

# AN INNOVATIVE RTP-BASED RESIDENTIAL POWER SCHEDULING SCHEME FOR SMART GRIDS

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## ABSTRACT

This paper proposes a Real-Time Pricing (RTP)-based power scheduling scheme as demand response for residential power usage. In this scheme, the Energy Management Controller (EMC) in each home and the service provider form a Stackelberg game, in which the EMC who schedules appliances' operation plays the follower level game, and the provider who sets the real-time prices according to current power usage profile plays the leader level game. The sequential equilibrium is obtained through the information exchange between them. Simulation results indicate that our scheme can not only save money for consumers, but also reduce peak load and the variance between demand and supply, while avoiding the "rebound" peak problem.

*Index Terms*— Smart grids, Demand response, Real-time price, Day-ahead price, Stackelberg game.

## 1. INTRODUCTION

Matching of supply and demand has always been a central challenge in designing and operating electricity networks. Traditional approaches of building enough generation and transmission capacity to meet peak load has resulted in substantial infrastructure that is idle for all but a few hours a year. Moreover, such peaking capacity tends to come from the grid's oldest and most polluting assets. To deal with steadily increasing demand in the future, electricity service providers will rely on demand response (DR) programs to encourage users to shift their loads away from peak times [1].

Real-time pricing (RTP) is considered as a very direct and efficient approach for DR [2]. With RTP, the service provider announces electricity prices on a rolling basis, i.e., the price for a given time period (e.g., an hour) is determined and announced before the start of the period (e.g., 15 minutes beforehand). With the development of smart metering technologies [1], which will enable secure, reliable, real-time, and two-way information exchange between consumers and their electricity service providers, these RTPs can be provided to consumers multiple times a day, hour, or even second. To handle the resulting data volume and decision making velocity, consumers will rely on energy management controllers

(EMCs) [3], which are devices or programs that use electricity prices and user preferences to modify power usage across a home or building. From the service provider's perspective, providing high frequency pricing updates to EMCs will enable better load shaping and thus better matching of volatile supply and demand. From the consumer's perspective, RTP will provide new opportunities to lower rates, provided that they (i.e., EMCs) make smart usage decisions.

In this paper, we propose a smart RTP-based power scheduling scheme for residential power usage using a Stackelberg game model. In this model, the provider plays the leader level game by setting the real-time price and the consumer's EMC, which schedules appliances in a home, plays the follower level game. The sequential equilibrium is obtained through a two-way information exchange enabled by some underlying communication infrastructure (e.g., the smart metering network). Results show that our scheme can alleviate peak load and reduce the variance between the actual demand and planned supply, which implies substantial cost and stability benefits for the service provider. At the same time, we show that our approach can enable benefits for consumers as well through reduced electricity bills.

The results and analysis in this paper differ from the related work in several aspects. In [4], consumers (i.e., EMCs) make decisions on their hourly aggregate power consumption using current RTPs. Since individual devices have start and end times that can be deterministic or random, or they may cycle on and off, hourly aggregate consumption values may not be easily mapped to how each device or appliance is operated. By contrast, in our scheme, EMCs schedule power consumption on an appliance-by-appliance basis. In [5, 6], the RTP scheme assumes that the power consumption of each appliance can be modified to optimize power allocation during each hour. This assumption is suitable for a few devices (e.g., electric vehicle batteries), but most end-use devices do not offer that level of flexibility. On the other hand, several devices do offer flexibility regarding when they are operated, and it is this "schedulability" of appliances that we exploit to demonstrate the power of RTPs and EMCs to collectively match demand with supply.

The remainder of the paper is organized as follows. Section II describes the system model. Section III formulates

and analyzes the proposed Stackelberg game. Section IV proposes algorithms for our RTP-based power scheduling scheme. Section V presents simulation results and we conclude our discussion in Section VI.

## 2. SYSTEM MODEL

We consider a residential power system which consists of a service provider and several consumers in a neighborhood, as shown in Fig. 1. The service provider buys electricity from the wholesale market and sells it to consumers. The EMC in each home interacts with the service provider through an underlying two-way communication network (e.g., the smart metering infrastructure). The EMC coordinates power use among in-home smart appliances; of particular interest in our model, it schedules the time of use of *schedulable* appliances within the home.

