Integrating Production and Transportation Logistics in a Supply Chain Environment: A Lagrangean Decomposition Approach

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INTEGRATING PRODUCTION AND TRANSPORTATION LOGISTICS IN A SUPPLY CHAIN ENVIRONMENT: A LAGRANGEAN DECOMPOSITION APPROACH

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ABSTRACT

Integrating management across different functional areas within a firm or across multiple supply segments is a main issue in supply chain coordination. In this paper, we consider the integration of production and transportation logistics. We focus our attention on manufacturing supply chains, where transportation constitutes the second largest cost component following the production operations. Although the benefit of integrating these two functions is intuitively obvious, the fundamental tradeoff between costs and level-of-service is not well understood. We propose a model that integrates production planning and transportation routing at the operational level. Our method allows optimization to reconcile the viewpoints from transportation and production planning. The process of optimization can be viewed metaphorically as a negotiation process between a set of interrelated production facilities and a thrid party logistic provider. We introduce basic production and transportation models that are tailored to this particular integration and show the value of the integration using a Lagrangean decomposition scheme.

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1. Introduction

Since the early 1990's, leading manufacturing firms start to recognize the cost- reduction potentials offered by supply chain coordination. Integration across multiple facilities, multiple levels of the production hierarchy, or across multiple segments in the value-added process, could bring significant cost improvement by utilizing resources more efficiently throughout the chain while improving customer service. The driving forces behind this strategic direction are the ever-increasing market pressure on cost efficiency and the rapid advances in the information technologies. In this paper, we will tackle a critical segment of supply chain coordination focusing on the integration of production and transportation planning.

Major cost factors encountered within a manufacturing supply chain includes production, transportation, inventory, and material handling costs. The portion of these cost factors within the total cost varies largely by industry. However, production cost is the largest of all in almost all the industries, followed by transportation and inventory cost (Chen (1997)). Determining factors for transportation cost includes delivery frequency and quantity. In-transit inventory cost might be another significant factor depending on the distance and distribution channel between the layers of the supply chain. Adopting a continuous replenishment policy within the supply chain might reduce the inventory while increasing the transportation cost due to underutilized transportation resources. On the other hand, a truckload replenishment policy would have the opposite effect on the inventory and transportation costs.

Within a supply chain, there are two major inventory classes at each production location; finished goods and component inventories. The cost incurred for these inventories are reflected to the customer at the end of the supply chain as an additional cost. These inventories are driven by production, transportation and end item demand. If production and transportation are well aligned and synchronized with end item demand, there will be minimum inventories in the system.

Historically, production and transportation planning have been dealt with separately both in industry and academia. In industry, a production plan is developed and then a transportation plan is worked out by either the transportation department of a company, or a third party logistics company, in order to make sure that the developed production plan is realized. The extent to which the transportation costs are considered in the production plan does not go beyond a simple evaluation of a few transportation channels. Materials requirements planning, the most common decision support tool for production planning in the industry, does not even consider the capacity restrictions of production resources, let alone the transportation cost. Parallel to industry practice, researchers in academia have approached the two problems separately. On production planning problems, there has been an enormous body of research for lot sizing models. The main tradeoff considered is between inventory carrying and setup costs. For the multi-level, multi-item, multi-period dynamic lot-sizing problem (MLMILP), most of the research focuses on the development of heuristics due to the computational requirements of practical size problems. On transportation planning problems, vehicle routing and scheduling has also been studied extensively in the literature, ranging from the basic traveling salesman problem to multi-vehicle pickup and delivery problem with time windows (m-PDPTW). But considering the two problems together, which we are attempting to do in this paper, seems to be a lacking research area.

Chandra and Fisher (1994) empirically show the value of integrating production and transportation decisions in an environment which involves a single production facility and multiple customers. They report gains ranging between 3% to 20% obtained by integrating production planning and vehicle routing in a heuristic manner. Hahm and Yano (1995a, 1995b) consider the economic lot and delivery scheduling problem which simultaneously determine production scheduling of a captive supplier and deliveries from that supplier to the customer. They propose optimization models that minimize the average transportation cost, inventory at both the customer and supplier sites, and setup costs. Thomas and Griffin (1996) provide a survey of various functional coordination, they categories existing literature into buyer-vendor coordination, production-distribution coordination, and inventory-distribution coordination. Vidal and Goetschalckx (1997) provide an extensive literature review focusing on strategic production-distribution coordination,

In this paper, we provide a somewhat different perspective focusing on the integration of production and transportation planning among multiple manufacturing facilities. These manufacturing facilities are interdependent and their production requirements are linked by product bill-of-material structures. We propose production planning and routing scheduling models that are tailored to integrated decision making across these facilities. To show the value of integration, we analyze the trade-off between the two models using a Lagrangean Decomposition scheme. Next, we breifly review the literature for the two main parts of the problem, namely MLMILP and m-PDPTW, as they relate to integrated decision making.

Multi-level multi-item dynamic capacitated lot sizing problem

MLMILP can be defined as follows. Given the external demand for end items over a time horizon of T periods, find a production plan that minimizes total inventory holding and setup costs under the following restrictions:

- Items can be produced only after all their predecessor components become available.
- Capacity of the resources is limited.
- No backlogging for the end items is allowed.

