITION 3. *$var(\tilde{Q}^c, 1, T+3) > var(Q^n, 1, T+3)$.*

Proof See Appendix 9.2. □

In addition to studying the impact of pricing strategies on order variability, we also want to investigate their impact on revenue. Let $R^n(1,t)$ and $R^c(1,t)$ represent the average revenue from period 1 to t under the naive and 1PC pricing strategies, respectively. Then

$$R^n(1,t) = \frac{1}{t}\sum_{i=1}^{t} p_i^n \min\{Q_i^n, c_i\} \tag{19}$$

$$R^c(1,t) = \frac{1}{t}\sum_{i=1}^{t} p_i^c \min\{Q_i^c, c_i\} \tag{20}$$

Since Q_i^c is difficult to analyze directly, so is $R^c(1,t)$. Therefore, we replace p_i^c and Q_i^c with $\tilde{p}_i^c$ and $\tilde{Q}_i^c$ in (20) and use the resulting quantity, $\tilde{R}^c(1,t)$, to approximate $R^c(1,t)$. Using this approximation, we can show that the 1PC pricing strategy always provides lower revenue:

PROPOSITION 4. $\tilde{R}^c(1,T+3) \le R^n(1,T+3)$.

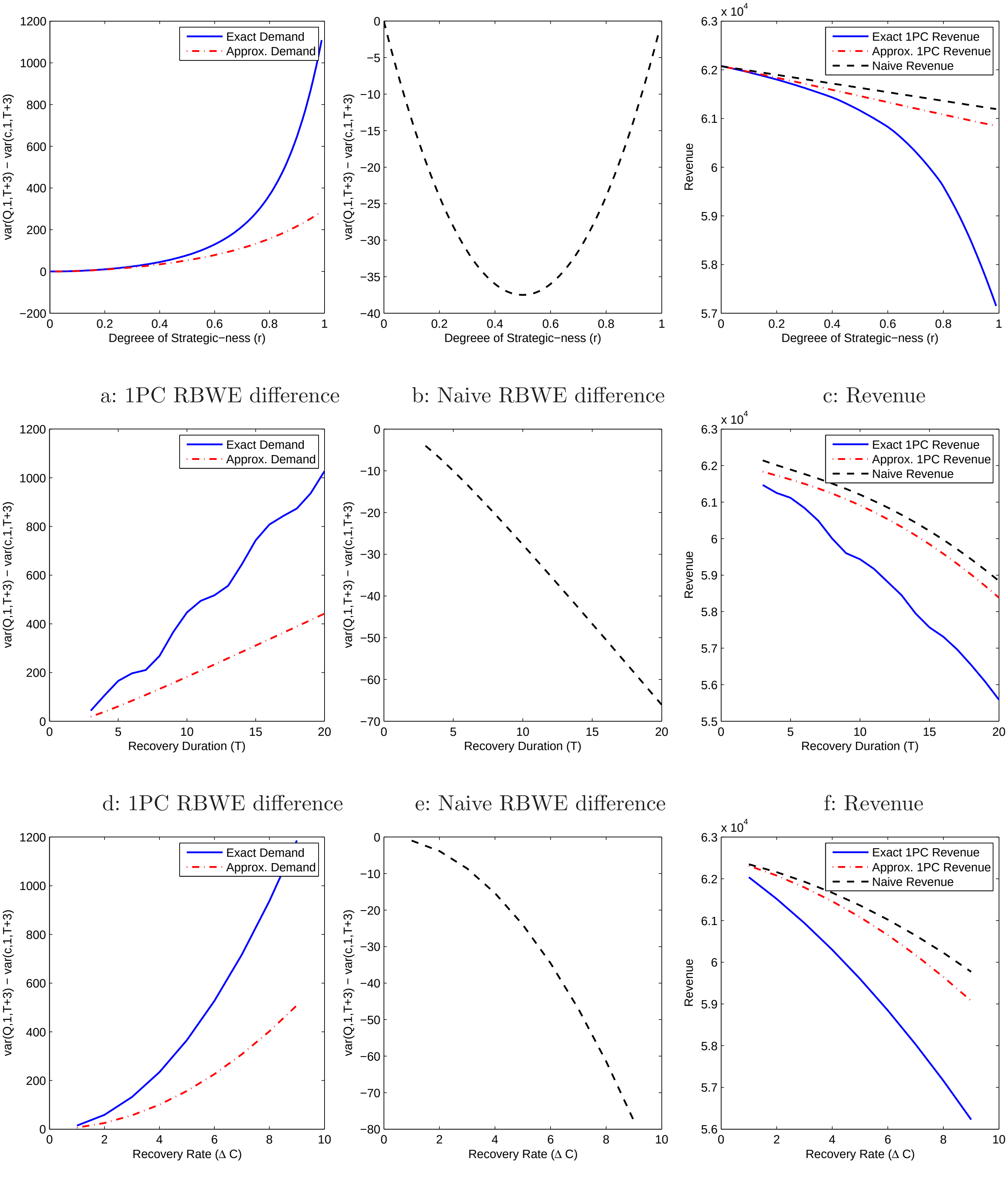
Proof See Appendix 9.2. □

We next use a numerical study to explore the differences between the two pricing strategies in more detail. Figure 3 contains nine subplots. The subplots in a given row all have the same independent variable; in rows 1–3, they are: the degree of strategic-ness (r), the recovery duration (T), and the recovery rate (Δc), respectively. The subplots in a given column all have the same dependent variable. Column 1 depicts the difference in variability between the order process and the capacity process for the 1PC pricing strategy under both the exact and approximate models. Column 2 depicts the difference in variability under the naive pricing strategy. Column 3 depicts the average revenue for the two pricing strategies. The default values of the parameters are $m = -1$, $b = 500$, $A = 60$, $B = 100$, $T = 9$, $\Delta c = 5$, $r = 0.8$.

