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Abstract

We study a capacity allocation problem with asymmetric information. We present a decentralized multi-unit capacity auction that elicits truthful information to optimally solve the coordination problem. This mechanism is strategyproof, individually rational, and efficient. We also investigate the value of improved demand information to guide planners toward a more educated decision about under what conditions forecast improvement techniques should be pursued.

1 Introduction

Manufacturing industries have to manage capacity effectively in order to make profits. However, for high-tech industries such as electronics, semiconductors and biotechnology, configuration and allocation of the capacity is particularly crucial since a significant amount of time and capital investment is needed to build the capacity. For instance, in semiconductor manufacturing, a wafer fab requires equipments that are worth millions of dollars each and a lead time of more than a year to be built. Although there is a significant manufacturing lead time, product life-cycle is short and products with old technology become obsolete very quickly. Furthermore, product demand is highly volatile and thus, it is difficult to get accurate forecasts.

In the high-tech industry, typically different decision entities manage customer demands and production resources. Each *Strategic Business Unit* (SBU) manages the customer demand for a specific product group that generally share the same manufacturing technology. *Product Managers* (PMs) are responsible from profitability of their SBUs and satisfaction of their customers. PMs are also accountable for the costs incurred by ineffective management of the capacity allocated to their SBUs. More capacity than customer demand results in inventory holding costs, while having to satisfy the demand with inadequate internal capacity leads to outsourcing costs. PMs have the most accurate information on demand of their own customers, as well as a very good understanding

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of possible market trend scenarios for the specific product group they are responsible for. Moreover, in general, PMs are rewarded based on the profits obtained by the corresponding SBUs.

On the other hand, capacity allocation decisions are made by capacity planners at the *Corporate Headquarters* (HQ). The total operating costs of all SBUs is the main concern for HQ to minimize. Therefore, HQ aims to allocate the available internal capacity among the SBUs in the most profitable way for the company. However, the reward structure among the PMs may give them the incentive to act selfishly. Furthermore, available internal capacity is usually scarce; mostly to prevent unnecessary investment on high-tech equipment in case demand does not turn out as high as expected. Also, having subcontractors is a widely utilized policy to reduce risk. However, considering that outsourcing is more expensive than internal capacity, PMs may exaggerate their demand forecasts in order to obtain a larger share of the internal capacity and decrease their costs by avoiding outsourcing.

Ideally, a centralized capacity allocation process would provide HQ the minimum systemwide cost. However, in order to solve a centralized optimization problem, the capacity planners need perfect information about the customer demand each SBU is facing. Sharing private information with HQ contradicts with PMs' own objectives in this case. Consequently, HQ is not guaranteed to be provided with truthful information in case the planners want to use a centralized solution procedure. Since HQ does not have the most up-to-date customer demand information, the company needs to implement a capacity allocation mechanism that would provide the PMs the incentive to reveal true customer demand information.

In this paper, we introduce an auction mechanism for allocating the internal capacity among SBUs to minimize the systemwide outsourcing and inventory holding costs from HQ's point of view. Our mechanism matches the centralized optimization solution using a decentralized approach. The incentive scheme embedded in our auction lets HQ to elicit bids of special characteristics from PMs without requiring them to share private customer demand information. We also investigate how volatility in demand for some product lines affect the bidding behavior of corresponding product managers, and how valuable better customer demand information is.

The remainder of the paper is organized as follows. We review the related literature in the next section. In Section 3, we define the capacity allocation problem and then propose an allocation auction in Section 4. We discuss value of improving demand information next, which is followed by experimental study and conclusions.

2 Literature Review

High-tech manufacturing companies experience difficulty in their capacity planning efforts. Building the capacity implies long lead times in this business environment. Therefore, decision makers are required to act way before the uncertainty in customer demand and production yield is resolved. However, the problem structure is not the only challenge that arises in capacity planning. The structure of decision making authority that involves different business planners with various objectives makes it even harder to manage capacity in high-tech manufacturing companies.

There is a vast literature on high-tech capacity planning problem. These studies differ from each other in methods they use to solve the problem and the assumptions on information structure. One of the two main groups of research on this subject include game-theoretic models that consider economic and behavioral strategies of decision makers. The other group of research use complex mathematical programming and stochastic models to optimize the system performance. In this section, we review studies in both these groups.

The strategic capacity planning problem has been studied by many researchers. This line of research is interested in obtaining good solutions to problems in building wafer fabrication plants, capacity expansion and tool procurement; while generally utilizing optimization based methods, heuristics and approximation schemes. Ahmed and Sahinidis (2003) formulate a stochastic tool procurement problem as a multi-stage stochastic integer program and use a linear programming based approximation scheme that guarantees feasible solutions. They also show that heuristic solution asymptotically converges to the optimal as the problem size increases. Karabuk and Wu (2002) also use a stochastic programming model to solve the capacity allocation problem under both demand and yield uncertainty. They decompose the scenario based mathematical model into marketing and manufacturing subproblems, and solve them in a decentralized manner via augmented Lagrangian based coordination mechanisms. They later build on this study to include capacity expansion decision to the problem (Karabuk and Wu, 2003).

Christie and Wu (2002) consider a multi-year, multi-stage stochastic programming model to optimally configure production technologies across multiple fabs. They deal with demand and capacity uncertainties via construction of a discrete scenario tree. Barahona et al. (2005) formulate a difficult two-stage mixed-integer stochastic programming model to solve the tool planning problem on an IBM semiconductor manufacturing line. Because of the extreme size of the model, they propose a heuristic approach to minimize the unmet demand. Swaminathan (2000, 2002) also introduces heuristic solution methods for the tool capacity planning problem in single and multi-period models, respectively. Recent work that we can count in optimization-based research in capacity planning for semiconductor industry include Hood et al. (2003), who consider a two-stage stochastic integer program and discrete demand scenarios to achieve a robust set of tools under budget constraints.

For a detailed survey of literature in strategic capacity planning in semiconductor industry, we refer the reader to the review by Geng and Jiang (2008) that provides relevant references to fundamental approaches in the area and recognizes recent methods in capacity planning as well as future research directions. Furthermore, an even more extensive survey by Wu et al. (2005) discusses capacity management in high-tech industry, with a focus of attention on capacity optimization in semiconductor manufacturing. Authors review many studies on strategic, tactical and operational level capacity planning activities. They present recent efforts in operations research, economics and game theory literature, while introducing several new research directions.

The mathematical programming based methods are successful in providing accurate solutions to large-scale, complex problems. However, they ignore incentives of participants that keep private information about model parameters and other data. Optimality of solution to a mathematical program is questionable when these decision makers are not necessarily truthful about their declarations. Next, we review game-theoretic models that coordinate different parties involved in capacity planning efforts.

Auctions has been one of the most frequently employed mechanisms as a tool to allocate resources. The winner and price of the resource in question is determined according to the bids that participants provide, using the specific rules of the auction. The seminal paper of Vickrey (1961) has made a revolution in auction theory by establishing a truth inducing mechanism in the form of a second-price auction. Introduction of VCG auction (see also Clarke, 1971; Groves, 1973) allowed sellers to achieve efficient allocations, which reward bidders that value the auctioned items the most. They are also strategy-proof, i.e, it is optimal to bid truthfully whatever the bids of other participants are. Note that another important property that is quite useful is incentive compatibility, which imposes truthful bidding as a Bayesian-Nash equilibrium. In one of the earlier studies, Leonard (1983) uses a generalized second-price auction mechanism to elicit honest preferences from agents in an assignment problem setting. A similar mechanism is used by Banks et al. (1989) to allocate multiple resources with demand or supply uncertainties. In this study, participants bid for portions of many resources with a single bid.

Multi-item combinatorial auctions are regularly used for selling licenses of radio or wireless spectrum. A large number of licenses are auctioned simultaneously and the value of a license depends on the other licenses that a buyer owns. Therefore, finding the revenue maximizing allocation is computationally difficult (Rothkopf et al., 1998). Allocation of airport slots (Rassenti et al., 1982), and supply chain procurement (Chen et al., 2005) are among many other environments that auctions are regularly used. For a more extensive review, we refer the reader to a survey of the combinatorial auction literature by De Vries and Vohra (2003).

In this study, we are more interested in use of auction mechanism as a medium of coordination. One of the relevant studies is by Kutanoglu and Wu (1999), who proposed combinatorial auction mechanisms to solve a job shop scheduling problem. Authors use the relationship between Lagrangean relaxation and a version of combinatorial auction to develop new auction protocols and the corresponding payment functions that iteratively coordinate jobs as a distributed scheduling mechanism. However, they assume that all agents participate in the auction without considering incentive compatibility. Ertogral and Wu (2000) solve a multi-facility production planning problem in a supply chain via an auction-theoretic mechanism while assuming truthful but selfish facility agents. They also take advantage of Lagrangean decomposition and create a conflict pricing structure that is updated in every iteration to reduce the inconsistency between solutions of facility submodels.

The information asymmetry between supply chain partners that are organized by a supply chain management within a decision making process is addressed by Feldmann and Muller (2003). The authors present an incentive mechanism based on the Groves scheme to make revelation of truthful private information possible. Dudek and Stadtler (2005) introduce a negotiation-based scheme for inter-organizational collaboration partners to modify order/supply patterns iteratively. The procedure can be performed in a decentralized manner between a buyer and a supplier, and a near optimal master plan for the supply chain is obtained. Stadtler (2006) reviews recent literature that is interested in collaborative planning schemes for inter-organizational supply chain management.

Karabuk and Wu (2005) consider a semiconductor manufacturing environment similar to ours, in which the headquarters gives product managers the authority of privately owning demand information on specific product groups that they manage. Capacity is allocated to product managers according to their reports on customer demand. These managers are rewarded through a bonus system based on their performance; therefore, they are motivated to misrepresent the demand information to gain benefits. The authors build an incentive scheme that elicits truthful private information by introducing a side payment, and achieve the system-optimum allocation.

In our study, we coordinate the decision makers with private information using a one-shot auction that maintains properties of truthful bidding and efficiency. The iterative auction mechanisms that are suggested in literature require the agents to continuously calculate and exchange transfer prices until a coordination is reached. This is not a practical solution method in reality. Furthermore, the structure of the winner determination problems or the mechanisms that calculate the transfer prices are not visible to the individual agents in general. In that case, the decision makers become more reluctant to act parallel to a black-box solution or to even participate at all. Our auction mechanism is clearly defined and every participant knows what to expect. We elicit honest bids from product managers and achieve the optimal solution of the centralized problem in a decentralized manner.

3 Model Formulation

The capacity planning of a high-tech manufacturer is two-fold. First is the *capacity expansion*, in which the company decides on the manufacturing technologies needed during the production horizon and their corresponding physical capacity levels. This is higher-level strategic planning by which the HQ basically builds the internal capacity for all product lines. Second is the *capacity configuration* that allocates facilities of certain technologies among SBUs with various product groups. In this paper, we focus on the capacity configuration problem assuming that HQ has already built the internal capacity. We look at this problem from HQ's point of view and try to achieve a good systemwide solution by coordinating all PMs through a capacity allocation mechanism. These PMs represent different SBUs of product groups, all of which will share a common internal capacity pool that HQ has built.

The overall objective is to meet customer demand with minimized total costs under asymmetric information. It is clear that a centralized optimization model would have given the minimum total cost if HQ had complete information on customer demand of all PMs. Therefore, the objective of HQ is to construct a decentralized capacity allocation mechanism that will produce a solution as close to the centralized optimal solution of the symmetric information case as possible.

In high-tech industry, capacity is typically expressed by number of product releases per planning period. Thus, capacity configuration (allocation) is determining the number of product releases for each product group during this time. When the internal capacity is insufficient, PMs can outsource capacity from external factories at additional costs. The lead time for outsourcing is negligible.

Since capacity building costs are very high, PMs cannot afford not utilizing the production resources. Therefore, when a PM is allocated more internal capacity then needed, the extra capacity is used toward meeting demand in the following periods. However, this causes PMs to incur inventory holding costs. PMs are accountable for outsourcing and holding costs accrued by their product groups, while HQ is liable for the total costs accrued by all PMs and can minimize it by optimal capacity configuration.

