Centralization versus Decentralization: Risk Pooling, Risk Diversification, and Supply Chain Disruptions

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ABSTRACT

We investigate optimal system design in a multi-location system in which supply is subject to disruptions. We examine the expected costs and cost variances of the system in both a centralized and a decentralized inventory system. We show that, when demand is deterministic and supply may be disrupted, using a decentralized inventory design reduces cost variance through the risk diversification effect, and therefore a decentralized inventory system is optimal. This is in contrast to the classical result that when supply is deterministic and demand is stochastic, centralization is optimal due to the risk-pooling effect. When both supply may be disrupted and demand is stochastic, we demonstrate that a risk-averse firm should typically choose a decentralized inventory system design.

Keywords: Inventory control; Risk; Supply chain disruptions

1 Introduction

As supply chains expand globally, supply risk increases. Classical inventory models have generally focused on demand uncertainty and established best practices to mitigate demand risk. However, supply risk can have very different impacts on the optimal inventory management policies and can even reverse what is known about best practices for system design.

In this paper, we focus on the impact of supply uncertainty on a multi-location system and compare two policies: centralization (stocking inventory only at a central warehouse) and decentralization (stocking inventory at multiple warehouses). Our analysis is a special case of One-Warehouse Multiple-Retailer (OWMR) system analysis; while most research on the OWMR model allows inventory to be held at both echelons, we allow inventory to be held at only one echelon in order to consider two opposing effects that can occur: risk pooling and risk diversification. The *risk pooling* effect occurs when inventory is held at a central location, which allows the demand variance at each retailer to be combined, resulting in a lower expected cost [Eppen, 1979]. The *risk diversification* effect occurs when inventory is held at a decentralized set of locations, which allows the impact of each disruption to be reduced, resulting in a lower cost variance [Snyder and Shen, 2006]. Whereas the risk-pooling effect reduces the expected cost but (as we prove) not the cost variance, the risk-diversification effect reduces the variance of cost but not the expected cost.

We prove that the risk diversification effect occurs in systems with supply disruptions. We also consider systems with both supply and demand uncertainty, in which both risk pooling and risk diversification have some impact, and numerically examine the tradeoff between the two. The risk mitigated through risk diversification is disruption risk or supply risk, whereas the risk mitigated through risk pooling is demand risk. We employ a risk-averse objective to minimize both risk sources and determine which effect dominates the system and drives the choice for optimal inventory system design.

Specifically, comparing centralized and decentralized inventory policies, we contribute the following:

- The exact relationship between optimal costs and inventory levels when demand is deterministic and supply is subject to disruption
- The exact relationship between optimal cost variances when:
  - demand is deterministic and supply is subject to disruption
  - demand is stochastic and supply is deterministic
- Formulations of the expected cost and cost variance when demand is stochastic and supply is subject to disruption

- Evidence that decentralization is usually optimal under risk-averse objectives

The remainder of the paper is organized as follows. In Section 2 we review the relevant literature. In Section 3 we analyze the risk-diversification effect in a multi-location system with deterministic demand and disrupted supply. We consider stochastic demand and deterministic supply in Section 4. In Section 5 we consider both demand uncertainty and supply disruption and again compare inventory strategies using a risk-averse objective to choose the optimal inventory design. We summarize our conclusions in Section 6. Proofs for all propositions and theorems are given in the Appendix.

2 Literature Review

Supply chain risk management has been widely studied ever since the concept of uncertainty was introduced into inventory theory. Uncertainty in supply chains is usually classified as either demand uncertainty or supply uncertainty. A simple model with demand uncertainty is the newsvendor problem, which determines the optimal order quantity or inventory level to minimizes the expected cost under stochastic demand in a single period for a single location. Eppen [1979] extends the newsvendor problem to a multiple-location model and shows that under demand uncertainty, a centralized inventory strategy provides risk-pooling benefits and reduces expected cost versus a decentralized strategy. Demand pooling is now a familiar idea in operations management and serves as a major instrument to protect against demand uncertainty. Corbett and Rajaram [2006] generalize Eppen's work to the case of non-normal, dependent demand and show that the magnitude of the risk-pooling effect increases when the demands are less positively dependent. Berman et al. [2011] analyze the benefit of inventory pooling in a multi-location newsvendor framework and show that the absolute benefit of risk pooling increases with demand variability.

There are several ways to implement demand pooling, including transshipments, postponement, and product substitution. Yang and Schrage [2009] explore the conditions that cause risk pooling to increase inventory under the setting of product substitution. Paterson et al. [2011] review the literature on transshipments as an analogue of inventory pooling. Alptekinoglu et al. [2013] propose a model of inventory pooling to meet differentiated service levels for multiple customers. In general, these forms of pooling leverage centralization so as to diminish the impact of demand uncertainty on supply chain performance, which can be thought of as a generalization of the risk-pooling effect studied by Eppen [1979] and by this paper. On the other hand, our work deviates from this stream of literature by considering supply uncertainty as well.

Supply uncertainty has been considered in several settings, including newsvendor [e.g., Dada et al., 2007, Tomlin, 2009] and EOQ [e.g., Berk and Arreola-Risa, 1994, Parlar and Berkin, 1991, Snyder, 2014] systems. The two most commonly considered forms of supply uncertainty are supply disruptions (in which supply is halted entirely for a stochastic amount of time) and yield uncertainty (in which the quantity delivered from the supplier is random). Chopra et al. [2007] and Schmitt and Snyder [2012] consider systems that have both supply disruptions and yield uncertainty. The literature on single-echelon (newsvendor) systems with both of these types of supply uncertainty is extensive, and we omit an exhaustive review here. The reader is referred to Atan and Snyder [2012] and Snyder et al. [2010] for reviews of the literature on inventory models with supply disruptions and Yano and Lee [1995] for a discussion of the literature on single-echelon systems with yield uncertainty. Multi-period models with stochastic demand and supply have been considered by Schmitt et al. [2010] and Tomlin [2006], among others, and we rely on several of their results in this paper. Schmitt et al. develop a closed-form approximate solution for the optimal base-stock level in the face of disruptions and stochastic demand. Tomlin investigates multiple strategies for coping with disruptions, including acceptance (doing nothing proactively), sourcing (using multiple suppliers), and inventory policies. These papers provide a foundation for our analysis, but our application to the OWMR model provides new insights on the impact of supply disruptions in complex systems.

Regarding the management of supply risk, Aydin et al. [2012] point out that decentralization, a commonly approach to mitigate supply risk, creates a misalignment of incentives between suppliers and buyers, competition among suppliers, competition among buyers, and asymmetric information among the supply chain parties, while Ellis et al. [2011] discusses the implementation of supply disruption risk research. Interested readers are referred to Babich et al. [2007], Yang et al. [2009], Tang and Kouvelis [2011] and Yang et al. [2012] for a closer discussion of competition versus diversification. Decentralization also provides flexibility for decision makers at the strategic level. For example, Yu et al. [2009] consides the effects of supply disruptions on the decision of whether to single- or dual-source. Wang et al. [2010] compare dual sourcing and process improvement to mitigate supply risk. Sawik [2011], Sawik [2013] and Qi [2013] discuss supplier selection when the supply is subject to disruptions.

Decentralization may be deployed not only occur among suppliers, but also in the inventory systems themselves. Snyder and Shen [2006] use simulation to study multiple complex inventory systems, including the OWMR system with supply uncertainty with inventory at a single echelon.

Their simulation results show that, under supply disruptions, expected costs are equal for centralized and decentralized systems, but the variance of the cost is higher in centralized systems. They call this the *risk-diversification* effect and suggest that it occurs because a disruption in a centralized system affects every retailer and causes more drastic cost variability. They conclude that risk diversification increases the appeal of inventory decentralization in a system with disruptions. In the context of multi-location inventory management, decentralization or diversification is leveraged to achieve lower cost variance instead of the expected cost, as in, e.g., Schmitt and Snyder [2012] and Tomlin [2006].

