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A Generalized Leading Indicator Forecasting Framework

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Abstract

In this report, we address the demand forecasting problem in dynamic, volatile high-tech markets, and develop a general leading indicator forecasting framework that perpetually reduces forecast variance through information updates, model combinations, and new demand observations over time. We incorporate a vast variety of dynamically evolving market information, which include indications of future demand trends, and become available throughout a product's lifecycle. We present a comprehensive theoretical setting that guarantees variance reduction when relevant data are filtered from these information sources and utilized as leading indicator signals in a consistent, systematic manner to update prior beliefs. The improvement that our framework provides in forecast accuracy is demonstrated through a case study.

1 Introduction

In recent years, managing demand and supply has become very challenging for the high-tech industry. Continuous improvement in technology allows for more frequent introduction of new products as well as creation of new segments everyday. Demand for these products –especially high-tech consumer electronics such as media players, video game consoles, mobile phones, and personal computers– is very volatile, caused by a variety of factors including but not limited to functionality, price, seasonality, and popularity. An increasing number of companies compete for their share of this dynamic market. Thus, throughout these frequent transitions of short-lifecycle product generations, adapting to changing market conditions and capability of improving forecasts continuously over time are essential characteristics for a company to effectively manage demand and react appropriately in a timely manner.

Forecasting also plays a critical role in operational activities. High-tech capacity is very expensive and production requires significant amount of time compared to the high-tech product lifecycle. Therefore, a supply shortage cannot be easily avoided by last-minute production ramp-ups, and

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eventually causes lost sales, decelerated adoption and even loss of market share. On the other hand, excessive capacity allocation results in expensive investments on resources that could be used for other products, and inventories that suffer holding costs or become useless. A company that is capable of generating accurate forecasts in such a dynamic market environment can prevent these operational costs and maintain its competitive edge.

In addition to the volatility caused by factors such as fast technology, competition, price moves, etc., the seasonality effects also plays an important role in creation of sharp spikes and troughs in demand pattern on a daily basis. Throughout the diffusion process, keeping right amount of supply at the right time supports strong acceptance and adoption of a product. Furthermore, gains and losses are amplified when seasonality is the most effective. Thus, accurately predicting spikes and troughs in demand is key to securing financial success of a product and sustaining profitability of the company.

During a product's lifecycle, an abundant variety of information that incorporate advanced indications of future demands become publicly available. We believe that this information can be used in forecasting after relevant data is filtered and processed in a consistent and systematic manner. In this study, we develop a generalized forecasting method that exploits miscellaneous dynamic market information in the form of leading indicators to continuously reduce forecast variance and improve accuracy throughout the demand lifecycle.

Technological forecasting literature has been very narrowly defined in terms of the models that can be employed to characterize the demand process, as well as the information sources and corresponding information processing methods that can be used to provide indications of future demand patterns. Our motivation is to extend the technological forecasting literature to provide forecasters with the capability of employing a large variety of information –that become available and change continuously as market evolves– as leading indicators, and the flexibility to work with any underlying lifecycle demand models to better model and project future demand signals. Our theoretical contribution will be broadening of the technological forecasting approach to a more comprehensive theoretical setting through generalizing:

1. Time series models used to characterize demand,
2. Leading indicators that provide advanced market information,
3. Means of information processing.

In reality, when generic forecasting methods perform poorly, judgment calls are frequently involved in the forecasting process. Apart from its benefits in terms of forecast accuracy, our framework also facilitates standardization of the forecasting process, making it consistent, systematic and repeatable.

In the next section, we review the relevant literature. In Section 3, we describe the problem of technological forecasting in high-tech markets, and explain how forecasters can take advantage of diverse information available throughout a product's lifecycle. In Section 4, we introduce a generalized leading indicator forecasting framework. Next, we present a case study with implementation of our forecasting model in Section 5, which is followed by concluding remarks in Section 6.

2 Literature Review

Rapid innovation processes and significant market competition are two main characteristics of the high-tech industry. These companies suffer from demand and supply management problems triggered by volatile customer demand. Some of the reasons for demand volatility can be listed as product variety, competition, seasonality and market condition. Considering the value of production capacity and market share in the high-tech industry, efficient management of resources is crucial to support customer demand with timely and adequate supply. This is only possible by accurate forecasting of customer demand.

During the last several decades, technological forecasting with growth models has been a popular approach to predict future demands for new products/technologies. When they were first introduced, these models predicted the rate of adoption for consumer durables very accurately. In these studies, diffusion of a new consumer product is modeled to follow a unimodal curve. After introduction of the new technology to the market, rate of diffusion increases until peak demand is reached. Then, diffusion rate decreases slowly as everyone in the population adopts the technology.

In order to predict future demand realizations, researchers have proposed various models that mainly differ in their cumulative diffusion profiles throughout the lifecycle of a product. The forecasting activity generally starts with choice of a growth model and parameter estimation through use of available information sources such as initial observations of demand and history of past product sales. Then, future demand is forecast using the adopted model and corresponding parameters. For extended surveys of technology growth models with detailed information about underlying assumptions, model parameters, and their classification, see Meade and Islam (1998) and Kumar and Kumar (1992).

The most well-known, widely used and extended growth model is introduced by Bass (1969). This model has provided accurate predictions on timing and magnitude of sales throughout the product lifecycle when it was initially tested on consumer durable goods. Since then, Bass model has been revised to consider additional features of technology diffusion such as pricing and advertisement, and used for diffusion forecasting in various markets including but not limited to retail, education, pharmaceutics and agriculture (Mahajan et al., 2001). However, demand characteristics for high-tech products differ from those markets in their volatility and shorter lifecycles caused by rapid innovation.

One of the studies that consider high-tech product diffusion is by Norton and Bass (1987), who build upon the Bass model to forecast successive generations of high-tech products in semiconductor industry. Krishnan et al. (1999) incorporate the effect of pricing moves into the Bass growth model to determine optimal pricing strategies for new products. Similarly, Kurawarwala and Matsuo (1996) employ a seasonal influence parameter to the Bass model to predict demand of a personal computer manufacturer. Modis and Debecker (1988) also analyze demand of computer manufacturers using an S-shaped logistic diffusion curve. Although literature suggests that numerous high-tech companies have used variations of the Bass model in their forecasting efforts (Bass, 1980), there are very few publications that consider business practitioners' perspective, and implementation with actual data. In one of those studies, Bass et al. (2001) use the Bass model to plan for launch of satellite TV technology.

Unfortunately, there is a shortcoming of these studies that stems from the inability of a single growth model (e.g., Bass model or one of its various modifications) to fit well to a diverse selection of demand lifecycles; which restricts the model to be used only for a specific product group. Demand for modern high-tech products is very volatile and seasonal. In addition, there is a substantial variety of high-tech markets with their own drivers and dynamics that should demonstrate different goodness of fit with respect to different models. We introduce a forecasting framework that allows for use of an unrestricted variety of models to accurately predict demand in any market.

Choice of the appropriate lifecycle model for a specific product has been one of the most challenging decisions that a forecaster faces. Parameters of particular models limit the different diffusion shapes that can be predicted. Hence, instead of relying on a single model, combining the intelligence of multiple models might be a better idea, since each one understands different aspects of the adoption process. The idea of combining forecasts has been utilized by many studies to diminish the risk of using a single model since Bates and Granger (1969) proposed in their influential paper. How to combine the forecasts of individual models as well as estimating the model parameters are implementation issues and researchers have suggested a vast number of different methods (Mahajan and Muller, 1979; Sultan et al., 1990; Makridakis and Winkler, 1983). Literature suggests that combination of forecasts outperforms forecasts of individual growth models (Meade and Islam, 1998), improves accuracy, and reduces variance of the forecasting errors (Timmermann, 2005). Our forecasting framework also employs various models to predict demand and then systematically combines them to reduce forecast variance while increasing accuracy.

Another weakness of the models in literature that we mentioned before lies in the information input used for forecasting. More specifically, the information that can be used is very narrowly defined and usually limited to demand observations and historical products. Meixell and Wu (2001) first proposed the concept of demand leading indicators, which were defined to be specific products in a cluster of products with similar properties (i.e., products that have the same functionality or share the same resources, etc.), whose demand signals provide advanced information on demand trends for all the other products in the cluster that are lagging in demand realization. Later, Wu et al. (2006) tested the suggested leading indicator products in a semiconductor manufacturing setting and verified the improvement in forecast accuracy. There are many different factors that affect how future demand would materialize, and fortunately, some of these factors are visible ahead of time. Through our forecasting framework, we incorporate a large variety of publicly or privately available information sources as leading indicators in a consistent and systematic way.

The other stream of literature that is related to our work uses Bayesian updating to revise demand forecast as new information becomes available. Oliver (1987) proposes a Bayesian model to predict growth and eventual market saturation of a recently introduced consumer-durable product. The author assumes that demand has a Binomial distribution, and it is revised based on the most recent number of new buyers to compute the predictive distribution of new buyers in the future. Zhu and Thonemann (2004) use past products to come up with candidate diffusion profiles, which they combine to forecast demand for a new product. They introduce an adaptive forecasting algorithm that uses actual demand information to update combination probabilities and distributions of Bass model parameters for these profiles, as new information becomes available. An empirical study at a PC manufacturer shows that the Bayesian algorithm outperforms several other forecasting methods.

More recently, Aytac and Wu (2008) were the first to introduce a demand characterization approach that uses advanced demand signals provided by leading indicator products to improve available information via utilization of multiple growth models and a Bayesian updating framework. We extend and improve the Bayesian forecasting framework by generalizing the underlying demand models that can be used (previously limited to 29 different technology growth models) to accurately characterize demand trend with any shape from any market. In our study, we also extend the definition of a leading indicator to include any source of up-to-date information with relevant data and indicative strength. The generalized definition broadens the capabilities of forecasters to harness a great variety of publicly available information to generate a forecast with improved accuracy.

3 Using Market Information in Forecasting

3.1 The Forecasting Problem in High-Tech Markets

Demand forecasts are critical to planning activities in high-tech businesses. Companies in these businesses need to generate accurate forecasts to build adequate manufacturing capacity, plan timely production schedules as well as effectively manage inventories and marketing operations. Moreover, financial consequences of poor planning are considerably severe in the high-tech industry since production resources are extremely expensive. To maintain profitability, a company should be capable of continuously and consistently identifying the demand process throughout products' lifecycles.

Inconveniently, high-tech markets incorporate volatility on a daily basis, while production processes take months. In fact, in general, the first several months of a demand forecast is only used to reprioritize work in process and reposition inventory. The production start is initiated by forecast for the following few months. This makes the accuracy of not only short term, but also mid term forecasts even more essential.

The volatility in demand originates from ever-changing, very competitive characteristic of the high-tech market. Some of the factors that play a role in demand volatility are fast moving technology that results in high turnover of product generations, competitors with a wide selection of products and diverse marketing activities (advertisements, promotions, etc.), economic conditions, and seasonality effects. Under the circumstances, the forecaster has the very challenging task of maintaining the quality of demand predictions by promptly recognizing and adapting to the changes in market conditions over time.

Implementations with actual data show that demand patterns for past products prove to be useless in indicating future demand patterns in dynamic markets. In addition, generic statistical models fail to predict new market trend ahead of time. Consequently, management's judgment plays an important role in the final forecast. The downside of such judgmental forecasts is that they are inconsistent, non-repeatable and time/effort-consuming.

Although forecasting in high-tech markets seems like a shot in the dark, a vast variety of information sources that contain advanced indications of future market trend materialize during the adoption process. These sources can be used to improve forecast quality after they are methodically filtered for relevant data. In this study, we develop a framework that continuously incorporates new, diverse market information (with advanced indications of future trends) into the forecasting process to gradually improve accuracy.

In the next section, we identify and describe some of the most apparent sources of information with potential of indicating future demand patterns ahead of time.

3.2 Indicative Information Sources

In this section, we present various categories of information sources that potentially comprise early indications of future demand patterns before they materialize in high-tech markets. Among possibly many others, the indicative potential of these sources may originate from advanced demand signals with logical, proven and strong connections to demand pattern of the product in question, market dynamics or product characteristics that prompt how the demand takes shape, as well as sophisticated inside/private knowledge or collective wisdom. Here, we present categories that are relevant to high-tech markets, however, for other products, forecasters can think of greater variety of categories from which to extract useful information.