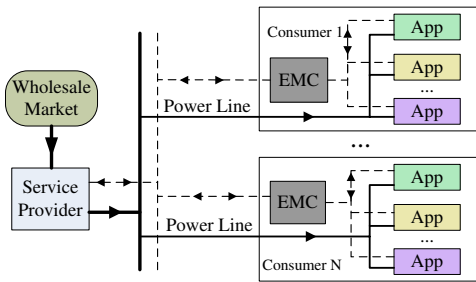


Fig. 1. The model of the residential power system

Time is divided into slots for scheduling and RTP updates. Let  $\mathcal{T}$  denote the set of time slots in a given time horizon, and  $\mathcal{N}$  denote the set of consumers, where  $T \triangleq |\mathcal{T}|$ ,  $N \triangleq |\mathcal{N}|$ . For each consumer  $n \in \mathcal{N}$ , let  $\mathcal{A}_n$  denote the set of schedulable appliances in the home. For appliance  $a \in \mathcal{A}_n$ , the operation duration is denoted by  $l_{n,a}$ , which will be set at the time of request, and the power usage for this duration is  $c_{n,a}$  kW.

## 3. STACKELBERG GAME MODEL ANALYSIS

We employ a Stackelberg game which is divided into two levels: the service provider plays the *leader level* game and EMCs play the *follower level* game.

### 3.1. EMC/Follower Level Decisions

The EMC aims to minimize the cost to the consumer for an appliance's usage. Its action is to determine the optimal start time  $s$  for a schedulable appliance that was requested to turn on at time slot  $t_0$ . Delaying the appliance to a cheaper price period will save money, but this delay itself also incurs an inconvenience cost. We assume each time slot of delay for appliance  $a \in \mathcal{A}_n$  implies a cost of  $\psi_{n,a}$  dollars. We assume the consumer specifies a maximum allowable delay of  $d_{n,a}$  time slots and the EMC is required to schedule all appliances within the horizon  $\mathcal{T}$ . Given the price vector  $\mathbf{\Pi} = \{\pi_1, \pi_2, \dots, \pi_T\}$  for the time horizon, the optimal scheduled

start time  $s^*$  is obtained by solving the following optimization:

$$\begin{aligned} \min_s \quad & (s - t_0)\psi_{n,a} + \sum_{r=s}^{s+l_{n,a}} \pi_r c_{n,a} \\ \text{s.t.} \quad & t_0 \leq s \leq t_0 + d_n, \quad s + l_{n,a} \leq T. \end{aligned} \quad (1)$$

The minus of the objective function above thus forms the utility function for the EMC.

### 3.2. Service Provider/Leader Level Decisions

The service provider sets the retail price  $\pi_t$ , which is the sum of the wholesale price  $\phi_t$  and the price gap  $\epsilon_t$ . In our model, the wholesale price affects EMCs' scheduling so that peak load is reduced while the price gap enables the actual load to be more close to the planned supply. The wholesale price  $\phi_t$  is defined as  $\phi_t = C_t(q_t)/q_t$ , where  $q_t$  in kW is the planned supply load for time slot  $t$  and forms the vector  $\mathbf{Q} = [q_1, q_2, \dots, q_T]$  for the time horizon  $\mathcal{T}$ .  $C_t(q_t)$  is the *cost function* [6], which we assume is an increasing and strictly convex function of  $q_t$ ; thus  $\phi_t$  is higher during high load periods than during low load periods. We can choose the cost function as  $C_t(q_t) = \alpha q_t^2$ , where the coefficient  $\alpha$  converts the cost to a monetary value. In this case,  $\phi_t = C_t(q_t)/q_t = \alpha q_t$ .

The price gap  $\epsilon_t$  is designed to influence the difference between the actual demand and the available supply. The service provider maintains a real-time load vector  $\mathbf{Z} = [z_1, z_2, \dots, z_T]$  in kW which tracks the aggregated load for the time horizon  $\mathcal{T}$ . We design  $\epsilon_t$  such that it is proportional to a function  $g(q_t, z_t, w)$ , which decreases with  $\delta_t = q_t - z_t$ , i.e., the larger  $\delta_t$ , the lower price gap  $\epsilon_t$  so that the EMC is more willing to schedule the appliance to operate during this period, and vice versa. In this paper, we adopt  $g(q_t, z_t, w)$  as<sup>1</sup>:

$$\epsilon_t \propto g(q_t, z_t, w) = \begin{cases} \frac{1}{(q_t - z_t)^w}, & \text{if } q_t > z_t \\ (z_t - q_t)^w, & \text{if } q_t < z_t \\ 1, & \text{if } q_t = z_t; \end{cases} \quad (2)$$

where  $w$  is the incentive factor.