From Figure 3, it is evident that the 1PC pricing strategy (exact and approximate) results in both greater customer order variability and lower average revenue than the naive strategy, suggesting that the naive strategy strictly dominates the 1PC one. Moreover, by comparing Figures 3.b and 3.c, one can see that, at least under the naive strategy, customer order variability is not necessarily correlated with average revenue; that is, the supply chain profit does not necessarily decrease as customer order variability becomes more severe.

We consider how the degree of customer strategic-ness affects the system by examining the first row of Figure 3. Since $r^*(9) = 0.0143$, the 1PC pricing strategy almost always produces the RBWE. The higher r is, the more pronounced the RBWE is. On the other hand, the naive pricing strategy produces the lowest customer order variability when $r = 0.5$, as Proposition 1 predicts. But the higher r is, the lower the average revenue both pricing strategies obtain. This suggests that the firm is worse off under both strategies as the customer becomes more strategic.

We also investigate how the capacity process affects the system's performance under both pricing strategies. In the second row of Figure 3, we consider the impact of T. The variability difference between the customer orders and the capacity is increasing in T under the 1PC pricing strategy



a: 1PC RBWE difference b: Naive RBWE difference c: Revenue

d: 1PC RBWE difference e: Naive RBWE difference f: Revenue

g: 1PC RBWE difference h: Naive RBWE difference i: Revenue

Figure 3 Comparison of Pricing Strategies

but decreasing in T under the naive pricing strategy. Notice that when T increases and Δc is fixed, both $var(Q^n, 1, T+3)$ and $var(c, 1, T+3)$ are increasing, but their difference is decreasing under the naive pricing strategy. However, the average revenue is always decreasing in T. From the last row of Figure 3, one can see that Δc has a similar impact on the variability difference and average revenue as T does. Thus, severe disruptions and long recovery processes hurt the system no matter which pricing strategy is implemented.

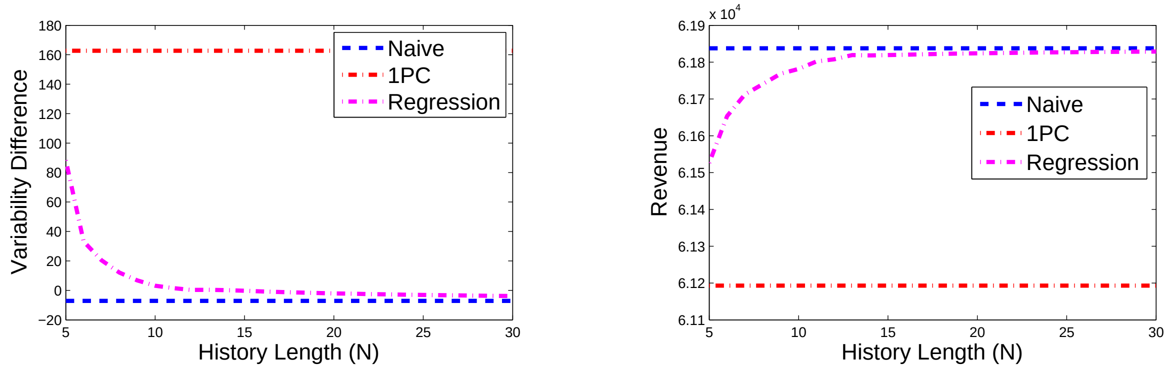
From Figure 3, one can see that, under the 1PC pricing strategy, our approximation of the customer's order quantity always results in an underestimate of the exact variability of customer orders (column 1) and an overestimate of the revenue (column 3). These two findings suggest the following conjecture.

Conjecture 1. *The approximation of the decision dynamic under the 1PC pricing strategy, given by* (15) *and Lemma 4, provides optimistic estimates of the actual system. That is, it always underestimates the variability of customer orders and overestimates the revenue.*

Moreover, the figure suggests that the accuracy of the approximation worsens as r, T, or Δc increase. Since our approximation uses a first-order approximation of p^c in (15), this approximation is more accurate when r is small. (18) shows that the highest order of T and $\triangle c$ among all parameters in the polynomial expression for r is 1 and 2. The larger T and $\triangle c$ are, the large the gap between the approximated and exact values is. However, even when the approximation is fairly inaccurate, the approximation still allows us to compare the naive and 1PC pricing strategies.

7. Regression Pricing Strategy

Both the naive and the 1PC pricing strategies use very little information about the history of the prices and customer orders. One may argue that the firm should use more historical data to compensate for not knowing the customer's underlying behavior under supply disruptions. In other words, by utilizing more historical data, the performance of the firm should be improved. However, this is only partially true. In this section, we provide a numerical study to compare the naive and 1PC pricing strategies with the regression pricing strategy, which incorporates the N most recent



a: Variability Difference b: Revenue

Figure 4 Variability Difference and Revenue vs. History Length (N)

data points of prices and customer orders, to demonstrate that an appealing approach with more data does not necessarily generate better results than a simple approach.

We computed the revenue and the variability difference between the customer orders and the capacity for each (r, T) pair, where r ranges from 0.1 to 0.9 in increments of 0.2 and T ranges from 3 to 11 in increments of 2. For each setting, we simulated initial conditions for the regression pricing strategy, as discussed in Section 3.3, 100 times. The other settings are the same as those in Section 6. In Figure 4, we plot the average values for each history length N.