This problem arises as a part of the planning steps within the widely adopted MRPII logic. There have been several formulations of the problem in the literature (Stadtler 1996). In the most general case, it has been shown that even finding a feasible solution to the problem is NP complete (Maes et al. 1991). Very few exact methods for the problem in the literature (Pocket et al. 1991, Chapman 1985) are restricted to problems of limited size. A majority of the research has been concentrated on heuristic approaches ranges from Lagrangean relaxation to tabu search. Zahorik et al. (1984) describe an optimization based heuristic approach, employing a 3-period network flow formulation of the problem, for a quite restrictive case with no setup cost or time. Billington et al. (1986) introduce a heuristic using Lagrangean relaxation. It is assumed that the capacity restriction exists only at one level in the BOM structure. They use a price-directed approach to reflect both the multilevel structure and the capacity limitations. Maes et al. (1991) address the complexity of finding feasible solutions to capacitated MLMILP and present three similar heuristics for the solution. Roll and Karni (1991) present a heuristic approach which consists of the application of eight different subroutines. These subroutines either convert an infeasible solution to a feasible one or improve a given solution. Maes and Van Wassenhove (1991) extend their ABC algorithm (Maes and Van Wassenhove 1986) for capacitated single level lot sizing problems to multilevel problems with serial BOM structure. Kuik et al. (1993) propose simulated annealing and tabu search in their search heuristics. They define a search neighborhood using the setup variables. They computationally show that these heuristics perform better on the average compared to those in Maes et al. (1991). Tempelmeier and Helber (1994) address four variants of a two-phase heuristic approach for the problem. In the first phase, the problem is decomposed in to a set of single level capacitated multi-item lot sizing problems and an ordering for the solution of these single level problems is determined. In the second phase, the single level problems are solved in the order determined in the first phase by using a heuristic due to Dixon and Silver (1981). Tempelmeier and Derstroff (1996) propose a method that Lagrangean relax the inventory and capacity constraints to obtain single item uncapacitated lot-sizing subproblems. Using a subgradient search algorithm and a feasibility restoration heuristic, they are able to produce very good results. Stadtler (1996) collectively presents five mixed integer programming formulations of the problem proposed in the literature including network based plant location and shortest route models. He compares these models in terms of problem size, integrality gap of LP relaxation, and computation time. Two recent survey papers regarding the problem are given by Drexl and Kimms (1997), and Kuik et al. (1994).

Conventional MLMILP models do not involve transportation time between production locations. Delivery of goods is generally assumed to be instantaneous. The only exception, to the best of our knowledge, is the model presented in Haq et al (1991), where the authors take transportation time into consideration. But a shortcoming of this model is that the transportation times are taken into account by utilizing the indices of variables and, therefore, they have to be integral multiples of the unit production planning period. This may significantly limit the potential benefit of integrating production and transportation.

Multiple vehicle pickup and delivery problem with time windows

The multiple vehicle pickup and delivery problem with time windows (m-PDPTW) can be stated as construction of optimal routes for a set of vehicles so as to satisfy a specified set of transportation requests. Each request is described by a pickup location, a delivery location, the amount of load to be transported, and pickup and delivery service time windows. Optimal routes here are ones that optimize a certain objective function. We give a mathematical formulation of the problem in section 2.3.

A typical objective for problems concerning transportation of goods is minimization of the total distance traveled by the vehicles or the total travel time. The minimization of the number of vehicles used or the associated total vehicle fixed cost might be considered together with the above objectives. The m-PDPTW that involves the transportation of people is called the dial-a-ride problem in the literature. Because drivers and vehicles are the most expensive parts in a dial-a-ride system, minimizing the number of vehicles to serve all the requests is commonly the objective. In dial-a-ride systems, minimizing client inconvenience is another usual objective. Client inconvenience is created by the pickups or deliveries that are performed either before or after the customer-prescribed time windows. m-PDPTW in the context of transportation of goods has not received the same extensive attention as some other vehicle scheduling problems. Almost all models presented in the literature assume the load size to be transported and the transportation time allowances as parameters. Jaw et al. (1986) address an insertion based heuristic algorithm for multiple vehicle dial-a-ride problems. The objective is to minimize a weighted combination of customer inconvenience and system cost. Bodin and Sexton (1986) present a cluster first and route second type approach for dial-a-ride type problems with multiple vehicles. In the first phase, clients are assigned to vehicles and a single vehicle dial-a-ride problem for each vehicle is solved to get an overall initial solution. This initial solution is improved in the second phase by reassigning clients with bad service to other vehicles. Dumas et al. (1989) develop a heuristic approach for the same type problems. Their heuristic uses the traditional cluster first route second approach but moves a part of the clustering problem into the routing problem. The main idea is to create route segments for small groups of requests, called mini-clusters, then stringing them together to find a solution. Mini-clusters are basically the route segments which starts and ends with an empty vehicle. Ioachim et al. (1991) illustrate a two phase heuristic approach which uses the mini-clustering idea. In the first phase, mini-clustering problem is solved. They use mathematical optimization techniques to define a set of mini-clusters. In the second phase, the problem of linking the mini-clusters to construct routes for vehicles is modeled and solved as m-TSPTW. Destrosiers et al. (1991) suggest a parallel insertion heuristic to create mini-clusters. They base their insertion criterion on the concept of neighboring requests which is defined as the set of request satisfying certain temporal or spatial proximity restrictions. Fisher et al. (1995) describe a heuristic approach for a truck-load type problem where vehicles can not carry more than one load at a time and, therefore, they go to the delivery location directly after visiting the pickup point. The approach utilizes a network flow based relaxation to find near feasible solutions. Then these solutions are made feasible using some simple heuristic techniques. The computational study involving some real problems shows that the approach resulted into solutions within 1% of the optimal.

An optimal algorithm for the problem in the context of transportation of goods is proposed by Dumas et al. (1991). They present a set partitioning formulation for m-

PDPTW and use a column generation solution procedure. Solving a linear relaxation of the set partitioning formulation, the authors use the column generation idea to introduce new columns corresponding to admissible routes. Columns in the set of admissible routes are generated as needed, as a solution to the constrained shortest path problem which defines the admissible routes. Ruland and Rodin (1997) explore the polyhedral structure of the dial-a-ride problems and develop four classes of valid inequalities. They use these inequalities in a branch-and-cut procedure to solve the problem. Savelsbergh and Sol (1995) describe the general characteristics of the problem and present a survey.

2. Problem Description and Formulation

2.1 Problem description

In a broad sense, we can state our problem as the integration of production and transportation planning in a supply chain environment so as to minimize the operational cost over these functions. A complementary relationship exists between the two: production facilities require dependable and cost-efficient delivery services to carry out production without excessive inventory, transportation providers rely on leveraging opportunities across multiple facilities for better utilization of their resources. To minimize their joint operational costs requires a basic understanding of the trade-off between costs and level-of-service.