We assume a constraint for the price gap which can be seen as the effect of either market competition or price caps set by some regulatory body. For an appliance's request at  $t_0$ , this constraint is  $\sum_{t=t_0}^T \epsilon_t = M_{t_0} = \sum_{t=t_0}^T \epsilon_0$ , where  $\epsilon_0$  is the comparable constant price gap in some (alternate) fixed rate pricing scheme. The price gap  $\epsilon_t$  is expressed in terms of  $g(q_t, z_t, w)$  as:

$$\epsilon_t = \frac{g(q_t, z_t, w)}{\sum_{t=t_0}^T g(q_t, z_t, w)} M_{t_0}. \quad (3)$$

The retail price  $\pi_t$  is then:

$$\pi_t = \phi_t + \epsilon_t = \alpha q_t + \frac{g(q_t, z_t, w)}{\sum_{t=t_0}^T g(q_t, z_t, w)} M_{t_0}. \quad (4)$$

<sup>1</sup>We round  $q_t$  and  $z_t$  to integer numbers in kW.

Given the scheduled start time  $s$ ,  $c_{n,a}$  and  $l_{n,a}$ , the service provider forms a power consumption vector  $\mathbf{P}_{n,a} = [p_{n,a}^1, p_{n,a}^2, \dots, p_{n,a}^t, \dots, p_{n,a}^T]$  in kW for this appliance, where

$$p_{n,a}^t = \begin{cases} c_{n,a}, & t \in [s, s + l_{n,a}) \\ 0, & t \in \mathcal{T} \setminus [s, s + l_{n,a}). \end{cases} \quad (5)$$

The real-time load vector is then updated as

$$\mathbf{Z}' = \mathbf{Z} + \mathbf{P}_{n,a} = [z'_1, z'_2, \dots, z'_T]. \quad (6)$$

The utility function of the service provider is defined as the gross profit,  $GP$ , which is  $\sum_{t=1}^T \pi_t \cdot p_{n,a}^t$ , minus the cost of this usage to the provider. This cost has two parts: One cost  $C_e$  comes from purchasing electricity for this appliance usage from the wholesale market, i.e.,  $C_e = \sum_{t=1}^T \phi_t \cdot p_{n,a}^t$ ; the other cost  $C_m$  is due to the ‘‘mismatch’’ between the actual load and planned supply caused by this appliance, i.e.,  $C_m = \beta \left[ \sum_{t=1}^T (q_t - z'_t)^2 - \sum_{t=1}^T (q_t - z_t)^2 \right]$ , where  $\beta$  is a coefficient that converts the cost to a monetary value.

The service provider will maximize its utility by choosing incentive factor<sup>2</sup>  $w$ ; this optimization problem can be written as

$$\max_w \sum_{t=1}^T \epsilon_t p_{n,a}^t - \beta \left[ \sum_{t=1}^T (q_t - z'_t)^2 - \sum_{t=1}^T (q_t - z_t)^2 \right]. \quad (7)$$

### 3.3. Equilibrium of the Stackelberg Game

For each  $w$  chosen by the provider, there is a corresponding retail price vector  $\mathbf{\Pi}^w$ . Using these price vectors, the EMC can determine an optimal start time  $s^w$  for the appliance under consideration. The sequential equilibrium for this game is the set of values  $(w^*, s^*)$  such that the optimal start time  $s^*$  corresponding to the price vector  $\mathbf{\Pi}^{w^*}$  maximizes the provider’s utility function, i.e. implying that  $w^*$  is optimal.