From Figure 4, one can see that by utilizing more historical data, the firm can reduce the variability of the customer's order process and increase its own profit. Although the least-squares fit requires at least two data points, the 1PC pricing strategy can be treated as a special case of the regression pricing strategy if the estimated short-run demand curve goes through only one data point. Thus, from Figure 4, it is clear that the regression pricing strategy is better than the 1PC pricing strategy in terms of both the customer's order variability and the firm's revenue. However, the regression pricing strategy is worse than the naive pricing strategy in both measures. On the other hand, the performance of the regression pricing strategy approaches that of the naive strategy as the history length, N, increases. The reason for this is that as N increases, the regression strategy uses more and more information from the underlying demand curve. Therefore, when N is large enough, the estimated short-run demand curve under the regression pricing strategy is essentially the equal to the underlying demand curve, which is the same curve used by the naive pricing strategy.

One may wonder why the regression pricing strategy does not perform better than seemingly less sophisticated ones. One implicit requirement for the least-squares method to be successful is that the short-run demand curve must be time-invariant, which is not true under our model of customer behavior. Moreover, more data cannot substitute for a better understanding of that behavior. No matter how much history the firm uses, it does not have much data to capture the customer's behavior in the first few periods of the disruption. With few data points available to describe the system status after a sudden change, the firm needs a better understanding of the customer's behavior under supply disruptions if it wishes to improve the effectiveness of its forecasting.

Ours is not the only study to demonstrate that sophisticated models with more input data may not outperform naive strategies. In the context of stock trading, DeMiguel et al. (2007) use simulation to show that it takes thousands of months' worth of data for the sample-based mean-variance strategy to outperform an equally weighted portfolio in the U.S. equity market. Stulz (2009) also identifies relying too much on historical data as one of ways to fail in risk management.

8. Conclusions and Future Research

In this paper, we investigate three pricing strategies for a firm with limited information about customer behavior under supply disruptions. The 1PC pricing strategy tries harder to model the customer demand process, but it does so inaccurately. We find that if the firm applies this strategy, it leads to a more volatile customer order process and lower revenue than the naive pricing strategy does. Moreover, although the regression pricing strategy accounts for the recent history of prices and order quantities, this more "advanced" strategy also fails to outperform the naive pricing strategy. Our study establishes how the strategic-ness of the customer and the capacity affect the RBWE, and it demonstrates that higher RBWE is usually associated with lower revenue.

Our research highlights the importance of understanding customer behavior under supply disruptions. Without accurate knowledge of customer behavior, it is impossible for the firm to develop an effective pricing strategy. No matter what pricing strategy is used, the pricing decision immediately after the supply disruption is important since the mismatch between demand and supply is most pronounced during that time. But many supply disruptions are infrequent events, which makes it difficult to observe customer behavior in the real world. Nevertheless, the firm needs to learn from its experience and that of other firms (Sheffi 2005). We also expect that in the future, laboratory experiments will be used to provide insights into customer behavior during disruptions by investigating players' reactions in a controlled environment.

The underlying customer demand-price curve remains fixed in the short run. However, in the long run, it may change over time if prices become too volatile. For example, people may replace their cars with more fuel-efficient ones, and the government may invest more in public transportation. Or customers might switch their allegiance from the firm to its competitors. When the firm makes pricing decisions, it must bear this long-term effect in mind.

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9. Technical Appendix

9.1. Random Capacity Whose Mean Follows Linear Recovery Process

In this section, we assume that the capacity is random during the disruption and recovery process. Let C_t be the random capacity in period t. Then we assume that

$$C_t = \begin{cases} c_t + \varepsilon_t, & 1 \le t \le T-1 \\ c_t, & \text{otherwise} \end{cases} \tag{21}$$

where c_t is defined in (1) and ε_t is an iid random variable with mean 0 and variance σ. Also we assume that once the capacity recovers to the normal state in period T, there is no variability in the capacity ($\varepsilon_t = 0$). Replacing c_t with C_t in (3)–(6), we obtain Q_t^n and Q_t^c under the random capacity process, which are random variables as well. We call c_t the mean of the capacity process and ε_t the noise of the capacity process in period t. Similarly, $E[Q_t]$ is called the mean of the customer's order process and $Q_t - E[Q_t]$ is the noise of the customer's order process in period t.

We need to modify the definition of variability in (9) and (10) to measure the variability of the capacity and customer's order process with the noise.

$$\begin{aligned} avg(Q,1,t) &= \frac{1}{t}\sum_1^t E[Q_i] \\ avg(C,1,t) &= \frac{1}{t}\sum_1^t E[C_i] \\ var(Q,1,t) &= \frac{1}{t}\sum_1^t E[(Q_i - avg(Q,1,t))^2] \\ var(C,1,t) &= \frac{1}{t}\sum_1^t E[(C_i - avg(C,1,t))^2] \end{aligned} \tag{22}$$

Similarly, we also have

$$\begin{aligned} var(Q,1,t) &= \frac{1}{t}\sum_1^t E[Q_i^2] - (avg(Q,1,t))^2 \\ var(C,1,t) &= \frac{1}{t}\sum_1^t E[C_i^2] - (avg(C,1,t))^2 \end{aligned} \tag{23}$$

Notice that $t \times var(C,1,t) = t \times var(c,1,t) + \sum_{t \le T-1} \sigma^2$.

We first discuss the impact of random capacity on the naive pricing strategy. Then we apply the same approximation for the 1PC pricing strategy under random capacity and investigate its impact as well.

Under the naive pricing strategy, the customer's order quantity is given by

$$Q_t^n = \begin{cases} A + r(T-1)\Delta c + (1-r)\varepsilon_1, & t = 1 \\ A + (t-1-r)\Delta c + (1-r)\varepsilon_t + r\varepsilon_{t-1}, & 2 \le t \le T-1 \\ A + (T-1-r)\Delta c + r\varepsilon_{T-1}, & t = T \\ B & \text{otherwise} \end{cases}$$

Similar to Lemma 1, we have

Lemma 7. 1. $avg(Q^n, 1, T+1) = avg(C, 1, T+1)$.