In this paper we consider supply and demand uncertainty in a multiple-demand-point system where inventory may be held at a centralized warehouse to mitigate the demand risk or at multiple warehouses to mitigate the supply risk. We assume that inventory may be held at only a single echelon in order to draw clear conclusions regarding centralization versus decentralization. We explore the implications of the risk diversification effect by developing an analytical model for the expected cost and cost variance of a multi-location system subject to supply uncertainty. We analytically prove the presence of risk diversification in this system, discuss its impact, and examine the system under uncertainty in both supply and demand. When demand is deterministic and supply is subject to disruptions, we determine the optimal inventory levels and costs. For that case and the case in which demand is stochastic and supply is deterministic, we quantify the cost variance. We combine supply disruptions and stochastic demand in a subsequent model and formulate the expected costs and cost variances. To consider the effect of cost variances in the decision making process, we adopt a risk-averse objective to incorporate the cost variance with the expected cost at the same time.

In the latter part of this paper, we consider risk-averse objectives for inventory optimization. Risk-aversion, a topic which is gaining momentum in the operations literature, has been considered in newsvendor models to mitigate demand uncertainty. For example, Eeckhoudt et al. [1995] show that order quantities decrease with increasing risk-aversion. Van Meighem [2007] considers resource diversification in newsvendor models with risk-averse objectives, advocating diversifying resource availability to protect against risk. Tomlin and Wang [2005] consider a single-period newsvendor setting with supply disruptions; they model loss-averse and conditional value-at-risk (CVaR) objectives when deciding between single- and dual-sourcing and between dedicated and flexible resource availability. Chen et al. [2007] consider multi-period inventory models with risk-aversion, modeling both replenishment and pricing decisions. Tomlin [2006] also considers a multi-period setting and employs a mean–variance approach to consider risk in a two-supplier system in which one supplier is subject to disruptions and the other is perfectly reliable but more expensive. We show that under a risk-averse objective function, the benefits of risk diversification typically surpass those of risk pooling and therefore decentralization is the optimal network configuration.

3 Risk Diversification Effect with Supply Disruptions

In this section we examine the impact of supply disruptions on both the centralized and the decentralized multi-location system. Before we present the models, we describe our assumptions and notation.

3.1 Assumptions and Notation

We consider a two-echelon inventory system where demand occurs according to the same distribution across multiple locations. We assume all locations follow a base-stock policy, and we compare policies for stocking inventory at the central warehouse only (centralized system) or at multiple warehouses (decentralized system). We assume the fixed ordering costs are zero for the all the following analysis. This enables us to place orders in every period regardless of any fixed order cost for a multi-period system. This is not unreasonable in internal distribution settings. We discuss what impact this might have in our conclusions. We also assume zero lead times since deterministic lead times would not impact optimal base-stock levels or order quantities, but would simply require that orders be placed exactly the lead time quantity in advance.

A holding cost of h per unit per period is incurred at the warehouse in the centralized system or at each of the warehouses in the decentralized system. The holding costs are evaluated after the demands are realized. Unmet demands are backordered, and a stockout penalty of p per unit per period is incurred in both systems.

The parameters, decision variables (base-stock levels at each location), and performance measures for the system are summarized in Table 1. Note that since S_D^* is defined as the optimal base-stock level for an individual warehouse in the decentralized system, the total inventory for that system is nS_D^*.

In both the centralized and decentralized systems, disruptions occur randomly at the inventory location(s); that is, disruptions occur at the central warehouse in the centralized system or at any of the multiple warehouses in the decentralized system. Disruptions occur independently at all locations in the system. Examples of disruptions of this nature could include disruption of an

Table 1: Notation

Notation	*Definition*
n	number of demand sets or locations; number of decentralized warehouses
p	penalty cost per period at each facility
h	holding cost per period at each facility
S^*	optimal base-stock level for an individual facility
S_D^*	optimal base-stock level for each warehouse in the decentralized system
S_C^*	optimal base-stock level for the warehouse in the centralized system
$E[C]$	expected cost for an individual facility
$E[C_D]$	expected cost for the decentralized system
$E[C_C]$	expected cost for the centralized system
$V[C]$	variance of the cost for an individual facility
$V[C_D]$	variance of the cost for the decentralized system
$V[C_C]$	variance of the cost for the centralized system

upstream supplier, failure of inbound transportation capabilities (e.g., a snowstorm or flood that shuts down rail or major roadways), transportation labor issues (e.g., a port strike), shipment quality issues (e.g., a bacterial contamination), or other major issues that interrupt material flow to a single location. In our model, there is no back-up source of material available (i.e., no cross-shipments or alternate sources) when a disruption occurs at a facility. If a disruption occurs at the central warehouse, then all downstream locations feel any resulting shortage equally. In both systems, disruptions "pause" the flow of supply to the disrupted stage, but any inventory at that stage may still be used during the disruption; therefore, disruptions affect a stage's supply-receiving function but not its demand-receiving function.

We model the disruptions as a random process governed by the pmf π_i and cdf $F(i)$, where i is the number of consecutive periods during which a given stage has been disrupted, as is typical in the supply disruption literature [e.g., Hopp and Liu, 2006, Tomlin, 2006]. Therefore π_i is the probability that a given stage has been disrupted for precisely i periods ($i \geq 0$) and $F(i)$ is the probability that it has been disrupted for i periods or fewer. Disruptions may be governed by a Markov chain or a more general process. The locations have identical and independent disruption processes that are stationary over time. After modeling the disruptions using this random process, we can evaluate the expected costs and cost variances for both the centralized and decentralized multi-location systems. We use an additional subscript s to denote the disrupted-supply model discussed in this section.

3.2 Mean and Variance of Optimal Cost

For a base-stock inventory policy at a single warehouse, the costs in a given period depend on the state of the system, defined as the number of consecutive periods for which it has been disrupted. The expected cost and optimal base-stock level for such a system are given by [see, e.g., Snyder and Shen, 2011]:

$$E^s[C] = \sum_{i=0}^{\infty} \pi_i [h(S^s - (i+1)d)^+ + p((i+1)d - S^s)^+] \tag{1}$$

$$S^{s*} = jd, \text{ where } j \text{ is the smallest integer such that } F(j-1) \geq \frac{p}{p+h} \tag{2}$$

Schmitt et al. [2010] discuss these results, demonstrating that the optimal base-stock level is an increasing step function of the newsvendor fractile and the disruption probability. As the cost of disruptions increases, the optimal solution increases by discrete jumps to the next whole period's worth of demand.

Incorporating these results in our analysis of the centralized and decentralized multi-location systems, we present the relationships between the inventory levels, expected costs, and cost variances for the various systems in the following theorem.

Theorem 1 *In the systems subject to supply disruptions, the optimal base-stock levels and performance measures in the centralized, decentralized, and single-facility setting are related as follows:*

1. $S_C^{s*} = nS_D^{s*} = nS^{s*}$
2. $E^s[C_C^*] = E^s[C_D^*] = nE^s[C^*]$
3. $V^s[C_C^*] = nV^s[C_D^*] = n^2V^s[C^*]$

Proof: See Appendix, Section A.2.

The theorem states that the total system inventory levels and expected costs are equal in the centralized and decentralized systems, but the cost variance is n times greater for the centralized system, suggesting that the decentralized system is preferred for a risk-averse decision maker. The reduction in the cost variance for a decentralized system while incurring the same level of the expected cost as in the centralized system is called the *risk diversification effect.* The risk diversification effects occurs when the supply is subject to disruption and demand is deterministic.

The intuition behind Theorem 1, i.e., the risk diversification effect, is as follows. Because the centralized system is just an aggregation of the demands from the decentralized system with the identical random disruption process, the total base-stock levels at optimality should be the same for the two systems. That is to say, the total inventory devoted to each of the n sets of customers is the same in both systems. Therefore each customer set faces the same number of stockout periods (effects of disruptions) on average in both systems due to the independence and homogeneity of the supply disruption processes. By defining the service level as the number of stockout periods on average, commonly known as the type-1 service level, another way to say the above is that identical disruption processes in both systems cause the customer set to experience the same service level. We state this formally in the following Corollary.

Corollary 2 *In the multi-location system subject to supply disruptions, if base-stock levels are set optimally, the distribution of consecutive periods for which a given warehouse is stocked out is identical for the centralized and decentralized systems.*

This same stockout distribution causes the expected holding and stockout costs to be the same in both systems. However, disruptions are less frequent and more severe in the centralized system than in the decentralized system due to the independence of the disruptions in the decentralized system and the aggregation of the demands in the centralized system. Therefore the centralized system incurs a greater cost variance.