3.1 Problem Setting

There is a set ($N = \{1, ..., N\}$) of PMs that are interested in being allocated internal capacity for products in their SBUs. We look at the single period problem and the main decision HQ has to make is the number of wafer starts, denoted by x_i, for product group (PM) i ($i \in N$). We assume that the customer demand is stochastic and distributed with respect to a Normal distribution. We use ξ_i to denote customer demand for product group i. PMs privately have the most accurate information on demand for products of their SBUs because they regularly communicate with the specific customers that are interested in buying. Furthermore, PMs are responsible for satisfying the customer demand for their products by using internal capacity or outsourcing.

Outsourcing and inventory positioning decisions are finalized by PMs after the customer demand is realized. Therefore, outsourcing and inventory decisions are modeled as recourse to the capacity allocation decision. We denote the amount of wafer starts outsourced by PM i with respect to demand realization ξ by variable $o_i(\xi)$. Similarly, $I_i(\xi)$ is the inventory decision. We use φ_i and h_i to denote unit costs associated with outsourcing and inventory variables, respectively. Finally, κ is the available internal capacity to be allocated.

3.2 Centralized Capacity Allocation

Under the assumption of information symmetry, i.e., if HQ has the same customer demand information as PMs, solution to the following centralized model would be optimal. The *centralized capacity allocation problem* is formulated as a stochastic program, and the objective is to minimize expected total costs. The realized total cost when demand is $\xi = (\xi_1, ..., \xi_N)$ and allocation is $x = (x_1, ..., x_N)$

is denoted by $Q(x, \xi)$ in the following model.

$$
\begin{array}{lrl}
(CP') & \texttt{Min} & E_\xi \left[Q(x,\xi) \right] \\
 & \texttt{s.t.:} & \sum_{i=1}^{N} x_i = \kappa \\
 & & x \geq 0,
\end{array}
$$

where

$$
\begin{aligned}
Q(x,\xi) \;=\; \texttt{Min} \quad & \sum_i \left(\varphi_i o_i(\xi) + h_i I_i(\xi) \right) && (1)\\
\texttt{s.t.:} \quad & o_i(\xi) - I_i(\xi) = \xi_i - x_i, \quad \forall i \in N && (2)\\
& o_i, I_i \geq 0, \quad \forall i \in N
\end{aligned}
$$

For the sake of simplicity, we discretize the probability distribution and represent the demand uncertainty in the industry by scenario sets, S_i for PM i. Each element $s_i \in S_i$ identifies a customer demand quantity with the corresponding probability of occurrence π_{s_i} ($\sum_{s_i \in S_i} \pi_{s_i} = 1$). Amount of customer demand for product group i in demand scenario s is given by D_i^s. A scenario for the allocation problem is represented by a vector of independent demand scenarios, $\vec{s} = (s_1, s_2, .., s_N) \in S$, consisting of a scenario from each PM. S is the set of all demand scenario combinations ($S = S_1 \times S_2 \times .. \times S_N$) and $\pi_{\vec{s}} = \pi_{s_1}\pi_{s_2}..\pi_{s_N}$.

In light of the above assumptions, we can formulate the deterministic equivalent of the above stochastic program, centralized capacity allocation problem (CP), using demand scenarios as follows.

$$
\texttt{Minimize} \quad z^c = \sum_{s \in S} \pi_s \left[\sum_{i=1}^{N} \left(\varphi_i o_i^s + h_i I_i^s \right) \right] \tag{3}
$$

The objective is to minimize total expected cost incurred by all SBUs over possible scenario realizations. Balance constraints (4) ensure that customer demand is satisfied by outsourcing when internal capacity allocation is inadequate. Excess internal capacity is positioned as inventory to be used to satisfy demand in the following periods.

$$
x_i + o_i^s - I_i^s = D_i^s \quad \forall s \in S \tag{4}
$$

We assume that the internal capacity is not sufficient to satisfy the total demand. If capacity was not scarce, PMs would be willing to share private information since they could get allocated exactly the amount of capacity they ask for. Constraint set (5) imposes HQ to allocate all of the internal capacity among PMs.

$$\sum_{i=1}^{N} x_i = \kappa \tag{5}$$

Although the centralized capacity allocation problem (CP) described by 3-5 provides the optimal solution to the problem from HQ's point of view, this model does not fit well with the actual information structure in the company since it neglects to consider local decision problems of PMs. From PMs' point of view, PMs have every incentive to alter demand information to get an advantage during capacity allocation, and a centralized model cannot provide systemwide optimal solution without truthful demand information. Consequently, HQ has to implement a decentralized allocation structure that: (a) respects private information, and (b) provides a solution that is close to the centralized optimal solution. In the next section, we present a decentralized capacity allocation model and characterize the marginal utility of each capacity unit.

3.3 Decentralized Capacity Allocation

We first look at what the local decision problems of PMs and HQ are. PMs' objective is to minimize outsourcing and inventory holding costs that their business units incur. However, the amount of wafer starts a PM is allocated is decided by HQ, and given as an input for the following optimization model, *PM's local problem* (PMP).

$$\texttt{Minimize} \quad z_i(x_i) = \textstyle\sum_{s_i \in S_i} \pi_{s_i} \left[\varphi_i o_i^{s_i} + h_i I_i^{s_i}\right] \tag{6}$$

$$\texttt{s.t.:} \qquad o_i^{s_i} - I_i^{s_i} = D_i^{s_i} - x_i \qquad \forall s_i \in S_i \tag{7}$$

$$I, o \geq 0$$

Once the allocation is made, the problem of PM i to minimize his expected cost is trivial, since inventory and outsourcing decisions are determined by simple recourse. Without involvement of PMs in allocation process, the allocation quantities cannot be optimal. HQ wants to minimize total expected costs for all PMs, however does not have the control on decision variables I, o or the customer demand data, D. Following is *HQ's local problem* (HQP).

$$\texttt{Minimize} \quad z_{HQ} = \textstyle\sum_{i \in N} z_i(x_i)$$

$$\texttt{s.t.:} \qquad \textstyle\sum_{i \in N} x_i = \kappa$$

$$x \geq 0$$

PMs and HQ decide on different decision variables that affect each other and the systemwide cost. Therefore, a coordination mechanism is needed to determine values for all decision variables together to achieve the globally optimal solution. Below we characterize the optimal values for inventory and outsourcing decision variables.

Proposition 1. *For each scenario s, at most one of the decision variables o^s and I^s can have a positive value in the optimal solution of PMP.*

Proof. Suppose that $\exists s \in S$ such that $I_i^s > 0$ and $o_i^s > 0$. Let $I_i^s \geq o_i^s$, without loss of generality. Define $\hat{I}_i^s = I_i^s - o_i^s$ and $\hat{o}_i^s = 0$. Note that $\hat{I}_i^s, \hat{o}_i^s$ still satisfy constraint 7 and provide a smaller objective value. Therefore, o^s and I^s cannot be positive at the same time.

□

We will use the above property to calculate the marginal utility of capacity units in Proposition 2.

Proposition 2. *The dual price, $d_i(x_i)$, when PM i is allocated x_i units is given by the term:*

$$\sum_{s:D_i^s<x_i} -\pi_s h_i + \sum_{s:D_i^s\geq x_i} \pi_s \varphi_i \tag{8}$$

Proof. Dual of PM i's problem is:

$$\begin{aligned} \texttt{Maximize} \quad & \textstyle\sum_{s\in S_i}(D_i^s - x_i)w_i^s \\ \texttt{st:} \quad & w_i^s \leq \pi_s\varphi_i, \quad \forall s \in S_i \quad (o_i^s) \\ & -w_i^s \leq \pi_s h_i, \quad \forall s \in S_i \quad (I_i^s) \end{aligned}$$

By the *complementary slackness theorem*, when the optimal value of a primal variable (o_i^s, I_i^s) is positive, the corresponding dual constraint is binding. Therefore, we can determine the value of dual variables associated with each scenario by investigating the relative magnitude of allocation quantity (x_i) and demand scenario (D_i^s).

$$I_i^s > 0, \quad \forall s \in S_i \mid D_i^s < x_i \quad \Rightarrow \quad w_i^s = -\pi_s h_i$$

$$o_i^s \geq 0, \quad \forall s \in S_i \mid D_i^s \geq x_i \quad \Rightarrow \quad w_i^s = \pi_s \varphi_i$$

Thus, the dual price corresponding to scenario s for PM i is either $-\pi_s h_i$ (for small demand scenarios) or $\pi_s \varphi_i$ (for greater demand scenarios). Then, the dual price when PM i is allocated x_i units, $d_i(x_i)$, is calculated as:

$$d_i(x_i) = \sum_{s:D_i^s < x_i} -\pi_s h_i + \sum_{s:D_i^s \geq x_i} \pi_s \varphi_i \tag{9}$$

□

Above term (dual price) is the expected marginal utility of an additional unit for PM i. In other words, as a risk neutral agent, PM i would be willing to pay at most this much to obtain an additional unit. The dual price depends on the current allocation quantity, x_i. Dual price is nonincreasing since the number of demand scenarios that are smaller than x_i either remains the same or increases as allocation quantity increases. Therefore, the negative term in Equation 9 becomes larger, while the positive term gets smaller as x_i increases. Figure 1 illustrates dual prices for a PM with the parameters: $\varphi = 4, h = 4, D = (25, 60, 80, 110), \pi = (0.2, 0.3, 0.4, 0.1)$.

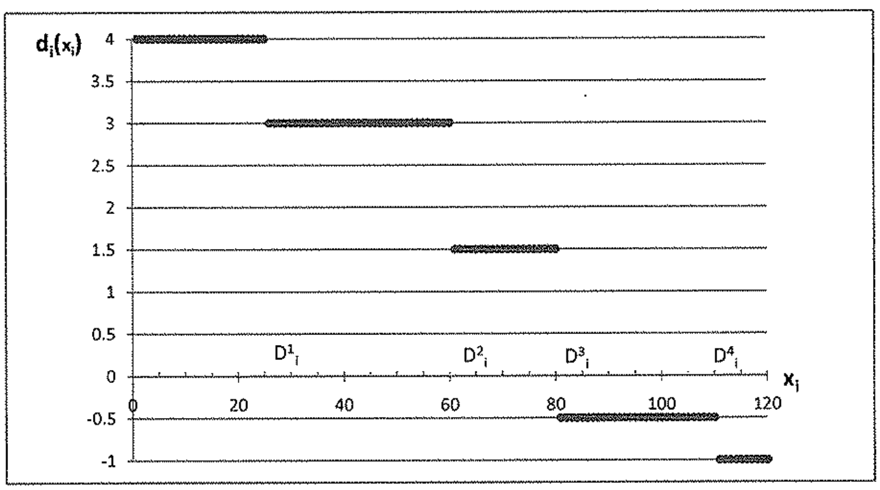


Figure 1: Nonincreasing dual prices

Next, we present an auction mechanism to allocate the capacity optimally among PMs.

4 A Decentralized Capacity Auction

In the following section we will explain the assumptions we use and present the order of events during capacity allocation. Then, we propose an auction mechanism that will reach the optimal solution from the HQ's perspective.

4.1 Assumptions and Order of Events

Capacity is allocated via an auction that is run by HQ. Any PM that wants a share from internal capacity has to bid for capacity units. Winners are determined according to the *allocation rule* dictated by the auction. Each winner makes a payment to the HQ, amount of which is determined by the *pricing rule*. Let O be the set of possible outcomes of the auction, and $z_i(o), p_i(o)$ be the total cost and auction payment incurred by agent i with respect to outcome $o \in O$. We assume that all participants are risk neutral. PMs have quasi-linear utilities, which correspond to the total of business costs and payments accrued, $u_i = -(z_i(o) + p_i(o))$. HQ values the capacity units at zero and total customer demand at all SBUs is always greater than available capacity. Below we present the order of events during the capacity allocation auction that is run by HQ.

Suppose that resource for one product release is the smallest capacity unit that a PM can be allocated. Let b_j^i denote the bid of PM i for the jth capacity unit he is allocated. Recall that although capacity units are homogeneous, bids are nonincreasing since the latter capacity units are used to satisfy less certain customer demand. At the beginning of the auction, each PM reveals his bid schedule to HQ, without observing bids of other PMs. Then HQ reveals the amount of capacity, κ, available for allocation. HQ runs a *losing-price auction*, allocates the capacity, and charges the PMs for the units they win. In the next section, we will explain the losing-price auction in detail.

