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A Quantum Interior Point Method for Semidefinite Optimization Problems

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Abstract

We present a provably convergent quantum interior point method for semi-definite and linear optimization problems. We provide explicit definitions for the Newton linear systems corresponding to the Nesterov-Todd and Alizadeh-Haeberly-Overton search directions, and show how to solve these symmetrized linear systems using a quantum computer. These results are based on recent progress in quantum linear system solvers and on the block-encoding technique. In the presence of quantum-accessible storage (also known as QRAM), we are able to obtain speedups in terms of the size of the problem, at the price of a worse dependence on the condition number. We compare the theoretical performance of classical and quantum interior point methods with respect to various input parameters, concluding that based on known lower bounds and the current state of the art, we do not anticipate a clear quantum speedup for the solution of classically-specified problems, for methods based on solving the Newton linear system.

1 Introduction

In this paper, we develop quantum interior point methods (QIPMs) for the solution of semidefinite optimization problems (SDOPs). Letting $c \in \mathbb{R}^m$, matrices $A^{(1)}, \dots, A^{(m)}, B \in \mathcal{S}^n$, where $\mathcal{S}^n$ is the subspace of $n \times n$ symmetric matrices, we consider the primal SDOP given as

$$z_P = \min_{x \in \mathbb{R}^m} \left\{ c^T x \mid \sum_{i \in [m]} x_i A^{(i)} \succeq B \right\}, \tag{1}$$

where $[m] = \{1, \dots, m\}$, $U \succeq V$ indicates that $U - V$ is a symmetric positive semidefinite matrix. We further assume that the matrices $A^{(1)}, \dots, A^{(m)}$ are linearly independent. Let $S \in \mathcal{S}$ denote the slack matrix of the primal problem, i.e.,

$$S = \sum_{i \in [m]} x_i A^{(i)} - B \succeq 0. \tag{2}$$