The relationships presented in Theorem 1 demonstrate the risk diversification effect in systems in which the supply is subject to disruption. If demand is deterministic, the risk from uncertain supply is mitigated in the decentralized system, as the lower variance for that system demonstrates. Moreover, expected costs for the centralized and decentralized systems are equal. Snyder and Shen [2006] demonstrate a similar result using simulation, though their model differs slightly from ours in that they assume that inventory at a given stage may not be used when that stage is disrupted. However, the risk pooling effect, which favors a centralized system, is observed when demand is stochastic and supply is deterministic. We present these results in the next section.

4 Risk Pooling Effect with Stochastic Demand

In this section, we present the risk pooling effect when the demand is stochastic but supply is deterministic. We consider the classical model discussed by Eppen [1979]. Following Eppen, consider two alternative system configurations: the centralized system and the decentralized system. In both systems, the supply is deterministic while the demand is stochastic. The demand is normally distributed with the same mean μ and variance σ^2 at each of the decentralized warehouses. Therefore, the demand at the central warehouse in the centralized system has mean $n\mu$ and variance $n\sigma^2$. We employ a superscript d to denote this model with stochastic demand. Optimal base-stock levels are the major concerns for both the centralized system and the decentralized system. We therefore review the results concerning the expected costs and their corresponding optimal base-stock levels presented by Eppen [1979]. Since we are interested in not only the average performance of the systems but also the stability of the systems, we evaluate the cost variances of the systems at optimality in Section 4.2. To the best of our knowledge, ours is the first study to consider the cost variance at optimality in these systems.

4.1 Expected Cost

The expected cost and optimal base-stock level for a single warehouse subject to stochastic demand following normal distribution $N(\mu, \sigma^2)$ are well known as:

$$E^d[C] = h(S^d - \mu) + \sigma(p+h)\Phi^1\left(\frac{S^d - \mu}{\sigma}\right) \tag{3}$$

$$S^{d*} = \mu + \sigma\Phi^{-1}\left(\frac{p}{p+h}\right) \tag{4}$$

where $\Phi^1(x) = \int_x^\infty (v-x)\varphi(v)dv$ is the standard normal loss function. The expected cost and optimal base-stock level here can be directly applied to a single warehouse in the decentralized system. For the centralized system, the demand follows a normal distribution $N(n\mu, \sigma_C^2)$, where $\sigma_C^2 = n\sigma^2$. Therefore, for the centralized system,

$$E^d[C_C] = h(S_C^d - n\mu) + \sigma_C(p+h)\Phi^1\left(\frac{S_C^d - n\mu}{\sigma_C}\right) \tag{5}$$

$$S_C^{d*} = n\mu + \sigma_C\Phi^{-1}\left(\frac{p}{p+h}\right) \tag{6}$$

Since the decentralized system functions as n single-warehouse systems, $E^d[C_D] = nE^d[C]$ and $S_D^{d*} = S^{d*}$. We have the following theorem:

Theorem 3 (Eppen, 1979) *The optimal expected costs and inventory levels for the centralized and decentralized multi-location systems subject to stochastic demand are as follows:*

$$E^d[C_D^*] = \sqrt{n}E^d[C_C^*] \tag{7}$$

$$nS_D^{d*} - S_C^{d*} = (n - \sqrt{n})\sigma\Phi^{-1}\left(\frac{p}{p+h}\right) \tag{8}$$

Proof: Given by Eppen [1979].

This theorem exhibits the benefits of centralization: by serving the warehouses' demand from a centralized inventory site, we pool the risk from demand uncertainty. As a result, the optimal expected cost for the centralized system is $\sqrt{n}$ times smaller for the decentralized system, and less inventory is required. Next, we evaluate the cost variance for both systems at optimality.

4.2 Cost Variance

The following theorem presents the relationship between the cost variances for the two systems.

Theorem 4 *At optimality, the cost variance for the centralized and decentralized multi-location systems subject to stochastic demand are equal:*

$$V^d[C_C^*] = V^d[C_D^*] \tag{9}$$

Proof: See Appendix, Section A.3.

This theorem claims that centralization reduces the expected cost of the system while not affecting the cost variance compared to a decentralized multi-location system, which is referred to as the *risk pooling effect*. This suggests that centralization is optimal under the presence of stochastic demand and deterministic supply.

We therefore obtain the following conclusions so far. Under stochastic demand and deterministic supply, centralization is optimal due to the risk pooling effect, which affects the expected cost only and has no impact on cost variance. In contrast, under supply disruptions and deterministic supply, decentralization is optimal due to the risk diversification effect, which affects the cost variance only and has no impact on the expected cost. A natural question is, when both demand uncertainty and disruptions are present, which system is optimal, i.e., which prevails: risk pooling or risk diversification? We address this question in the next section.

5 System with Supply Disruptions and Stochastic Demand

In this section we compare the cost means and variances for centralized and decentralized multi-location systems in which each warehouse is subject to stochastic demand and the whole system is subject to disrupted supply. We assume that when supply is not disrupted, the yield is deterministic. We use a superscript b to denote this model since both supply and demand are stochastic. We formulate the expected costs and cost variances in Sections 5.1 and 5.2, respectively. We perform numerical analysis to compare the performance of the two systems in Section 5.3. Finally, in Section 5.4, we introduce a mean–variance objective that allows us to choose optimally between the two systems when risk aversion is accounted for. We summarize our findings in Section 5.5.

5.1 Expected Costs

At a single warehouse, a single period's demand is denoted D and is distributed as $N(\mu, \sigma^2)$, while the total demand in i periods is distributed as $N(i\mu, i\sigma^2)$. The cost for a single warehouse subject to normally distributed demand and disrupted supply is given by Schmitt et al. [2010] as:

$$\begin{aligned} E^b[C] &= \sum_{i=1}^{\infty} \pi_{i-1} \left(h \int_{-\infty}^{S^b} (S^b - id) f_i(id) dd + p \int_{S^b}^{\infty} (id - S^b) f_i(id) dd \right) \\ &= \sum_{i=1}^{\infty} \pi_{i-1} \left(h(S^b - i\mu) + \sigma\sqrt{i}(p+h)\Phi^1 \left(\frac{S^b - i\mu}{\sigma\sqrt{i}} \right) \right) \end{aligned} \tag{10}$$

Schmitt et al. [2010] prove that $E^b[C]$ is convex and argue that the optimal base-stock level cannot be found in closed form because $\Phi\left(\frac{S-i\mu}{\sigma\sqrt{i}}\right)$ appears in the first-order condition for $i = 0, 1, \ldots, \infty$.

Using (10), we derive the expected costs for the two systems in the following Proposition.

Proposition 5 *When demand is normally distributed at the warehouses and supply is subject to disruptions, the expected costs for the centralized and decentralized multi-location systems are:*

$$E^b[C_D] = n \sum_{i=1}^{\infty} \pi_{i-1} \left(h(S_D^b - i\mu) + \sigma\sqrt{i}(p+h)\Phi^1 \left(\frac{S_D^b - i\mu}{\sigma\sqrt{i}} \right) \right) \tag{11}$$

$$E^b[C_C] = \sum_{i=1}^{\infty} \pi_{i-1} \left(h(S_C^b - in\mu) + \sigma\sqrt{in}(p+h)\Phi^1 \left(\frac{S_C^b - in\mu}{\sigma\sqrt{in}} \right) \right) \tag{12}$$

Proof: See Appendix, Section A.4.

Clearly, for the decentralized system, $S_D^{b*} = S^{b*}$. For the centralized system, we have confirmed numerically that $S_C^{b*} \neq nS^{b*}$ in general, as in the deterministic-demand model in Section 3. Unlike that model, S_C^{b*} is neither consistently greater than nor consistently less than nS^{b*} when demand is stochastic. As a result, the expected cost for the centralized system at optimality is not consistently less than in the decentralized system as suggested by the risk pooling effect. However, we demonstrate numerically in Section 5.3 that the centralized system has a lower expected cost than the decentralized system in most of the instances we test, which confirms that, when demand is stochastic, the risk pooling effect usually holds (i.e., the centralized system has lower expected cost), even when supply is subject to disruption.