Following are various quantitative and qualitative information sources that we saw potential for providing advanced indications of high-tech demand. We try to give a brief explanation to each source and provide examples.

3.2.1 Demand signals with logical, proven and strong connections

Product Drivers:

A product driver is a software or a type of media that is used by a particular product to increase its utility. Some examples of product drivers can be listed as video games, e-books, mobile applications, movies and mp3s. Demand for product drivers may provide indications of demand for the associated product, and increasingly so if the association is exclusive.

Complementary Products:

A complementary product is an item that is required to accompany the main product for its proper functioning. An example could be a chipset, which needs to be attached to the central processing unit (CPU) in a computer. Chipsets are cheaper and less sophisticated products and generally shipped to the OEMs months ahead of the CPUs. Since there is a constant, known attach rate between chipset and CPU, demand for chipset is a good indicator of demand for CPU.

Service Partners:

A service partner provides a subscription based (assistance) service to operate the product in question. For instance, iPhone demand can be linked to its exclusive service partner AT&T's 3G data plan subscriptions.

3.2.2 Market dynamics that shape the demand

Market Share:

Market share is a company's proportion of total sales realized in the market among all competitors. For example, Sony, Microsoft and Nintendo share the video game console market, while Apple, RIM, LG, Nokia and Samsung are sharing the mobile phone market. Market share usually has little variability since it takes significant effort (such as advertisements and promotions) to build up market share. Hence, market share can be useful in determining future demand for a company if future demand for the total market or competitors are known.

Economic Index:

Economic indices are leading indicators of future economic activity, derived from a combination of important economic variables. Leading Economic Index (LEI) and Business Cycle Indicators (BCI) are two examples of economic indices. These signals can be useful to identify economic downturns or boosts, which affect purchasing power of potential customers, and thus, product demand.

Networking Effects:

Products that offer social networking capabilities are expected to have a faster adoption rate because of the amplified imitation (word of mouth) effect. Several examples are the video game consoles that offer online gameplay networks such as Xbox Live and Playstation Network, as well as TVs with Facebook. Number and the rate of change of the subscribers to these networks may indicate the demand for the corresponding products.

Technology Adoption Stage:

Technology adoption stage describes how mature and affordable the overall technology is. HDTVs, blu-ray players and 3DTVs are examples of high-tech products that go through a process of improvement in technical features and a decrease in price from the first time they are introduced to market until the technology is replaced with a new generation. Consequently, models that are introduced later in technology adoption stage attract larger market potential than the earlier models.

3.2.3 Product characteristics that shape the demand

Functionality:

Functionality identifies a specific product's capabilities that may improve the product's use, appeal, and market potential. Motion sensor controlled game consoles, audio in e-readers, video for iPod, wireless capability in CPUs, and GPS for mobile phones can be listed as examples of innovative additions to functionality of products in various product categories.

Price Points:

Price points refer to the set of prices that is selected for a product to have from its introduction to market until its end of life. Lower price points lead to larger market potential and price cuts are expected to boost the demand. It is easier to draw a connection between price and demand if the product has significant price volatility.

Quality:

Quality is a perceptual and subjective characteristic that describes superiority of a product. Given other attributes are the same, a product with higher quality is expected to attract higher demands. However, it is difficult to measure the quality of a product as well as the relationship between quality and demand.

3.2.4 Inside/private knowledge or collective wisdom

Public Opinion:

Public opinion represents recent, up-to-date information about the popularity of a product. Frequency of Google searches for a product (Google Trends), consumer market surveys, wishlists, bestsellers are potential information sources for this category. This information can be used as an indicator of upcoming demand for the particular product.

Experts' Forecasts:

Experts' forecasts can be demand forecasts that are generated by credible professionals with market experience or inside knowledge, as well as a collective projection of somewhat knowledgeable group of people. Forecasts provided by market research companies such as EEDAR, and prediction markets (e.g., simExchange) that use wisdom of crowds to predict future demands are examples for this category.

In addition to the ones listed above, forecasters can also exploit other forms of information available internally and externally, from micro to macro level. The selection of categories with useful information may change with respect to the product in question. Furthermore, the predictive quality of a source may vary at different stages of a product's lifecycle. Thus, it is possible to see diverse groups of information being used throughout the lifecycle. Notice that some of the information sources (Sections 3.2.1 and 3.2.4) are at the product level and in a time series format, while there are others at macro level (market share, economic index), or in qualitative format (functionality, quality). Thus, it is imperative to systematically filter these information into a unique, useful format.

In the next section, we present a demand forecasting framework that incorporates dynamically evolving market information to perpetually reduce forecast variance and increase accuracy.

4 A Generalized Leading Indicator Forecasting Framework

In this section, we present a framework for demand forecasting in high-tech markets. Our framework generalizes the narrowly structured technological forecasting approach to broader dimensions through (1) utilizing unlimited demand models, (2) incorporating vast variety of dynamically evolving market information as leading indicators, and (3) information processing that systematically extracts relevant data to be used for perpetually reducing the variance.

Below, we explain all three aspects of our general framework in detail. More specifically, we answer the following associated questions: How can we model the demand process given actual observations to date, and how can this model be used to project future demands? What approach can we use to incorporate advanced demand signals into our model? What is the effect of this information on the future demand projections? How can we integrate information obtained from a variety of sources, and how can we adjust our model as new observations and new information materialize over time?

4.1 Modeling the Underlying Demand Process

Maintaining an effective and efficient supply strategy is only possible with successful demand forecasting. Demand forecasts are used to decide on and build adequate manufacturing resources for all products, and then to reposition amongst them throughout the products' lifecycles. Importance of forecasting is even more emphasized in the high-tech industry since expensive resources lead to huge operational costs when they are not managed efficiently (we refer the reader to Chapter 4).

Forecasters can investigate past demand realizations to obtain some information about the underlying lifecycle demand process. In fact, time series forecasting has been used widely to estimate future demands based on past demand patterns. If a time series model that exhibits strong goodness-of-fit to observed demand data can be found, prediction of lifecycle demand can be made by forward projecting this underlying model. Time series forecasting assumes that the relationship between demand process and the model would continue into the future. Therefore, the lifecycle forecast constitutes a realistic basis for future demand patterns, unless there is new evidence/information that shows the demand pattern would deviate from what the underlying time series model suggests. In that case, the lifecycle forecast should be updated/revised considering new information. An advantage of this approach is that through modeling the demand process using certain time series, a forecaster can reduce infinitely many future demand scenarios to manageable finite number of possibilities.

Below, we present a systematic technique to describe and predict lifecycle demand using various time series models with verifiable logical and mathematical connections to the demand pattern. The lifecycle forecast obtained here constitutes a *prior distribution* of actual demand process.

Prior Demand Distribution

A lifecycle demand forecast based on past demand observations can be generated in two steps. The first step is determining the underlying time series model (with corresponding parameters). After the underlying model is chosen, the time series is projected onto future periods in the second step.

Let $\Theta(T)$ denote actual demand observations up to time T, in time series format ($\Theta(T) = \{X(1), X(2), ..., X(T)\}$). Then, given available information at time T, $\Theta(T)$, the estimate of demand that will be observed τ periods later is obtained through the following procedure:

1. Fit a time series model $F_k(t) = \{F_k(1), ..., F_k(T)\}$ to demand data $\Theta(T)$, and determine corresponding model parameters. A nonlinear least squares method with a goodness-of-fit measure

(e.g., mean squared error) is suitable for the fitting process.

2. Estimate demand for τ periods ahead by forward projecting the fitted time series model (i.e., $\hat{X}_k(T+\tau|\Theta(T)) = F_k(T+\tau)$).

Then, the actual demand in period $T+\tau$ is expressed as:

$$\begin{aligned} X(T+\tau) &= \hat{X}_k(T+\tau|\Theta(T)) + \epsilon(T+\tau|\Theta(T)) \\ &= F_k(T+\tau) + \epsilon(T+\tau|\Theta(T)), \end{aligned}$$

where $\hat{X}_k(T+\tau|\Theta(T))$ is the estimate of demand observed in period $T+\tau$ projected by time series model k, and $\epsilon(T+\tau|\Theta(T))$ is the estimation error.

We assume that the estimation error is random and normally distributed with mean zero. The uncertainty of demand projections originates from the nonlinearity of the fitting process and the uncertainty in estimations of underlying time series model parameters. We also assume that projection of a time series model ($F_k(T+\tau)$) is an unbiased forecast of actual demand $X(T+\tau)$. These assumptions are conventional in the literature and follows the assumptions used in Aytac and Wu (2008).

Consequently, actual demand at $T+\tau$ can be represented by a normal random variable with a mean projected by the time series model k ($F_k(T+\tau)$) and variance σ_k^2:

$$\tilde{X}_k(T+\tau) \sim N(\hat{X}_k(T+\tau|\Theta(T)), \sigma_k^2)$$

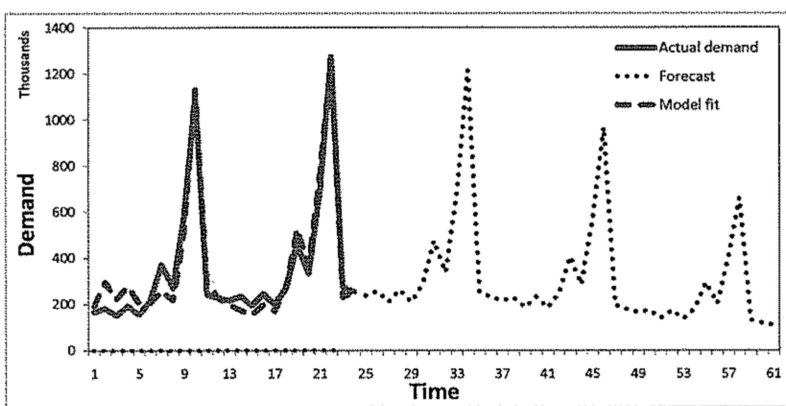

Figure 1: Fitting a time series model and then forecasting lifecycle demand

The time series projection ($\tilde{X}_k$) we describe in this section represents a *prior distribution* for the actual demand. For capacity planning purposes, demand information in terms of a distribution is more useful since point estimates are not a hundred percent accurate. Notice that the confidence on the demand information is greater when the variance is smaller. Therefore, via our forecasting framework, we aim to continuously reduce the demand variance by incorporating advance predictions of future demand and combining diverse information sources. Next, we introduce leading indicators and corresponding demand distribution samples, which we use to update the prior distribution.

4.2 Incorporating Dynamically Evolving Market Information

The framework we present in this paper perpetually reduces forecast variance by incorporating a diverse selection of dynamic market information into the forecasting process. The demand signal extracted from market information can be used in place of the actual demand signal to predict following periods' demand ahead of time. In the following sections, we explain how different information sources can be molded into leading indicator signals and utilized to increase the forecast quality throughout various steps of our framework.

4.2.1 Leading Indicators

A leading indicator is a pattern in the form of a time series that predicts the demand pattern of a new product before it materializes. The purpose of using leading indicators is to get informed about the recent market condition and how the trend is changing. The additional information provided by leading indicators help reduce the variance of possible future demand scenarios (Figure 2).