2.

$$var(Q^n, 1, T+1) - var(C, 1, T+1) = r(r-1)\frac{T-1}{T+1}(T\Delta^2 + 2\sigma^2). \tag{24}$$

Proof By utilizing the fact that ε_t is iid, $E[\varepsilon_t^2] = \sigma^2$ and $E[\varepsilon_t] = 0$, one can follow the same procedure as in Lemma 1 to get the desired results. □

Lemma 7 indicates that the RBWE always exists unless $r = 0$. By comparing Lemma 1 and Lemma 7, we can decompose the variability difference between the random capacity and the customer order process into two parts. One is the variability difference between their mean processes. The other is the variability difference of their noise parts. Under the naive pricing strategy, both parts are less than 0, which means the naive pricing strategy results in not only a more stable mean of the customer order process but also less noise in the order process.

Under the 1PC pricing strategy, we apply the same approximation as in Section 5 to get the approximate customer order quantity under random capacity as well.

$$\begin{aligned}
\tilde{Q}_1^c &= (1-r)A + rB + (1-r)\varepsilon_1 \\
\tilde{Q}_2^c &= (1+r-r^2)A + (1-r)\Delta c + (r^2-r)B + (1-r)\varepsilon_2 + (2r-r^2)\varepsilon_1 \\
\tilde{Q}_3^c &= (1+r^2)A + (2-r^2)\Delta c - r^2 B + (1-r)\varepsilon_3 + (2r-r^2)\varepsilon_2 - (r-2r^2)\varepsilon_1 \\
\tilde{Q}_t^c &= c_t + (1-r)\varepsilon_t + (2r-r^2)\varepsilon_{t-1} - (r-2r^2)\varepsilon_{t-2} - r^2\varepsilon_{t-3}, \text{ for } 4 \le t \le T-1 \\
\tilde{Q}_T^c &= B + (2r-r^2)\varepsilon_{T-1} - (r-2r^2)\varepsilon_{T-2} - r^2\varepsilon_{T-3} \\
\tilde{Q}_{T+1}^c &= B + r\Delta c - (r-2r^2)\varepsilon_{T-1} - r^2\varepsilon_{T-2} \\
\tilde{Q}_{T+2}^c &= B + r^2\Delta c - r^2\varepsilon_{T-2}
\end{aligned}$$

Then, similar to Lemma 5, we have

Lemma 8. 1. $avg(\tilde{Q}^c, 1, T+3) = avg(C, 1, T+3)$.

2.

$$d(r,T,\Delta c) = \frac{2\Delta c^2}{T+3}[T^2(r^4 - r^3 + r^2) + T(-r^4 + r^3 - r^2) + r^4 - r^2 - r] + \frac{2(T-1)}{T+3}\sigma^2 r(3r^2 - 4r^2 + 3r - 1). \quad (25)$$

Proof By utilizing the fact that ε_t is iid, $E[\varepsilon_t^2] = \sigma^2$ and $E[\varepsilon_t] = 0$, one can follow the same procedure as in Lemma 5 to get the desired results. □

By comparing Lemma 5 and Lemma 8, we can decompose the variability difference between the random capacity and the customer order process into two parts as well. If $r \geq 0.5944$, the noise of the random capacity is amplified in the customer order process, otherwise it is dampened. Also note that if we assume $\sigma < \Delta c$, that is, the magnitude of the capacity noise does not exceed the recovery rate, then the first element associated with Δc in (25) dominates the second element associated with σ^2. Thus, all the insights in Theorem 2 and Proposition 2 still hold for the 1PC pricing strategy in the presence of random capacity.

9.2. Proofs

Lemma 4

For $t = 1$, we have

$$\begin{aligned}\tilde{Q}_1^c &= \frac{(1-r)\tilde{p}_1^c + r\tilde{p}_0^c - b}{m} \\ &= \frac{(1-r)mc_1 + (1-r)b + rmB + rb - b}{m} \\ &= (1-r)c_1 + rB \\ &= (1-r)A + rB\end{aligned}$$

For $t = 2$, we have

$$\begin{aligned}\tilde{Q}_2^c &= \frac{(1-r)\tilde{p}_2^c + r\tilde{p}_1^c - b}{m} \\ &= \frac{(1-r)m(c_2 + rc_1) + (1-r)b(1+r) - (1-r)rP + rmc_1 + rb - b}{m} \\ &= (1-r)c_2 + (2r - r^2)c_1 + (r^2 - r)\frac{P-b}{m}\end{aligned}$$

$$= (1-r)c_2 + (2r-r^2)c_1 + (r^2-r)B$$

$$= (1-r)(A+\Delta c) + (2r-r^2)A + (r^2-r)B$$

$$= (1+r-r^2)A + (1-r)\Delta c + (r^2-r)B$$

For $t=3$, we have

$$\begin{aligned}
\tilde{Q}_3^c &= \frac{(1-r)\tilde{p}_3^c + r\tilde{p}_2^c - b}{m} \\
&= \frac{(1-r)(m(c_3 + r(c_2-c_1)) + b) + r(m(c_2+rc_1) + b(r+1) - rP) - b}{m} \\
&= (1-r)c_3 + (2r-r^2)c_2 - (r-2r^2)c_1 - r^2\frac{P-b}{m} \\
&= (1-r)c_3 + (2r-r^2)c_2 - (r-2r^2)c_1 - r^2B \\
&= (1-r)(A+2\Delta c) + (2r-r^2)(A+\Delta c) - (r-2r^2)A - r^2B \\
&= (1+r^2)A + (2-r^2)\Delta c - r^2B
\end{aligned}$$