5.2 Cost Variance

We present the cost variance for arbitrary base-stock levels (S_D^b in the decentralized system and S_C^b in the centralized system, respectively) in the following Proposition.

Proposition 6 *When demand is normally distributed at the warehouses and supply is subject to disruptions, the cost variances for the decentralized and centralized multi-location systems are:*

$$\begin{aligned} V^b[C_D] &= n\Bigg[\sum_{i=1}^{\infty}\pi_{i-1}\Bigg((h^2{S_D^b}^2 - 2S_D^b i\mu + i\sigma^2 + i^2\mu^2) + 2i\sigma^2(p^2-h^2)\Phi^2\left(\frac{S_D^b - i\mu}{\sigma\sqrt{i}}\right)\Bigg) - \\ &\quad \left(\sum_{i=1}^{\infty}\pi_{i-1}\left(h(S_D^b - i\mu) + \sigma\sqrt{i}(p+h)\Phi^1\left(\frac{S_D^b - i\mu}{\sigma\sqrt{i}}\right)\right)\right)^2\Bigg] \qquad (13) \\ V^b[C_C] &= \sum_{i=1}^{\infty}\pi_{i-1}\Bigg(h^2({S_C^b}^2 - 2S_C^b ni\mu + ni\sigma^2 + n^2i^2\mu^2) + 2ni\sigma^2(p^2-h^2)\Phi^2\left(\frac{S_C^b - ni\mu}{\sigma\sqrt{ni}}\right)\Bigg) - \\ &\quad \left(\sum_{i=1}^{\infty}\pi_{i-1}\left(h(S_C^b - ni\mu) + \sigma\sqrt{ni}(p+h)\Phi^1\left(\frac{S_C^b - ni\mu}{\sigma\sqrt{ni}}\right)\right)\right)^2, \qquad (14) \end{aligned}$$

where $\Phi^1(\cdot)$ is the standard normal loss function and $\Phi^2(\cdot)$ is the standard normal second-order loss function.

Proof: See Appendix, Section A.5.

We have confirmed numerically that it is not always true that $V^b[C_C^*] \geq V^b[C_D^*]$ (as the risk diversification effect would suggest). However, we demonstrate numerically in Section 5.3 that this inequality holds for most of the instances we tested. Thus, when supply may be disrupted, the risk diversification effect usually holds (i.e., the decentralized system has lower cost variance), even when demand is stochastic.

5.3 Numerical Study

In this section, we perform a numerical study to compare the performance of the two systems when supply disruptions and stochastic demand are present simultaneously. More specifically, we model the supply disruptions as a two-state, discrete-time Markov process with disruption probability α and independent recovery probability β. (That is, the probability of being disrupted [not disrupted] next period given that the system is not disrupted [disrupted] in the current period is α [β].)

The steady-state probabilities for the disruption process are given by [Schmitt et al., 2010]:

$$\pi_0 = \frac{\beta}{\alpha+\beta}\ ;\ \pi_i = \frac{\alpha\beta}{\alpha+\beta}(1-\beta)^{i-1}, i \geq 1 \qquad (15)$$

Table 2: Parameter Levels

Variable	Values
α (failure probability)	0.001, 0.005, 0.025, 0.01, 0.02, 0.05, 0.08, 0.1, 0.15, 0.2, 0.25, 0.3
β (recovery probability)	0.5, 0.75, 0.85, 0.90, 0.95, 0.99
σ (demand st. dev.)	5, 10, 15, 20, 25, 50
$z = p/(p+h)$ (newsvendor fractile)	0.50, 0.60, 0.65, 0.68, 0.71, 0.74, 0.77, 0.80, 0.83, 0.86, 0.89, 0.92, 0.95, 0.98, 0.99, 0.995

In Section 5.3.1, we vary the input parameters independently to determine the impact of each on the performance of the two systems. In Section 5.3.3 we examine the impact of the disruption characteristics on the systems.

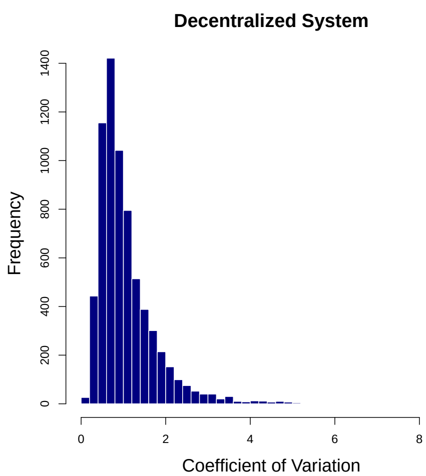
5.3.1 Impact of Parameters

We conducted a full-factorial experiment to determine the effect of the input parameters on the optimal base-stock levels and the cost mean and variance at optimality in the two multi-location systems. In this section, we are interested in how the parameters affect the performance metrics for a given set of customers. Therefore, we look at a system that has 6 customer sets, i.e., $n = 6$. Without loss of generality, we fix μ to be 150 and h to be 1 and then vary the single-location standard deviation, σ, and the stockout penalty, p, because the model is sensitive to changes in the ratios σ/μ and $z = p/(p+h)$ (the newsvendor fractile) rather than to changes in the individual parameters. In addition, we vary the disruption and recovery parameters α and β. Table 2 lists the values tested for each parameter. The total number of parameter combinations is $12 \times 6 \times 6 \times 16 = 6,912$.

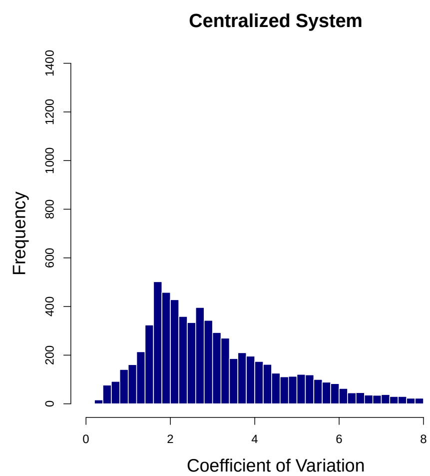
For each combination of parameters, we optimized the expected cost functions (11) and (12) numerically to solve for the optimal base-stock levels in both systems. For each instance, we evaluate the total stock levels (nS_D^{b*} or S_C^{b*}), the expected costs at optimality $E^b[C^*]$, the cost variances at optimality $V^b[C^*]$, and the coefficient of variation (CV) of the cost at optimality, given by $\rho^b(C^*) = \frac{\sqrt{V^b[C^*]}}{E^b[C^*]}$. Table 3 summarizes the numerical results for the 6,912 instances we tested. The decentralized system has a higher total stock level at optimality in 87% of the instances tested. This occurs because of the discrete nature of disruptions. If demand is deterministic, then the optimal base-stock level is always an integer multiple of the demand (see Section 3.2), and demand stochasticity perturbs these discrete base-stock levels only slightly. Since the total standard deviation of demand is smaller in the centralized system, the jumps are less smooth (closer to the deterministic-demand solution) and may jump slightly above the optimal decentralized system levels.

Decentralized	Frequency	(%)
Stocks More	6011	86.96%
Higher Cost	6901	99.84%
Lower Variance	6909	99.96%
Lower CV	6909	99.96%

Table 3: Decentralized system stocks more total inventory units, incurs higher expected total costs $E^b[C^*]$ and lower cost variances $V^b[C^*]$, and therefore lower coefficient of variation $\rho^b[C^*]$, in most of the numerical experiments.



(a)



(b)

Figure 1: Coefficient of variation $\rho^b[C^*]$ distribution for (a) decentralized system and (b) centralized system.

The decentralized system has a higher expected cost at optimality for 99% of the instances tested, which confirms the presence of the risk-pooling effect in that centralization reduces expected cost even with supply disruptions. In contrast to the higher expected cost, the decentralized system has lower cost variance/coefficient of variation in cost for more than 99% of the instances, which reveals the presence of the risk-diversification effect in that decentralization reduces the cost variance even under demand stochasticity. The histograms in Figure 1 provide a distributional view of the CV of the costs in decentralized and centralized systems. The range of CV for the decentralized system is from 0 to 3.5 while the range of CV for the centralized system is from 0 to 9, indicating that the cost variance is much higher in the centralized system.