4.2 The Losing-price Auction

Given the bid schedules of all participating PMs, the auctioneer (HQ) awards the highest κ bids. When there is a tie for being the highest bid and there is not enough capacity, a PM is randomly chosen to be awarded the capacity unit. Below is the winner determination problem, in which y_j^i is the binary variable that denotes whether PM i is awarded his jth capacity unit or not.

$$\begin{aligned} \texttt{Maximize} \quad & \textstyle\sum_{i \in N} \sum_{j \in \kappa} y_j^i b_j^i \\ \texttt{st:} \quad & \textstyle\sum_{i \in N} \sum_{j \in \kappa} y_j^i \leq \kappa \\ & y \in \{0,1\} \quad \forall i,j \end{aligned}$$

For the units awarded to him, a winner PM pays to the HQ a total price equal to the sum of a number of highest competing bids that did not win. The number of units a winner pays for is equal to the number of units he is awarded. This is a Vickrey-based payment scheme that is used widely in literature to induce truth telling; especially in multi-unit VCG auctions. However, the above winner determination problem is solved N+1 times in a VCG auction: one with all PMs to determine the allocation, and N times each without a PM, for pricing; making it computationally very demanding. Instead, we propose an algorithm to allocate the capacity units and calculate the payments in a single pass.

The algorithm uses the bid schedules and available capacity as input and there is no information update from outside during the auction. This auction algorithm is designed as repeated single-unit auctions. First κ rounds of the single-unit auction are run to determine the winners of every capacity unit available for allocation. We call these *allocating auctions.* Then, the following rounds are used to determine the prices the winners should pay, and therefore called *pricing auctions.*

During allocating auctions are run, in every round, the highest bidder wins a unit. Winner's bid is updated using the bid schedule he provided before. The auction is repeated κ times to determine all of the winners. Then, during pricing auctions, a winning bid does not win any units but determines the payments of winners in allocating auctions. Let bidder i win n_i units ($\sum_i n_i = \kappa$). Then, winner i pays the sum of first n_i winning bids of competitors during the pricing auctions. The single-pass auction algorithm used here is easy to adapt.

Proposition 3. *The running time of losing-price auction is O(mn), where m is the number of capacity units to be allocated, and n is the number of bidders.*

Proof. Please find the details of the losing-price auction algorithm in the Appendix.

□

Since the auction algorithm can be run in $O(mn)$ time, and no information exchange is needed during the auction, this allocation mechanism is very practical to implement. However, the size of bid schedules and practical computational complexity of the losing-price algorithm are actually even less because the bid for a certain unit is different from the previous one only if a new scenario is defined for demand above that quantity. Thus, number of different bids depend on the number of scenarios. We can assume that number of different scenarios, S, is a fixed, finite integer. Therefore, each PM makes the bid calculation only S times.

In parallel to this, the computational complexity of the losing-price algorithm can be rephrased as $O(Sn)$, where n represents the number of PMs. Once a bid is announced winner, it is obvious that all consecutive bids until next demand scenario are equal and also winner. Therefore, the search space is significantly reduced by a factor of S/κ. Apart from ease of implementation, this auction mechanism also has additional favorable properties such as *strategy-proofness, individual rationality,* and *efficiency.*

Theorem 1. *Truthful bidding is a dominant strategy for each PM in a losing-price auction.*

Proof. In this auction, a bidder i that wins n_i units pays the highest n_i losing bids of other bidders. Thus, the payment of an agent is the same as long as he wins the same number of units, whether his bids are the same or not. Given a player i, and his true value (v_j^i) for j^{th} capacity unit (without loss of generality) he won, let HL be the highest losing bid of other agents, and LW be the lowest winning bid of other agents ($LW > HL$).

Case 1. If $v_j^i > HL$: When player bids v_j^i, his utility from obtaining this unit will be $u_i > 0$. If player i lies by bidding $b_j^i > v_j^i$ or $v_j^i > b_j^i > HL$, he still wins the unit. His total payment does not change because the total number of units he wins remains the same. On the other hand, if he lies by bidding $b_j^i < HL$, he does not win the unit and loses the positive utility previously obtained from that unit. Therefore, PM is not better off by changing his bid from being equal to his true valuation.

Case 2. If $v_j^i < HL$: If player bids v_j^i, his utility will be $u_i = 0$ since he will not win. If player i lies by bidding $b_j^i > LW > HL > v_j^i$, he wins the unit, but his payment increases by the bid he eliminated, LW. Thus, his utility will be negative.

$\square$

Consequently, bidding true valuations maximize the utility. Using $n_i(\vec{b})$ as the capacity units won as a function of the bid schedule $\vec{b}$, we can write total utility function as:

$$\sum_{j=1}^{n_i(\vec{b})} \left(v_j^i - p_j^i\right) \tag{10}$$

which is maximized by a truthful bidding strategy, $\vec{b} = \vec{v}$.

Theorem 2. *Losing-price auction is an individually rational mechanism.*

Proof. Winners pay the highest competing bid of other players for each item they win. Therefore, payments are always lower than the bids when the bidders bid truthfully. Moreover, a player that does not win does not pay.

□

Theorem 3. *Losing-price auction is efficient.*

Proof. Any mechanism with an efficient allocation rule is said to be efficient. In this auction, the capacity is allocated to the bidder that values it the most. Furthermore, the bidding strategy is truth-telling. Therefore, the allocation rule maximizes the social welfare, i.e., it is efficient.

□

Next, we find the optimal bidding policy of a PM in a losing-price auction, given the properties listed above.

Theorem 4. *Bidding dual prices (marginal utilities) is optimal for PMs.*

Proof. PMs are utility maximizers, and the utility of PM i is given by $u_i = (-z_i - p_i)$. Note that $z_i(x_i) = z_i(0) - \sum_{j=0}^{x_i-1} d_i(j)$. Then, after the auction, utility of PM i can be written as $u_i(n_i) = -z_i(0) + \sum_{j=0}^{n_i-1} d_i(j) - \sum_{j=0}^{n_i-1} p_j^i$. Consequently, in order to maximize the utility, a PM needs to maximize $\sum_{j=0}^{n_i-1}(d_i(j) - p_j^i)$.

Note that the dual prices are fixed for each bidder and independent from the bids. Moreover, the pricing rule for losing-price auction suggests that the total payment for units won does not change as long as the number of units won is the same, whatever the winning bids are. Thus, PM i only needs to maximize the number of units won, n_i, while making sure that dual price is greater than the payment for each unit won, $d_i(j) \geq p_j^i, \forall i \leq n_i$.

Using the strategy-proofness of the pricing rule, replacing true valuations with dual prices in Equation 10, we conclude that $\sum_{j=1}^{n_i(\vec{b})} \left(d_i(j) - p_j^i\right)$ is maximized by bidding dual prices, $\vec{b} = \vec{d}$.

□

We presented the allocation and pricing rules for the decentralized auction as well as the optimal bidding policies of participating PMs. Following is a numerical example to illustrate workings of the losing-price auction algorithm.

4.3 Numerical Example

We assume that there are four participants, $PM1 - PM4$, each of whom has private information on customer demand scenarios with their associated probabilities. The parameter values used in the example are given in Table 1. In order to establish capacity scarcity, we choose $\kappa = 10$.

	D_1	π_1	D_2	π_2	D_3	π_3	D_4	π_4	h	φ
$PM1$	1	0.2	3	0.5	5	0.3			1	3
$PM2$	2	0.4	4	0.6					1	4
$PM3$	1	0.15	2	0.25	3	0.35	4	0.25	2	4
$PM4$	2	0.35	4	0.45	5	0.2			1	4

Table 1: Example parameters

First, each PM calculates their bids for capacity units. Since this is a losing-price auction, PMs' optimal policy is to bid marginal utilities. Below, we present calculation of the bids.

Bid Schedules:

Bid schedule for PM1:

unit	calculation	bid
1	3x(0.2+0.5+0.3)	3
2..3	3x(0.5+0.3)-1x0.2	2.2
4..5	3x0.3-1x(0.2+0.5)	0.2

Bid schedule for PM2:

unit	calculation	bid
1..2	4x1	4
3..4	4x0.6-1x0.4	2

Bid schedule for PM3:

unit	calculation	bid
1	4x1	4
2	4x0.85-2x0.15	3.1
3	4x0.6-2x0.4	1.6
4	4x0.25-2x0.75	−0.5

Bid schedule for PM4:

unit	calculation	bid
1..2	4x1	4
3..4	4x0.65-1x0.35	2.25
5	4x0.2-1x0.8	0

Above are the bid schedules PMs will submit at the beginning of the auction. Each PM starts by bidding for the first unit. As they win units the winning bid is replaced with the bid for the consecutive unit. When there is a tie for the highest bid, a PM is randomly chosen.

Auction:

Rounds of the losing-price auction are provided in Table 2. Auction's allocating rounds last for 10 rounds because of the κ. In round 1, all PMs bid their bids for first unit. The highest bidder is randomly chosen among PMs 2,3, and 4. In round 2, PM2 updates his bid by replacing it with his bid for unit 2. However, the bids are same for the first two units for PM2. He still is the highest bidder. In round 3, PM2's bid becomes 2, and highest bidders are PM3 and PM4. Randomly PM3 is chosen, and his bid is updated to be 3.1 in round 4. The auction continues in this manner until all 10 units are allocated. The bold bids represent the winner in a given round.

After 10 rounds, the auction continues as usual. However, differently, winning bids are noted with italic letters and they are used to determine the payments of PMs for the units they won above the line (boldface bids). Each PM has to make a payment to the HQ for the winnings. Starting from the highest (winning in pricing round) italic bid, each PM pays competitors' highest losing bids. Number of losing bids a PM has to pay for is equal to the total number of capacity units he wins.

The payment calculation for each PM is as follows:

PM1: Payment for 2 units: 2+2=4

PM2: Payment for 2 units: 2.2+1.6=3.8

round	*PM 1*	*PM 2*	*PM 3*	*PM 4*
1	3	**4**	4	4
2	3	**4**	4	4
3	3	2	**4**	4
4	3	2	3.1	**4**
5	3	2	3.1	**4**
6	3	2	**3.1**	2.25
7	**3**	2	1.6	2.25
8	2.2	2	1.6	**2.25**
9	2.2	2	1.6	**2.25**
10	**2.2**	2	1.6	0
11	*2.2*	2	1.6	0
12	0.2	*2*	1.6	0
13	0.2	*2*	1.6	0
14	0.2	0	*1.6*	0

Table 2: Rounds of losing-price auction.

PM3: Payment for 2 units: 2.2+2=4.2

PM4: Payment for 4 units: 2.2+2+2+1.6=7.8

4.4 Cost of Decentralized Solution

We have been able to show that losing-price auction satisfies individual rationality, strategy-proofness and efficiency. Furthermore, we also established that optimal bidding policy for a PM is bidding the dual prices. Now, we want to determine how close the result of this auction will be to the optimal solution of the centralized capacity allocation problem. The gap between optimal solutions of CP and losing-price auction represents the cost of decentralized solution. In a real setting, HQ would want to consider this gap and implement the auction only if the cost of decentralized solution has a reasonable magnitude.

Let $z^c(\vec{x})$ be the objective value of CP when $\vec{x} = (x_1, x_2, ..., x_N)$ is the capacity allocation vector and $z_i(x_i)$ be the objective value of PMP when PM i is allocated x_i units. Below lemma shows that there is no gap between the optimal results of centralized and decentralized solutions.

Lemma 1.

$$\sum_{i=1}^{N} z_i(x_i) = z^c(\vec{x}), \forall \vec{x}$$

Proof.

$$
\begin{aligned}
z^c(\vec{x}) &= \sum_{s\in(\Pi S_i)} \pi_s \sum_{i=1}^{N} [h_i I_i^s + \varphi_i o_i^s] \\
&= \sum_{i=1}^{N} \left(\sum_{s_i=1}^{S_i} \pi_{s_i} \left(\sum_{s_{-i}\in(\Pi S_{-i})} \pi_{s_{-i}} \right) [h_i I_i^s + \varphi_i o_i^s] \right) \\
&= \sum_{i=1}^{N} \left(\sum_{s_i=1}^{S_i} \pi_{s_i} [h_i I_i^s + \varphi_i o_i^s] \right) \\
&= \sum_{i=1}^{N} z_i(x_i).
\end{aligned}
$$

□

Above lemma shows that for any allocation vector $\vec{x}$, total objective function value of local decision problems is equal to the centralized problem's objective value. Next, we will show that minimizing one of them will also provide us the minimum for the other. In other words, we will show that decentralized losing-price auction provides the optimal solution to the centralized problem.