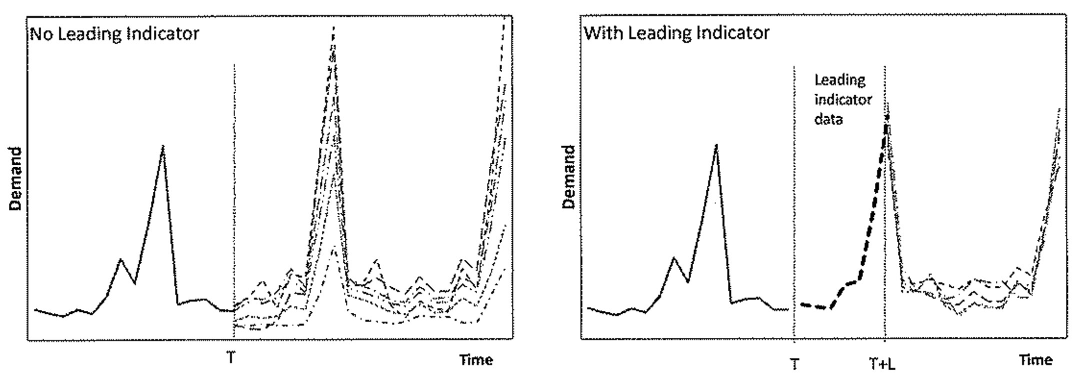

Figure 2: Demand scenarios with and without a leading indicator

Previous theory on technological forecasting with leading indicators was built in a very narrow context. The concept of demand leading indicators was first proposed by Meixell and Wu (2001). Later, leading indicators were used in other studies and implementations (Wu et al., 2006; Aytac and Wu, 2008) as they were originally defined. In all of these, the idea was to find a product that has a demand pattern with high correlation to the demand pattern of a new product, and to use the remaining pattern (which is already observed) of the leading indicator product to predict the future demand of the new product, assuming the high correlation would continue during upcoming periods. However, our implementation experience has confirmed that in the high-tech industry (e.g., consumer electronics), historical information usually fails to indicate recent market dynamics since there are too many continuously changing factors driving the demand. Even in very few instances that a high correlation is identified between the new product and the past product for part of the pattern that has materialized, the relationship did not continue in the future, basically because recent market condition is much different than it was when the old product was in demand months ago.

Consequently, we generalize the context of leading indicators to include any publicly or privately available up-to-date information that contains advanced indication of market dynamics and allows for drawing verifiable connections with future demand pattern in a consistent manner, in the form of a time series. The first distinction of the generalized model pertains to the use of up-to-date information, as historical demand signals are not good indicators of new market trends. Secondly and most importantly, we generalize the framework to allow for the use of public information that is vastly available through Internet, various media and other sources of data. This new interpretation of leading indicators revolutionizes the way forecasters select and utilize information in the forecasting framework by introducing a plethora of data to use and improve accuracy via increased number of indicators employed. The new interpretation also lets outsiders or competitors to generate intelligent forecasts for a wide range of products without any inside information.

Transforming Market Information Into Leading Indicators:

Leading indicators are used to extend observed demand data. Therefore, they are required to be in time series format as well as an unbiased estimator of the actual demand. Some market information such as ship-aheads, expert forecasts and partners' data may already be in the required format. However, there are other information that should be transformed into the leading indicator format.

There is not one standard transformation procedure, and it depends on the information source.

For macro level information such as market forecast, a top-down approach can be used to attain a specific product's demand series using the market share information. On the other hand, for micro level information such as field sales intelligence (demand forecast for a particular customer at a particular geography) or design wins (customer specific preproduction design projects), a bottoms-up approach is useful via aggregation of the data to find the total demand series. Once the information is transformed into time series format, the indicative power of the series can be measured by correlation between the series and the observed demand signal. The higher the correlation is, the greater the indicative power should be.

Any information source with indicative power can be used as a leading indicator. However, these sources may have some degree of bias for a variety of reasons. Although noise in the indicator information is acceptable and it is generally canceled by use of multiple leading indicators, bias may shift the mean of the estimate and increase the forecast variance (Aytac and Wu, 2008). Nevertheless, the bias in the leading indicator information can be corrected over time as the forecaster notices the deviation and the learning mechanism takes effect. For instance, there may be very high correlation between an advanced signal and the demand signal, but the forecaster 'learns' over time that one signal consistently outputs greater values than the other (which indicates existence of bias). High correlation is a good measure for determining whether a certain signal is a decent leading indicator, thus, advanced signal can be used as a leading indicator after the bias is corrected by accounting for (reducing as necessary) the overage.

As explained above, all information sources should be encoded into time series format as unbiased estimators to be useful in the forecasting framework. It is possible to verify through standard hypothesis testing if the leading indicators are unbiased estimates. If biases are identified, forecaster should correct the bias before using the indicator data. Although there are diverse methods to unbias, one of the most basic and useful procedures is regression, details of which is provided below.

Algorithm 1. Unbias a leading indicator

1. *At the beginning of time* $t+1$*, let* $X_1, X_2, ..., X_t$ *be the time series of the demand data realized and let* $I_1, I_2, ..., I_t, ..., I_{t+k}, ..., I_{t+L}$ *be the time series of leading indicators, where* L *is the time lag between the realization of product demand and the collection of leading indicators for each point* t*. Linearly regress the* t *actual demand onto the* t *leading indicators to get the following relationship:* $X_t = \alpha \cdot I_t^2 + \beta \cdot I_t + \gamma$.

2. *The relationship established in Step 1 is then used to unbias the leading-indicator data for future time periods. Let I'_{t+k} be the unbiased leading indicator for future period $t+k$; then $I'_{t+k} = \alpha \cdot I^2_{t+k} + \beta \cdot I_{t+k} + \gamma$.*

4.2.2 Samples of Demand Distribution

At time T, we extend available demand information $\Theta(T)$ with time series of leading indicator from period $T+1$ to $T+L$. The objective here is to increase the accuracy of the estimation via the information obtained from the indicator. Then, a time-series model is fitted to the demand data extended by leading indicator j by L periods, $\Theta^j(T+L)$. Assuming that data from leading indicator are unbiased estimators of the actual data over $T+1$ through $T+L$, the projection provided by the fitted model onto periods starting with period $T+L+1$ constitute an unbiased likelihood (sample) for the forecast that would have been produced using observed (actual) demand information $\Theta(T+L)$.

When there are m different leading indicators, we obtain m independent samples of lifecycle projections. These independent samples are then combined to generate the *sampling distribution.* Note that the convex combination of m normal random variables is also a normal random variable, and it can be summarized as:

$$\tilde{X}_k(T+\tau) \sim N\left(\sum_{j=1}^{m} w_j \hat{X}_{kj}\left(T+\tau|\Theta^j(T+L)\right), \sum_{j=1}^{m} w_j^2 \hat{\sigma}_{kj}^2\right), \tag{1}$$

where $\hat{X}_{kj}\left(T+\tau|\Theta^j(T+L)\right)$ is the lifecycle projection of model k using the data extended by leading indicator j for L additional periods, $\hat{\sigma}^2_{kj}$ is the corresponding projection variance, and w_j is the combination weight for projection with respect to leading indicator j.

As we describe above, our framework benefits from the intelligence (information) of several leading indicators through combining them. In the following section, we discuss which combination method (combination weights) should be chosen to attain the greatest possible reduction in forecast variance.

4.2.3 Combining Information

The general idea of combining information (i.e., forecasts) has been a widely used practice among technological forecasters to incorporate diverse intelligence elements and cancel out the noise inherent in various models. Nevertheless, both the models to combine and the combination technique may differ with respect to the problem in consideration. In our case, we want to combine models that represent lifecycle demand for technology products. However, the combination method that we use can affect the outcome. More specifically, it is important to decide (i) whether various models should be combined sequentially or collectively, (ii) in which order they should be combined if sequentially, and (iii) what should be the combination weights. Below, we try to answer these questions, considering that the aim of our work is to decrease the forecast variance.

Let information source i suggests that a future demand observation is distributed with respect to $N(\mu_i, \sigma_i^2)$. Let $\bar{\mu}$ and $\bar{\sigma}^2$ denote mean and variance after the combination, respectively.

Theorem 1. *Given that sources of information are independent and normally distributed, the following are true:*

i. Using combination weights inversely proportional to information variances minimizes the combined variance.

ii. Combining information sequentially or collectively provides the same result when the specified weights are used.

Proof.

First, we consider collective (simultaneous) combination of information. We show that using combination weights that are proportional to the precision of information minimizes the combined variance. Proof follows from Dickinson (1972):

Let $\boldsymbol{w} = (w_1, .., w_m)'$ be the column vector of combination weights (w_i for information source i), and $\boldsymbol{V}$ be the diagonal covariance matrix for information sources ($i = 1, ..m$). Then, variance of the linear combination is denoted by $\boldsymbol{w}'\boldsymbol{V}\boldsymbol{w}$, and minimized to find the optimal combination weights as follows:

$$Min \quad \boldsymbol{w}'\boldsymbol{V}\boldsymbol{w}$$

$$s.t. \quad \boldsymbol{w}'\mathbf{1} = 1$$

where $\mathbf{1}$ is an mx1 column vector of ones. Using Lagrangean relaxation, we get:

$$Min \;\; \boldsymbol{w}'\boldsymbol{V}\boldsymbol{w} - 2\lambda\left(\boldsymbol{w}'\mathbf{1} - 1\right)$$

After taking derivatives with respect to w and λ, we have:

$$\boldsymbol{V}\boldsymbol{w} = \lambda\mathbf{1}, \qquad \boldsymbol{w}'\mathbf{1} = 1$$

Multiplying both sides with $\boldsymbol{V}^{-1}$, we obtain $\boldsymbol{w} = \boldsymbol{V}^{-1}\lambda\mathbf{1}$. The transpose of this term is $\boldsymbol{w}' = \mathbf{1}'\lambda\boldsymbol{V}^{-1'}$. Substituting this term into the equation $\boldsymbol{w}'\mathbf{1} = 1$, we get $\mathbf{1}'\lambda\boldsymbol{V}^{-1}\mathbf{1} = 1$. Consequently, we find:

$$\lambda = \left(\mathbf{1}'\boldsymbol{V}^{-1}\mathbf{1}\right)^{-1}$$

Then, $\boldsymbol{w} = \boldsymbol{V}^{-1}\lambda\mathbf{1} = \boldsymbol{V}^{-1}\left(\mathbf{1}'\boldsymbol{V}^{-1}\mathbf{1}\right)^{-1}\mathbf{1}$, which can be explicitly written as:

$$w_i = \frac{1/\sigma_i^2}{\sum_{j=1}^{m} 1/\sigma_j^2}$$

Notice that $0 \leq w_i \leq 1, \forall i$ and $\sum_{i=1}^{m} w_i = 1$. Therefore, above weights are also optimal for minimum convex combination variance.

Next, we consider sequential combination of information. Sequential combination of m information sources can be represented in terms of $m-1$ pairwise collective combinations. For instance, three sources can be combined sequentially in two steps by first combining sources 1 and 2; and then combining the combination (of 1 and 2) with source 3. We first investigate sequential combination with weights that are inversely proportional to information variances. Note that the collective combination variance with these weights is given by:

$$\begin{aligned}
\boldsymbol{w}'\boldsymbol{V}\boldsymbol{w} &= \left(\boldsymbol{V}^{-1}\mathbf{1}\left(\mathbf{1}'\boldsymbol{V}^{-1}\mathbf{1}\right)^{-1}\right)'\boldsymbol{V}\left(\boldsymbol{V}^{-1}\mathbf{1}\left(\mathbf{1}'\boldsymbol{V}^{-1}\mathbf{1}\right)^{-1}\right) \\
&= \left(\mathbf{1}'\boldsymbol{V}^{-1}\mathbf{1}\right)^{-1}\left(\mathbf{1}'\boldsymbol{V}^{-1}\mathbf{1}\right)\left(\mathbf{1}'\boldsymbol{V}^{-1}\mathbf{1}\right)^{-1} \\
&= \left(\mathbf{1}'\boldsymbol{V}^{-1}\mathbf{1}\right)^{-1} \\
&= \frac{1}{\sum_{i=1}^{m}\sigma_i^{-2}},
\end{aligned}$$

which means the reciprocal of the combination variance can be found by adding the reciprocals of variances of the combined information sources. Then, for two sources, combined variance is equal to $\bar{\sigma}_{1,2}^{-2} = \sigma_1^{-2}+\sigma_2^{-2}$; and for three $\bar{\sigma}_{(1,2),3}^{-2} = \bar{\sigma}_{1,2}^{-2}+\sigma_3^{-2} = \sigma_1^{-2}+\sigma_2^{-2}+\sigma_3^{-2}$. It follows that, the reciprocal of the new combination variance can be found by adding the reciprocal of variance of the newest sample to the reciprocal of the combination variance from the previous step ($\bar{\sigma}^{-2} = \sigma_1^{-2}+...+\sigma_k^{-2}, \forall k \leq m$). Consequently, the combination sequence does not affect the final combination variance.