Besides the overall results from the numerical study, we are interested in how individual parameters affects the results. Figure 2 plots the relative difference in $E^b[C^*]$, $V^b[C^*]$ and $\rho^b[C^*]$ between the decentralized and the centralized system for different parameters. We found that the difference in expected cost is more sensitive to changes in parameters than the difference in cost variance is. The benefits of risk pooling/centralization are enhanced when σ increases, z decreases, α decreases, or β increases. That is to say, the expected cost reduction from centralization is increased if high uncertainty in demand is observed, backlogging incurs low penalty cost, disruptions in supply do not occur frequently, or the recovery rate is high. The intuition for these observation is as follows. The risk pooling effect is more pronounced when the demand uncertainty is high, as is evident from the model in Section 4. When the recovery rate is high, or the supply disruptions are not frequent, the supply suffers low uncertainty, as in the deterministic supply model in Section 4, and therefore risk pooling benefits. When the penalty cost is low, the cost of depleting the stocks during disruptions is low, which has the same effect as low disruption rate, and therefore risk pooling benefits.

We found the cost variance is, in general, lower for the decentralized system, confirming the risk diversification effect. Although the difference in cost variance is not sensitive to parameter changes, their relative impacts on the system configuration decision, which are reflected by the coefficients of variation, are visible. The risk diversification effect is more pronounced when σ decreases, z increases, α increases, or β decreases, which are the opposite conditions for the risk pooling effects to be visible. We can therefore use similar arguments as before to interpret such results.

5.3.2 Impact of Number of Demand Locations

We next present the impact of the number of demand locations, n, on the differences between the centralized and decentralized systems. Instead of varying n independently, we fix the total demand $n\mu$ to be 900 and the total demand standard deviation $\sqrt{n}\sigma$ to be 60, and then vary the number of demand locations n to obtain the demand distribution at individual facilities. This setting best mimics the situation in which the total demand in the market is stable and the demand at each individual facility depends on the number of facilities built. We tested eight scenarios consisting of the various combinations of the following: disruption rate α is low/high, recovery rate β is low/high and newsvendor fractile z is low/high. Figure 3 summarizes the differences in expected cost and cost variances for the two systems.

As is expected, an increase in the number of facilities amplifies the risk pooling effect as well as the risk diversification effect. The risk pooling effect is most amplified when the disruption rate α is low and the newsvendor fractile z (or the penalty cost p) is low. The underlying intuition falls

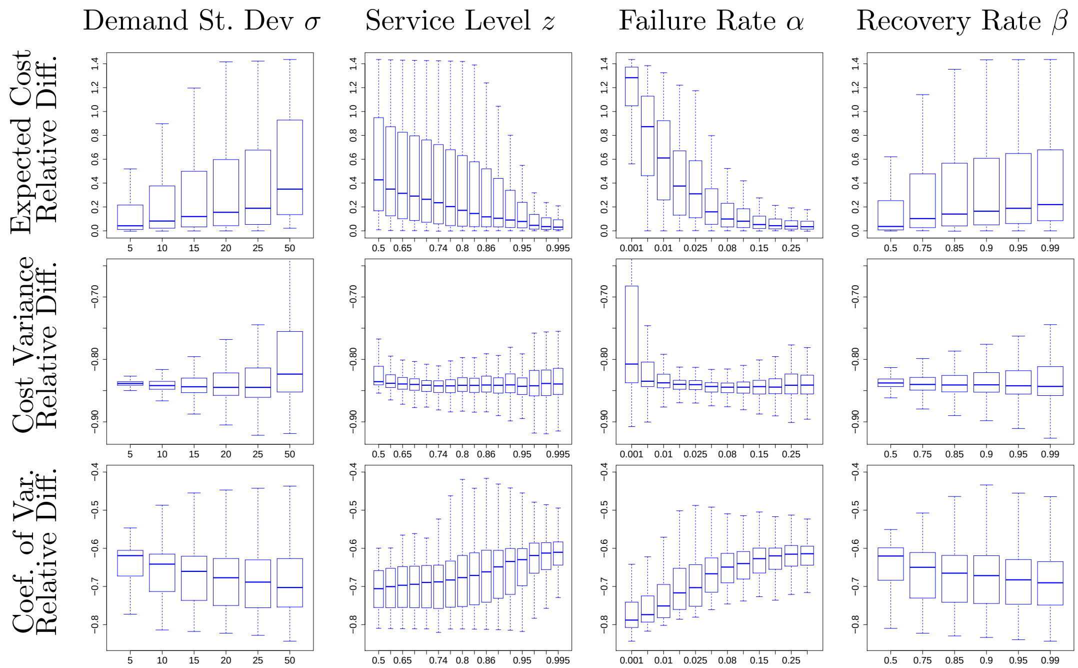


Figure 2: Relative difference in expected cost $(\frac{E^b[C_D]-E^b[C_C]}{E^b[C_C]})$, cost variance$(\frac{V^b[C_D]-V^b[C_C]}{V^b[C_C]})$, and coefficient of variation $(\frac{\rho^b[C_D]-\rho^b[C_C]}{\rho^b[C_C]})$ of decentralized and centralized systems at optimality.

into the same argument as in Section 5.3.1. But the reduction in expected cost from risk pooling is smaller in scale when the disruption rate is high or the penalty cost is high. In contrast, the reduction in cost variance due to the risk diversification effect is visible in all scenarios. As a result, as long as the supply disruptions are frequent or the stockout penalty is high, risk-diversification is more pronounced than risk-pooling. We therefore can conjecture that a risk-averse decision maker will be in favor of a decentralized system, which is confirmed by the implementation of the mean–variance objective in Section 5.4.

5.3.3 Impact of Disruption Profile

Two pairs of (α, β) values may result in the same fraction of periods disrupted, defined as

$$1 - \pi_0 = \frac{\alpha}{\alpha + \beta}, \tag{16}$$

but still have very different characteristics. For example, if α and β are both close to 0, then disruptions are rare but long, while if they are both close to 1, then disruptions are frequent but short. Snyder and Tomlin [2008] refer to the frequency/duration characteristics of a disruption as

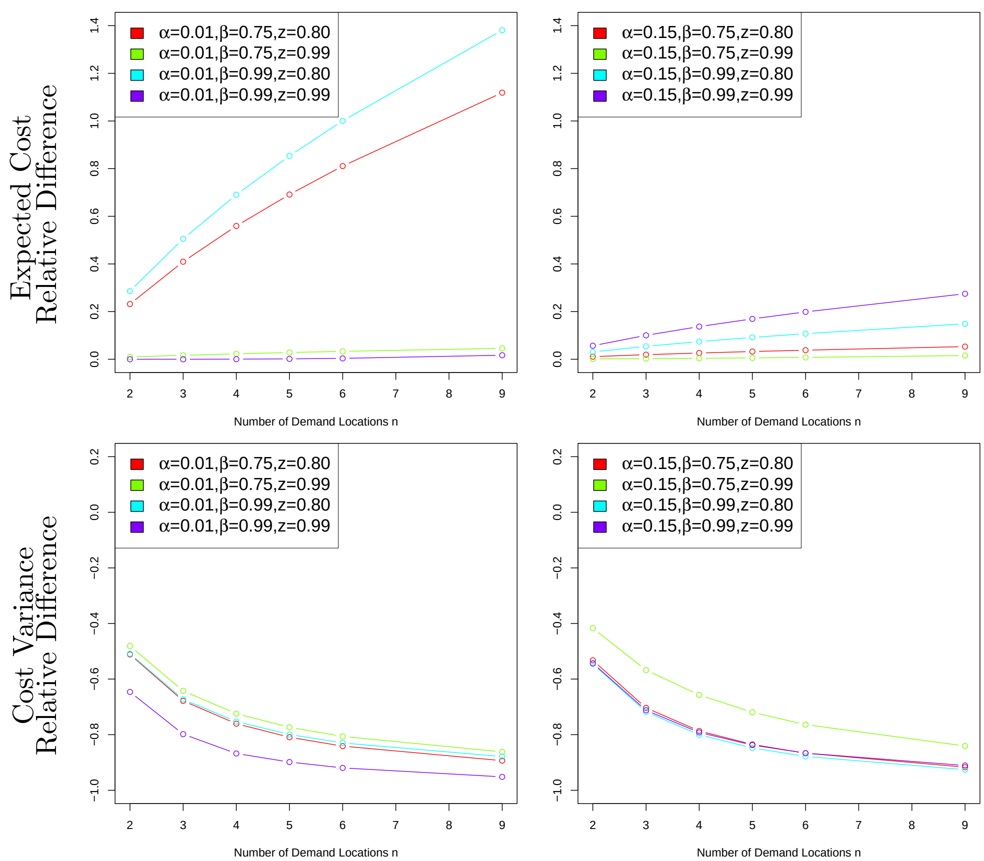


Figure 3: Risk pooling (expected cost reduction) and risk diversification (cost variance reduction) of centralized and decentralized systems under low/high disruption rate, low/high recovery rate, and low/high stockout penalty.

the *disruption profile*. In this section, we study the effect of the disruption profile on the optimal base-stock levels and performance measures of the two systems.