Theorem 5 (Nemhauser and Wolsey (1988)). *The following statements are equivalent:*

i. A (the constraint matrix) is TU.

ii. For every $J \subseteq A = \{1, ..., n\}$ (columns of the constraint matrix), there exists a partition J_1, J_2 of J such that

$$\left| \sum_{j\in J_1} a_{ij} - \sum_{j\in J_2} a_{ij} \right| \leq 1 \quad for \quad i = 1, ..., m.$$

Proof. The proof can be found in Nemhauser and Wolsey (1988), pg. 543.

□

As long as the optimal solution to the optimization problem CP is integral, the auction replicates the results since it only considers the integer solutions. Therefore, in order to conclude that the auction mechanism always achieves the centralized solution, we have to show that the optimal solution to the centralized problem is always integral.

It is known that the solution of a problem is integral if all extreme points of the polyhedron we are searching the solution in is integral. We also know that a totally unimodular constraint matrix with an integer right hand side vector provides an integral polyhedron. The characterization of total unimodularity that is presented in Theorem 5 helps us in identifying the constraint matrix of CP.

Proposition 4. *Constraint matrix of CP is totally unimodular.*

Proof. See Appendix for a partition algorithm that satisfies Theorem 5.

□

Lemma 2. *Optimal solution to CP is integral if available capacity* (κ) *and demand scenarios* (D^s) *are integer.*

Proof. The constraint matrix of CP is *totally unimodular* by Proposition 4. Therefore, all extreme points of the polyhedron are integral if the right hand side vector is integral.

□

Theorem 6. *The losing-price auction terminates with the optimal solution to the centralized capacity planning problem, CP.*

Proof. Let z^{c^*}, and $\vec{x}^* = (x_1^*, x_2^*, ..., x_N^*)$ be the optimal objective function value and optimal solution to CP, respectively. Then, by Lemma 1, $\sum_{i=1}^N z_i(x_i^*) = z^{c^*}$.

Suppose that $\vec{x}^* \neq \arg\min_x \sum_{i=1}^N z_i(x_i)$. We know that the auction finds the optimal allocation that minimizes $\sum_{i=1}^N z_i(x_i)$, since it is efficient. Let $\hat{\vec{x}}$ be the optimal allocation. So, $z^c(\vec{x}^*) = \sum_{i=1}^N z_i(x_i^*) > \sum_{i=1}^N z_i(\hat{x}_i) = z^c(\hat{\vec{x}})$, which is a contradiction.

□

Consequently, we have shown that the solution to the decentralized allocation auction we introduced in this paper always matches the optimal solution to the centralized problem. The losing-price auction has less information requirements, solves the problem in a decentralized manner and it is easy to implement. However, the crucial point is, there is no gap between the solutions of centralized problem and the decentralized auction.

Next, we investigate the two period model, and show that the same bidding strategy still enables matching the optimal centralized solution in a decentralized manner.

4.5 Two-period Capacity Allocation Model

We have shown that losing-price auction and the corresponding bidding strategy of PMs enable HQ to obtain the optimal solution in a single period environment. Next, we want to see whether it is still possible to match the optimal solution to CP in multi-period problems. We assume that customer demand is observed in two ($T = 2$) consecutive periods. PMs compete for the internal capacity, κ_t, that HQ builds for each period t. The allocation is done before any demand is observed, and in a single pass. The remaining assumptions are the same as the single-period model. Following is the two-period analogue of the centralized allocation problem (CP2):

$$
\begin{array}{lll}
\texttt{Minimize} & z^c = \sum_{s \in S} \pi_s \left[\sum_{i=1}^{N} \sum_{t=1}^{T} \left(\varphi_i o_{i,t}^s + h_i I_{i,t}^s \right) \right] & \\
s.t.: & o_{i,1}^s - I_{i,1}^s = D_{i,1}^s - x_{i,1} & \forall i \in N, s \in S \\
& o_{i,2}^s + I_{i,1}^s - I_{i,2}^s = D_{i,2}^s - x_{i,2} & \forall i \in N, s \in S \\
& \sum_{i=1}^{N} x_{i,t} = \kappa_t & t \in \{1,2\}
\end{array}
$$

We propose that the optimal solution of the CP2 can be obtained in a decentralized way, using a similar bidding structure to what PMs employ in losing-price auction. Bidding dual prices (marginal utilities) for capacity units provides the optimal solution. However, second period's bids depend on the allocation for the first period. This is because first period allocation affects the inventory passed to the second period. Consequently, there is a different bid schedule in second period for every different possible allocation decision in first period.

The calculation of bid schedule for period 1 is straightforward, and follows directly from the dual of CP2 and the complementary slackness theorem:

$$
\sum_{s: D_{i,1}^s \geq x_{i,1}} \pi_s \varphi_i + \sum_{s: \sum_t D_{i,t}^s \geq x_{i,1}} \pi_s (\varphi_i - h_i) + \sum_{s: \sum_t D_{i,t}^s < x_{i,1}} \pi_s (-2h_i) \tag{11}
$$

Following is the calculation of bid schedules for period 2, in which PMs also consider the inventory left from period 1. The first term of the summation describes scenarios that would lead to outsourcing in period 2, while the second term is for the cases with an inventory at the end of last period.

$$
\sum_{s: \left[(D_{i,1}^s \geq x_{i,1}) \& (D_{i,2}^s \geq x_{i,2}) \right] or \left[(D_{i,1}^s < x_{i,1}) \& (\sum_t D_{i,t}^s \geq \sum_t x_{i,t}) \right]} \pi_s \varphi_i + \sum_{s: otherwise} \pi_s (-h_i) \tag{12}
$$

Since the bid schedule for period 2 depends on period 1 allocation, $x_{i,1}$; running sequential allocation auctions for each period cannot provide the optimal solution. Therefore, we need to run a combinatorial auction that consider allocation decision for two periods together. Consequently, the bid schedules should define the bid for every allocation combination. Since there are two periods, the size of the bid schedule is quadratic with respect to available internal capacity. However, we would expect the size to increase polynomially as the number of periods in the model increases. The winner determination problem can easily be solved via dynamic programming, and the auction result matches the optimal solution of CP2 via the same argument in single-period problem.

In the next section, we study the bidding strategy and auction performance of PMs with demand volatility.

5 Value of Demand Information

In this section, we will investigate how volatility in demand for some product lines affect the bidding behavior of corresponding product managers. One of the advanced techniques that help better understand the demand parameters was presented by Wu et al. (2006). *The leading indicator engine* identifies a number of products that are adopted first by the market, and provides advanced customer demand information for the rest of the products. The additional information obtained by leading indicator products significantly reduces the planner's perception of demand volatility. Consequently, it is possible to obtain better demand information when additional investment is made in one of the many advanced statistical tools available such as the leading indicator engine. We will also investigate whether the improvement in the outcome of the capacity auction is worth the investment in advanced information.

5.1 Model and Assumptions

We will continue to use the notation and assumptions in Section 4.1. We still assume that the customer demand is stochastic and distributed with respect to a Normal distribution, and the problem formulation is represented by model CP$'$. We want to remind the reader further that PMs have private information about demand from their own customers, in a single-period model. They have to satisfy the demand using internal capacity or outsourcing. HQ allocates the available internal capacity via a losing-price auction, therefore PMs have to submit discrete bid schedules in the auction. In order to portray capacity scarcity, the available internal capacity is assumed to satisfy the relationship: $\kappa < \sum_i x_i^*$, where x_i^* is the number of capacity units with a positive marginal utility for PM i. Throughout this section we assume that PMs have a correct understanding of the mean demand, but a more accurate information on demand variability can be obtained by additional investment in advanced statistical analysis tools.

The allocation decision, x, has to be finalized before the customer demand is realized. After demand is known, outsourcing and inventory decisions are made in the second stage. Using the constraints (2), the optimal outsourcing and inventory quantities can be computed by the following rules once the allocation is made and demand is realized.

$$o_i^*(\xi_i) = max(\xi_i - x_i, 0) \tag{13}$$

$$I_i^*(\xi_i) = max(x_i - \xi_i, 0) \tag{14}$$

Replacing equations (13) and (14) in the total cost function (1), the value of the recourse can be written as:

$$\begin{aligned} R(x) &= \sum_i \left(E_{\xi_i}\left[\varphi_i \, max(\xi_i - x_i, 0) + h_i \, max(x_i - \xi_i, 0)\right]\right) \\ &= \sum_i \left(\int_{-\infty}^{x_i} h_i(x_i - \xi_i) dF(\xi_i) + \int_{x_i}^{\infty} \varphi_i(\xi_i - x_i) dF(\xi_i)\right) \\ &= \sum_i \left((h_i + \varphi_i) \cdot \left(\int_{-\infty}^{x_i} F(\xi_i) d\xi_i\right) + \varphi_i E\left[\xi_i\right] - \varphi_i x_i\right) \end{aligned}$$

These type of problems are generally solved by using standard nonlinear programming techniques. In fact, it is possible to separate this problem to N subproblems of each PM. If the optimal solution for each one of them is found and the feasibility constraints are satisfied, then it is also the optimal solution for the centralized problem. Let $Ri(x_i)$ be the expected costs of PM i. Then the optimal allocation of PM i, from PM i's perspective can be found by evaluating the first order derivative of $R_i(x)$.

$$R_i(x_i) = (h_i + \varphi_i) \cdot \left(\int_{-\infty}^{x_i} F(\xi_i) d\xi_i\right) + \varphi_i E\left[\xi_i\right] - \varphi_i x_i \tag{15}$$

$$\frac{\partial}{\partial x_i} R_i(x_i) = (h_i + \varphi_i) F(x_i) - \varphi_i \tag{16}$$

Then, the optimal solution is $x_i^* = F^{-1}(\frac{\varphi_i}{\varphi_i + h_i})$. However, we assume in this study that the available internal capacity, κ, is less than the total of optimal capacity requirements of all PMs.

PMs that participate in the capacity allocation auction are going to bid marginal utilities, as we have discussed in the previous chapter. The marginal value of an additional capacity unit (x^{th} unit) for agent i, before the demand is realized, can be determined by the term $R_i(x-1) - R_i(x)$. Since the demand is distributed with respect to Normal(μ, σ^2), we will use the following relationship:

$$\int_{a_1}^{a_2} F(x)dx = \int_{\frac{a_1-\mu}{\sigma}}^{\frac{a_2-\mu}{\sigma}} \Phi(y)\sigma dy, \quad where$$
$$\Phi(y) = \frac{1}{2}\left[1 + erf\left(\frac{y}{\sqrt{2}}\right)\right]$$

Consequently, the marginal utility $u(x)$ gained from allocation of individual unit x is given by:

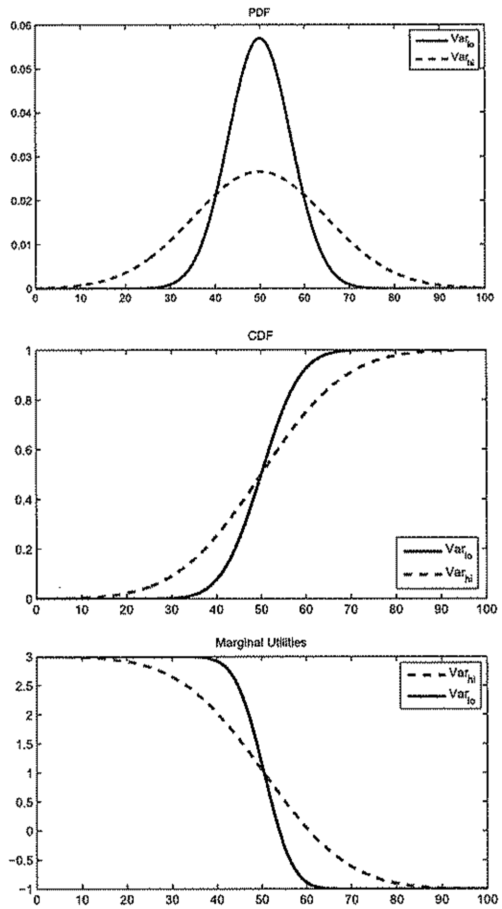
$$\begin{aligned} u(x) = R_i(x-1) - R_i(x) &= \varphi_i - (h_i + \varphi_i)\int_{x-1}^{x} F(\xi)d\xi \\ &= \varphi_i - (h_i + \varphi_i)\int_{\frac{(x-1)-\mu}{\sigma}}^{\frac{x-\mu}{\sigma}} \frac{1}{2}\sigma\left[1 + erf\left(\frac{y}{\sqrt{2}}\right)\right] dy \end{aligned}$$

Unfortunately, neither one of the cumulative normal distribution and error function (erf) can be expressed in terms of finite operations. Therefore, they have to be either computed numerically or approximated. Note that the cumulative distribution function is increasing. Thus, the marginal utilities are decreasing. This is a reasonable finding since every additional capacity unit is used to satisfy a less certain customer demand.