Note that if information is sequentially combined (through $m-1$ pairwise convex combination steps with $w_1 + w_2 = 1$, $0 \leq w_i \leq 1, i = 1,2$ at each step), the final combination is a convex combination of m information sources. Note also that both sequential and collective combination yield the same combination variance, $\frac{1}{\sum_{i=1}^{m} \sigma_i^{-2}}$. Therefore, no other sequential combination weights can provide a lower combination variance.

□

To reduce the forecast variance as much as possible, we will use the weights suggested in Theorem 1 to combine information throughout our study. In the next section, we describe the combination of information samples.

4.2.4 Combining Sampling Information

During this step of the framework, we combine the m independent samples of lifecycle projections, which are obtained by projection of models fitted to demand data extended by m different leading indicators. Combination of these independent samples generates the *sampling distribution.* Next lemma describes the mean and variance of the distribution obtained by combining m leading indicator based samples.

Lemma 1. *A sampling distribution obtained from m information samples that are distributed with respect to $N\left(\hat{X}_i\left(T+\tau|\Theta^i(T+L)\right), \sigma_i^2\right)$ is also normally distributed with mean $\bar{\mu}$ and variance $\bar{\sigma}^2$, where:*

$$\bar{\mu} = \sum_{i=1}^{m} \frac{\sigma_i^{-2}}{\sum_{j=1}^{m} \sigma_j^{-2}} \hat{X}_i(T+\tau|\Theta^i(T+L)), \qquad \bar{\sigma}^2 = \sum_{i=1}^{m} \left(\frac{\sigma_i^{-2}}{\sum_{j=1}^{m} \sigma_j^{-2}}\right)^2 \sigma_i^2.$$

Proof. The result follows from use of Equation 1 with combination weights proportional to precisions of samples ($w_i = \frac{1/\sigma_i^2}{\sum_{j=1}^m 1/\sigma_j^2}$).

$\square$

Proposition 1. *Forecast variance of the sampling distribution is smaller than the variance of any leading indicator based lifecycle projection used in the combination. The sampling variance decreases as the number of leading indicators used in combination increases.*

Proof. The result follows from Lemma 1 and Theorem 1.

$$\bar{\sigma}^2 = \sum_{i=1}^m \left(\frac{1/\sigma_i^2}{\sum_{j=1}^m 1/\sigma_j^2}\right)^2 \cdot \sigma_i^2 = \sum_{i=1}^m \frac{1}{\sigma_i^2 \cdot \left(\sum_{j=1}^m 1/\sigma_j^2\right)^2} = \frac{1}{\sum_{j=1}^m 1/\sigma_j^2} < \sigma_i^2, \forall i \in \{1,..,m\}.$$

$\square$

To sum up, the sampling distribution incorporates various leading indicator based information while reducing the variance. Next, we introduce the update of prior distribution with the sampling distribution to reduce the forecast variance even more.

4.2.5 Bayesian Update and Posterior Demand Distribution

Aim of this procedure is to improve the demand information of the prior distribution by incorporating advanced demand signals featured by the sampling distribution in a systematic manner. Bayesian updating of normally distributed prior with sampling distribution that also follows normal distribution results in a normally distributed posterior forecast.

Lemma 2. *Bayesian update of the prior distribution (with mean $\hat{X}_k(T+\tau|\Theta_T)$ and variance σ_k^2) with the sampling distribution obtained from m information samples (with mean $\bar{X}_k(T+\tau|\Theta_{T+L})$ and variance $\bar{\sigma}_k^2$) results in a normally distributed posterior forecast:*

$$\tilde{X}_k(T+\tau) \sim N\left(\mu_k', \sigma_k'^2\right), \textit{ where}$$

$$\mu_k' = \frac{1/\sigma_k^2}{1/\sigma_k^2 + 1/\bar{\sigma}_k^2}\hat{X}_k(T+\tau|\Theta_T) + \frac{1/\bar{\sigma}_k^2}{1/\sigma_k^2 + 1/\bar{\sigma}_k^2}\bar{X}_k(T+\tau|\Theta_{T+L}), \textit{ and } \sigma_k'^2 = \frac{\sigma_k^2\bar{\sigma}_k^2}{\sigma_k^2 + \bar{\sigma}_k^2}.$$

Proof. The proof is provided by Lee (1989) and presented in the Appendix.

$\square$

Proposition 2. *Forecast variance after the Bayesian update is less than both the variance of the prior distribution and the variance of the sampling distribution.*

Proof. The result follows from Lemma 2 and Proposition 1.

□

Consequently, the information update provided by use of leading indicators reduces the forecast variance. Also notice that as the number of unbiased leading indicator estimates that are used increases, the variance of sampling distribution decreases, which leads to a smaller posterior variance.

4.2.6 Combination of Posterior Forecasts

Posterior distribution incorporates diverse leading indicator information with prior beliefs on the demand forecast, using estimates that are projected based on a particular lifecycle pattern. However, it is not clear which lifecycle pattern is the best choice to model the demand diffusion. In fact, if a model that fits the best to the demand data set would also always project with the least error, the choice would have been obvious.

However, a good fit to observed demand data does not always translate to an accurate demand projection because of the inherent uncertainties. As a result, in our framework, after several posterior distributions are generated using various underlying lifecycle models, they are combined to further decrease the forecast variance and improve accuracy. The combination strategy introduced in Theorem 1 is used to minimize the variance.

Lemma 3. *When n different posterior forecasts $(\tilde{X}_k(T+\tau))$ that are distributed with respect to $N\left(\mu_k^{'}, \sigma_k^{'2}\right)$ are combined using weights inversely proportional to forecast variance, the final forecast is also normally distributed with mean $\tilde{\mu}$ and variance $\tilde{\sigma}^2$, where:*

$$\tilde{\mu} = \sum_{k=1}^{n} \frac{\sigma_k^{'-2}}{\sum_{j=1}^{n} \sigma_j^{'-2}} \mu_k^{'}, \qquad \tilde{\sigma}^2 = \sum_{k=1}^{n} \left(\frac{\sigma_k^{'-2}}{\sum_{j=1}^{n} \sigma_j^{'-2}} \right)^2 \sigma_k^{'2}.$$

Proof. The result follows from use of Equation 1 with combination weights proportional to precisions of posterior forecasts ($w_k = \frac{1/\sigma_k^2}{\sum_{j=1}^{n} 1/\sigma_j^2}$).

□

Proposition 3. *Combining posterior forecasts obtained from different lifecycle models using weights that are proportional to their precision (inverse variance) yields a final forecast variance that is smaller than forecast variance of each individual posterior forecast in the combination.*

Proof. The result follows from Lemma 3 and Proposition 1. □

Consequently, our framework reduces the forecast variance perpetually through incorporating leading indicator information into a sampling distribution, Bayesian updating the prior forecast, and finally combining the posterior forecasts. Next, we present the Generalized Leading Indicator Forecasting Algorithm, components of which have been explained in previous sections.

4.3 Generalized Leading Indicator Forecasting Algorithm

An initial forecast information (prior distribution) is obtained by forward projecting the underlying lifecycle model, which has been selected (i.e., fitted) based on up-to-date demand realizations. The underlying model is fitted again, this time using up-to-date demand realizations plus advance demand signals from a leading indicator, and then, projected into future. This projection is used to update the initial forecast via the Bayesian statistical framework. When multiple leading indicators are available, a methodical combination of all samples (sampling distribution) are used to update the initial forecast. This procedure is replicated for every model that is employed to describe the diffusion process. The updated forecasts (posterior distribution) are later combined to generate the final forecast. Next, we outline the Generalized Leading Indicator Forecasting Algorithm in detail.

Algorithm 2. Generalized Leading Indicator Forecasting

Input at time T:

- *Actual demand observations* $\Theta(T) = \{X(1), .., X(T)\}$.
- *Indicative information sources* $I_1, .., I_m$.
- *Time series* $(F_k(t), k \in K)$ *to model the underlying demand process.*

begin

For each *information source* $I_i, i \in M$, ***do:***

    Information Processing:

    begin $\{|M|$ passes$\}$

        Extract relevant data from the information source in time series format.

        Unbias the signal and prepare as leading indicator *series* $l_i(t), i \in M$.

    end; $\{|M|$ passes$\}$

For each *time series model* $F_k(t), k \in K$, ***do:***

    begin $\{|K|$ passes$\}$

        Modeling the Underlying Demand Process:

            Estimate parameters for model k by fitting demand observations $\Theta(T)$.

Project demand series from $(T+1)$ *to* $(T+\tau)$ *using parameters for model* k*;*
Add the demand series to the prior distribution.

Incorporating Dynamically Evolving Market Information:
For each *leading indicator* $l_i, i \in M$*,* ***do:***
begin $\{|M|$ passes$\}$
Use leading indicator l_i *to extend* $\Theta(T)$ *by* L_i *periods;*
Estimate parameters for time series F_k *by fitting to data* $\Theta(T+L_i)$*.*
Project demand from $(T+L_i)$ *to* $(T+\tau)$ *using parameters for model* F_k*;*
Add the demand series to the sampling distribution.
end; $\{|M|$ passes$\}$
Perform Bayesian updates using prior *and* sampling distributions.
Obtain posterior dist. *with respect to underlying time series model* F_k*.*
end; $\{|K|$ passes$\}$

For each *time series model* $k \in K$*,* ***do:***

Combining Multiple Demand Models:
begin $\{|K|$ passes$\}$
Add the posterior forecast with respect to model k *to the* final forecast.
end; $\{|K|$ passes$\}$
end;

The above algorithm is used continuously throughout the lifecycle. As new demand observations become available or as the leading indicator information evolves with the market, beliefs on how the diffusion pattern is going to unfold are updated by reestimating model parameters and combination weights. As we explained before in Section 4.2.5, the forecast variance is perpetually reduced by the Bayesian update –increasingly so with greater number of unbiased leading indicators– and combinations. Selection of leading indicators being used can change for different products and at different points during the planning horizon since the predictive quality of an indicator may vary with respect to time, product, etc.

In the next section, we present the study by Aytac and Wu (2008), which constitutes a special case in our framework.

4.3.1 Previous Research as Special Cases of the Generalized Agorithm

Earlier studies in technological forecasting literature have been very narrowly structured in terms of the models that can be used to represent the demand process, the variety of information that can be incorporated, and the way that information is processed. We want to emphasize that the forecasting framework we introduce is very general in terms of the mentioned aspects, so that any demand process can be easily modeled and forecasted while taking advantage of a vast variety of information available throughout a product's lifecycle for continuous refinement of accuracy. In this section, we present a stream of technological forecasting literature that constitutes a special case in our framework. We anticipate that the theoretical contribution of our study will become even more apparent when the reader experiences how this framework has been generalized from a special case to a very flexible and capable tool with broader dimensions.

In a high-tech manufacturing setting, the idea of utilizing advanced indicators of future demand patterns was first introduced by Meixell and Wu (2001). The authors propose that demand patterns of some products may incorporate similarities with demand patterns of other products that are introduced later (lagging in demand realization) and have comparable functionality or resource requirements. Using correlation between demand patterns of leading and lagging products as a measure, Wu et al. (2006) identify leading indicator products in a semiconductor manufacturing environment. They project future demands for lagging products assuming that the correlation between the demand for the leading indicator and the demand for the particular product should continue into the future.

Leading indicator products were employed again later in a more streamlined study by Aytac and Wu (2008). They exclusively use growth curves to model the demand pattern, and utilize leading indicator data to extend the available information. Authors use the projections based on actual demand data extended with leading indicators to update their earlier projection, which only

Notice that the model introduced by Aytac and Wu (2008) constitutes a special case in our framework since:

1. The underlying demand process is modeled exclusively by growth curves.

2. Leading indicator information is limited to demand patterns of earlier products.

3. Indicator information is processed in a single way; through linearly regressing leading pattern over the predicted pattern, and then forward projecting.

Dynamic high-tech markets such as consumer electronics implicate very volatile demand processes. The demand trend changes rapidly in a continuous manner. Therefore, in general, demand patterns of older products have inadequate indicative quality. Our model allows for use and processing of every available information source with indicative quality. Furthermore, with our framework, any time series model can be utilized to represent the underlying demand process, so that the seasonality and volatility can be modeled more accurately.