Traditionally production and transportation functions are separated using large inventory buffers. But there is a constant market pressure for firms to reduce the inventory in their production channels and many industries now operate in a just-in-time manner. It is therefore a pressing issue for firms to explore much closer coordination between production and transportation functions. We consider our problem in the following context: a manufacturing supply chain has multiple production facilities which coordinate their productions following a general product structure. Pick-up and delivery services among the facilities are carried out by a third-party logistics carrier. To explore the potentials for cost saving, we will assume that the logistics carrier is capable of dynamic routing and scheduling of their transportation resources given up-to-date production requirements of all facilities. We use a MLMILP model to represent main production planning decisions across these facilities. And we use the multiple vehicle pickup and delivery model with time windows (m-PDPTW) to characterize the carrier's routing and scheduling decisions.

Both MLMILP and m-PDPTW need to be modified for this purpose. The former model assumes all deliveries are instantaneous while the latter assumes fixed loads to be picked up and delivered and there is a fixed travel time allowance for each request. Since we are trying to integrate two planning activities, we will be seeking the size of the loads and the travel time allowances that are mutually beneficial to both production and transportation. This results in generalized versions of MLMILP and m-PDPTW models.

2.2 Production planning model

Notation used in the model is as follows :

Variables ;

Q_{it} : Amount of production of item i in period t.

I_{it} : Inventory of item i as finished good at the end of period t.

CI_{ijt} : Inventory of item i as subassembly to be used in the production of item j at the end of period t.

PS_{it} : Production start time of item i in period t in terms of time buckets.

SL_{ijt} : Size of load of item i which is sent in period t for the production of item j.

V_{it} : Binary setup variable for item i in period t.

Parameters;

hc_i : Inventory holding cost of item i per period when item i is regular inventory.

sc_i : Setup cost for item i.

$deltah_i$: Percent increase over hc_i in the inventory holding cost of item i when item i becomes subassembly inventory.

ini_i : Initial inventory of item i.

$inci_{ij}$: Initial subassembly inventory of item i for the production of item j.

st_i : Setup time for item i in time buckets.

pt_i : Production time for item i in time buckets.

trt_{ij} : Traveling time between production location i and j in time buckets.

cp_i : Amount of capacity consumed when one unit of item i is produced.

cs_i : Amount of capacity consumed when a setup for item i is performed.

lc_i : Production location where item i is produced.

cap_{it} : Capacity of resource i in period t.

dem_{it} : Demand for item i in period t.

a_{ij} : Number of item i required to produce one unit of item j.
per : Length of a production planning period in time buckets.
M : A large number.

Sets;
T : Set of planning periods.
I : Set of items.
N_i : Set of indices of the immediate successor items of item i in the product structure.
RS : Set of resources
RS_i : Set of items that are produced at resource i.
L : Set of pairs of indices (i, j) representing the links between items in the product structure.

Following is the production planning model we propose:

$$Min \sum_{t\in T}\left(\sum_{i\in I}\left(hc_i I_{it} + sc_{it} V_{it} \right) + \sum_{(i,j)\in L} (1+deltah_i) hc_i CI_{ijt} \right) \quad (1)$$

$subject\,to$:

$$I_{it-1} + Q_{it} - I_{it} - dem_{it} - \sum_{j\in N_i} SL_{ijt} = 0 \qquad \forall i \in I, \forall t \in T \backslash \{1\} \quad (2)$$

$$ini_i + Q_{i1} - I_{i1} - dem_{i1} - \sum_{j\in N_i} SL_{ij1} = 0 \qquad \forall i \in I \quad (3)$$

$$CI_{ijt-1} + SL_{ijt-1} - CI_{ijt} - a_{ij} Q_{jt} = 0 \qquad \forall (i,j) \in L, \forall t \in T \quad (4)$$

$$inci_{ij} - CI_{ij1} - a_{ij} Q_{i1} = 0 \qquad \forall (i,j) \in L \quad (5)$$

$$(t-1)per + trt_{lc_i lc_j} \leq PS_{jt} + st_j + CI_{ijt-1}(pt_j / a_{ij}) \qquad \forall (i,j) \in L, \forall t \in T \backslash \{1\} \quad (6)$$

$$(t-1)per \leq PS_{it} \qquad \forall i \in I, \forall t \in T \quad (7)$$

$$PS_{it} + st_i + pt_i Q_{it} \leq t\,per \qquad \forall i \in I, \forall t \in T \quad (8)$$

$$\sum_{i\in RS_j} (cp_i Q_{it} + cs_i V_{it}) \leq cap_{jt} \qquad \forall j \in RS, \forall t \in T \quad (9)$$

$$Q_{it} \leq M\, V_{it} \qquad \forall i \in I, \forall t \in T \quad (10)$$

$$Q_{it} \geq 0,\ FI_{it} \geq 0,\ PS_{it} \geq 0, V_{it} \in \{0,1\}, \qquad \forall i \in I, \forall t \in T \qquad (11)$$

$$SL_{ijt} \geq 0,\ \ CI_{ijt} \geq 0, \qquad \forall (i,j) \in L, \forall t \in T \qquad (12)$$

The production planning model has two important features that make it possible to link to transportation models. First, it involves load size, component inventory and regular inventory variables. Load size variables are for the amount of an item sent from one location to another in a period. Component inventory variables represent the inventory of a particular item at a production location to be used in the production of another item which is produced at this location. Regular inventory is the inventory of an item that is held at the production location of the item. Second, it considers the transportation time as a parameter. The model uses two different time scales: time scale for production planning periods (in weeks or months), for which we decide the production amounts and load sizes, and time scale for production and transportation time (in minutes or hours). Hereafter, we will call this second time scale the time bucket.

In the objective function of the model we include inventory holding and setup costs. We assume that the production cost does not change over time for the sake of simplicity. We assign different per unit inventory holding cost to I_{it} and CI_{ijt} since we consider the transportation of goods as a value-added operation and, consequently, per unit inventory holding cost for CI_{ijt}'s will be somewhat higher.