Using backward induction [7], the sequential equilibrium can be solved analytically. On the other hand, we can simplify the optimization problem in (7) by discretizing the set of feasible values of  $w$  as  $\mathcal{W} = \{w_1, w_2, \dots, w_W\}$ , where  $W \triangleq |\mathcal{W}|$ . Then the optimization problem in (7) can be rewritten as

$$\max_{w \in \mathcal{W}} \sum_{t=1}^T \epsilon_t p_{n,a}^t - \beta \left[ \sum_{t=1}^T (q_t - z'_t)^2 - \sum_{t=1}^T (q_t - z_t)^2 \right]. \quad (8)$$

In this simplified model, the sequential equilibrium  $(s^*, \hat{w}^*) \in \mathcal{S} \times \mathcal{W}$  can be achieved through information exchange taking advantage of the two-way communication network.

## 4. RTP-BASED POWER SCHEDULING SCHEME

In a neighborhood scenario which contains  $N$  EMCs and thus  $\sum_{n=1}^N |\mathcal{A}_n|$  appliances, the service provider can use its

<sup>2</sup>There is a trade-off in selecting  $w$ : Larger  $w$  provides more incentive for shifting the current appliance’s load to a period with a larger  $\delta_t$  value (thus reduces the mismatch cost  $C_m$ ). But larger  $w$  also reduces the money charged to the consumer, and therefore the gross profit.

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### Algorithm 1 Executed by the service provider

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- 1: Initialization.
  - 2: **repeat**
  - 3:   **if** receive request signal from EMC  $n$  for app  $a$  **then**
  - 4:     **for**  $w=w_1$  to  $w_W$  **do**
  - 5:       Compute the price vector  $\mathbf{\Pi}^w$  using (4).
  - 6:       Send  $\mathbf{\Pi}^w$  to EMC  $n$ .
  - 7:       **for all** start time  $s^{*,w}$  received **do**
  - 8:         Solve (8) to find the optimal  $\hat{w}^*$ .
  - 9:       **end for**
  - 10:      Send the  $\hat{w}^*$  to EMC  $n$ , and update  $\mathbf{Z}$  as (6).
  - 11:     **end for**
  - 12:   **end if**
  - 13: **until** The end of the day
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### Algorithm 2 Executed by EMC $n$

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- 1: Initialization.
  - 2: **if** consumer  $n$  has a request for appliance  $a$  at  $t$  **then**
  - 3:   Send the request signal to service provider.
  - 4:   **for all** price vector  $\mathbf{\Pi}^w$  received **do**
  - 5:     Solve (1) to find optimal  $s^{*,w}$  for each  $w$ .
  - 6:     Send  $s^{*,w}$  to the service provider.
  - 7:   **end for**
  - 8:   **if** receive the optimal  $\hat{w}^*$  from provider **then**
  - 9:     Select  $s^* = s^{*,\hat{w}^*}$  and schedule the appliance  $a$ .
  - 10:   **end if**
  - 11: **end if**
- 

pricing scheme to influence the aggregate load from all these scheduled appliances. For example, in day-ahead pricing (where the prices for each time period in the next day are set a day in advance), each home’s EMC can schedule appliance usage to avoid high load (and thus high cost) periods. However, since local neighboring EMCs may make similar usage decisions, the aggregate load for the neighborhood may not experience peak load reduction since a ‘‘rebound’’ peak may appear during what was supposed to be a low load (i.e., low cost) period. Our scheme, presented here as set of interactive algorithms (Algorithm 1 for the provider, Algorithm 2 for the EMC<sup>3</sup>), alleviates this problem by updating the price according to the real-time load vector  $\mathbf{Z}'$ . In this way, a new appliance’s scheduled use will affect future prices and thus the scheduling decisions for future appliances.

## 5. PERFORMANCE EVALUATION

We simulate a neighborhood consisting of 80 consumers, and each consumer has 3 schedulable appliances, i.e., dishwasher, clothes dryer and clothes washer, controlled by the EMC. We assume  $l_{n,a}$  has an exponential distribution with mean of  $\bar{l}_{n,a}$ . We assume homogeneous end-use consumers, i.e.,  $c_{n,a} = c_a$ ,  $d_{n,a} = d_a$ ,  $\psi_{n,a} = \psi_a$ , and  $\bar{l}_{n,a} = \bar{l}_a$ . In this simulation, we

<sup>3</sup>Both provider and EMC can compute and receive data simultaneously.