In the next section, we implement our framework in a case study to forecast demand for Xbox 360 video game console.

5 Case Study: Forecasting Demand for Xbox 360

Our forecasting framework can be employed to forecast product demand in various high-tech industries. These products are known to have volatile demand caused by fast-changing, dynamic markets. Our framework provides accurate forecasts by utilizing diverse sources of public or private information. In this section, we describe our experience in implementing our generalized forecasting model to project customer demand for Xbox 360 video game console.

Xbox 360 was introduced into the market in November, 2005; and we continue to observe strong demand for the product. The monthly unit sales data is provided in the Appendix (Table 4), and also plotted in Figure 3. Earlier generations of video game consoles indicate that the typical demand lifecycle for these products is about seven or eight years long. We aim to implement our forecasting methodology throughout the lifecycle of the product, assuming we were forecasting the upcoming demand every time actual demand observations were made. In other words, starting with the market introduction, every month, we fit our lifecycle models to actual U.S. demand observations (plus leading indicators), and project upcoming months' demand. Then, we compare our forecast with the actual realization of the demand, and illustrate the benefit obtained from employing the forecasting framework.

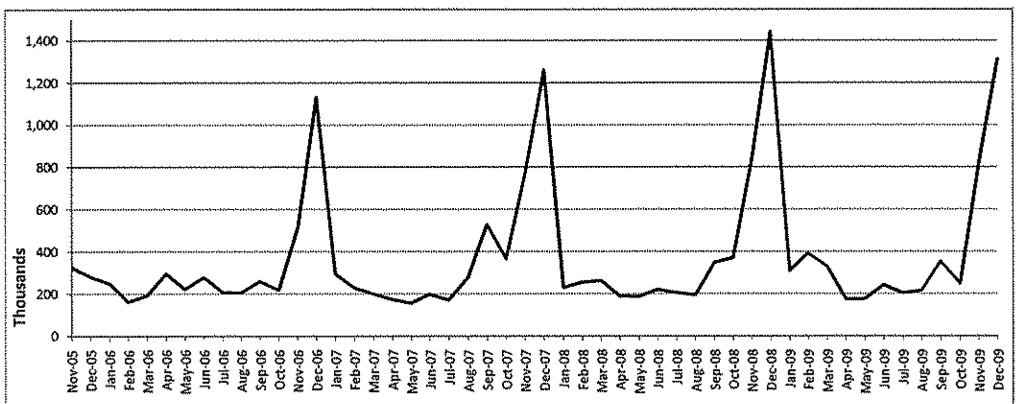

Figure 3: Xbox 360 monthly U.S. sales provided by NPD Group

5.1 Leading Indicators

Leading indicators have two important functions in our framework. First and the most important, they provide advanced intelligence about changing market dynamics. This helps the forecaster to reduce future demand scenarios and correspondingly the forecast variance. Second, they extend the set of demand observations needed to fit the models, which is valuable to obtain confident fits especially earlier in the demand lifecycle. In this section, we describe the specific leading indicators that we use in the case study in detail.

Although there is vast amount of public information available during a product's diffusion, most of the information is not recorded and preserved for use in the future. Because of that, for a variety of information sources, we had some difficulty retrieving data consistently available going back up to five years. Even with the mentioned inconvenience, we were able to find an adequate selection of leading indicators that help reduce the variance and increase the accuracy of the demand forecast through their use in our framework.

To use as a leading indicator, we filter each information source into an unbiased estimator of future demands in time series format. Below, we explain the process in detail for each information source. We also describe the reasons why some information were not suitable for use in this case.

1. Market Share:

Estimating the aggregate demand for the whole market of products is easier than it is for any particular product. Market demand is more predictable because it is unaffected by competition or substitution among its components and individual demand fluctuations cancel out. We notice that video game consoles have very close market introduction times such that the different generations of demand lifecycles are clearly visible and also can be easily forecasted (Figure 4). The sixth generation consoles are Sony Playstation 2, Microsoft Xbox and Nintendo Gamecube.

Once an aggregate forecast is obtained, the demand for a particular product can be estimated by accounting for its market share. Market share is generally a fairly steady ratio that has been established through years of sales and competitive efforts (e.g., advertising, pricing, quality, selection, etc.) against the other products/companies. Since console market is very competitive, marketing activities of different companies (price cuts, promotions, advertisements) happen simultaneously and thus, are less likely to affect their shares. Therefore, we expect a steady market share, especially as we move towards demand maturity.

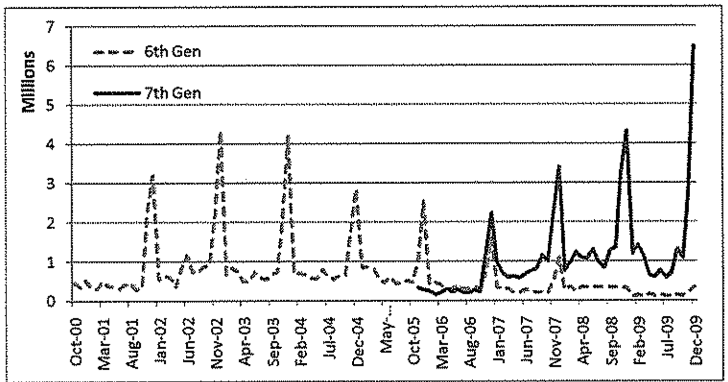

Figure 4: Demand for 6th and 7th generation video game consoles

There are three products in the seventh generation video game console market: Microsoft Xbox 360 (Xbox), Sony Playstation 3 (PS3), and Nintendo Wii (Wii). We notice that Wii is rather different than the other two products in terms of technology, purpose, gameplay, and available media, while PS3 and Xbox are pretty similar in their properties and target audience, causing total demand for Xbox and PS3 to be more predictable.

At any given time t, we systematically compute the leading indicator for future Xbox demand via the following steps:

i. Estimate total combined demand for PS3 and Xbox ($D_{t+\tau}^{PS3+Xbox}$) for future periods ($\tau = 1, 2, 3, ..$).

ii. Determine the expected market share of Xbox (s^{Xbox}).

iii. Obtain the leading indicator for Xbox demand at period $t + \tau$ using formula: $s^{Xbox} * D_{t+\tau}^{PS3+Xbox}$.

The total demand projection in Step i is obtained by fitting a time series model (S.R. Bass in particular; see Table 1 in Section 5.2 for the time series formula) to available demand data (Figure 5) and then extrapolating the fitted model onto upcoming periods. During Step ii, the market share of Xbox is calculated using a weighted moving average of recent market share information

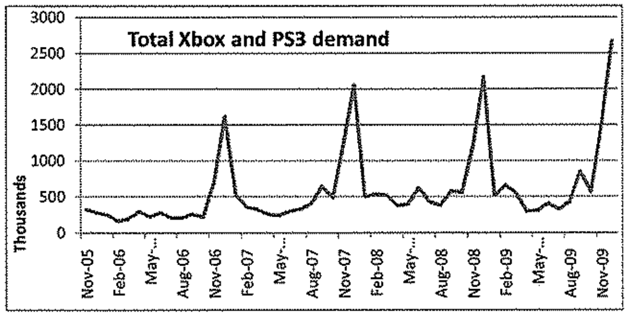

Figure 5: Total demand for Xbox and PS3

($s_{t+1} = 0.2s_t + 0.3s_{t-1} + 0.5s_{t-2}$, where s_t is the share of Xbox demand in total demand for Xbox and PS3 for month t). Although we observe fluctuations at the beginning of the lifecycle, market share stabilizes as the adoption matures (Figure 6). Finally, we compute the leading indicator values for Xbox in Step iii.

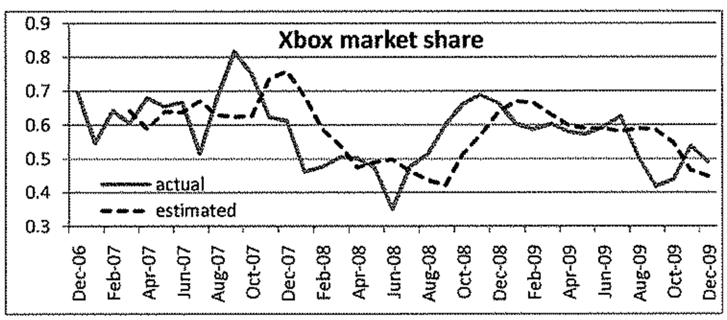

Figure 6: Actual and estimated market share for Xbox

2. Google Trends:

In recent years, the Internet has become the most efficient and perhaps the leading source for obtaining information. Moreover, online sales now constitute a significant portion of all sales in America. It is very practical for customers to compare technical specifications, price, and selection of products available through the Internet before purchasing, especially for high-tech products.

Google Trends is a website that documents how frequently a topic has been searched over time. The website provides up-to-date data about Google search traffic (from the geographies of choice) in weekly intervals, for any given subject. The data can also be interpreted as 'the popularity of a subject over time'. Thus, we believe that potential adopters of a video game console are likely to search the Internet for information on the product, and a high correlation between search frequency and demand signal is expected. This information source can be very useful since it reflects up-to-date public opinion and market trend in dynamic markets.

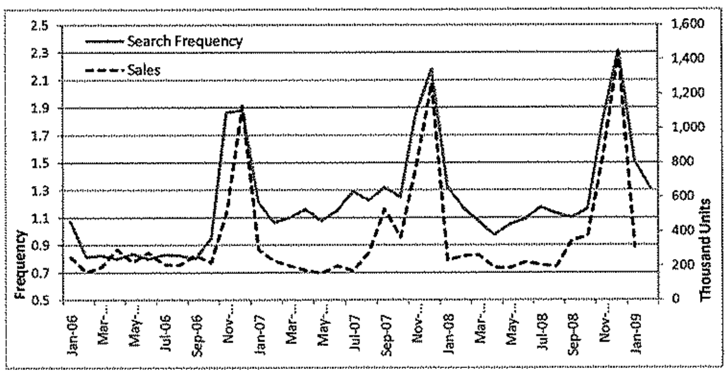

Figure 7: Search frequency and unit sales plotted on separate axes

Consequently, we obtained the weekly search frequency data for the term 'Xbox 360' (Table 7 in the Appendix) to check whether it provides an indicative signal. We plotted the average monthly search frequency for the term 'Xbox 360' and actual demand for Xbox 360 on separate axes (Figure 7). The correlation between the two series are calculated to be 0.878, and the similarity between trends of the two signals are easily visible in the figure. Therefore, we decide to use this information source to compute a leading indicator time series for future Xbox demand.

Since the time lag between trend signal and actual demand is 1-2 weeks, we estimate the search trend for the following months ($t+1$,$t+2$,etc.) by extrapolating the available frequency data set. In particular, we fitted an S.R. Bass time series model to data up to t, and then forward projected to obtain the following periods' data. In addition, to use as an unbiased leading indicator of Xbox demand, we rescale and unbias the frequency data via the procedure presented below, every time a new observation materializes (every month t):

Let D_t denote the actual demand realized in month t, and X_t denote the indicative time series (search frequency data) corresponding to month t. Then,

i. Determine the underlying time series model by fitting to $X_1, X_2, ..., X_t$. Use established time series parameters to project following months' frequency data $\hat{X}_{t+1}, \hat{X}_{t+2}, ...$

ii. Determine the extent of bias in the frequency series by regressing the series $D_1, D_2, ..., D_t$ onto $X_1, X_2, ..., X_t$, using a quadratic regression formula :

$$D_t = a \cdot X_t + b \cdot X_t^2 + c$$

iii. Use the relationship obtained in previous step to unbias and rescale the frequency data projected in step i, and obtain leading indicator ($I_{t+\tau}$) for τ months ahead ($\tau = 1, 2, ..$):

$$I_{t+\tau} = a \cdot \hat{X}_{t+\tau} + b \cdot \hat{X}_{t+\tau}^2 + c$$

Below, we plot actual demand and corresponding leading indicator series computed one-month ahead of time (Figure 8).