Constraint set (2) is the regular inventory balance constraint. If an item is an end item the expression involving the variable SL_{ijt} in this constraint set will drop off the equation since there will be no upper layer to send the item. Instead demand of the end item will be directly satisfied. If the item is not an end item then the demand expression will drop off the equation since we assume there is no external demand for the component items. The summation involving SL_{ijt} represents the total amount of component sent to one upper layer in the product structure. Constraint set (4) represents the inventory balance restriction for component inventories. The initial conditions regarding the constraint sets (2) and (4) are taken care of by constraint sets (3) and (5) respectively. Constraint set (6) states that the new load of component items must arrive before the current component inventories are consumed by the production. An assumption regarding this constraint is that the time at which we send a load of component items associated with period t is the start of the period $t+1$ which is represented by $t\,per$ in the constraint. The expression involving CI_{ijt-1} in this constraint yields the time period that it takes the production to consume the component inventory coming from previous period. Constraint set (7) simply ensures that the start time of production in a period must be greater than the start time of the period. Constraint set 8 is to make sure that production in a period ends before the end of the period. The capacity restriction is enforced by constraint set (9). Constraint set (10) is to set the binary setup variable to one when the associated amount of production variable takes a value greater than zero.

In this model, the production amount in a period is limited in three ways. First, it is restricted by the availability of subassemblies in terms of arrival time and quantity. Second, production must take place within the start-end time limits of a period. Third, total production associated with a resource should not exceed the available capacity of the resource. Here, it is implicitly assumed that a resource can process multiple items simultaneously provided that the capacity restriction on total production across items is not violated. In this formulation we considered the travel times between locations, $trt_{lc_i lc_j}$, as parameters to the model. As we discussed in the problem description section, we will explain the model that takes these travel times as variables in section 2.4.

2.3 Transportation planning model

As a base model for the transportation planning problem, we present the formulation given in Dumas et al. (1991) for the multiple vehicle pickup and delivery problem with time windows. We closely follow the explanations in this paper as we describe the model. This formulation assumes a single depot and a homogenous fleet of vehicles.

We explain the notation and the way that the model is constructed together. Given that the problem involves n requests, let node i and node $n+i$ represent the pickup and delivery location of the i^{th} request, respectively, in a network. In this network, different nodes obviously may represent the same physical location. Adding node zero and node $2n+1$, our network has the node set $N = \{0, 1, 2, .., n, n+1, n+2, .., 2n, 2n+1\}$. Pickup and delivery nodes are included in the sets P^+ and P^- respectively where $P^+ = \{1, 2, .., n\}$ and $P^- = \{n+1, n+2, .., 2n\}$. $P = P^+ \cup P^-$ is the set of the nodes other than the depot nodes.

Request i requires that d_i units be shipped from node i to node $n+i$ and pickup and delivery must be within the time windows $[a_i, b_i]$ and $[a_{n+i}, b_{n+i}]$ respectively. Let also

$[a_0, b_0]$ and $[a_{2n+1}, b_{2n+1}]$ denote the departure time and the arrival time back to the depot respectively. $V = \{1, 2, .., |V|\}$ represents the set of vehicles indexed by v. For each pair of i, j in N, let trt_{fc_i,fc_j} denote the travel time and c_{fc_i,fc_j} travel cost associated with going from the physical location of node i, fc_i, to physical location of node j, fc_j. s_i is the service time at node i.

There are three variables used to construct the model: binary flow variables X_{ij}^v, $v \in V$, $i, j \in N$, $i \neq j$, the time variables TS_i, $i \in P$, and TS_0^v, TS_{2n+1}^v, $v \in V$ and the load variables Y_i, $i \in P$. X_{ij}^v is equal to 1 if vehicle v goes from node i to node j and equal to zero otherwise. TS_i is the time at which service at node i starts. TS_0^v and TS_{2n+1}^v represent the times at which vehicle v leaves and comes back to the depot respectively. Total load on a vehicle right after it leaves a node is represented by Y_i. At the start, vehicles are assumed to be empty, i.e. $Y_0 = 0$.

There is a cost function $g(Y_i)$ in the objective which might be excluded from the formulation depending on the problem characteristics. $g(Y_i)$ denotes a non decreasing function of the total load on a vehicle just after it leaves node i, $i \in P$. This function takes the load factor into consideration in calculation of the travel cost.

Summary of the notation :

X_{ij}^v : binary flow variables which are equal to 1 if vehicle v goes from node i to node j and equal to zero otherwise

TS_i : the time at which service at node i starts

TS_0^v and TS_{2n+1}^v : the times at which vehicle v leaves and comes back to the depot respectively

Y_i : total load on a vehicle right after it leaves a node i

d_i : number ofunits to be shipped from node i to node $n+i$

s_i : the service time at node i

fc_i : physical location of node i

$[a_i, b_i]$: service time window for node i

trt_{fc_i,fc_j} : the travel time associated with going from the physical location of node i, fc_i, to physical location of node j, fc_j

c_{fc_i,fc_j} : the travel cost associated with going from the physical location of node i, fc_i, to physical location of node j, fc_j

The pickup and delivery model will serve as the transportation model in the remainder of this report. Following is the formulation of the pickup and delivery problem with time windows.

$$Min \sum_{v\in V}\sum_{i\in N}\sum_{j\in N} g(Y_i)\, c_{ij}\, X_{ij}^v \qquad (13)$$