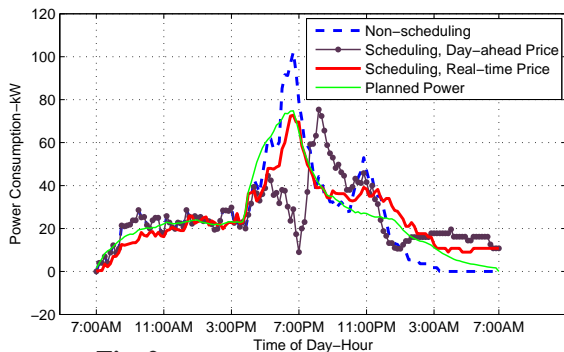
further assume a time horizon of one day, starting from 7:00 AM until the 7:00 AM the next day. Appliances have an on-peak period from 5:00 PM to 8:00 PM, during which time they are requested by consumers with higher probability. We divide time into 10 minute scheduling/pricing slots and use the appliance parameters given in Table 1.

**Table 1.** Appliances' parameters for each home

Appliance	$c_a$ (kW)	$d_a$ (hr)	$\psi_a$ (\$/hr)	$\bar{l}_a$ (hr)
Dishwasher	1.8	6.0	0.10	3.0
Clothes Dryer	3.4	4.0	0.25	1.0
Clothes Washer	0.4	2.0	0.40	0.5

### 5.1. Benefits to Service Provider

Fig. 2 compares non-scheduling aggregate demand for the neighborhood to the load using our RTP-based scheduling scheme. For reference, we also plot the planned supply curve. Without RTP-based power scheduling, the peak load is 102.2 kW at 6:50 PM, while our scheme has a 28.9% lower peak load of 72.8 kW. At the same time, our scheduled consumption curve is much closer to the planned supply curve, i.e., the deviation of the non-scheduled demand curve to the supply curve is 28.3%, which reduces to 16.1% under our scheme.



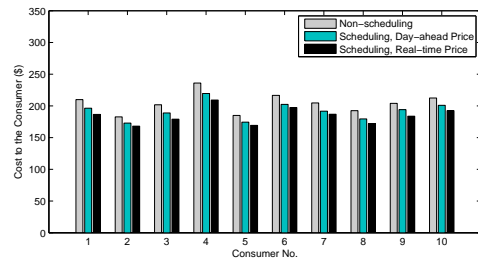
**Fig. 2.** One-day power usage comparison.

In Fig. 2 we also compare demand curve for our scheme with the demand curve using day-ahead pricing, where each local EMC makes decisions based on the same day-ahead prices. From the plot, we can see that at 6:50 PM, where there is an original peak load, both our real-time pricing and the day-ahead pricing schemes reduce demand by 28.9% and 71.7%, respectively. However, at 8:20 PM, the demand curve for day-ahead pricing exceeds the planned supply load by 81.7%, thereby producing a “rebound” peak. In contrast, our scheme has no such rebound effects.

### 5.2. Benefits to Consumers

Next, we examine the benefits of our proposed scheme for consumers. To do so, we plot in Fig. 3, the costs (using the objective function in (1)) for 10 random consumers for our scheme, the non-scheduling approach and scheduling based on day-ahead prices. Compared to the non-scheduled curve, consumers see a 9.46% reduction in cost for our scheme; while for the day-ahead prices scheme, cost saving is only

5.84%. When considering only electricity costs (i.e., electricity bills), the savings of our proposed scheme is 24.22% over the non-scheduled case, compared to 14.63% savings for the day-ahead scheme.



**Fig. 3.** Money saving evaluation (100 days).

Our simulation results further show that these cost reductions for the consumers are consistent across the neighborhood of homes. That is, based on a 100-day average, we observed that the standard deviation in the electricity bills across the 80 homes is within 2.5% of the average electricity bill for the same consumption level. This demonstrates the fairness of our pricing scheme from the consumer's perspective.

## 6. CONCLUSION

In this paper, we propose a real-time RTP-based power scheduling scheme as a demand response mechanism for residential electric power consumption. A Stackelberg game model is formulated to analyze the interaction between a consumer's EMC and the service provider. Our scheme can reduce peak load and the mismatch between actual load and planned supply, while avoiding a rebound peak. Using EMCs, consumers can exploit the proposed real-time prices to reduce electricity bills.

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