For $t>3$, we have

$$\begin{aligned}
\tilde{Q}_t^c &= \frac{(1-r)\tilde{p}_t^c + r\tilde{p}_{t-1}^c - b}{m} \\
&= (1-r)(c_t + r(c_{t-1} - c_{t-2})) + r(c_{t-1} + r(c_{t-2} - c_{t-3})) \\
&= (1-r)c_t + (2r-r^2)c_{t-1} - (r-2r^2)c_{t-2} - r^2c_{t-3}
\end{aligned}$$

For $4 \le t \le T$, we have

$$\begin{aligned}
\tilde{Q}_t^c &= (1-r)c_t + (2r-r^2)c_{t-1} - (r-2r^2)c_{t-2} - r^2c_{t-3} \\
&= (1-r)(A+(t-1)\Delta c) + (2r-r^2)(A+(t-2)\Delta c) \\
&\quad -(r-2r^2)(A+(t-3)\Delta c) - r^2(A+(t-4)\Delta c) \\
&= A + (t-1)\Delta c \\
&= c_t
\end{aligned}$$

Finally,

$$\tilde{Q}_{T+1}^c = (1-r)c_{T+1} + (2r-r^2)c_T - (r-2r^2)c_{T-1} - r^2c_{T-2}$$

$$= (1-r)B + (2r-r^2)B - (r-2r^2)(B-\Delta c) - r^2(B-2\Delta c)$$

$$= B + r\Delta c$$

$$\tilde{Q}^c_{T+2} = (1-r)c_{T+2} + (2r-r^2)c_{T+1} - (r-2r^2)c_T - r^2 c_{T-1}$$

$$= (1-r)B + (2r-r^2)B - (r-2r^2)B - r^2(B-\Delta c)$$

$$= B + r^2\Delta c$$

For $t \geq T+3$, since $c_t = c_{t-1} = c_{t-2} = c_{t-3} = B$, we have

$$\tilde{Q}^c_t = (1-r)c_t + (2r-r^2)c_{t-1} - (r-2r^2)c_{t-2} - r^2 c_{t-3}$$

$$= B$$

□

Lemma 5

Part 1:

We can rewrite $\tilde{Q}^c_t$ for $t \in \{1,2,3,T+1,T+2\}$ as

$$\begin{aligned}
\tilde{Q}^c_1 &= A + r(B-A) \\
\tilde{Q}^c_2 &= A + \Delta c - r\Delta c + (r^2-r)(B-A) \\
\tilde{Q}^c_3 &= A + 2\Delta c - r^2\Delta c - r^2(B-A) \\
\tilde{Q}^c_{T+1} &= B + r\Delta c \\
\tilde{Q}^c_{T+2} &= B + r^2\Delta c
\end{aligned} \tag{26}$$

From (26), it is clear that

$$\sum_{t=1,2,3,T+1,T+2} \tilde{Q}^c_t = \sum_{i=1,2,3,T+1,T+2} c_t.$$

By Lemma 4, we have $\tilde{Q}^c_t = c_t$ for $4 \leq t \leq$ T and $t \geq T+3$. Then the result immediately follows.

Part 2:

From part 1 and (10),

$$var(\tilde{Q}^c,1,T+3)-var(c,1,T+3)=\frac{\sum_{i=1}^{3}((\tilde{Q}_i^c)^2-c_i^2)+\sum_{i=T+1}^{T+2}((\tilde{Q}_i^c)^2-c_i^2)}{T+3}. \tag{27}$$

From (26),

$$\begin{aligned}
&(\tilde{Q}_1^c)^2+(\tilde{Q}_2^c)^2+(\tilde{Q}_3^c)^2+(\tilde{Q}_{T+1}^c)^2+(\tilde{Q}_{T+2}^c)^2-(A^2+(A+\Delta c)^2+(A+2\Delta c)^2+2B^2)\\
=&[A+r(B-A)]^2\\
&+[A+\Delta c+(r^2-r)(B-A)-r\Delta c]^2\\
&+[A+2\Delta c-r^2(B-A)-r^2\Delta c]^2\\
&+(B+r\Delta c)^2+(B+r^2\Delta c)^2\\
&-A^2-(A+\Delta c)^2-(A+2\Delta c)^2-2B^2.
\end{aligned}$$

After further simplification, we have

$$\begin{aligned}
&(\tilde{Q}_1^c)^2+(\tilde{Q}_2^c)^2+(\tilde{Q}_3^c)^2+(\tilde{Q}_{T+1}^c)^2+(\tilde{Q}_{T+2}^c)^2-(A^2+(A+\Delta c)^2+(A+2\Delta c)^2+2B^2)\\
=&2rA(B-A)+r^2(B-A)^2\\
&+2(r^2-r)(A+\Delta c)(B-A)-2r(A+\Delta c)\Delta c+(r^2-r)^2(B-A)^2+r^2\Delta c^2\\
&-2(r^3-r^2)(B-A)\Delta c\\
&-2r^2(A+2\Delta c)(B-A)-2r^2(A+2\Delta c)\Delta c+r^4(B-A)^2+r^4\Delta c^2+2r^4(B-A)\Delta c\\
&+2rB\Delta c+r^2\Delta c^2\\
&+2r^2B\Delta c+r^4\Delta c^2\\
=&r\left[2A(B-A)-2(A+\Delta c)(B-A)-2(A+\Delta c)\Delta c+2B\Delta c\right]\\
&+r^2\left[(B-A)^2+2(A+\Delta c)(B-A)+(B-A)^2+\Delta c^2+2(B-A)\Delta c-2(A+2\Delta c)(B-A)\right.\\
&\left.-2(A+2\Delta c)\Delta c+\Delta c^2+2B\Delta c\right]\\
&+r^3\left[-2(B-A)^2-2(B-A)\Delta c\right]\\
&+r^4\left[(B-A)^2+(B-A)^2+\Delta c^2+2(B-A)\Delta c+\Delta c^2\right]
\end{aligned}$$