Subject to

$$\sum_{v\in V}\sum_{j\in N} X_{ij}^v = 1, \qquad i \in P^+ \qquad (14)$$

$$\sum_{j\in N} X_{ij}^v - \sum_{j\in N} X_{ji}^v = 0, \qquad i \in P, v \in V \qquad (15)$$

$$\sum_{j\in P^+} X_{0j}^v = 1, \qquad v \in V \qquad (16)$$

$$\sum_{i\in P^-} X_{i,2n+1}^v = 1, \qquad v \in V \qquad (17)$$

$$\sum_{j\in N} X_{ij}^v - \sum_{j\in N} X_{j,n+i}^v = 0, \qquad i \in P^+, v \in V \qquad (18)$$

$$TS_i + s_i + trt_{fc_i, fc_{n+i}} \le TS_{n+i}, \qquad i \in P^+ \qquad (19)$$

$$X_{ij}^v = 1 \Rightarrow TS_i + s_i + trt_{fc_i, fc_j} \le TS_j, \qquad i, j \in P, v \in V \qquad (20)$$

$$X_{0j}^v = 1 \Rightarrow TS_0^v + trt_{fc_0, fc_j} \le TS_j, \qquad j \in P^+, v \in V \qquad (21)$$

$$X_{i,2n+1}^v = 1 \Rightarrow TS_i + s_i + trt_{fc_i, fc_{2n+1}} \le TS_{2n+1}^v, \qquad i \in P^-, v \in V \qquad (22)$$

$$a_i \le TS_i \le b_i, \qquad i \in P \qquad (23)$$

$$a_0 \le TS_0^v \le b_0, \qquad v \in V \qquad (24)$$

$$a_{2n+1} \le TS_{2n+1}^v \le b_{2n+1}, \qquad v \in V \qquad (25)$$

$$X_{ij}^v = 1 \Rightarrow Y_i + d_i = Y_j, \qquad i \in P, j \in P^+, v \in V \qquad (26)$$

$$X_{ij}^v = 1 \Rightarrow Y_i - d_{j-n} = Y_j, \qquad i \in P, j \in P^-, v \in V \qquad (27)$$

$$X_{0j}^v = 1 \Rightarrow Y_0 + d_j = Y_j, \qquad j \in P^+, v \in V \qquad (28)$$

$$d_i \leq Y_i \leq D, \qquad i \in P^+ \qquad (29)$$

$$Y_0 = 0 \qquad (30)$$

$$X_{ij}^v \in \{0,1\}, \qquad i,j \in N, v \in V \qquad (31)$$

The objective function of the formulation represents the total travel cost of the vehicles. Constraint set (14) enforces that each pickup node is visited once by one of the vehicles. Constraint set (15) ensures that if a vehicle enters a node it must exit it. The first visit by all vehicles must be to a pickup node and the last visit must be to a delivery node. These two conditions are stated in constraint sets (16) and (17) respectively. Constraint set (18) makes sure that if a vehicle visits a pickup node then it must visit the associated delivery node. Pickups must come before deliveries. This is enforced by constraint set (19). Constraint sets (20)-(22) describe the compatibility requirements between routes and schedules, while constraint sets (23)-(25) are the time windows constraints. Constraint sets (26)-(28) express the compatibility requirements between routes and vehicle loads, while constraint set (29) is the capacity constraint.

Any route in a solution to the model will be elementary, i.e. without cycles due to constraint sets (20)-(22) since these constraints enforce increasing service times, TS_i's, at the nodes of the route. As a special structure in the formulation, constraint sets (14)-(17) and (29) form a multi commodity flow problem. Another special structure is that, for a given vehicle, constraint sets (15)-(25) describe a path starting and ending at the depot under pairing constraints (18), precedence constraints (19), time window constraints (20)-(25) and capacity constraints (26)-(30). That is, this structure represents a constrained shortest path problem. Dumas et al. (1991) utilize this structure in their algorithm to find optimal solutions.

2.4 Linkage between the two models

There are three variables/parameters that link the production planning and the transportation planning models. These are load sizes, SL_{ijt}'s, travel time allowances, for which we give notation later, and service time windows, $[a_i, b_i]$'s. In our perception of the problem, we will assume that the service time windows are fixed as parameters in both models and consider only load sizes and the travel time allowances as the variables that link two models. In this section, we will modify both models so that they will determine the linking variables independently. In other words, these modified models will give solutions that are best in terms of their own objectives. We will describe an approach in Section 3 to simultaneously determine the linking variables which optimize the total cost over both models.

We can solve the production and transportation planning problems in a sequential fashion. We first solve the production planning model and obtain load sizes and travel time allowances. Using these load sizes and travel time allowances we solve the transportation model. As a result we get a complete solution consisting of production and transportation plans. This solution is dominated by the production planning part. It will provide an upper bound to the optimal solution of the integrated model since it is just a feasible solution to the integrated model. There might exist another solution which involves much less transportation cost and not much higher production cost compared to sequential solution and is better in terms of total cost. For a solution to be optimal in our problem setting, it needs to be determined by considering the cost factors of both models simultaneously.

Modifications to the production planning model

In order to integrate the planning decisions in the two models, we need to make some further modifications to models. For the production planning part, one of the modifications necessary will be in constraint set (6). Let $R_{lc_ilc_j}$ denote the travel time allowance variable for going from lc_i, the production location for product i, to lc_j, the production location for product j, and tw the time window length parameter. $R_{lc_ilc_j}$ is a multiplication factor representing the travel time allowance in terms of the number of direct travel time periods. Actual travel time allowance in time buckets is given by the term $trt_{lc_ilc_j}R_{lc_ilc_j}$. We assume that tw is the same for all the pickups and deliveries in the problem in order to simplify the illustrations. We replace constraint set (6) in the production model with the following constraint:

$$(t-1)per + 2tw + trt_{lc_i lc_j} R_{lc_i lc_j} \leq PS_{jt} + st_j + CI_{ijt-1}(pt_j / a_{ij}) \qquad \forall (i,j) \in L, \forall t \in T \setminus \{1\} \qquad (32)$$

In this constraint, total travel time for the items includes travel time allowance and both pickup and delivery time windows. The implicit assumption here is that we make production planning based on the worst case scenario where the items are always delivered to one upper layer at the end of the delivery service time window. Hence, total travel time is considered to be $2tw + trt_{lc_i lc_j} R_{lc_i lc_j}$ in the constraint.

Because of the assumption we made about the sending time of the loads in the production model we fix pickup time windows to be at the beginning of the periods. Delivery time windows will be dependent on the value that $R_{lc_i lc_j}$ will take. We also add the following constraint to put lower and upper bounds on $R_{lc_i lc_j}$.

$$lwrb \leq R_{lc_i lc_j} \leq urb \qquad \forall (i,j) \in L \qquad (33)$$

where $lwrb$ and urb represent the lower and upper bounds, respectively.