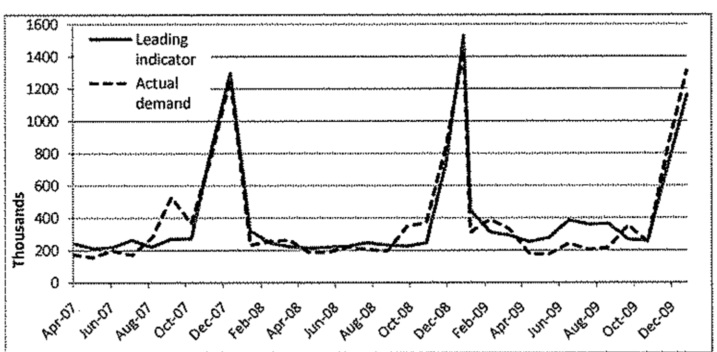

Figure 8: Unbiased leading indicator series and actual demand series

3. Experts' Forecasts:

For popular high-tech products such as video game consoles, it is possible to find a wealth of sources that publicly or privately publish their prediction of future demands. Market research companies such as NPD Group and Electronic Entertainment Design and Research (EEDAR), investment or consulting firms such as Wedbush Morgan and also manufacturers like Microsoft and Sony are only a few examples of entities with inside knowledge that regularly publish their forecasts. It is also possible to find forecasts of outsiders as well as websites (simExchange) that use wisdom of crowds to predict upcoming demand.

We compiled two leading indicator series from the information sources we mentioned before. The time lag between actual demand and forecast data is one month. Therefore, leading indicator data for the first month ahead is readily available. The leading indicator data for following months $(t+2, t+3, ..)$ are estimated via the same extrapolation technique used for Google Trend data (Step i, followed by $I_{t+\tau} = \hat{X}_{t+\tau}$). The leading indicator series are plotted with actual demand in Figure 9. We also present experts' forecasts in the Appendix (Table 5).

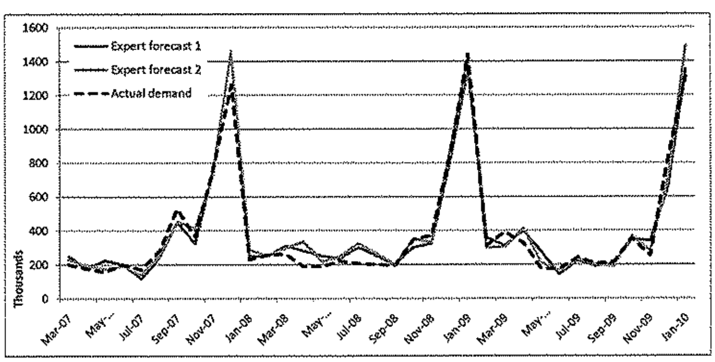

Figure 9: Experts' forecasts as leading indicators

There are other sources of information that we considered as potential leading indicators; however, after detailed analysis, their indication quality turned out to be neither adequate nor consistent. For instance, pricing moves are usually expected to affect the demand (i.e., a price cut usually boosts the demand), and can be incorporated into the demand lifecycle model (Krishnan et al., 1999).

However, in our case, pricing moves are very rare and simultaneous with the competitors, avoiding us to be able to distinguish their effect. Similarly, vastness and variety of information on video game sales (including data on console-exclusives and pre-orders) make it very difficult to determine the relationship between demand and specific product drivers. We could not find accurate numbers on the members and growth of social (gameplay) network associated with the product, while the economic indicators happen to work poorly with such economy-proof products in dynamic markets. As a result, we use four different leading indicator series in our case study. Next, we describe the implementation of the Generalized Leading Indicator Forecasting Framework.

5.2 Implementation

One of the critical characteristics of the demand for video game consoles is high seasonality. Black friday and back-to-school promotions as well as the demand surge during the holidays cause severe spikes in demand in addition to the typical volatility. The models that ignore the seasonality have been shown to fit to the demand signal poorly and result in inaccurate predictions.

It is possible to find examples of models with adjustments for seasonality. For instance, Kurawarwala and Matsuo (1996) modified the Bass diffusion model by replacing the time variable (t) with the expression $\int_0^t \alpha_\tau d\tau$ that rescales the time axis, where α_τ is the seasonality parameter. Among diverse time series models that could be chosen to model the underlying demand process, our initial tests show that a seasonally rescaled diffusion model provides satisfactory fits to game console demand signal. Therefore, using the same idea, we modified four other diffusion models for seasonality and used in our study. Our tests showed that seasonally rescaled trend curve models (e.g., (all seasonally rescaled (S.R.)) Simple logistic, Weibull, Bass, Extended logistic, and Log-reciprocal) demonstrated the best fit to the game console demand. In Table 1, we describe the lifecycle models that we use to characterize the demand process in our forecasting framework. Note that α_i is the seasonality parameter for month i ($\alpha_1 = 1$; $\alpha_i \geq 0$, $i = 2, .., T$; $\alpha_i = \alpha_{i-12}$, $i = 13, .., T$).

S.R. Simple Logistic	$X_t = \frac{1}{1+c \cdot exp(-b \sum_{i=1}^{t} \alpha_i)} + \epsilon_t, b > 0, c > 0$
S.R. Weibull	$X_t = 1 - exp\left(-\left(\frac{\sum_{i=1}^{t} \alpha_i}{a}\right)^b\right) + \epsilon_t, a > 0, b > 1$
S.R. Bass	$X_t = \frac{1-exp(-(p+q)\sum_{i=1}^{t} \alpha_i)}{1+\frac{q}{p} exp(-(p+q)\sum_{i=1}^{t} \alpha_i)} + \epsilon_t, p > 0, q > 0$
S.R. Extended Logistic	$X_t = \frac{1-c\frac{p}{q} exp(-(p+q)\sum_{i=1}^{t} \alpha_i)}{1+c exp(-(p+q)\sum_{i=1}^{t} \alpha_i)} + \epsilon_t, p > 0, q > 0$
S.R. Log-reciprocal	$X_t = exp\left(-\frac{1}{b\sum_{i=1}^{t} \alpha_i}\right) + \epsilon_t, b > 0$

Table 1: Time-series models used in case study

We coded the generalized forecasting algorithm using Visual Basic for Applications (VBA) and implemented in Microsoft Office Excel. We used Excel Solver to fit each of the five time series models to (1) demand data set to obtain prior distributions, and to (2) demand data sets extended with each of the four different leading indicators to obtain the sampling information. After updating the prior distributions with sampling information separately for each lifecycle model, we combine the resulting forecasts to obtain the final forecast. Since 12 different seasonality parameters are required to be estimated, the initial demand data set to fit the time series models can be no shorter than one year. Thus, we replicate the forecast each month between March 2007 and December 2009.

In our study, the length of the period that demand data set is extended with leading indicators varies from one to three months. We analyze the results in four separate cases, which correspond to 3, 6, 9, and 12 months ahead forecast performances, respectively. In the following section, we present the results of the case study. More specifically, we illustrate the improvement in accuracy that is obtained at each step throughout the forecasting framework.

5.3 Results

This section includes the results of our case study, in which we forecast future U.S. demand for Xbox 360 every month, going back up to thirty months. Our forecasting framework reduces the forecast variance at every intermediate step through updates and combinations of information. Consequently, an improvement in forecast accuracy is expected. Features that improve the accuracy of the forecast can be listed as: 1) Time series model that fits well to highly seasonal demand process, 2) Advance information provided by leading indicators through a Bayesian update, 3) Combining intelligence of different time series that model the underlying demand process, and 4) Updating the model over time (month-to-month) as new demand data materialize.

After each improvement step undertaken by our forecasting framework, we record and compare the interim forecast with the actual demand data and calculate the mean absolute percentage error (MAPE) using the formula:

$$MAPE_t = \frac{|\hat{D}_t - D_t|}{D_t}$$

where $\hat{D}_t$ and D_t are estimated and actual demand values at time t, respectively. Then, we calculate the average MAPE throughout the period of the demand lifecycle that we generate forecasts ($avg\ MAPE = \sum_{t=1}^{T} MAPE_t$, March 2007 being $t = 1$ and December 2009 being T).

In Table 2, after each step of the algorithm, for each lifecycle model, we present the average MAPE, and the improvement in accuracy with respect to the previous step. This table represents the case that actual demand data set is extended for two months by leading indicators. Data for Simple model refers to the forecast results obtained by using the corresponding time series model without any seasonal adjustment. Prior forecast represents the lifecycle projection with seasonally rescaled models, while Posterior forecast refers to the results after Bayesian update with advance

		S. Log.		Weibull		Bass		Ext. Log.		Log-rec.	
		avg	imp	avg	imp	avg	imp	avg	imp	avg	imp
3 mo.	Simple model	0.757		0.676		0.797		1.129		0.512	
	Prior forecast	0.245	67.6%	0.256	62.1%	0.226	71.6%	0.300	73.4%	0.535	-4.5%
	Posterior forecast	0.219	10.8%	0.246	4.0%	0.216	4.8%	0.252	15.9%	0.348	35.1%
	Combined forecast	0.212	3.2%	0.212	13.8%	0.212	1.7%	0.212	16.0%	0.212	39.1%
6 mo.	Simple model	0.724		0.637		0.768		1.138		0.469	
	Prior forecast	0.254	64.9%	0.254	60.2%	0.231	69.9%	0.277	75.6%	0.454	3.2%
	Posterior forecast	0.238	6.4%	0.248	2.3%	0.227	2.0%	0.265	4.6%	0.369	18.8%
	Combined forecast	0.216	9.2%	0.216	13.0%	0.216	4.8%	0.216	18.5%	0.216	41.5%
9 mo.	Simple model	0.661		0.585		0.716		1.092		0.456	
	Prior forecast	0.264	60.0%	0.246	58.0%	0.234	67.3%	0.265	75.8%	0.410	10.2%
	Posterior forecast	0.249	5.8%	0.242	1.4%	0.231	1.6%	0.255	3.8%	0.365	11.0%
	Combined forecast	0.210	15.8%	0.210	13.5%	0.210	9.1%	0.210	17.7%	0.210	42.5%
12 mo.	Simple model	0.616		0.550		0.681		1.026		0.455	
	Prior forecast	0.277	55.1%	0.243	55.9%	0.231	66.2%	0.266	74.1%	0.375	17.7%
	Posterior forecast	0.259	6.3%	0.240	1.2%	0.226	1.8%	0.242	8.9%	0.345	7.8%
	Combined forecast	0.202	22.2%	0.202	15.9%	0.202	10.9%	0.202	16.8%	0.202	41.6%

Table 2: Improvement throughout forecasting with 2 month leading ind. extension

information. Last row of data is obtained after forecasts associated with all five time series models are combined to improve intelligence.

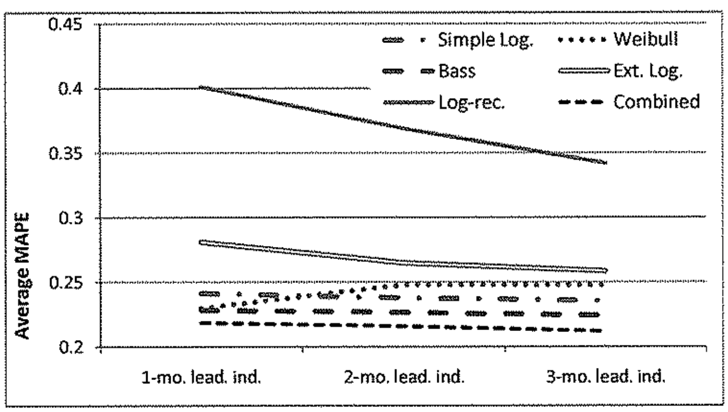

Figure 10: Average MAPE for Posterior and Combined forecasts

Forecast results for the remaining cases can be found in the Appendix (Table 6). We can see that the framework perpetually increases the forecast accuracy, which is an expected result of the forecast variance reduction. Next, we illustrate the average error for individual lifecycle model forecasts as well as the combined forecast (Figure 10). This figure represents the case that we measure forecast error 6 months into the future. However, the results are similar for 3, 9, and 12-month cases. We can see that the combination of all posterior forecasts consistently outperforms each individual posterior forecast,and provides the most accurate estimates independent of the market information usage level.

Notice that our framework also performs another information update over time, by revising the time series models' fits (and parameters), taking newly materialized demand observations into account. The improvement in accuracy provided by this update is not represented in previous results.