5.2 Information Update

With the assistance of statistical analysis techniques such as use of leading indicators, it is possible to obtain a better understanding of customer demand. As a result of the analysis, some uncertainty over the demand information can be eliminated. These statistical tools help the agents obtain a more accurate demand forecast by improving market knowledge and providing a smaller standard deviation perception. We will refer to such a procedure of using statistical tools as the information update on standard deviation of customer demand. Note that, before deciding to use these additional tools, the company has to find out whether the benefit obtained is worth the required investment.

In this section, we omit the actual procedure of information update and only deal with the resulting decrease in the standard deviation. The relationship between the methods to be used during the updating procedure and the amount of decrease in standard deviation is the topic of another study. Let the decrease in standard deviation after the analysis be Δ. In order to help visualize the effect of information update, below we plotted probability density, cumulative distribution and marginal utility functions with respect to normally distributed demand. The figures correspond to a product segment with a demand mean of 50, unit outsourcing cost of 3, and holding cost of 1.



The probability density function with the same mean and lower variance is distributed closer to the mean with respect to the one with higher variance.

The cumulative distribution function is used to calculate the marginal utilities with respect to the following formula:

$$u(x) = \varphi_i - (h_i + \varphi_i) \int_{x-1}^{x} F(\xi) d\xi$$

PMs will use the marginal utilities at the integral quantities as their bids in the auction.

Figure 2: Distribution of demand and marginal utility

Figure 2 shows how the customer demand and marginal utilities are distributed before and after information update.

In order to understand the effects of more certain information on customer demand, we need to compare the results of allocation mechanism with and without information update, everything else remaining the same. Furthermore, another objective is to draw a relationship between the amount of uncertainty that is eliminated, which is represented by Δ, and the amount of resulting expected cost change. If there is a certain distribution of the utility gain as a result of information update, knowing it would be very helpful. The information about these relationships will provide a better idea to a PM before making the decision on how much effort to consume to improve own perception of demand volatility.

In our setting, there are two PMs, i and j, competing for internal capacity. We will let these

PMs to compete in two independent capacity auctions that are exactly the same in terms of the rules and available capacity. The customer demand faced by these PMs are also same in both auctions. We assume that PM j has the same information on the demand for his product segment in both auctions. On the other hand, we assume that PM i has better information on his demand in *auction 2* than he had in *auction 1*, via reducing the demand variance (as it is perceived) by use of statistical tools. Table 3 provides the details of our setting. The comparison of the outcomes of these two auctions for PM i will provide us an understanding on the effects of better information.

		Auction 1	*Auction 2*
PM i	Mean	μ_i	μ_i
	std. dev.	σ_i	$\sigma_i - \Delta$
PM j	Mean	μ_j	μ_j
	std. dev.	σ_j	σ_j
available	capacity	κ	κ

Table 3: Demand information used by PMs in each auction setting

Perceived demand distribution for PM i as well as the marginal utilities in auctions 1 and 2 are represented in Figure 2. The functions denoted by Var_{hi} corresponds to PM i in auction 1, while Var_{lo} corresponds to PM i in auction 2 (after the information update). We can also see that the marginal utility drops below zero after x_i^* (calculated by Equation 16). When the capacity allocated to a PM is a lot more than the mean demand, the expected marginal utility becomes negative for additional units since it is almost certain that those units will not be used to satisfy the demand and a unit holding cost will be incurred. Let $\bar{q}$ represent the number of units with a positive marginal utility. Then, each PM bids up to $\bar{q}$ units in an auction. Since we assume that there is not enough internal capacity to meet the demand of all PMs ($\kappa < \bar{q}_i + \bar{q}_j$), all available capacity will be allocated among the PMs, each getting not more than $\bar{q}$ units.

5.3 Bidding Behavior and Auction Results

A PM can win at most $\bar{q}$ capacity units in the allocation auction since he never bids for units with negative marginal utilities. Consequently, we can use two different cases to analyze the outcome of an auction. The number of capacity units that PM i wins is either less than or equal to the mean demand or more than the mean but no more than $\bar{q}$. Although we do not talk about the outcome for PM j, note that when PM i wins x_i units, PM j wins $\kappa - x_i$ units. Let x_i^1 and x_i^2 denote the capacity PM i wins in auctions 1 and 2, respectively.

Next, we investigate what differences to expect in the outcome of auction 2 after the demand update. The following two cases for the result of auction 1 are exhaustive in terms of covering all possible outcomes that PM i may face before the update.

Case 1: PM i wins at most μ_i units in auction 1 ($x_i^1 \in [0, \mu_i]$)

Note that the bids of PM j remains the same in both auctions. In auction 2, PM i wins at least as much as x_i^1 since he bids higher in auction 2 up to μ_i. However, he will get no more than μ_i since the bids were higher in auction 1 above μ_i and he did not win those units. Consequently, we can say that $x_i^2 \in \left[x_i^1, \mu_i\right]$. Refer to Figure 3 for a representation of bids of PM i in auctions 1 and 2, as well as corresponding winnings.

Case 2: PM i wins more than μ_i units in auction 1 ($x_i^1 \in (\mu_i, \bar{q}_i]$)

After the update, PM i will still win at least as much as μ_i for sure, since he bids more in auction 2 up to μ_i. However, he will win no more than x_i^1 because the bids are lower in auction 2 above μ_i. Consequently, we can say that $x_i^2 \in \left[\mu_i, x_i^1\right]$, as you can also see in Figure 4.

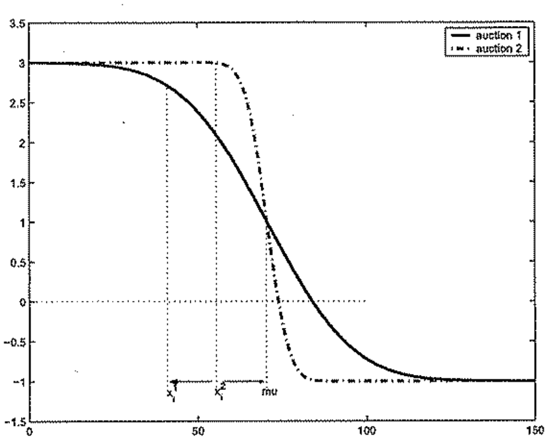


Figure 3: Case 1

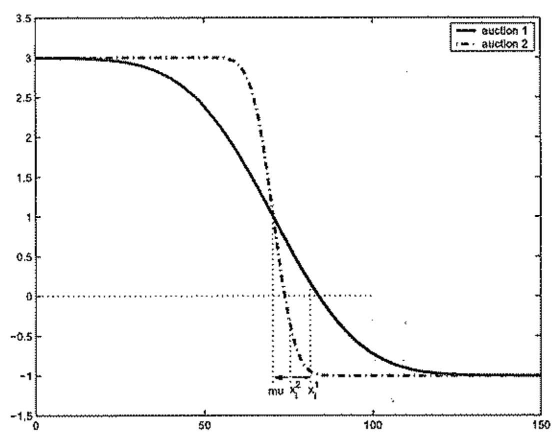


Figure 4: Case 2

We can summarize the above relationship between information update and the corresponding auction outcome in the following proposition.

Proposition 5. *The following are true for the bids of PM i:*

1. *PM i bids higher for up to μ units in auction 2 than he would without the update in auction 1.*

2. *PM i bids lower for capacity above μ units in auction 2 than he would bid in auction 1.*

3. *If PM i can win at most μ units in auction 1, he cannot win more than μ units in auction 2. However, in auction 2, he wins at least as much as he can win in auction 1.*

4. *If PM i wins more than μ units in auction 1, he also wins at least μ units in auction 2. However, in auction 2, he cannot win more than what he wins in auction 1.*

In order to see a difference in the allocation quantities after the information update, the change in standard deviation must be large enough to either decrease the lowest winning bid of the agent below the highest losing bid of opponents for a unit loss, or increase the highest losing bid of the agent over lowest winning bid of opponents for a unit gain. Consequently, we define a threshold standard deviation change, $\hat{\Delta}$, below which there is no difference in allocation quantity. Although there is no change in quantity when the decrease in standard deviation is less than $\hat{\Delta}$, the expected cost actually changes since we update the beliefs on marginal utilities (bids). Below, we present the two cases with result of the auction remaining the same after the update (since $\Delta < \hat{\Delta}$). Note that the bid function of the opponent is not depicted in the figures.

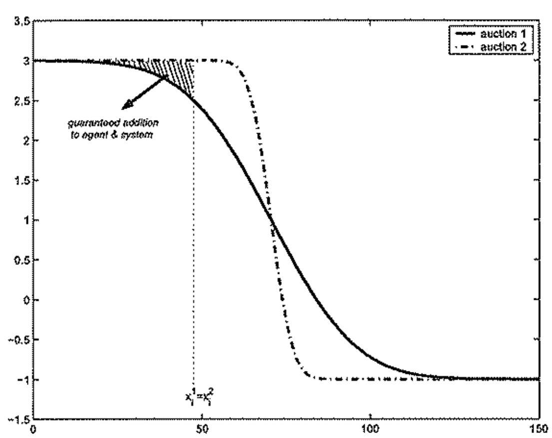


Figure 5: $\Delta < \hat{\Delta}$: Case 1

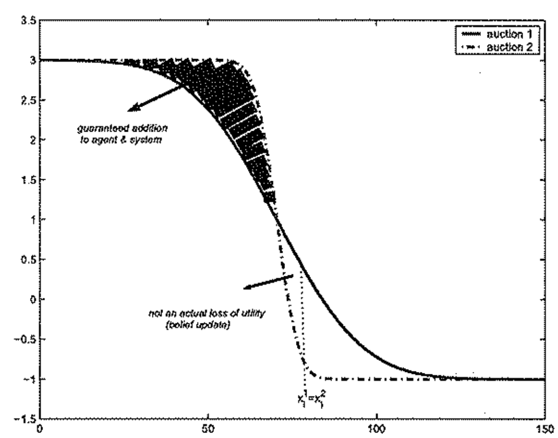


Figure 6: $\Delta < \hat{\Delta}$: Case 2

The shaded region represents the amount of utility that the PM adds to his expectation as a result of the information update. Before, PM was not aware of the fact that those units were actually more valuable than he thought they were. However note that, although we shade the whole region for a better visualization, the actual utility recovery is the total of differences at the integral bid points in these regions. Whenever we mention these regions in the remaining parts of this study, we will be referring to the utilities at integral bids. We will use A to denote this shaded region before

μ. Next theorem uses region A to show that expected costs decreases as Δ is increased up to the allocation-changing threshold.

Theorem 7. *Cost is a monotone decreasing function of* Δ *while* $\Delta < \hat{\Delta}$.

Proof. We have to look at the result of auction 2, when the result of auction 1 is in either one of two different cases we mentioned before:

Case 1: Since there is no quantity change until $\hat{\Delta}$ is reached, we update our utility function as delta increases. The shaded region in Figure 5 is the total marginal utility the agent finds out that he will also be getting. Then, utility in region A can be represented by the following term:

$$(h+\varphi)\sum_{y=1}^{x_i^1}\left[\int_{y-1}^{y}\left(F_i^1(\xi)-F_i^2(\xi)\right)d\xi\right]$$

It is trivial to see that $F_i^1 > F_i^2$ below μ_i, since both are Normal cdf with the same mean. Consequently, A is nonzero and increasing with Δ.