Modifications to the transportation planning model

In the production model, load size variables have three indices, i, j, t. Since each load corresponds to a request in the transportation model and each pickup is represented by a single index, we need to convert each triply indexed set i, j, t to a single pickup node index. In order to do this, we first define predecessor sets for the items as follows:

PR_j : Set of items needed to produce item j.

That is, each i, j, t triple, $j \in I, i \in PR_j, t \in T$, represents a load variable. The following expression converts the three indices to a unique single pickup node index:

$$i, j, t \square l = \sum_{k=1}^{j-1} |PR_k|(|T|-1) + (ord(i)-1)(|T|-1) + t \qquad (34)$$

where l is the pickup node index. For a given i, j, t, the term $\sum_{k=1}^{j-1} |PR_k|(|T|-1)$ gives the number of the pickup node indices used to represent the load size variables until the previous item, $j-1$. There will be $|T|-1$ unique pickup node indices in the transportation model for each pair of i, j. We use $|T|-1$ here since there will be no load send in the last period. Therefore, the total number of indices in the transportation model for load variables involving the predecessor items of the items $1, 2, 3, .., j-1$ will be $\sum_{k=1}^{j-1} |PR_k|(|T|-1)$. $ord(i)$ in this expression is the rank of item i in the predecessor set that item i belongs to. There will be $(ord(i)-1)(|T|-1)$ indices used for load variables associated with the predecessors of item j that come before item i in the set PR_j. Finally, t will distinguish the load variables that have the same i, j indices. We illustrate this conversion in an example presented in Appendix A.

In this way we obtain a unique pickup node index for each load variable. We will use this conversion procedure in our modifications to the transportation model. We first add the following constraint for number of units to be shipped:

$$d_l = SL_{i,j,t} \qquad \forall (i,j) \in L, \forall t \in T \backslash \{|T|\} \qquad (35)$$

l is as we defined in equation (34). Next we add several sets of constraints involving and restricting $SL_{i,j,t}$'s, the load size variables. These constraints are analogous to flow conservation constraints in network models and basically state the fact that inputs to an item node in the product structure from each incoming arc by time period t must be greater than or equal to the total output across all the outgoing arcs by period $t+1$. This one period gap between input and output is due to the assumption that any load send in period t will arrive at the upper layer in period $t+1$. Let E denote the set of end items. We add the following constraint sets to the transportation model for end items:

$$inci_{ij} + \sum_{t=1}^{T_0-1} SL_{ijt} \geq a_{ij} \left[\sum_{t=1}^{T_0} dem_{jt} - ini_j \right] \qquad \forall j \in E, \forall i \in PR_j, \forall T_0 \in T \backslash \{1\} \qquad (36)$$

$$inci_{ij} \geq a_{ij} \left(dem_{j1} - ini_j \right) \qquad \forall j \in E, \forall i \in PR_j \qquad (37)$$

Let B denote the items that are neither end item nor the lowest layer item in the product structure. For these middle level items, we replace the demand term in equations (36) and (37) with the total amount of load that is sent from the item node to all successor item nodes in the upper layer.

$$inci_{ij} + \sum_{t=1}^{T_0-1} SL_{ijt} \geq a_{ij}\left[\sum_{t=1}^{T_0} \sum_{k\in N_j} SL_{jkt} - ini_j\right]$$

$$\forall j \in B, \forall i \in PR_j, \forall T_0 \in T\backslash\{1\} \qquad (38)$$

$$inci_{ij} \geq a_{ij}\left[\sum_{k\in N_j} SL_{jkt} - ini_j\right] \qquad \forall j \in B, \forall i \in PR_j \qquad (39)$$

Since we assume there is an infinite supply of subassemblies for the lowest level items we will not add similar constraints for these items. By adding constraint sets (35), (36), (37), (38), and (39), we make sure that the load sizes, d_l or SL_{ijt}'s, which are determined by the transportation model with no regard to the production related costs, satisfy the demands and obey the restriction of the conservation of item flows we described above. We can see here that some of the load variables can take zero value in a solution. Since only the pickup points whose associated load variable has positive value need to be visited by a vehicle, we replace constraint set (14) with the following constraint;

$$M\left[\sum_{v\in V} \sum_{j\in N} X_{ij}^v\right] \geq d_i, \qquad i \in P^+ \qquad (40)$$

Where M is a large number. Another modification we need to make to the transportation model concerns the time windows. We had both pickup and delivery time windows fixed as parameters in our previous transportation model. In the modified model, we will also have pickup time windows fixed as parameters but the delivery time windows will depend on the travel time allowances variable. We represented this variable as $R_{lc_i lc_j}$ in the modified production model. First we define some parameters for the modified transportation model. Let $a_l = tper$ where t comes from the triple i, j, t which is used to find l. a_l designates the start time of the pickup time window for pickup node l and this time is basically the end of the period that the load variable SL_{ijt} is associated with. Let also $fc_l = lc_i$ and $fc_{l+n} = lc_j$ where again i, j comes from the triple i, j, t which is used to find l. Here n represents the total number of total requests. fc_l is the pickup location associated with the pickup node l and fc_{l+n} is the delivery location of this pickup which corresponds to the delivery node $l+n$. These pickup and delivery locations are the production locations of items i, j, i.e. lc_i, lc_i. We replace the constraint set (23) in the transportation model with the following ones:

$$a_l \leq TS_l \leq a_l + tw \qquad l \in P^+ \qquad (41)$$

$$a_{i-n} + tw + R_{fc_{i-n}fc_i}\, trt_{fc_{i-n}fc_i} \leq TS_i \leq a_{i-n} + 2tw + R_{fc_{i-n}fc_i} \quad i \in P^- \qquad (42)$$

Constraint set (41) imposes the pickup time window restriction which is considered to be a fixed parameter. Constraint set (42) describes the delivery time window in terms of travel time allowance variable $R_{fc_{i-n}fc_i}$. As we did for the modified production model, we put upper and lower bounds on $R_{fc_i fc_j}$ by adding the following constraint;

$$lwrb \leq R_{fc_i fc_j} \leq urb \qquad i, j \in N \qquad (43)$$

After all these modifications, we finally have a transportation model which involves the load sizes and the travel time allowances as variables, like the modified production model, in contrast to our previous transportation model which considers these two variables to be fixed parameters. In the following section we will explain a Lagrangean decomposition approach that utilizes the modified production and transportation models to make integrated planning decisions.