	Prior not updated					Prior updated over time				
avg MAPE	Slog	Wei	Bass	Elog	Log-R	Slog	Wei	Bass	Elog	Log-R
3-mo.	0.459	0.526	0.307	0.357	0.554	0.245	0.256	0.226	0.300	0.535
6-mo.	0.433	0.497	0.294	0.354	0.504	0.254	0.254	0.231	0.277	0.454
9-mo.	0.435	0.497	0.294	0.350	0.491	0.264	0.246	0.234	0.265	0.410
12-mo.	0.438	0.496	0.291	0.339	0.491	0.277	0.243	0.231	0.266	0.375

Table 3: Forecast error comparison for prior forecast with and without update over time

To assess the magnitude of this improvement, at every period throughout the demand lifecycle, we use the same model parameters (estimated in the first forecast run) to generate the prior forecast. Then, we calculate the average MAPE (for 3 to 12 months ahead) and compare the results with the updated (over time as demand materializes) prior forecasts' average MAPE. We can see in Table 3 that updating the lifecycle model as new information is obtained leads to significant improvement in accuracy.

The final measure we want to mention is the accuracy of the forecast at peak demand points. This is an important measure because the magnitude of a demand peak is a crucial knowledge. At these stages of the lifecycle, both losses and gains are amplified. At a peak, lost demand in case of a shortage could permanently decrease the market share. The timing of the demand peaks are pretty well known since the seasonality effects are consistently in play in high-tech consumer markets. On the other hand, the magnitude of the demand surge is not always easy to predict.

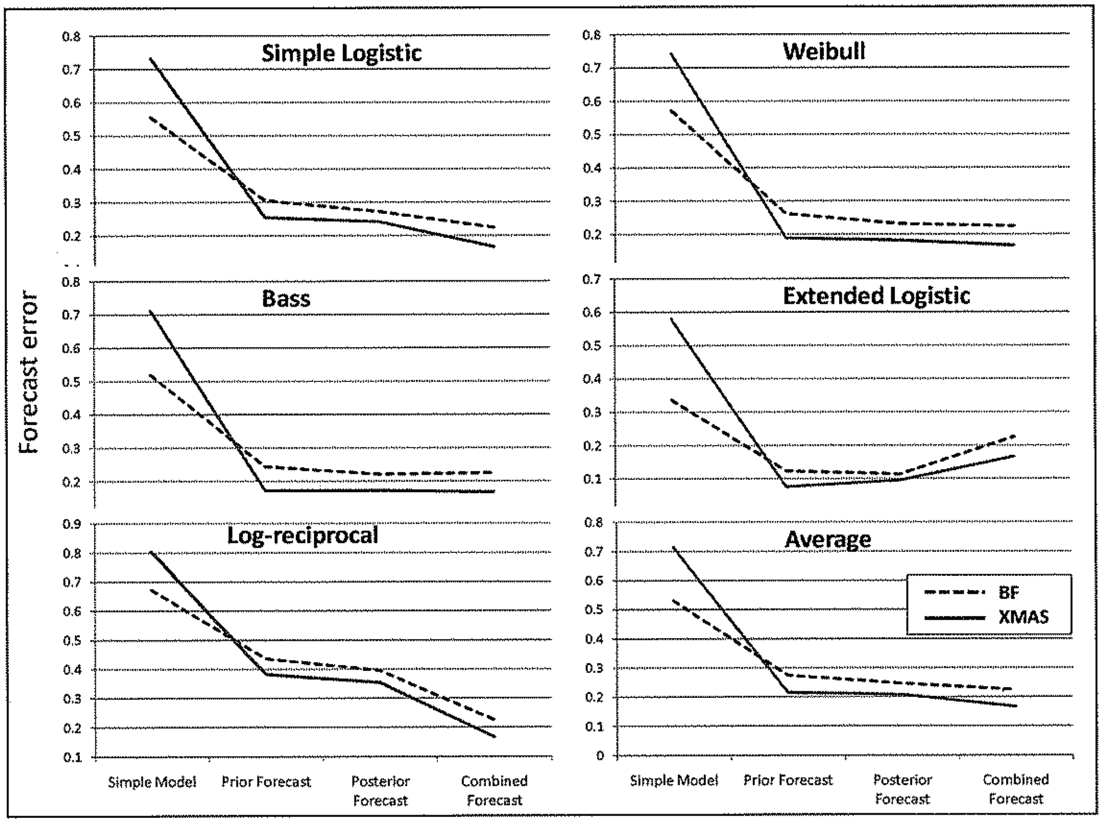

Figure 11: Forecast accuracy at peak demand

We recognize that there are two significant demand peaks for this product in any given year. These are Black-friday (BF), and Christmas (XMAS) seasons with corresponding demand surges in November, and December, respectively. We aim to illustrate how accurate our framework is in forecasting the demand magnitude during these peaks. To do this, for each lifecycle model, we calculate the accuracy at different forecast intelligence levels. In Figure 11, we present the average error for November and December demand forecast. On average and for each individual lifecycle model, the forecast error of our method is drastically lower then a simple method's. In general, the accuracy perpetually improves as the amount of information incorporated into the forecast gradually increases.

6 Conclusions

In this chapter, we introduced a general forecasting framework that provides accurate forecasts of future demands in highly volatile markets. Our framework contributes to the technological forecasting literature by generalizing the demand models, leading indicator information sources, and the information processing methods that can be used to generate lifecycle demand estimates.

Previously, leading indicators were very narrowly defined to consist of demand patterns of past products. We extend this definition to include a vast variety of dynamically evolving, publicly or privately available indicative market information with consistent and verifiable connections to a product's demand. After these information sources are systematically processed and filtered for relevant data, they are used to reduce the forecast variance.

We also generalize the models that can be used to characterize the lifecycle demand process. Using our framework, a forecaster can utilize any time series model to reduce possible future demand scenarios. We establish necessary theoretical foundations implying that reductions in forecast variance is guaranteed when unbiased leading indicator information is used to revise prior estimations, as well as when several projections are combined to improve intelligence. A case study is presented to illustrate the perpetual improvement in forecast quality when our framework is used.

In reality, judgment calls are a significant part of the forecasting process, especially when demand is very volatile and regular forecasting methods perform poorly. In addition to improved accuracy, our framework standardizes the forecasting methodology; making the process systematic, repeatable, practical, less time and energy consuming.

We expect that the improvements in forecast accuracy should translate to increased revenues by means of greater on-time deliveries and decreased operational costs through more efficient planning and utilization of expensive resources and inventories. We believe that using this framework within an integrated demand-supply model is a promising direction of research, in which, potentially more direct relationships to operational and cost measures can be identified.

Another research direction that would add value to our framework is systematical assessment of the strength of different information sources. Planners would benefit greatly from knowledge on how good or noisy various information sources are, and how differently these sources should be filtered and unbiased. In addition, implementation of this framework for all information sources that we presented would be useful in the assessment process. Finally, testing for different type of products in other industries, using diverse kinds of time series models and information sources is an interesting application direction.

References

Aytac, B., S. D. Wu. 2008. Characterization of demand for short-lifecycle technology products. Tech. rep. ISE Dept. Lehigh University, Bethlehem, PA.

Bass, F. M. 1969. A new product growth model for consumer durables. *Management Science* **15**(5) 215–227.

Bass, F. M. 1980. The relationship between diffusion rates, experience curves, and demand elasticities for consumer durable technological innovations. *Journal of Business* **53**(3) S51–S67.

Bass, F. M., K. Gordon, T. L. Ferguson, M. L. Githens. 2001. DIRECTV: Forecasting diffusion of a new technology prior to product launch. *Interfaces* **31**(3) S82–S93.

Bates, J. M., W. J. Granger. 1969. The combination of forecasts. *Operational Research Quarterly* **20**(4) 451–468.

Dickinson, J. P. 1972. The combination of short term forecasts. *Proceedings of University of Lancaster Forecasting Conference*.

Krishnan, T. V., F. M. Bass, D. C. Jain. 1999. Optimal pricing strategy for new products. *Management Science* **45**(12) 1650–1663.

Kumar, U., V. Kumar. 1992. Technological innovation diffusion: The proliferation of substitution models and easing the user's dilemma. *IEEE T. Eng. Manage.* **39**(2) 158–168.

Kurawarwala, A. A., H. Matsuo. 1996. Forecasting and Inventory Management of Short Life-cycle Products. *Operations Research* **44**(1) 131–150.

Lee, P. M. 1989. *Bayesian statistics: an introduction.* Oxford.

Mahajan, V., E. Muller. 1979. Innovation diffusion and new product growth models in marketing. *Journal of Marketing* **43** 55–68.

Mahajan, V., E. Muller, F. M. Bass. 2001. New-product diffusion models: A review and directions for research. *Journal of Marketing* **54** 1–26.

Makridakis, S., R. L. Winkler. 1983. Averages of forecasts: some empirical results. *Management Science* **29**(9) 987–996.

Meade, N., T. Islam. 1998. Technological forecasting – Model selection, model stability, and combining models. *Management Science* **44**(8) 1115–1130.

Meixell, M. J., S. D. Wu. 2001. Scenario analysis of demand in a technology market using leading indicators. *IEEE Transactions on Semiconductor Manufacturing* **14**(1) 65–75.

Modis, T., A. Debecker. 1988. Innovation in the computer industry. *Technological Forecasting and Social Change* **33** 267–278.

Norton, J. A., F. M. Bass. 1987. A Diffusion Theory Model of Adoption and Substitution for Successive Generations of High-Technology Products. *Management Science* **33**(9) 1069–1086.

Oliver, R. M. 1987. A Bayesian Model to Predict Saturation and Logistic Growth. *Journal of the Operational Research Society* **38**(1) 49–56.

Sultan, F., J. U. Farley, D. R. Lehmann. 1990. A meta-analysis of applications of diffusion models. *Journal of Marketing Research* **27** 70–77.

Timmermann, A. 2005. *Handbook of Economic Forecasting*, chap. Forecast Combinations. North Holland.

Wu, S. D., B. Aytac, R. T. Berger, C. A. Armbruster. 2006. Managing Short-Lifecycle Technology Products for Agere Systems. *Interfaces* **36**(3) 234–247.

Zhu, K., U. W. Thonemann. 2004. An adaptive forecasting algorithm and inventory policy for products with short life cycles. *Naval Research Logistics* **51** 633–653.

Appendix

Proof. [Lemma 2]

Let us simplify the notation by referring to future demand realization $X(T+\tau)$ by X, denoting the prior distribution $\left(\hat{X}_k(T+\tau|\Theta_T), \sigma_k^2\right)$ by (μ_0, φ_0), and likelihood variance $\bar{\sigma}_k^2$ by φ.

Suppose that your prior beliefs on future demand realization X can be expressed in terms of a normal distribution:

$$X \sim N(\mu_0, \varphi_0)$$

and suppose also that you have a likelihood, distributed normally with mean X equal to the variable of interest:

$$Y \sim N(X, \varphi).$$

Then,

$$p(X) = (2\pi\varphi_0)^{-1/2} \, exp\left\{-(X-\mu_0)^2/2\varphi_0\right\}$$
$$p(Y|X) = (2\pi\varphi)^{-1/2} \, exp\left\{-(Y-X)^2/2\varphi\right\}$$

and hence

$$\begin{aligned} p(X|Y) &\propto p(X)\,p(Y|X) \\ &= (2\pi\varphi_0)^{-1/2} \, exp\left\{-(X-\mu_0)^2/2\varphi_0\right\} \text{ x } (2\pi\varphi)^{-1/2} \, exp\left\{-(Y-X)^2/2\varphi\right\} \\ &\propto exp\left\{(-1/2)\,X^2\left(\varphi_0^{-1}+\varphi^{-1}\right) + X\left(\mu_0/\varphi_0 + Y/\varphi\right)\right\} \end{aligned}$$

Then, we can write

$$p(X|Y) \propto exp\left\{(-1/2)\, X^2/\varphi_1 + X\mu_1/\varphi_1\right\}$$

where

$$\varphi_1 = \frac{1}{\varphi_0^{-1}+\varphi^{-1}} \text{ and } \mu_1 = \varphi_1\left(\mu_0/\varphi_0 + Y/\varphi\right)$$

After adding constant $(-1/2)\,\mu_1^2/\varphi_1$ into the exponent, it follows that:

$$p(X|Y) = (2\pi\varphi_1)^{-1/2}\, exp\left\{-\left(X-\mu_1\right)^2/2\varphi_1\right\},$$

which means the posterior density is

$$N\left(\mu_1^2, \varphi_1\right)$$

which translates to posterior variance $\sigma_k'^2 = \frac{\sigma_k^2\bar{\sigma}_k^2}{\sigma_k^2+\bar{\sigma}_k^2}$ and posterior mean that is weighted mean of prior and likelihood data mean, weights being inversely proportional to their respective variances.