Case 2: A covers all μ_i units. Similar to the previous case, the region is nonzero and increasing as the standard deviation is reduced by up to $\hat{\Delta}$.

□

Next, we analyze the effect of a decrease in variance when the decrease is large enough ($\Delta > \hat{\Delta}$) to change the outcome of the auction. It would be interesting to show that cost is still a decreasing function of Δ. In that case a bidder is always better off by improving the demand information. Below, we illustrate how the expected cost is affected when auction outcome is different after the update.

We can see in Figure 7 that there is never a loss of utility in Case 1, but as in Figure 8, it is possible to lose utility in Case 2 since the agent loses units. Let B denote the utility loss in this case. This loss in Case 2 happens when a decrease in standard deviation more than threshold $\hat{\Delta}$ causes the bidder to realize that he was overbidding for capacity units above mean. When the bidder decreases his bids for those units, he loses some of them to the opponent in auction 2. In the following theorem, we characterize the threshold decrease in standard deviation after information update. We use $b^j_{x_j^1+1}$

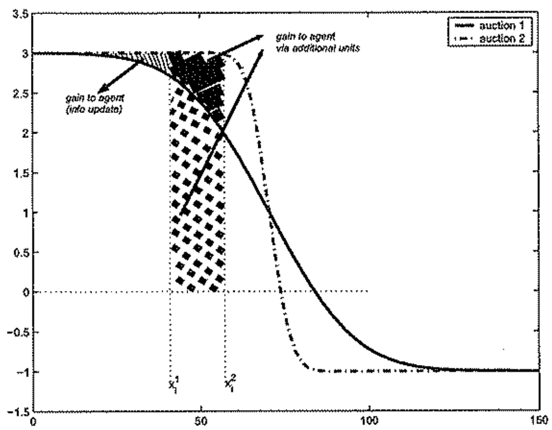


Figure 7: $\Delta > \hat{\Delta}$: Case 1

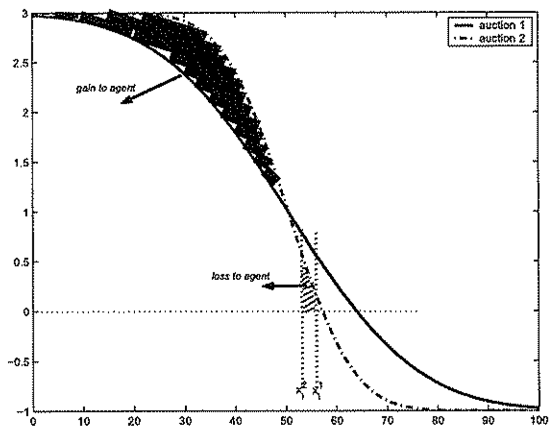


Figure 8: $\Delta > \hat{\Delta}$: Case 2

to denote the highest losing competing bid in Case 2, and $b^j_{x^1_j}$ for the lowest winning competing bid in Case 1.

Theorem 8. *The threshold decrease, $\hat{\Delta}$, in standard deviation that changes the result of the auction is the smallest Δ that satisfies:*

1. *Case 1:*

$$\frac{2\left(\varphi - b^j_{x^1_j}\right)}{h+\varphi} \geq \int_{x^1_i}^{x^1_i+1} \left(1 + erf\left(\frac{\xi-\mu}{(\sigma-\Delta)\sqrt{2}}\right)\right) d\xi$$

2. *Case 2:*

$$\frac{2\left(\varphi - b^j_{x^1_j+1}\right)}{h+\varphi} \leq \int_{x^1_i-1}^{x^1_i} \left(1 + erf\left(\frac{\xi-\mu}{(\sigma-\Delta)\sqrt{2}}\right)\right) d\xi$$

Proof. Please see the Appendix for the derivation. □

When competing winning bids are very high, it is possible to have a situation that no magnitude of Δ is enough to change the allocation quantity. Although there is a monotonic increase of utility, there is also a limit to how much utility can be gained by information update. It is obvious that the best case scenario is having perfect information, i.e., $\sigma = 0$ ($\Delta = \sigma$). Then, every capacity unit up to μ is worth φ, and worth $-h$ after that. Let the PM win x units without information update.

Then, the following is true about the limit on utility gain:

Case 1: When the allocation quantity does not change, the maximum utility that can be gained is given as:

$$(h+\varphi)\int_0^{x_i^1} F(\xi)\,d\xi$$

This amount is also the minimum guaranteed utility gain with perfect information. Furthermore, if the PM gains additional units, his utility gain will be larger. The upper bound on the utility gain is reached when μ units are won with the perfect information, and is calculated as follows:

$$\left(\mu - x_i^1\right)\varphi + (h+\varphi)\int_0^{x_i^1} F(\xi)\,d\xi$$

Case 2: PM can only lose units in this case. Therefore, the maximum utility gain is reached with perfect information, when PM is left with only μ units. The utility gain is composed of the recovered losses and marginal utility belief updates. The upper bound on utility gain in Case 2 is given as:

$$\left(x_i^1 - \mu\right)h + (h+\varphi)\int_0^{\mu} F(\xi)\,d\xi$$

Consequently, there is a limit to the amount of utility gain, and it is reached when standard deviation is reduced to zero. However, the gain is diminishing as the bidder gets closer to having perfect information. Also remember that the amount of improvement a PM can see with better information depends on the allocation quantity, x. For example in Case 1, if x is very close to μ, the maximum amount a PM can gain with better information is very limited with respect to a small x.

The relationship between change in delta and utility gain, i.e., the diminishing effect of improvement on demand information is presented for different allocation quantities in Figure 9. In this figure, mean demand is 30 and utility gain is depicted for a bidder (when he is allocated 5, 15 and 25 units, separately) as standard deviation is reduced from 15 to 0. In addition, Figure 10 illustrates how bids change as the standard deviation changes ($\Delta_1 > \Delta_2 > \Delta_3$). Although in general the bids are more significantly affected getting closer to the mean, we can see that the change in the bids is diminishing immediately around μ. Therefore, it is difficult to win additional units if a bidder is already awarded very close to μ number of capacity units.

Next, we will look at the change of cost in Case 2, when the decrease in standard deviation is greater than $\hat{\Delta}$. As Δ increases, the uncertainty on the customer demand decreases. Therefore,

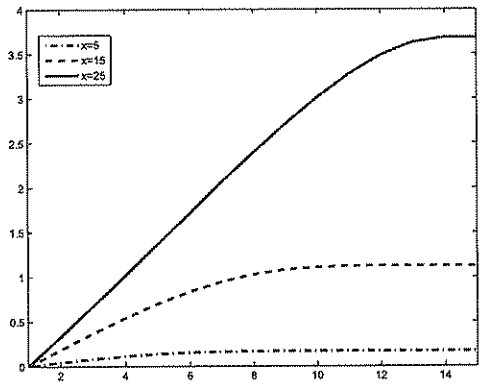


Figure 9: Diminishing gain for various x

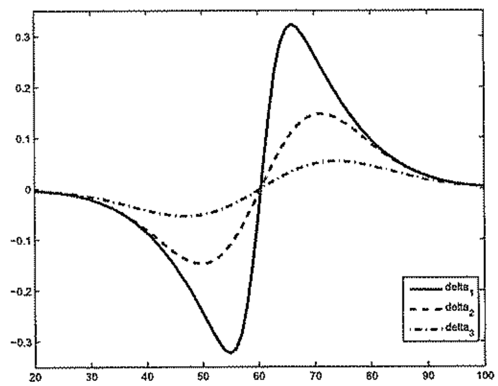


Figure 10: Bid difference for various Δ

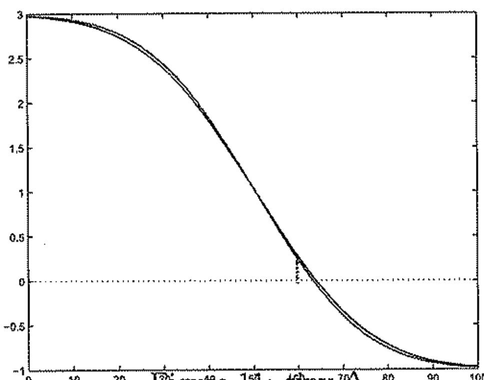


Figure 11: tiny Δ

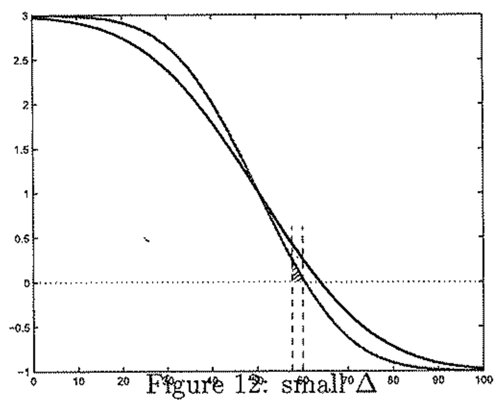


Figure 12: small Δ

the agent realizes that he has been valuing some of the capacity units more than their actual value. As the standard deviation is decreased to be very close to zero, the agent even realizes that he has valued some of the negative-value items at positive values, so his expected cost starts decreasing by correctly reassigning his expected utilities. In Figures 11- 14, we show the utility loss and utility recovery as Δ increases.

The question of whether or not a possible loss of units in Case 2 can cause an overall loss of utility still remains. If it can, then demand update is not always beneficial. If the loss (region B) depicted in the first two charts above is larger than the gain in region A, then there is a utility loss overall. In order to understand the situation better, we conduct the following numerical analysis. We use an initial demand distribution of Normal(200, 30^2). Then, for different levels of x_i^1, we reduce the standard deviation by steps of 0.25 all the way below to zero. At each step, we chart the utility

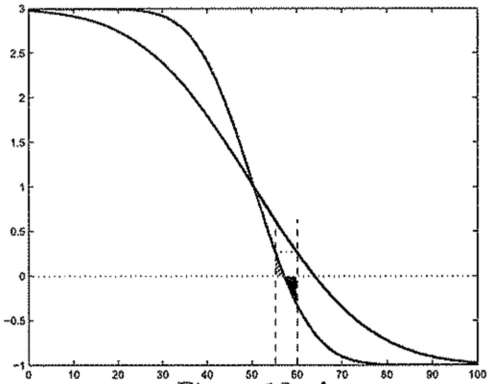


Figure 13: Δ

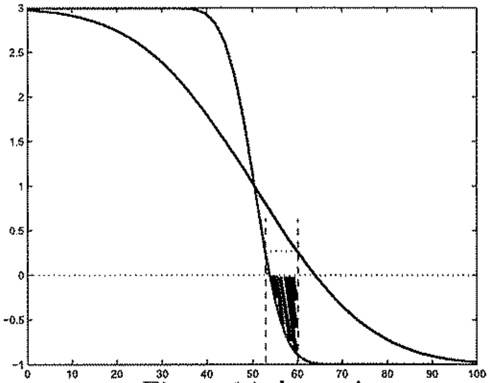


Figure 14: large Δ

gain in region A, and the utility loss in region B. Furthermore, the number of units won and lost in the auction depends on the bids of the competing PM. Therefore, we make an assumption on competing bids to analyze the effect of decrease in standard deviation. We assume that the highest losing competing bid lies in the middle of PM i's bids $b_{x_i^1}, b_{x_i^1+1}$. We also assume that all bids that drop below $b^j_{x_j^1+1}$ are lost to PM j. So, we are analyzing the worst case scenario. Without this second assumption, it would be more difficult to lose units, causing the loss of utilities to happen later (probably allowing enough gain in region A to guarantee overall improvement).