3. Integrating production and transportation planning using Lagrangean Decomposition

We now demonstrate the value of integrating production and transportation decisions using Lagrangean Decomposition. First we will consider the case where the travel time allowance variables are fixed and try to make integrated decisions about load variables. Then, we will consider the case where the load variables are fixed by the production model and the integrated decisions involve only the time allowance variables. That is, we will deal with the linking variables separately in order to see their effects separately.

Lagrangean decomposition was first proposed by Guignard and Kim (1987a, b). The technique has been applied to several discrete optimization problems (Guignard and Rosewein (1989), Millar and Yang (1992), Fumero and Vercellis (1997)). It assumes a MIP problem has a constraint set which consists of several special structures. It is possible to define a set of subproblems, one for each special structure in the original constraint set. Each subproblem uses a "copy" of the original variable set. The constraint that the copies of the variable have to be equal in the final solution, called the coupling constraint, is

Lagrangean relaxed and reflected in the objectives of the subproblems. Unlike Lagrangean relaxation which removes a part of the constraint set, Lagrangean decomposition retains all of the original constraints.

Using the framework provided by Lagrangean Decomposition, we first envision our production-transportation problem as an integrated problem involving a production and a transportation components. We then decompose this integrated model into production and transportation subproblems. This viewpoint allows us to use the mechanism of Lagrangean Decomposition to link the production and the transportation models. The fact that we did not actually need to construct a integrated (unsolvable) model should be obvious. We define the following notation:

P : integrated model
P_P : production model
P_T : transportation model
f_P : Objective function of the production model
f_T : Objective function of the transportation model
C_p : constraint set of the production model
C_T : constraint set of the transportation model
$SL1_{ijt}$: load variable of the production model (Amount of item i sent for the production of item j in period t)
$SL2_{ijt}$: load variable of the transportation model
$SL1$: vector of the load variables of the production model
$SL2$: vector of the load variables of the transportation model
$R1_{ij}$: travel time allowance variable of the production model (Number of direct travel time periods allowed to carry goods from location i to location j)
$R2_{ij}$: travel time allowance variable of the transportation model
$R1$: vector of the travel time allowance variables of the production model
$R2$: vector of the travel time allowance variables of the transportation model
λ_{SL} : vector of multipliers for the load variables
λ_{R} : vector of multipliers for the travel time allowance variables
$v(.)$: optimal value for problem (.)

We define problem P as follows :

$P : min\ f_P + f_T$

$$
\begin{aligned}
& subject\,to \\
& C_P, C_T, \\
& SL1_{ijt} = SL2_{ijt}, \qquad\qquad \forall\, i, j, t \\
& R1_{ij} = R2_{ij}, \qquad\qquad \forall\, i, j, t
\end{aligned}
$$

The last two equalities are the coupling constraints to which we apply Lagrangean relaxation to get the Lagrangean decomposed model, i.e. $(LD)_{\lambda_{SL},\lambda_{\mathrm{R}}}$, as follows :

$$
\begin{aligned}
(LD)_{\lambda_{SL},\lambda_{\mathrm{R}}} \quad & : Min\,\{f_P + f_T + \lambda_{SL}(SL2 - SL1) + \lambda_{\mathrm{R}}(R2 - R1) \;\; s.t. \;\; C_P, C_T\,\} \\
& : \Big\{ Min\,\{f_P - \lambda_{SL}\,SL1 - \lambda_{\mathrm{R}}\,R1 \;\, s.t. \;\, C_P\} + Min\,\{f_T + \lambda_{SL}\,SL2 \\
& + \lambda_{\mathrm{R}}\,R2 \;\, s.t. \;\, C_T\,\} \Big\}
\end{aligned}
$$

We define the following problems :

$$
\begin{aligned}
& P_{PP} : \;\; Min\,\{f_P - \lambda_{SL}\,SL1 - \lambda_{\mathrm{R}}\,R1 \,|\, C_P\,\} \\
& P_{TT} : \;\; Min\,\{f_T + \lambda_{SL}\,SL2 + \lambda_{\mathrm{R}}\,R2 \;|\, C_T\,\}
\end{aligned}
$$

The Lagrangean decomposition dual problem (LDD) is defined as follows :

$$
LDD : \qquad \underset{\lambda_{SL},\lambda_{\mathrm{R}}}{Max} \;\; v((LD)_{\lambda_{SL},\lambda_{\mathrm{R}}})
$$

Subgradient search algorithm to solve the Lagrangean decomposition dual

We present a subgradient search algorithm to solve LDD and to get upper bounds as the iterations progress. We define the following additional notation to describe the algorithm:

UB^j : Upper bound found in iteration j
LB^j : Lower bound found in iteration j
UB^* : Current best upper bound
γ^j : Scale factor at iteration j. It is set to 2 initially and reduced by half if the lower bound does not improve in four iterations.
u^j : Step size of the subgradient search.

The subgradient search algorithm we apply is as follows :

Step1 : Set $j = 1$. Initialize λ_{SL}, λ_{R}, γ^j, and UB^*.
Step2 : Solve P_{PP} and P_{TT}. Set $LB^j = v(P_{PP}) + v(P_{TT})$.
Step3 : Determine UB^j. Set $UB^* = min\{UB^j, UB^*\}$.
Step4 : Determine the duality gap as follows :

$$\theta = \frac{2\,(UB^* - LB^j)}{UB^* + LB^j} * 100$$

Stop if $j > iteration\ limit$ or $\theta < \varepsilon$ where ε is the prespecified tolerance. Otherwise go to step 5.