We generated twelve groups of (α, β) pairs. Within each group, the (α, β) pairs have the same percentage down-time (16) but a different disruption profile. For each (α, β) pair, we vary the disruption rate α in eight levels if a valid corresponding β can be found. In all instances, we set $\sigma = 25$ and $p = 4$ (newsvendor fractile $z = 80\%$) and, as before, set $n = 6$, $h = 1$ and $\mu = 150$. Figure 4 presents the CV difference $\frac{\rho[C_D^*]-\rho[C_C^*]}{\rho[C_C^*]}$, which captures the relative impact of cost variance with respect to expected cost, for the centralized and decentralized systems. The figure suggests that the disruption profile has a significant impact on the performance measures.

The difference in CV slightly increases when the percent down-time decreases. Moreover, the difference in CV increases when the disruption rate increases for given percent down-time. This implies that the risk-diversification effect is more pronounced when the disruptions are frequent/short rather than rare/long. These results confirm results by Snyder and Tomlin [2008], Tomlin [2006], and others that suggest that rare/long disruptions are more difficult to plan for than frequent/short ones. In other words, when disruptions are more frequent, the risk pooling effect is less pronounced and the risk diversification effect is more pronounced.

5.4 Optimal Risk-Averse Design

We have shown that for the OWMR system with both supply disruptions and stochastic demand, inventory decentralization reduces the cost variance via the risk-diversification effect while centralization reduces the expected cost via the risk-pooling effect. It is natural to ask which effect is more pronounced, and therefore which system a risk-averse decision maker should prefer.

5.4.1 Mean–Variance Objective

We employ the classical mean–variance approach, minimizing the following objective function:

$$(1-\kappa)E[C] + \kappa V[C] \tag{17}$$

where $\kappa \in [0, 1]$. The larger κ is, the more risk-averse the decision maker is. Many other objectives for risk-averse decision making have been proposed in the finance and operations literature. We chose to focus on the mean–variance objective primarily for analytical tractability. Numerical studies using the Conditional Value-at-Risk (CVaR) objective produced similar insights, but a more formal theoretical analysis was precluded by the increased complexity of the objective function.

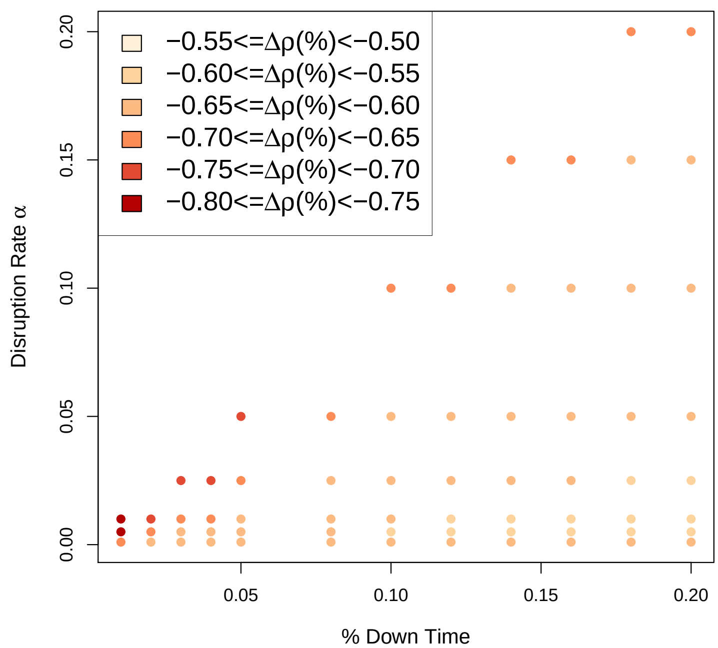


Figure 4: Reduction in coefficient of variation via decentralization increases when disruption rate α increases for different disruption profiles.

We optimized (17) numerically using a line-search procedure. This function is neither convex nor concave for all values of κ. There are values of κ and S_b such that (17) is convex and values for which it is not convex. By testing the function numerically, we found that for $\kappa \leq 0.05$, the range we used in our tests below, (17) is convex at nearly every value of S^b, with the exception of $S^b \approx 0$. We also found that the function is convex for all $S \geq \mu$, in the instances we tested, and this range typically includes the optimal base-stock level. Moreover, the second derivative of (17) is positive at the solution found by our optimization procedure for every instance we tested. Therefore, we are confident that our results represent the globally optimal solutions for each instance.

5.4.2 Numerical Study

One might argue that, because the variance term typically has a higher order of magnitude than the expected cost term, the mean-variance objective would artificially favor the decentralized system, since it has a lower cost variance. Therefore, we chose to minimize the mean-standard deviation objective $(1-\kappa)E[C]+\kappa\sqrt{V[C]}$ instead of the mean-variance objective $(1-\kappa)E[C]+\kappa V[C]$. (Our numerical tests, not given in detail here, confirm that the mean-variance objective gives similar results.) Our numerical study shows that even for very small values of κ, the decentralized system is almost always optimal under the mean-standard deviation objective $(1-\kappa)E[C]+\kappa\sqrt{V[C]}$. As long as disruption rate α is high enough (exceeds 0.01) or the recovery rate β is no better than 0.80 ($\beta = 1$ corresponds to immediate response), the decentralized system is always optimal. This implies that if the firm is subject to either frequent supply disruption or slow disruption response, than decentralized system configuration should be preferred for a risk-averse decision maker.

In order to investigate the conditions under which centralization would be an optimal choice, we consider the case where the supply is fairly stable. We set $n = 6$, $\mu = 150$, $\sigma = 25$, $\alpha = 0.001$, and $\beta = 0.95$, resulting in a percent down-time of 1.05%. Figure 5 plots the regions in which each system is optimal for $\kappa \in [0,1]$ and $p/(p+h) \in (0,1)$ (keeping h fixed to 1, i.e., $p \in (0,\infty)$) and a zoom-in when the newsvendor fractile is greater than 0.5, i.e., $p \in [1,\infty)$. The region in which the centralized system is optimal is barely visible for the choice of high service level / high stockout penalty. Clearly, the centralized system is optimal only for very risk-neutral decision makers operating under low service levels: for $\kappa \geq 0.08$ and $p/(p+h) \geq 0.5$, the decentralized system is almost always optimal. When $\kappa = 0.5$, i.e., when the decision maker places the same importance on the expected cost and the cost variation (standard deviation), centralization is optimal only if the stockout penalty is under 0.2, i.e., centralization is optimal only if the service level is set to be