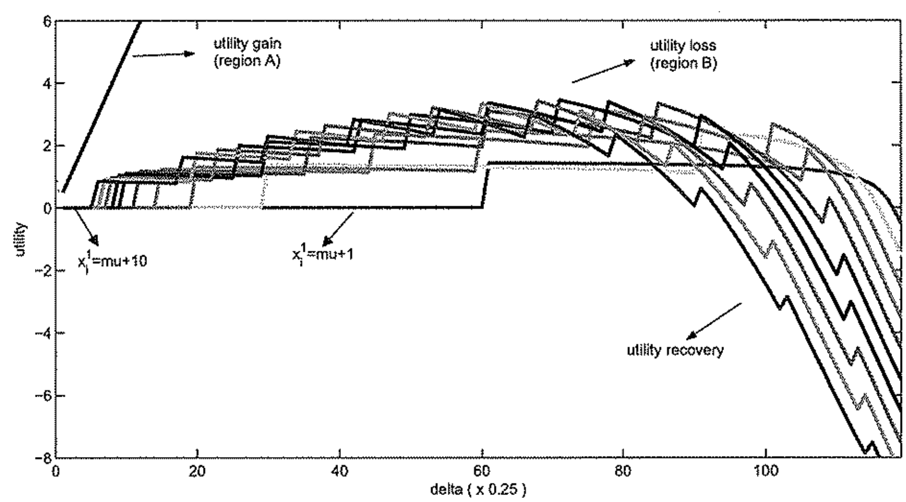


Figure 15: Gain and loss of utility with demand update in Case 2 for different allocation quantities

There is no loss of utility until the decrease in standard deviation reaches an amount high enough to decrease $b^i_{x_i^1}$ below the highest losing competing bid. We can see from Figure 15 that the gain of utility in region A is more than the loss of utility in region B for any Δ value under used assumptions. However, a loss is not theoretically impossible. If the competing bid is ϵ lower than $b^i_{x_i^1}$, while $\epsilon \to 0$ it is possible to have small enough Δ such that $A < b^i_{x_i^1}$. On the other hand, an overall utility improvement is almost certain if the demand update is not very ineffective or the competing bids are not too close. Under these reasonable assumptions, lower variance always leads to a cost decrease.

We can make following observations about Case 2:

- The larger x_i^1 is, the easier (quicker) it is to lose units and start recovering utility in auction 2 while Δ increases.
- The gain from region A is very steep such that even the competing bids are not separated enough, only a small decrease in standard deviation would be sufficient to guarantee an overall cost decline.

Proposition 6. *When $x_i^1 \leq \mu$, lower variance always leads to a cost decrease, and cost is a monotone decreasing function of Δ. When $x_i^1 > \mu$, lower variance always leads to a cost decrease if competing bids are separate enough.*

Proof. In Case 1, the utility can only monotonically increase since both utilities for each unit and the obtained number of units are monotonically increasing as Δ increases. In Case 2, the decrease in cost is guaranteed only if competing bids are separated, as suggested in our numerical analysis. The monotonicity is doubtful, because of the spikes in utility loss for smaller Δ.

$\square$

Our analysis on the value of demand information suggests that in problems with competitive agents and scarce internal capacity availability, accurate customer demand information is the most valuable since it is expected that agents fulfill less than mean demand by internal capacity allocation. In this case, expected cost of an agent is a monotone decreasing function of better information, i.e., decrease in perceived standard deviation of customer demand. On the other hand, when the capacity is fairly abundant, the stronger the competition is, small improvements in demand information is less important. However, even in this case, beyond a rather small threshold improvement in standard deviation information, more accurate information leads to a better auction outcome for the agents.

6 Computational Experiment

6.1 Data Generation

In order to verify the findings of our study and investigate the factors that might affect the outcome of the auction for PMs, we randomly generated a large number of test problems that represent various instances of complexity and size levels. Specifically, after the experimental study, we aim to answer the following questions: Does better customer demand information help? Under what conditions it helps the most / has no effect? Do experimental results agree with previous findings? Is the analysis still valid if any of the assumptions are relaxed?

We used a number of different factors to design our study. Each factor is experimented at a number of different levels to obtain a detailed analysis of the system. The first experimental factor we use is the number of demand scenarios (SC) each PM has. We assume that these scenarios represent a normally distributed customer demand. The mean demand for PM i, $\bar{d}_i$, is generated from a uniform distribution, i.e., $\bar{d}_i \sim U(a - b/2, a + b/2)$, using uniform random parameters a and b. Another experimental design factor is the coefficient of variation (CV), which we use to define the standard deviation for the demand distribution. A smaller CV represents a PM with better customer demand information, since the variance of the distribution is smaller. Using these parameters, then, the customer demand for PM i in scenario s is given as $d_i^s = F^{-1}\left(\frac{1}{2 \cdot SC}[2s-1]\right)$, where F is the cumulative normal distribution for the customer demand distributed as $N \sim (\bar{d}_i, \bar{d}_i \cdot CV)$. Creating scenarios in this way helps us generate problem instances that effectively simulate normally distributed demand.

Level of capacity scarcity is also an experimental factor. A low internal capacity causes a stronger competition by limiting the number of units a PM can win to gain utility. On the other hand, a higher capacity to share may reduce the competition and decrease the importance of better demand information. The available internal capacity is generated from the following uniform distribution: $\kappa \sim U(LB \cdot \bar{D}, UB \cdot \bar{D})$. In this formulation, $\bar{D}$ is the total mean demand and calculated as $\sum_i \bar{d}_i$. The actual parameters that we use are multipliers of total mean demand (LB, UB), the lower and upper bounds on the interval from which we sample available capacity, κ.

Unit outsourcing and inventory holding costs are as important as customer demand while determining the bids for the auction. The marginal utilities of capacity units depend on these two parameters the most. Therefore, we consider them as experimental design factors. However, it is enough to specify the ratio of one cost to another. The unit outsourcing cost is generated from the interval (φ_L, φ_H). The holding cost is then found by multiplying the outsourcing cost by the ratio factor, r. The final experimental factor is the number of PMs (PM). This factor affects the capacity scarcity, level of competition, level of payments, and computational complexity all at the same time. A summary of all parameters that are used to create the test problems can be found in Table 4.

Parameter	Value Distribution	Parameter	Value Distribution
a	$U(300, 600)$	π_s	$1/SC^{PM}$
b	$U(50, 150)$	φ_i	$U(5, 7)$
$\bar{d}_i$	$U(a - b/2, a + b/2)$	h_i	$r \cdot \varphi_i$
d_i^s	$d_i^s = F^{-1}\left(\frac{1}{2 \cdot SC}[2s-1]\right)$	κ	$U(LB \cdot \bar{D}, UB \cdot \bar{D})$
$\bar{D}$	$\sum_{i,s} \pi_s d_i^s$		

Table 4: Parameters used to generate the test problems

6.2 Experimental Design

In order to understand the value of better customer demand information, we compare the auction outcomes of a particular PM with and without better information. In order to achieve this, we run the allocation auction twice; the second one being different than the first only by the demand scenarios of a single PM. The agent with index number PM is the subject of our experiment. In auction 1, we generate a problem instance, run the auction, and obtain the results. In auction 2, we regenerate demand scenarios of only agent PM by multiplying the CV by a random variable distributed with respect to $U(u_L, u_H)$. We leave the data for the other PMs as they were generated in auction 1. This way, in auction 2, agent PM has a lower demand variability that can be thought as better customer demand information. We may call this an information update on customer demand of agent PM. We aim to find the value of better knowledge by comparing the auction outcomes of agent PM with and without the update, all other parameters remaining the same.

By designing this experiment, we want to test mainly how problem size, capacity scarcity, better customer demand knowledge, and the cost structure can affect the outcome. Number of PMs, number of scenarios, capacity scarcity, demand variability, and unit costs are the experimental factors that we use in our study. We created a model consisting of 486 factor combinations, each being repeated for 20 replications, with a total of (3x3x3x2x3x3x20) 9720 runs. Refer to Table 5 for a summary of experimental factors used in the study and the levels they are tested at.

Factor	Explanation	Levels		
PS	Number of agents	2	4	8
SC	Number of scenarios	2	4	8
CV	Coefficient of variation	0.1	0.3	0.5
(LB, UB)	Bounds: capacity	$(0.3, 0.4)$	$(0.7, 0.8)$	$(1.0, 1.2)$
r	Cost ratio	0.25	0.75	
(u_L, u_H)	Bounds: info update	$(0.1, 0.2)$	$(0.4, 0.5)$	$(0.7, 0.8)$

Table 5: Experimental design

6.3 Experimental Results

Our previous findings suggested that better customer demand information is expected to help a product manager unless there is a very specific unusual condition. Proposition 6 points out that an agent may lose some utility only if a very small improvement in standard deviation causes the agent to lose units because the competing bids are very close. Therefore, the chances of being worse off at the end of an auction while having better demand information is very slim. In fact, the experimental results support this theory. We have found in only one problem instance that an agent lost some utility. Moreover, this problem instance had the lowest level of information improvement, just as the theory suggests. As you can see in Figure 16, among all problem instances that we looked at, 68.12% of them resulted with a utility improvement, while 31.88% of the results were indifferent. Although there is almost no risk to lose utility, being indifferent is not very desirable either. This is because the information improvement requires an additional effort.

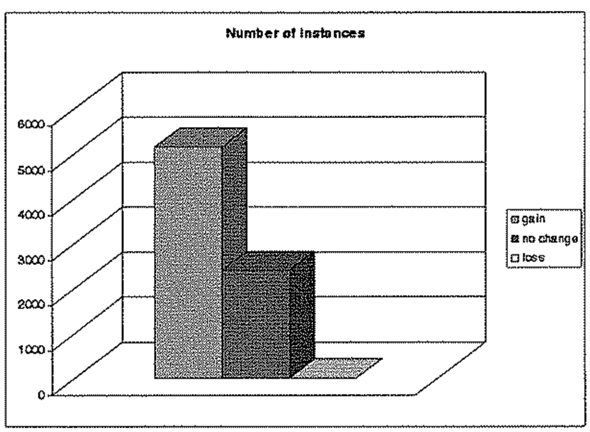


Figure 16: Problem instances with utility gain, indifference, loss

Remember that previously, our theorems used the assumption that bids are calculated using the normal distribution itself. Therefore, any magnitude of improvement affected the outcome. However, during the experimental study, we use demand scenarios to simulate a normal distribution. Thus, the number of different bids are limited to number of scenarios, and the problems are not very sensitive to minor improvements in demand information. For example, although the utility gain is a monotonic increasing function of Δ in theory; in this experiment, the utility gain in region A is not always accounted for. However, as Figure 17 shows, our results indicate that the higher the number of scenarios is, the greater the number of instances with utility improvement. Consequently, the theory still works without the assumption of a perfect normal distribution, but, the results are better as the number of scenarios increase and the normal assumption is simulated in a better way.

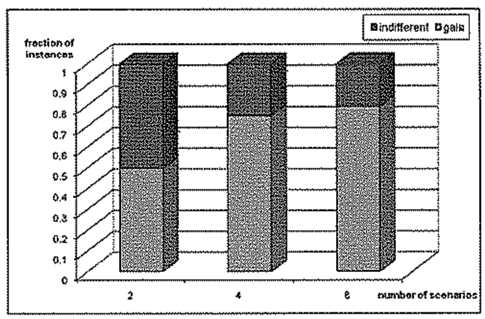


Figure 17: Effect of number of scenarios

Next, we will investigate whether some specific factor levels are more helpful than others. Figure 18 illustrates the relationship between factors and the utility gain. If we can identify such levels, agents can make wiser decisions of using their resources to get better information. When an agent knows that the market conditions suggests they are in helpful factor levels, it is an easy decision to invest in getting better demand information.

Our analysis shows that all of the factors that we tested in our experiment are significant. We see that higher number of PMs and scenarios provide more improvement. Moreover, a higher CV provides an opportunity to improve, resulting with greater gains. The quality of information update is also crucial in getting significant improvements. It is useless to invest in demand update if the update will not be helpful enough. Finally, very scarce internal capacity seems to prevent utility gain.