□

	2005	2006	2007	2008	2009
Jan		250,000	294,000	230,000	309,000
Feb		161,000	228,000	255,000	391,000
Mar		192,000	199,000	262,000	330,000
Apr		295,000	174,000	188,000	175,000
May		221,000	155,000	187,000	175,000
Jun		277,000	198,000	220,000	241,000
Jul		206,000	170,000	205,000	203,000
Aug		205,000	277,000	195,000	215,000
Sep		259,000	528,000	347,000	353,000
Oct		218,000	366,000	371,000	250,000
Nov	326,000	511,000	770,000	836,000	820,000
Dec	281,000	1,130,000	1,260,000	1,440,000	1,310,000

Table 4: U.S. Xbox 360 unit sales provided by NPD Group

Month	Expert 1	Expert 2	Month	Expert 1	Expert 2
Mar-07	228000	250000	Aug-08	201000	200000
Apr-07	194800	175000	Sep-08	350000	300000
May-07	174000	225000	Oct-08	333000	325000
Jun-07	191000	200000	Nov-08	859200	800000
Jul-07	137000	115000	Dec-08	1420000	1350000
Aug-07	255000	240000	Jan-09	299000	360000
Sep-07	452600	450000	Feb-09	302500	310000
Oct-07	412000	325000	Mar-09	413100	400000
Nov-07	728000	750000	Apr-09	210000	275000
Dec-07	1461000	1450000	May-09	160000	140000
Jan-08	288000	250000	Jun-09	230000	225000
Feb-08	250000	250000	Jul-09	200000	190000
Mar-08	296000	310000	Aug-09	190000	205000
Apr-08	335000	280000	Sep-09	370000	350000
May-08	216000	250000	Oct-09	280000	340000
Jun-08	248700	235000	Nov-09	700000	650000
Jul-08	325000	300000	Dec-09	1490000	1350000

Table 5: One month ahead experts' forecasts for Xbox 360

1-month Lead. Ind.		S. Log.		Weibull		Bass		Ext. Log.		Log-rec.	
		avg	imp	avg	imp	avg	imp	avg	imp	avg	imp
3 mo.	Simple model	0.757		0.676		0.797		1.129		0.512	
	Prior forecast	0.245	67.6%	0.256	62.1%	0.226	71.6%	0.300	73.4%	0.535	-4.5%
	Posterior forecast	0.224	8.8%	0.220	14.0%	0.218	3.7%	0.274	8.6%	0.420	21.6%
	Combined forecast	0.213	4.6%	0.213	3.1%	0.213	2.1%	0.213	22.2%	0.213	49.2%
6 mo.	Simple model	0.724		0.637		0.768		1.138		0.469	
	Prior forecast	0.254	64.9%	0.254	60.2%	0.231	69.9%	0.277	75.6%	0.454	3.2%
	Posterior forecast	0.241	5.1%	0.230	9.5%	0.228	1.4%	0.281	-1.3%	0.401	11.7%
	Combined forecast	0.218	9.4%	0.218	4.8%	0.218	4.2%	0.218	22.2%	0.218	45.5%
9 mo.	Simple model	0.661		0.585		0.716		1.092		0.456	
	Prior forecast	0.264	60.0%	0.246	58.0%	0.234	67.3%	0.265	75.8%	0.410	10.2%
	Posterior forecast	0.253	4.3%	0.231	6.0%	0.232	0.9%	0.280	-5.6%	0.380	7.1%
	Combined forecast	0.213	16.0%	0.213	8.0%	0.213	8.5%	0.213	24.0%	0.213	44.1%
12 mo.	Simple model	0.616		0.550		0.681		1.026		0.455	
	Prior forecast	0.277	55.1%	0.243	55.9%	0.231	66.2%	0.266	74.1%	0.375	17.7%
	Posterior forecast	0.265	4.1%	0.231	4.6%	0.229	0.6%	0.273	-2.7%	0.354	5.5%
	Combined forecast	0.204	23.3%	0.204	12.0%	0.204	11.2%	0.204	25.5%	0.204	42.5%

3-month Lead. Ind.		S. Log.		Weibull		Bass		Ext. Log.		Log-rec.	
		avg	imp	avg	imp	avg	imp	avg	imp	avg	imp
3 mo.	Simple model	0.757		0.676		0.797		1.129		0.512	
	Prior forecast	0.245	67.6%	0.256	62.1%	0.226	71.6%	0.300	73.4%	0.535	-4.5%
	Posterior forecast	0.217	11.6%	0.244	4.7%	0.214	5.6%	0.236	21.3%	0.280	47.7%
	Combined forecast	0.210	3.5%	0.210	14.0%	0.210	1.9%	0.210	11.3%	0.210	25.2%
6 mo.	Simple model	0.724		0.637		0.768		1.138		0.469	
	Prior forecast	0.254	64.9%	0.254	60.2%	0.231	69.9%	0.277	75.6%	0.454	3.2%
	Posterior forecast	0.235	7.3%	0.248	2.3%	0.224	3.1%	0.259	6.8%	0.342	24.8%
	Combined forecast	0.212	10.0%	0.212	14.5%	0.212	5.5%	0.212	18.0%	0.212	38.0%
9 mo.	Simple model	0.661		0.585		0.716		1.092		0.456	
	Prior forecast	0.264	60.0%	0.246	58.0%	0.234	67.3%	0.265	75.8%	0.410	10.2%
	Posterior forecast	0.245	7.3%	0.243	1.3%	0.230	2.0%	0.254	4.0%	0.354	13.5%
	Combined forecast	0.207	15.6%	0.207	14.8%	0.207	9.9%	0.207	18.6%	0.207	41.6%
12 mo.	Simple model	0.616		0.550		0.681		1.026		0.455	
	Prior forecast	0.277	55.1%	0.243	55.9%	0.231	66.2%	0.266	74.1%	0.375	17.7%
	Posterior forecast	0.251	9.1%	0.238	2.0%	0.225	2.5%	0.238	10.4%	0.342	8.7%
	Combined forecast	0.199	21.0%	0.199	16.4%	0.199	11.6%	0.199	16.7%	0.199	41.9%

Table 6: Improvement throughout forecasting with 1 and 3 month leading indicator extension

2006								2007							
week	frq	week	frq	week	frq	week	frq	week	frq	week	frq	week	frq	week	frq
Jan 1	1.5	Apr 2	0.9	Jul 2	0.9	Oct 1	0.85	Jan 7	1.4	Apr 1	1.35	Jul 1	1.4	Oct 7	1.25
Jan 8	1.05	Apr 9	0.85	Jul 9	0.8	Oct 8	0.9	Jan 14	1.25	Apr 8	1.2	Jul 8	1.4	Oct 14	1.2
Jan 15	1	Apr 16	0.8	Jul 16	0.85	Oct 15	0.95	Jan 21	1.15	Apr 15	1.1	Jul 15	1.3	Oct 21	1.2
Jan 22	0.9	Apr 23	0.75	Jul 23	0.8	Oct 22	0.95	Jan 28	1.05	Apr 22	1.05	Jul 22	1.2	Oct 28	1.35
Jan 29	0.9	Apr 30	0.7	Jul 30	0.8	Oct 29	1.1	Feb 4	1.05	Apr 29	1.1	Jul 29	1.15	Nov 4	1.45
Feb 5	0.8	May 7	0.85	Aug 6	0.8	Nov 5	1.3	Feb 11	1.05	May 6	1.05	Aug 5	1.2	Nov 11	1.6
Feb 12	0.8	May 14	0.85	Aug 13	0.8	Nov 12	1.7	Feb 18	1.15	May 13	1.05	Aug 12	1.25	Nov 18	2.4
Feb 19	0.85	May 21	0.8	Aug 20	0.9	Nov 19	2.65	Feb 25	1	May 20	1.05	Aug 19	1.25	Nov 25	1.95
Feb 26	0.8	May 28	0.85	Aug 27	0.8	Nov 26	1.8	Mar 4	1	May 27	1.15	Aug 26	1.2	Dec 2	1.95
Mar 5	0.8	Jun 4	0.8	Sep 3	0.8	Dec 3	1.55	Mar 11	0.95	Jun 3	1.1	Sep 2	1.2	Dec 9	1.8
Mar 12	0.8	Jun 11	0.8	Sep 10	0.75	Dec 10	1.6	Mar 18	1.05	Jun 10	1.1	Sep 9	1.15	Dec 16	1.85
Mar 19	0.85	Jun 18	0.8	Sep 17	0.8	Dec 17	1.7	Mar 25	1.4	Jun 17	1.2	Sep 16	1.2	Dec 23	3.1
Mar 26	0.85	Jun 25	0.8	Sep 24	0.85	Dec 24	2.8			Jun 24	1.2	Sep 23	1.65	Dec 30	2.2
						Dec 31	1.75					Sep 30	1.4		
2008								2009							
week	frq	week	frq	week	frq	week	frq	week	frq	week	frq	week	frq	week	frq
Jan 6	1.45	Apr 6	0.9	Jul 6	1.15	Oct 5	1.05	Jan 4	1.7	Apr 5	1.25	Jul 5	1.4	Oct 4	1.2
Jan 13	1.35	Apr 13	0.9	Jul 13	1.3	Oct 12	1.15	Jan 11	1.5	Apr 12	1.2	Jul 12	1.4	Oct 11	1.25
Jan 20	1.3	Apr 20	0.9	Jul 20	1.15	Oct 19	1.2	Jan 18	1.45	Apr 19	1.1	Jul 19	1.4	Oct 18	1.25
Jan 27	1.2	Apr 27	1.2	Jul 27	1.1	Oct 26	1.25	Jan 25	1.4	Apr 26	1.05	Jul 26	1.45	Oct 25	1.25
Feb 3	1.15	May 4	1.1	Aug 3	1.15	Nov 2	1.35	Feb 1	1.3	May 3	1.1	Aug 2	1.45	Nov 1	1.5
Feb 10	1.15	May 11	1.05	Aug 10	1.15	Nov 9	1.5	Feb 8	1.25	May 10	1.1	Aug 9	1.5	Nov 8	1.85
Feb 17	1.25	May 18	1	Aug 17	1.1	Nov 16	1.85	Feb 15	1.4	May 17	1.15	Aug 16	1.45	Nov 15	1.65
Feb 24	1.15	May 25	1.05	Aug 24	1.05	Nov 23	2.4	Feb 22	1.3	May 24	1.3	Aug 23	1.4	Nov 22	2.25
Mar 2	1.1	Jun 1	1	Aug 31	1.2	Nov 30	1.95	Mar 1	1.35	May 31	1.5	Aug 30	1.35	Nov 29	1.8
Mar 9	1.05	Jun 8	1.1	Sep 7	1.1	Dec 7	1.75	Mar 8	1.25	Jun 7	1.4	Sep 6	1.3	Dec 6	1.7
Mar 16	1.05	Jun 15	1.1	Sep 14	1.1	Dec 14	1.9	Mar 15	1.3	Jun 14	1.55	Sep 13	1.25	Dec 13	1.8
Mar 23	1.1	Jun 22	1.1	Sep 21	1.05	Dec 21	2.95	Mar 22	1.25	Jun 21	1.4	Sep 20	1.3	Dec 20	2.55
Mar 30	1.05	Jun 29	1.15	Sep 28	1.15	Dec 28	2.65	Mar 29	1.15	Jun 28	1.4	Sep 27	1.2	Dec 27	2.45

Table 7: Google Trends weekly search frequency data