$$= -2r\Delta c^2 + r^2\left[2(B-A)^2 + 2(B-A)\Delta c - 2\Delta c^2\right] + r^3\left[-2(B-A)^2 - 2(B-A)\Delta c\right]$$

$$+r^4\left[2(B-A)^2 + 2(B-A)\Delta c + 2\Delta c^2\right]$$

$$= 2[(-r\Delta c^2 + (T^2-T-1)r^2\Delta c^2 - (T^2-T)r^3\Delta c^2 + \Delta c^2(T^2-T+1)r^4]$$

$$= 2\Delta c^2[T^2(r^4-r^3+r^2) + T(-r^4+r^3-r^2) + r^4 - r^2 - r]$$

□

Lemma 6

Part 1:

From (18), it is clear that $r=0$ is a root of $d(r,T,\Delta c)$. We need to show that there is only one other real root with respect to r.

When $d(r,T,\Delta c)=0$, we must have

$$0 = (T^2-T+1)r^4 - (T^2-T)r^3 + (T^2-T-1)r^2 - r$$

$$= (a_3r^3 + a_2r^2 + a_1r^1 + a_0)r$$

for appropriately defined $a_0,\ldots,a_3 > 0$ since $T>0$. We need to show there is only one real root of $a_3r^3 + a_2r^2 + a_1r^1 + a_0 = 0$. The roots of a cubic equation can be expressed as

$$\alpha_1 = s + t - \frac{a_2}{3a_3}$$
$$\alpha_2 = -\frac{s+t}{2} - \frac{a_2}{3a_3} + \frac{\sqrt{3}}{2}(s-t)i$$
$$\alpha_3 = -\frac{s+t}{2} - \frac{a_2}{3a_3} - \frac{\sqrt{3}}{2}(s-t)i$$

where

$$s = \sqrt[3]{v + \sqrt{u^3+v^2}}$$
$$t = \sqrt[3]{v - \sqrt{u^3+v^2}}$$

in which

$$u = \frac{3a_3a_1 - a_2^2}{9a_3^2}$$

$$v = \frac{9a_3a_2a_1 - 27a_3^2a_0 - 2a_2^3}{54a_3^3}.$$

We will show that $u^3 > 0$ for all $T \geq 3$. Since $v^2 \geq 0$, this implies that $s > t$, and therefore that α_2 and α_3 are complex; it then follows that α_1 is the only real root.

First note that

$$\begin{aligned} u &= \frac{3a_3a_1 - a_2^2}{9a_3^2} \\ &= \frac{3(T^2-T+1)(T^2-T-1)-(T^2-T)^2}{9(T^2-T+1)^2} \\ &= \frac{2}{9} - \frac{4(T^2-T)+5}{9(T^2-T+1)^2}. \end{aligned}$$

When $T \geq 3$, $T^2 - T$ is a positive increasing function. Since $\frac{4x+5}{9(x+1)^2}$ is a decreasing function when x is positive, u is an increasing function of T. We can let $T = 3$ and ∞ to get the following bounds on u:

$$\frac{23}{147} \leq u \leq \frac{2}{9}.$$

Hence, $d(r, T, \Delta c)$ has only one real root (in addition to $r = 0$).

Part 2:

Let

$$f(T,r) \equiv d(r,T,\Delta c)/r = (T^2-T+1)r^3 - (T^2-T)r^2 + (T^2-T-1)r - 1.$$

Define $r^*(T) \neq 0$ such that $f(T, r^*(T)) = 0$. From Part 1, there is only one nonzero real root. We can get $r^*(3) \approx 0.2547$ numerically.

Then by the Implicit Function Theorem, we have

$$\begin{aligned} (r^*)'(T) &= -\frac{\frac{\partial}{\partial T} f(T,r^*)}{\frac{\partial}{\partial r^*} f(T,r^*)} \\ &= -\frac{(2T-1)((r^*)^3-(r^*)^2+r^*)}{3(T^2-T+1)(r^*)^2 - 2(T^2-T)r^* + (T^2-T-1)}. \end{aligned}$$

Since $r^*(3) \approx 0.2547$, it follows that $r^*(3)' < 0$. We will show that $(r^*)'(T) \leq 0$ for $T \geq 3$; that is, $r^*(T)$ is a monotonically decreasing function of T.

Suppose, for a contradiction, that there exists $T_1 > 3$ such that $(r^*)'(T_1) > 0$. We will treat $r^*(T)$ and $(r^*)'(T)$ as though their domains are $\mathbb{R}_+$ rather than $\mathbb{Z}_+$. Since $(r^*)'(T)$ is a continuous function of T and $(r^*)'(3) < 0$, there exists $3 < T_2 < T_3 < T_1$ such that $(r^*)'(T_2) = 0$ and $(r^*)'(T) > 0$ for $T \in (T_2, T_3]$. Then $r^*(T_2)$ must equal 0, since $r = 0$ is the only real root of $r^3 - r^2 + r = 0$, which is the only part of the numerator of $r^*(T)'$ that can equal zero. Since $(r^*)'(T) > 0$ for $T \in (T_2, T_3]$ and $r^*(T_2) = 0$, then $r^*(T) > 0$ for $T \in (T_2, T_3]$. Moreover, there exists $T_4 \in (T_2, T_3]$ such that $r^*(T) < 0.2$ for all $T \in (T_2, T_4)$, which implies that for $T \in (T_2, T_4)$, $3(T^2 - T + 1)(r^*)^2 - 2(T^2 - T)r^* + (T^2 - T - 1) > 0$. Moreover, $(r^*)^3 - (r^*)^2 + r^* > 0$. Hence, $(r^*)'(T) < 0$ for all $T \in (T_2, T_4)$, a contradiction since we know $(r^*)'(T) > 0$ for all $T \in (T_2, T_3]$. Therefore, $(r^*)'(T) \leq 0$ for $T \geq 3$. Thus $0.2547 \geq r^*(T) \geq \lim_{T\to\infty} r^*(T) = 0$ for $T \geq 3$. □