Below, we present how the utility gain is distributed. A histogram of all results are given in Figure 19, while the distribution of instances with a utility gain is more clear in Figure 20. Our

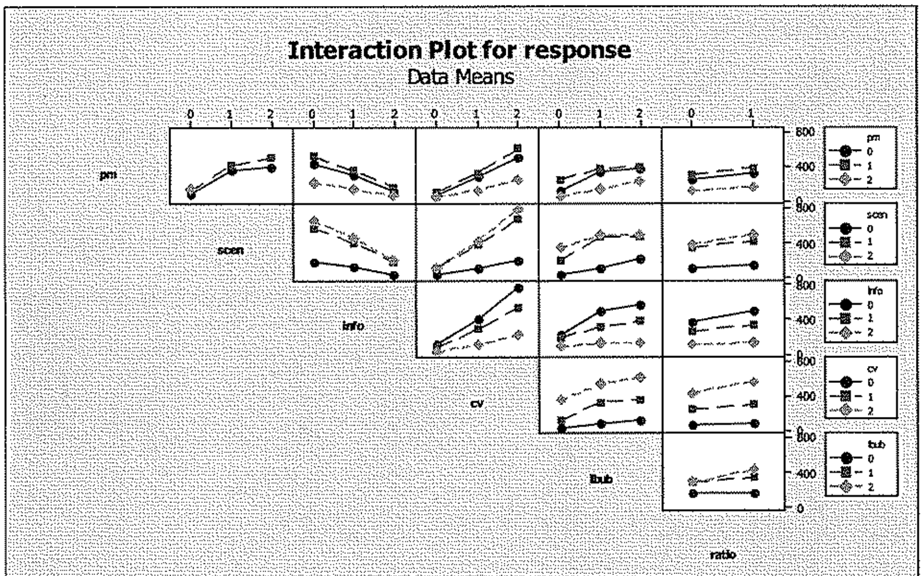


Figure 18: Factorial design

study indicates that scarce capacity levels, ineffective demand information updates and initial good information on customer demand are the factors that prevent utility gain with better information. When the instances with any of these factor levels are ignored, 92.8% of the problems yield a gain in utility when the agent has better demand information.

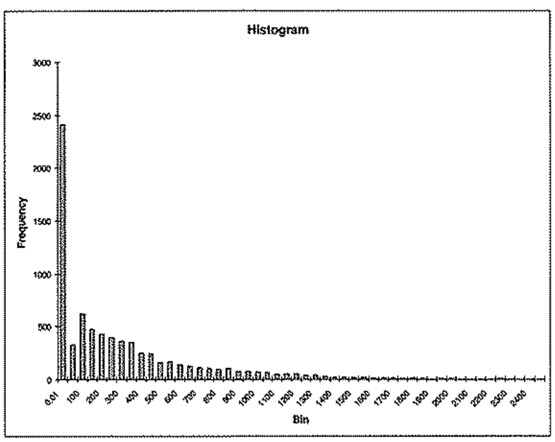


Figure 19: All instances

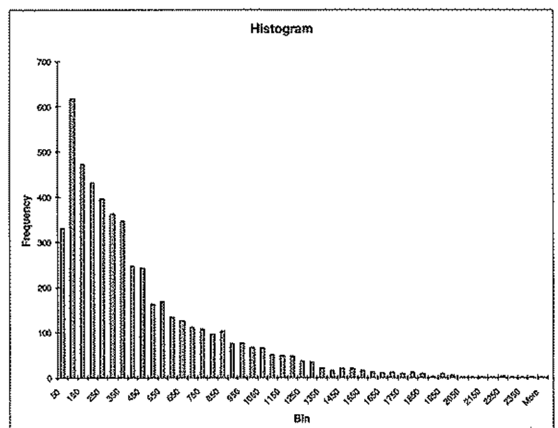


Figure 20: All instances with gain

In addition, if we also ignore the instances with the lowest number of demand scenarios, all of the problems yield a positive utility gain. Mean utility gain is very high for this selection of factor levels, and the distribution of the results are given in Figure 21 below.

The experimental results have confirmed our findings throughout Section 5. We were able to see

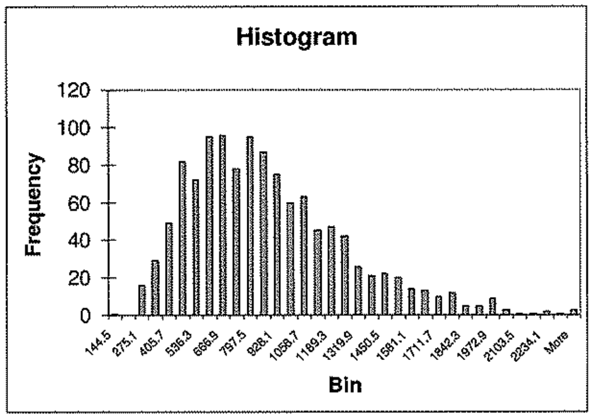


Figure 21: Distribution of utility improvement (for favorable factor levels)

that efforts for understanding market behavior and trying to eliminate some of the volatility possessed by customer demand help product managers better assess the value of capacity. Consequently, the expected outcome of capacity allocation auction is ordinarily much higher with better information. We also identified in which cases better information works more efficiently, and in which it does not.

Conducting the above numerical study would be very helpful for a PM with real data. This analysis provides a good idea of how much additional utility can be obtained by an improvement on customer demand information. An informed PM can easily select the appropriate factor levels and obtain significantly accurate results, which he can compare with cost of demand information update.

7 Conclusions

Our research for the first part of this report is motivated by the capacity allocation problem within a semiconductor manufacturer. PMs have the most accurate information on customer demand for particular product groups they are responsible for. Therefore, HQ allocates the available internal capacity based on PMs' reports. When PMs have their own local objectives and capacity is scarce, as it is in our problem setting; PMs may lie in their reports to gain advantage in allocation. Note that centralized optimization methods assume that model parameters and data are truthfully available, and thus, systemwide optimum cannot be guaranteed in this case under asymmetric information.

We propose a decentralized multi-unit capacity allocation auction that elicits bid schedules of special characteristics from PMs. This allocation mechanism is strategyproof, individually rational, and efficient. Most importantly, we show that the result of the auction matches the optimal solution to the centralized model with truthful inputs. Our auctioning algorithm respects private information by not forcing any information revelation and it is very easy to implement. The mechanism is transparent to all participants, i.e., it does not involve black-box calculations. It is also efficient in terms of achieving a result in one shot and not requiring multiple iterations of information exchange. Finally, the auction is capable of these without sacrificing the optimality.

In multi-period allocation problems, it is possible to utilize similar bidding schemes to obtain the optimal solution in a decentralized way. Although the losing-price auction does not work in this case, we claim that a straightforward combinatorial auction is sufficient. For even more complicated allocation problems that may involve additional constraints and lack total unimodularity, the losing-price auction can be used with the basic problem setting to obtain an initial reference allocation, which is helpful for strategic planning and budgeting purposes.

A better understanding of customer demand is helpful to a PM in planning capacity. There is a wide variety of tools available to help PMs estimate demand better. However, an analysis of a cost/value relationship is generally neglected before utilizing such tools. Using the auction mechanism presented here, we provide a study that measures the potential added value in allocation process. More accurate demand information can lead to additional profits for an SBU and for the whole company. We expect this analysis to guide planners toward a more educated investment decision.

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Appendix

Algorithm 1. The losing-price auction algorithm *[Proposition 3]:*

```
foreach capacity unit t = 1..κ do
    j = 0;
    foreach bidder i = 1..n do
        if b(i) > j then
            j = b(i);
            w = i;
        end
    end
    no-of-wins(w)=no-of-wins(w)+1;
    b(w)=buy(w,no-of-wins(w)+1);
end
foreach capacity unit t = κ + 1..2κ do
    j=0;
    foreach bidder i = 1..n do
        if b(i) > j then
            j = b(i);
            w = i;
        end
    end
    foreach bidder i = 1..n do
        if i≠w then
            if won(i)≤ no-of-wins(i) then
                pay(i)=pay(i)+j;
                won(i)=won(i)+1;
            end
        end
    end
end
```

During allocation auctions, for each capacity unit, the algorithm simply checks all the bids, rewards the highest bidder, and updates his bid. Given the allocation quantities, during pricing, algorithm continues to find the highest bidder for each unit after κ, and assigns the highest bid as a payment to all competitors (up to the number of units they won). Therefore, the algorithm runs on the order of $O(mn)$.

Proof. [Proposition 4] The structure of the constraint matrix of the centralized problem (CP) is given by the following matrix:

$$
\left(\begin{array}{c|c|c|c|c}
x_1\, x_2\ \cdot\ x_N & o_1^1\, o_1^2\ \cdot\ o_1^s\, I_1^1\, I_1^2\ \cdot\ I_1^s & o_2^1\, o_2^2\ \cdot\ o_2^s\, I_2^1\, I_2^2\ \cdot\ I_2^s & \cdot\ \cdot\ \cdot & o_N^1\, o_N^2\ \cdot\ o_N^s\, I_N^1\, I_N^2\ \cdot\ I_N^s \\
\hline
\begin{matrix} 1 & & & \\ 1 & & & \\ \vdots & & & \\ 1 & & & \end{matrix} &
\begin{matrix} 1 & & & & -1 & & & \\ & 1 & & & & -1 & & \\ & & \ddots & & & & \ddots & \\ & & & 1 & & & & -1 \end{matrix} & & & \\
\hline
\begin{matrix} & 1 & & \\ & 1 & & \\ & \vdots & & \\ & 1 & & \end{matrix} & &
\begin{matrix} 1 & & & & -1 & & & \\ & 1 & & & & -1 & & \\ & & \ddots & & & & \ddots & \\ & & & 1 & & & & -1 \end{matrix} & & \\
\hline
\begin{matrix} & & \ddots & \end{matrix} & & & \ddots & \\
\hline
\begin{matrix} & & & 1 \\ & & & 1 \\ & & & \vdots \\ & & & 1 \end{matrix} & & & &
\begin{matrix} 1 & & & & -1 & & & \\ & 1 & & & & -1 & & \\ & & \ddots & & & & \ddots & \\ & & & 1 & & & & -1 \end{matrix} \\
\hline
\begin{matrix} 1 & 1 & \cdot & 1 \end{matrix} & & & &
\end{array}\right)
$$

In the above constraint matrix A, let Q be the set of first N columns, i.e., the columns corresponding to decision variable x. Also let R be the set of remaining columns. Then, a column of $J \subseteq A$ is either in Q, or in R. Below, we provide an algorithm to partition the columns of any J such that Theorem 5 is satisfied.

Algorithm 2.

1. ***Input:*** *J: any subset of columns of A.*
2. ***Output:*** *Two partitions of columns; J_1 and J_2.*
3. *For columns i in Q (there may be less than N):*
   - *Assign odd-numbered columns to J_1, and even numbered columns to J_2 (after renumbering the columns).*
4. *For columns j in R:*
   (a) *If there is a column i in Q that has a nonzero entry in the same row that column j has a nonzero entry*
      i. *if the nonzero entry in column j is positive*
         - *assign j to the partition that i is not assigned to.*
         - *assign the column with the negative entry in the same row randomly*
      ii. *else*
         - *assign j to the same partition that i is assigned to.*
   (b) *else*
      i. *assign all columns that have a nonzero entry in the same row that column j has a nonzero entry to the same partition*

Consequently, we can conclude that the constraint matrix of CP is totally unimodular. □

Proof. [Theorem 8] In Case 1, change in allocation can happen when bid for $x_i^1 + 1^{th}$ unit tops the smallest winning bid of competitors:

$$\begin{aligned} u(x+1) &\geq b^j_{x^1_j} \Leftrightarrow \\ \varphi - (h+\varphi)\left(\int_x^{x+1} F(\xi)\, d\xi\right) &\geq b^j_{x^1_j} \Leftrightarrow \\ \varphi - (h+\varphi)\left(\int_x^{x+1} \frac{1}{2}\left[1 + erf\left(\frac{\xi-\mu}{\hat{\sigma}\sqrt{2}}\right)\right] d\xi\right) &\geq b^j_{x^1_j} \end{aligned}$$

where $\hat{\sigma} = \sigma - \hat{\Delta}$. Consequently, the smallest $\hat{\Delta}$ that satisfies the below inequality is the threshold change in standard deviation that results an allocation increase for bidder i.

$$\frac{2\left(\varphi - b^j_{x^1_j}\right)}{h+\varphi} \geq \int_x^{x+1} \left[1 + erf\left(\frac{\xi-\mu}{\sigma\sqrt{2}}\right)\right] d\xi$$

Case 2 considers losing a single capacity unit. The derivation of $\hat{\Delta}$ is similar, thus, we leave it to the reader.

$\square$