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Exponentially More Precise Tomography for Quantum Linear System Solutions via Iterative Refinement

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Abstract

We present an Iterative Refinement (IR) scheme to improve the error dependence of algorithms that rely on a quantum linear systems algorithm (QLSA) followed by state tomography. Existing QLSAs depend polylogarithmically on the inverse precision, but the naive approach to extract a QLSA solution with tomography incurs a polynomial dependence on the inverse precision. Our IR scheme reduces this error dependence to polylogarithmic, by solving a sequence of related linear systems with fixed precision. With quantum-accessible classical storage (QRAM) and classically available data, our IR scheme solves a $d \times d$ linear system $Mx = z$ and outputs an ϵ-precise solution in time $\widetilde{\mathcal{O}}_{d,\kappa_M,\frac{1}{\epsilon}}(d\kappa_M + ds)$, where s is the maximum number of nonzeros found in any row of M, κ_M is an upper bound on the condition number of the coefficient matrix M, and the $\tilde{\mathcal{O}}$ suppresses polylogarithmic factors. This approach leads to an exponential improvement in the precision parameter of quantum interior point methods for semidefinite optimization, reducing their complexity to $\widetilde{\mathcal{O}}_{d,\kappa_M,\frac{1}{\epsilon}}\left(\sqrt{n}\left(n^2\kappa + n^4\right)\right)$ if the matrices are of size $n \times n$, where κ is an upper bound on the condition numbers of the coefficient matrices of the Newton linear systems that arise during the course of the algorithm.

1 Introduction

It is widely believed that quantum computers can efficiently solve some problems that do not admit efficient classical algorithms. There are many examples of quantum speedups [37, 21]. The latest advances in the size and capabilities of noisy-intermediate scale quantum (NISQ) devices [36] have added further momentum to the development of theory and algorithms for quantum computing, with the overarching goal of evidencing a quantum advantage.

Recently, a considerable amount of attention has been devoted to quantum linear algebra; with a particular interest in a class of methods that use Hamiltonian simulation subroutines to prepare a quantum state $|x\rangle$ that is proportional to the solution of the linear system

$$Mx = z, \tag{1}$$

given a matrix $M \in \mathbb{R}^{d\times d}$ and a vector $z \in \mathbb{R}^d$. In the standard setting, M has a known condition number κ_M and at most s nonzero entries per row. Moreover, it is generally assumed we have oracles which provide access to the entries of M, and the ability to prepare a quantum state $|z\rangle$ that is proportional to the right hand side vector z. Research into this subfield began with the work of Harrow, Hassidim, and Lloyd [22], who proposed what has come to be known as the HHL algorithm for solving the *quantum linear systems*

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