Proposition 2

Part 1: Follows from (18).

Part 2:

$$\begin{aligned}
\frac{\partial d(r,T,\Delta c)}{\partial T} &= \frac{2\Delta c^2}{T+3}\Bigg[-\frac{(r^4-r^3+r^2)T^2+(-r^4+r^3-r^2)T+r^4-r^2-r}{T+3} \\
&\quad +2(r^4-r^3+r^2)T-(r^4-r^3+r^2)\Bigg] \\
&= \frac{2\Delta c^2}{(T+3)^2}[(r^4-r^3+r^2)T^2+4(r^4-r^3+r^2)T-3(r^4-r^3+r^2)-(r^4-r^2-r)] \\
&= \frac{2\Delta c^2}{(T+3)^2}[(r^4-r^3+r^2)T^2+4(r^4-r^3+r^2)T-4(r^4-r^3+r^2)+r+2r^2-r^3] \\
&= \frac{2\Delta c^2}{(T+3)^2}[(r^4-r^3+r^2)(T^2+4T-4)+r+2r^2-r^3] \\
&\geq 0
\end{aligned}$$

□

Proposition 3

From (12), (18), and the fact that $Q_t^n = c_t$ for $t \geq T+1$, for $r \in (0,1)$, we have

$$var(\tilde{Q}^c, 1, T+3) - var(Q^n, 1, T+3) = \Delta c[(2T^2-2T+2)r^4-(2T^2-2T)r^3+(T^2-T-2)r^2+(T^2-T-2)r].$$

Using similar logic as in the proof of Theorem 2, one can show that $var(\tilde{Q}^c, 1, T+3) - var(Q^n, 1, T+3)$ only has two real roots. One is 0; let r^* represent the other root. We have $r^*(3) \approx -0.3789$. Similarly to the proof of Lemma 6, we have

$$r^*(T)' = -\frac{(2T-1)(2(r^*)^3-2(r^*)^2+r^*+1)}{6(T^2-T+1)(r^*)^2-4(T^2-T)r^*+(T^2-T+2)} \leq 0$$

Thus $r^*(T) < 0$ for $T \geq 3$. Since the the coefficient of r^4 is positive, $var(\tilde{Q}^c, 1, T+3) > var(Q^n, 1, T+3)$. $\square$

Proposition 4

From (3), (11), (15) and (16), it is clear that the revenue in the first period is the same under both pricing strategies. We begin with the second period.

$$\begin{aligned}
p_2^n &= mc_2 + b = m(A + \Delta c) + b \\
Q_2^n &= A + (1-r)\Delta c \\
\tilde{p}_2^c &= mc_2 + b + rmc_1 + br - P \\
&= p_2^n - rm(T-1)\Delta c \\
\tilde{Q}_2^c &= A + (1-r)\Delta c + (r^2 - r)(T-1)\Delta c \\
&= Q_2^n + (r^2 - r)(T-1)\Delta c
\end{aligned}$$

Since both $\tilde{Q}_2^c$ and Q_2^n are less than $c_2 = A + \Delta c$, the difference in revenue in the second period is

$$\begin{aligned}
&\tilde{Q}_2^c \tilde{p}_2^c - Q_2^n p_2^n \\
&= p_2^n (r^2 - r)(T-1)\Delta c - Q_2^n mr(T-1)\Delta c - m(r^3 - r^2)(T-1)^2 \Delta c^2 \\
&= [m(A + \Delta c) + b](r^2 - r)(T-1)\Delta c - [A + (1-r)\Delta c]mr(T-1)\Delta c \\
&\quad - m(r^3 - r^2)(T-1)^2 \Delta c^2 \\
&= mA\Delta c(r^2 - r)(T-1) + m\Delta c^2 (r^2 - r)(T-1) + b\Delta c(r^2 - r)(T-1) \\
&\quad - mA\Delta c(T-1)r - m\Delta c^2 (r - r^2)(T-1) - m\Delta c^2 (r^3 - r^2)(T-1)^2 \\
&= mA\Delta c(T-1)(r^2 - 2r) + m\Delta c^2 [-(r^3 - r^2)(T-1)^2 + 2(r^2 - r)(T-1)] \\
&\quad + b\Delta c(r^2 - r)(T-1) \\
&= mA\Delta c((T-1)r^2 + (-2T+2)r) + m\Delta c^2 (-(T-1)^2 r^3 + (T^2 - 1)r^2 + (-2T+2)r) \\
&\quad + b\Delta c(r^2 - r)(T-1)
\end{aligned}$$

Next we examine the third period.

$$p_3^n = mc_3 + b = m(A + 2\Delta c) + b$$

$$
\begin{aligned}
Q_3^n &= A + (2-r)\Delta c \\
\tilde{p}_3^c &= mc_3 + b + rm\Delta c \\
&= p_3^n + rm\Delta c \\
\tilde{Q}_3^c &= A + (2-r)\Delta c - r^2T\Delta c + r\Delta c \\
&= Q_3^n + (-r^2T + r)\Delta c
\end{aligned}
$$

