Industrial and
Systems Engineering

Accurately Solving Linear Systems with Quantum Oracles

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ISE Technical Report 23T-006

LEHIGH UNIVERSITY

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January 4, 2023

Abstract

We present an Iterative Refinement (IR) scheme to improve the error dependence of algorithms that rely on a quantum linear systems algorithm (QLSA) followed by state tomography. Existing QLSAs depend polylogarithmically on the inverse precision, but the naive approach to extract a QLSA solution with tomography incurs a polynomial dependence on the inverse precision. Our IR scheme reduces this error dependence to polylogarithmic, by solving a sequence of related linear systems with fixed precision. With quantum-accessible classical storage (QRAM) and classically available data, our IR scheme solves a $d \times d$ linear system $Mx = z$ and outputs an ϵ-precise solution in time $\tilde{\mathcal{O}}(d\kappa_M + ds)$, where s is the maximum number of nonzeros found in any row of M, κ_M is an upper bound on the condition number of the coefficient matrix M, and the $\tilde{\mathcal{O}}$ suppresses polylogarithmic factors. This approach leads to an exponential improvement in the precision parameter of quantum interior point methods for semidefinite optimization, reducing their complexity to $\tilde{\mathcal{O}}\left(\sqrt{n}\left(n^2\kappa + n^4\right)\right)$ if the matrices are of size $n \times n$, where κ is an upper bound on the condition numbers of the coefficient matrices of the Newton linear systems that arise during the course of the algorithm.

1 Comment

This paper is revised deeply and the proposed algorithm, its complexity analysis, and all theoretical and numerical results are available in another paper with different title as follows.

Exponentially More Precise Tomography for Quantum Linear System Solutions via Iterative Refinement, *Mohammadhossein Mohammadisiahroudi, Brandon Augustino, Ramin Fakhimi, Giacomo Nannicini, Tamás Terlaky.*

2 Acknowledgement

This work is supported by Defense Advanced Research Projects Agency as part of the project W911NF2010022: *The Quantum Computing Revolution and Optimization: Challenges and Opportunities*.

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