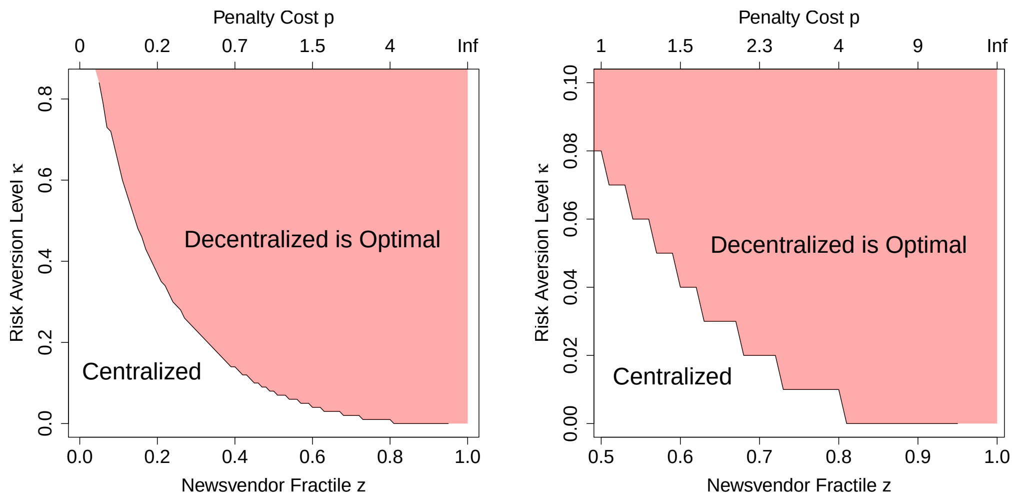


Figure 5: Left: Optimal system configuration under 1.05% down time with various choices of newsvendor fractile z / stockout penalty p and risk aversion level κ. Right: A zoom-in of the left figure for $z \geq 0.5$.

very low. The intuition behind this result can be reasoned as follows. When stockouts are relatively expensive, i.e., z is low, the impact of supply risk on the cost is small compared to the demand risk, and therefore the firm should centralize in order to mitigate the demand risk through the risk pooling effect. When stockouts are relatively expensive, i.e., z is high, the variability dominates the objective, and the firm therefore should decentralize in order to mitigate the supply risk due through the risk diversification effect.

The numerical studies suggest that the magnitude of the cost variance dominates the expected cost for a risk-averse decision maker, as was implied in Figure 1. Therefore, a risk-averse decision maker will always prefer a decentralized system configuration when the uncertainty in supply reaches a certain level.

5.5 Summary of Numerical Comparisons

For the multi-location system subject to both demand uncertainty and supply disruptions:

- The risk-diversification effect exists, resulting in lower cost variance in the decentralized system
  - The risk-diversification effect is more pronounced when disruptions are more frequent, longer and more costly
- The risk-pooling effect exists, resulting in lower expected cost in the centralized system

  - An increase in the number of locations amplifies the risk-pooling effect as well as the risk-diversification effect
  - The magnitude of the risk-pooling effect is less than that of the risk-diversification effect unless the supply disruptions are rare or the stockout penalty is low
- The number of demand locations affects the optimal solutions
  - The risk-pooling effect does not always result in lower total base-stock levels
  - The risk-pooling effect is more pronounced when disruptions are less frequent and shorter, or when demand is more variable
- Disruption profiles affect the optimal solutions
  - For the same percent down-time, more frequent disruptions result in more pronounced risk-diversification effect
  - Rare/long disruptions are more difficult to plan for than frequent/short ones through decentralization
- For a risk-averse firm, the decentralized system is typically the optimal inventory design
  - This holds unless service level (newsvendor fractile), κ value, or failure probabilities are very low

In conclusion, a risk-averse decision maker will prefer a decentralized system unless supply uncertainty is very low or the penalty cost is low, because the magnitude of risk-diversification effect dominates the risk-pooling effect in most of the cases.

6 Conclusions

In this paper, we consider a multi-location system with both supply and demand uncertainty. We investigate the risk-diversification effect, in which the expected cost is the same in the centralized and decentralized systems but the cost variance is smaller in the decentralized system. The intuition behind this effect is that by distributing inventory at multiple sites, the impact of any one disruption is smaller, even though each site is still affected by the same number of disruptions. The firm benefits from not putting all its eggs in one basket; although the same number of eggs may be destroyed, they are not all destroyed at once. We proved that the risk-diversification effect occurs in multi-location systems with supply disruptions.

In contrast, the classical risk-pooling effect prevails under demand uncertainty; under this effect, the expected cost is smaller in the centralized system, and we prove that the variance is equal in the two systems. We showed numerically that, when disruptions and demand uncertainty are both present, both effects occur to a certain extent, but that in most cases the risk-diversification effect strongly dominates the risk-pooling effect. Therefore, a decentralized system is optimal when disruptions and demand uncertainty are both present, except when the service level (newsboy fractile) is very low, the firm is very risk neutral, and/or the system is very reliable.

Natural extensions to this research can be made by relaxing the assumptions in our models. For example, we assumed zero fixed ordering costs. An argument could be made that this ignores an additional cost benefit from centralization; synchronizing orders could reduce the total fixed costs. However, as we have shown, risk diversification would suggest that a risk-averse firm should *not* synchronize orders if individual shipments may be disrupted at the supplier.

Another extension could relax our assumption of independently distributed disruptions. Eppen [1979] shows that demand correlation affects the magnitude of the risk pooling effect; positively correlated demand decreases the risk pooling benefits of centralization and negatively correlated demand increases them. A similar study could be conducted for supply uncertainty. We would expect that positively correlated supply disruptions would decrease the risk diversification benefits of decentralization, whereas negatively correlated disruptions would increase those benefits.

Another obvious extension to our work could address the assumption that inventory can only be held at one echelon of the supply chain, which we made to simplify our analysis and clarify our conclusions. Of course, centralized and decentralized systems are not the only choices available to a firm operating in a multi-location environment; the firm may also choose a hybrid system in which inventory is held at both echelons. Such a system may well provide a desirable balance between the risk-diversification and risk-pooling effects. These systems are significantly harder to analyze (as the literature on multi-location system attests), but an investigation of these two competing effects in this context is an important avenue for future research.

7 Acknowledgments

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A APPENDIX: PROOFS

A.1 Loss Functions

Let X be a random variable with pdf $f(x)$ and cdf $F(x)$. The loss function for X is given by:

$$G(x) = \int_x^\infty (t-x)f(t)dt = \int_x^\infty (1-F(t))dt \tag{18}$$

and the second-order loss function is:

$$H(x) = \frac{1}{2}\int_x^\infty (t-x)^2 f(t)dt = \int_x^\infty G(t)dt. \tag{19}$$

Let $\Phi^1(z)$ be the standard normal loss function and $\Phi^2(z)$ as the standard normal second-order loss function.

The following lemma presents properties we use in the proofs below. The proof of the lemma is omitted; it follows from well known results concerning loss functions [e.g., Axsäter, 2000, Zipkin, 2000].

Lemma 7 *1. Let X have cdf F and loss function G. Then for any function $\theta(x)$,*

$$\frac{d}{dx}G(\theta(x)) = -\theta'(x)[1-F(\theta(x))]. \tag{20}$$

2. If X is normally distributed with mean μ and variance σ^2 and $f(x)$ is its pdf, then

$$\int_S^\infty (x-S)^2 f(x)dx = 2\sigma^2\Phi^2\left(\frac{S-\mu}{\sigma}\right). \tag{21}$$

A.2 Proof of Theorem 1, Section 3.2

A.2.1 Optimal Base-Stock Levels

$S_D^{s*} = S^{s*}$ since each warehouse acts as a single-facility system. Moreover, from (2) we know that $S^{s*} = jd$, where j is the smallest integer such that $F(j-1) \geq \frac{p}{p+h}$. Similarly, in the centralized system, the optimal base-stock level is $S_C^{s*} = jnd$ for the same j (since the definition of j does not depend on the demand). Therefore, $S_C^{s*} = nS^{s*}$.

A.2.2 Expected Costs at Optimality

In the decentralized system, the total expected cost is simply the sum of the single-facility costs, so $E^s[C_D] = nE^s[C]$.

In the centralized system, the optimal expected cost at the centralized warehouse, using nS^{s*} in place of S_C^{s*} in (1), is given by:

$$
\begin{aligned}
E^s[C_C^*] &= \sum_{i=1}^{\infty} \pi_i [h(nS^{s*} - (i+1)nd)^+ + p((i+1)nd - nS^{s*})^+] \\
&= n\sum_{i=1}^{\infty} \pi_i [h(S^{s*} - (i+1)d)^+ + p((i+1)d - S^{s*})^+] \\
&= nE^s[C^*] = E^s[C_D^*] \qquad (22)
\end{aligned}
$$

A.2.3 Cost Variances at Optimality

For the decentralized system, since each warehouse operates independently, the variance of the cost is equal to the sum of the single-warehouse variances, or $V_s[C_D] = nV_s[C]$.