Clearly, both $\tilde{Q}_3^c$ and Q_3^n are less than $c_3 = A + 2\Delta c$. Then the difference in revenue in the third period is

$$
\begin{aligned}
&\tilde{Q}_3^c\tilde{p}_3^c - Q_3^n p_3^n \\
&= p_3^n(-r^2T + r)\Delta c + Q_3^n rm\Delta c + m\Delta c^2(-r^3T + r^2) \\
&= [m(A + 2\Delta c) + b](-r^2T + r)\Delta c + [A + (2-r)\Delta c]rm\Delta c \\
&\quad + m\Delta c^2(-r^3T + r^2) \\
&= mA\Delta c(-r^2T + r) + m\Delta c^2(-2r^2T + 2r) + b\Delta c(-r^2T + r) \\
&\quad + mA\Delta cr + m\Delta c^2(2r - r^2) + m\Delta c^2(-r^3T + r^2) \\
&= mA\Delta c(-r^2T + 2r) + m\Delta c^2(-r^3T - 2r^2T + 4r) + b\Delta c(-r^2T + r)
\end{aligned}
$$

Then for $4 \leq t \leq T$, we have $\min\{\tilde{Q}_t^c, c_t\} = c_t$, $Q_t^n = c_t - r\Delta c$, and $\tilde{p}_t^c = p_t^n + mr\Delta c$. Thus

$$
\begin{aligned}
&\sum_{t=4}^{T}\left(\min\{\tilde{Q}_t^c, c_t\}\tilde{p}_t^c - Q_t^n p_t^n\right) \\
&= \sum_{t=4}^{T}\left((Q_t^n + r\Delta c)(p_t^n + mr\Delta c) - Q_t^n p_t^n\right) \\
&= \sum_{t=4}^{T}\left(Q_t^n mr\Delta c + p_t^n r\Delta c + m\Delta c^2 r^2\right) \\
&= \sum_{t=4}^{T}\left((A + t\Delta c - (1+r)\Delta c)mr\Delta c + [m(A + t\Delta c - \Delta c) + b]r\Delta c + m\Delta c^2 r^2\right) \\
&= \sum_{t=4}^{T}\left(2mAr\Delta c - 2m\Delta c^2 r + b\Delta cr + 2tmr\Delta c^2\right) \\
&= mA\Delta c(2T-6)r + m\Delta c^2(-2T+6)r + b\Delta c(T-3)r + m\Delta c^2(T+4)(T-3)r
\end{aligned}
$$

$$= mA\Delta c(2T-6)r + m\Delta c^2(T^2-T-6)r + b\Delta c(T-3)r$$

When $t = T+1$, we have $\min\{\tilde{Q}^c_{T+1}, c_{T+1}\} = B$, $Q^n_{T+1} = B$, and $\tilde{p}^c_{T+1} = p^n_{T+1} + mr\Delta c$. Thus

$$\min\{\tilde{Q}^c_{T+1}, c_{T+1}\}\tilde{p}^c_{T+1} - Q^n_{T+1}p^n_{T+1}$$

$$= mA\Delta cr + m\Delta c^2(T-1)r$$

For $t > T+1$, $\min\{\tilde{Q}^c_t, c_{T+1}\} = Q^n_t = B$ and $\tilde{p}^c_{T+1} = p^n_{T+1} = P$. Then

$$\begin{aligned}
&\sum_{t=1}^{T+3}\Big(\min\{\tilde{Q}^c_t, c_t\}\tilde{p}^c_t - Q^n_t p^n_t\Big)\\
&= mA\Delta c((T-1)r^2 + (-2T+2)r) + m\Delta c^2(-(T-1)^2r^3 + (T^2-1)r^2 + (-2T+2)r)\\
&\quad + b\Delta c(r^2-r)(T-1)\\
&\quad + mA\Delta c(-r^2T+2r) + m\Delta c^2(-r^3T - 2r^2T + 4r) + b\Delta c(-r^2T + r)\\
&\quad + mA\Delta c(2T-6)r + m\Delta c^2(T^2-T-6)r + b\Delta c(T-3)r\\
&\quad + mAr\Delta c + m\Delta c^2(T-1)r\\
&= mA\Delta c[(T-1)r^2 + (-2T+2)r + (-r^2T+2r) + (2T-6)r + r]\\
&\quad + m\Delta c^2[-(T-1)^2r^3 + (T^2-1)r^2 + (-2T+2)r - r^3T - 2r^2T + 4r\\
&\quad + (T^2-T-6)r + (T-1)r]\\
&\quad + b\Delta c[(r^2-r)(T-1) - r^2T + r + (T-3)r]\\
&= mA\Delta c[-r^2-r]\\
&\quad + m\Delta c^2[(-T^2+T-1)r^3 + (T^2-2T-1)r^2 + (T^2-2T-1)r]\\
&\quad + b\Delta c[-r^2-r]\\
&= m\Delta c^2[(-T^2+T-1)r^3 + (T^2-2T-1)r^2 + (T^2-2T-1)r] - \Delta c(r^2+r)(mA+b)
\end{aligned}$$

Since $T \geq 3$ and $0 \leq r < 1$, $(-T^2+T-1)r^3 + (T^2-2T-1)r^2 + (T^2-2T-1)r \geq 0$. Moreover, since $m < 0$ and $mA + b > 0$, $\sum_{t=1}^{T+3}\Big(\min\{\tilde{Q}^c_t, c_t\}\tilde{p}^c_t - Q^n_t p^n_t\Big) < 0$. $\square$