Now, for a single warehouse,

$$
E^s[(C^*)^2] = \sum_{i=1}^{\infty} \pi_i \left[h^2 \left((S^{s*} - (i+1)d)^+\right)^2 + p^2 \left(((i+1)d - S^{s*})^+\right)^2\right] \qquad (23)
$$

In the centralized system, using $S_C^{s*} = nS^{s*}$,

$$
\begin{aligned}
E^s[(C_C^*)^2] &= \sum_{i=1}^{\infty} \pi_i \left[h^2 \left((nS^{s*} - (i+1)nd)^+\right)^2 + p^2 \left(((i+1)nd - nS^{s*})^+\right)^2\right] \\
&= n^2 \sum_{i=1}^{\infty} \pi_i \left[h^2 \left((S^{s*} - (i+1)d)^+\right)^2 + p^2 \left(((i+1)d - S^{s*})^+\right)^2\right] \\
&= n^2 E^s[(C^*)^2] \qquad (24)
\end{aligned}
$$

Thus, using (22) and (24),

$$
\begin{aligned}
V^s[C_C^*] &= E^s[(C_C^*)^2] - E^s[C_C^*]^2 = n^2 E^s[(C^*)^2] - (nE^s[C^*])^2 = n^2(E^s[(C^*)^2] - E^s[C^*]^2) \\
&= n^2 V^s[C^*] \qquad (25)
\end{aligned}
$$

Thus $V^s[C_C^*] = nV^s[C_D^*]$. $\square$

A.3 Proof of Theorem 4, Section 4.2

At optimality, the centralized and decentralized base-stock levels satisfy

$$
\frac{S_D^{d*} - \mu}{\sigma} = \Phi\left(\frac{p}{p+h}\right) = \frac{S_C^{d*} - n\mu}{\sigma_C}. \qquad (26)
$$

Let

$$
X \equiv \Phi\left(\frac{p}{p+h}\right) \qquad (27)
$$

Using (21) from Lemma 7,

$$\begin{aligned}
E^d[C^2] &= h^2 \int_{-\infty}^{S^d} (S^d - d)^2 f(d) dd + p^2 \int_{S^d}^{\infty} (d - S^d)^2 f(d) dd && (28)\\
&= h^2 \int_{-\infty}^{\infty} ((S^d)^2 - 2S^d d + d^2) f(d) dd + (p^2 - h^2) \int_{S^d}^{\infty} (d - S^d)^2 f(d) dd && (29)\\
&= h^2((S^d)^2 - 2S^d \mu + \sigma^2 + \mu^2) + 2\sigma^2 (p^2 - h^2) \Phi^2 \left(\frac{S^d - \mu}{\sigma} \right) && (30)
\end{aligned}$$

At optimality, we have

$$E^d[(C^*)^2] = h^2((S^{d*})^2 - 2S^{d*}\mu + \sigma^2 + \mu^2) + 2\sigma^2(p^2 - h^2)\Phi^2(X) \tag{31}$$

In addition, from (3),

$$E^d[C^*] = h(S^{d*} - \mu) + \sigma(p + h)\Phi^1(X). \tag{32}$$

Therefore,

$$\begin{aligned}
V^d[C^*] &= E^d[(C^*)^2] - E^d[C^*]^2 \\
&= h^2((S^{d*})^2 - 2S^{d*}\mu + \sigma^2 + \mu^2) + 2\sigma^2(p^2 - h^2)\Phi^2(X) \\
&\quad - \left[h(S^{d*} - \mu) + \sigma(p + h)\Phi^1(X) \right]^2 \\
&= h^2((S^{d*})^2 - 2S^{d*}\mu + \sigma^2 + \mu^2) + 2\sigma^2(p^2 - h^2)\Phi^2(X) \\
&\quad - \left[h^2((S^{d*})^2 - 2S^{d*}\mu + \mu^2) + 2\sigma h(p + h)(S^{d*} - \mu)\Phi^1(X) + \sigma^2(p + h)^2\Phi^1(X)^2 \right] \\
&= h^2\sigma^2 + 2\sigma^2(p^2 - h^2)\Phi^2(X) - 2\sigma h(p + h)(S^{d*} - \mu)\Phi^1(X) - \sigma^2(p + h)^2\Phi^1(X)^2 \\
&= \sigma^2 \left[h^2 + 2(p^2 - h^2)\Phi^2(X) - 2h(p + h)X\Phi^1(X) - (p + h)^2\Phi^1(X)^2 \right] && (33)
\end{aligned}$$

Let Y equal the sum of the terms inside the brackets. Note that Y is a function of X, not of σ or μ alone; therefore, Y is the same for an individual warehouse or for the centralized warehouse in the centralized system.

Since the decentralized system functions as n individual warehouses, we have

$$V^d[C^*_D] = nV^d[C^*]. \tag{34}$$

Applying (33),

$$V^d[C^*_C] = \sigma^2_C Y = n\sigma^2 Y = nV^d[C^*] = V^d[C^*_D]. \tag{35}$$

as desired. □

A.4 Proof of Proposition 5, Section 5.1

$E^b[C_D]$ is simply equal to $nE^b[C]$. Similarly, $E^b[C_C] = nE^b[C]$, but with the pooled demand standard deviation equal to $\sigma\sqrt{n}$ and the demand mean equal to $n\mu$. □

A.5 Proof of Proposition 6, Section 5.2

Using (21) from Lemma 7, we get

$$\begin{aligned}
E^b[C^2] &= \sum_{i=1}^{\infty} \pi_{i-1} \left(h^2 \int_{-\infty}^{S^b} (S^b - id)^2 f_i(id)dd + p^2 \int_{S^b}^{\infty} (id - S^b)^2 f_i(id)dd \right) \\
&= \sum_{i=1}^{\infty} \pi_{i-1} \left(h^2 \int_{-\infty}^{\infty} ((S^b)^2 - 2S^b id + i^2 d^2) f_i(id)dd + (p^2 - h^2) \int_{S^b}^{\infty} (id - S^b)^2 f_i(id)dd \right) \\
&= \sum_{i=1}^{\infty} \pi_{i-1} \left(h^2((S^b)^2 - 2S^b i\mu + i\sigma^2 + i^2\mu^2) + 2i\sigma^2(p^2 - h^2)\Phi^2 \left(\frac{S^b - i\mu}{\sigma\sqrt{i}} \right) \right) \qquad (36)
\end{aligned}$$

$$\begin{aligned}
V^b[C] &= E^b[C^2] - (E^b[C])^2 \\
&= \sum_{i=1}^{\infty} \pi_{i-1} \left(h^2((S^b)^2 - 2(S^b)i\mu + i\sigma^2 + i^2\mu^2) + 2i\sigma^2(p^2 - h^2)\Phi^2 \left(\frac{S^b - i\mu}{\sigma\sqrt{i}} \right) \right) - \\
&\quad \left(\sum_{i=1}^{\infty} \pi_{i-1} \left(h(S^b - i\mu) + \sigma\sqrt{i}(p + h)\Phi^1 \left(\frac{S^b - i\mu}{\sigma\sqrt{i}} \right) \right) \right)^2 \qquad (37)
\end{aligned}$$

In the decentralized system, the total variance is just the sum of the individual-warehouse variances since each retailer operates independently. In the centralized system, the total demand seen by the centralized warehouse is distributed as $N(ni\mu, ni\sigma^2)$. Therefore,

$$\begin{aligned}
V^b[C_C] &= \sum_{i=1}^{\infty} \pi_{i-1} \Bigg(h^2((S_C^b)^2 - 2S_C^b ni\mu + ni\sigma^2 + n^2 i^2 \mu^2) + \\
&\quad 2ni\sigma^2(p^2 - h^2)\Phi^2 \left(\frac{S_C^b - ni\mu}{\sigma\sqrt{ni}} \right) \Bigg) - \\
&\quad \left(\sum_{i=1}^{\infty} \pi_{i-1} \left(h(S_C^b - ni\mu) + \sigma\sqrt{ni}(p + h)\Phi^1 \left(\frac{S_C^b - ni\mu}{\sigma\sqrt{ni}} \right) \right) \right)^2 \qquad (38)
\end{aligned}$$

□