Step5 : Update multipliers λ_{SL}, λ_{R} and step size u^j. Set $j = j + 1$ and go to step 2.

θ in step 4 is the ratio of the difference between current best upper bound and the current lower bound to the average of these two values.

As we stated earlier, we will apply Lagrangean decomposition on two sets of linking variables separately. Step 3 and step 5 of the subgradient search algorithm will differ depending on which set of variables we consider.

Load variable case :

In this case we basically fix the $R1$ and $R2$ variables (defined in section 3.2) to a prespecified value and set the multipliers in λ_{R} to zero during entire iterations. We update the multipliers λ_{SL} and step size in step 5 as follows :

$$\lambda_{SL}^{j+1} = \lambda_{SL}^{j} + u^j(SL2 - SL1)$$

where u^j is given by :

$$u^j = \frac{\gamma^j(UB^* - LB^j)}{\sum_t\sum_i\sum_j(SL1_{ijt} - SL2_{ijt})^2}$$

We find the upper bound in step 3 as follows. After solving P_{PP} and P_{TT} to get LB^j, we take the values of $SL1_{ijt}$'s from the solution of P_{PP} and fix the values of load variables to these values both in P_P and P_T and solve these problems. The upper bound, UB^j, is :

$$UB^j = v(P_P) + v(P_T)$$

where the load variables are fixed as described.

Travel time allowance variable case :

First, we solve the production model P_P using average values for $R1$ variables to obtain a solution. The average values for $R1$ are the average of upper and lower bounds of $R1$. The values of the load size variables both in P_{PP} and P_{TT} are set to the values obtained from this first solution of P_P during all iterations. The multipliers λ_{SL} are set to zero during entire iterations.

We update the multipliers and step size in a similar manner to the previous case as follows:

$$\lambda_R^{j+1} = \lambda_R^j + u^j(R2 - R1)$$

where u^j is given by :

$$u^j = \frac{\gamma^j(UB^* - LB^j)}{\sum_i \sum_j (R1_{ij} - R2_{ij})^2}$$

We find the upper bound in step 3 in the following way. After solving P_{PP} and P_{TT} to get LB^j, we take the values of $R2_{ij}$'s from the solution of P_{TT} and set the travel time allowance variables both in P_P and P_T to these values and solve these problems. The upper bound, UB^j, is given by :

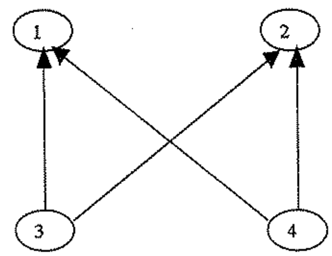
$$UB^j = v(P_P) + v(P_T)$$

where the travel time allowance variables are fixed as described.

4. The Value of Integration: Computational Results

To demonstrate the value of integrating production and transportation planning we apply the above Lagrangean decomposition scheme on a small size problem with four items that are produced in four facilities. Four planning periods are considered. We implemented the

Lagrangean Decomposition scheme using AMPL with the CPLEX solver. The product structure of the problem is depicted in the following figure:



We assume that there are two vehicles available to provide transportation to the system. There is also a depot location from which the vehicles are dispatched. The production period is assumed to be on the order of weeks while the production and transportation time units are assumed to be in minutes.

Other parameters of the problem, such as inventory holding and setup costs, distances between locations, and travel time between locations, are generated randomly using uniform distributions. The experiments are divided into two parts: 1) Those that involve load variables and 2) Those that involve travel time allowance variables.

The results we obtained for the load variable case are summarized in Table 1.

Table 1. Computational Results Using Load Variables as the Linkage

Prob. Type	Unt. Trv. Cost	% change in Pro. Pl.	% change in Trn. Pl.	Sequential solution	Best UB	Best LB	% Gap	% Gain
I	100	0	175.32	160556	160556	160077	.29	0
I	150	175.32	0	175856	175630	175076	.31	.13
I	200	175.32	0	191156	185330	185330	0	3.04
I	250	175.32	0	206446	190530	195030	0	5.53
II	200	41.02	90.54	102806	100822	83406	18.9	1.93
II	250	83.97	91.35	116556	111964	110957	.9	4.02

Problem types I and II are the same except that the setup cost in type II is 1/4 of those in type I and the unit travel time in problem type II is 1/2 of those in problem type I. Unit travel time is the time to travel one unit of distance. We expected that problem type II will have more balanced final solutions than the case where the production or transportation part dominates the final solutions. Type II problems required more computational effort than type I. We solved the same problems (either problem type I or II) using different unit travel costs. We set the iteration limit to 40 and quit the iterations if the optimal solution is found. The sequential solutions in column 5 are the ones obtained using the sequential approach where we first solve the production model and then the transportation model by taking the load variables of the production solution fixed. These solutions are taken as the initial upper bound in the subgradient search. % change in the production and transportation plans are given in columns 3 and 4. This percentage is defined as follows:

$$\% \text{ change} = 100 \text{ x} \left\{ \frac{\sum |\textit{independent solutions} - \textit{final solutions}|}{\sum (\textit{ independent solutions })} \right\}$$

The independent solutions are the load variable values in the solutions obtained by both models at the first iteration of the subgradient search when the multipliers are all zero. That is, these are the solutions that are best in terms of respective objectives of the two models with no regard to the objective of the other model. The final solutions are the load variables in the solution corresponding to the best upper bound found by subgradient search. The table also includes best lower bounds, % gap between best upper bound and best lower bounds, which is denoted by θ in the subgradient search algorithm above, and the % gain of the best upper bound over the sequential solution. As expected, we observe in % change columns that as the unit transportation cost increases the best upper bound solution, or integrated solution in other words, becomes the solution dictated by the transportation model and that type II problems gave more compromised solutions than type I problems. % gain or the value of integration also increases as the unit transportation cost increases, which is a trend one could anticipate.

We used only problem type I for travel time variable case. Table 2 summarizes the solutions we obtained.

Table 2. Computational Results using Travel Time Variables as the Linkage

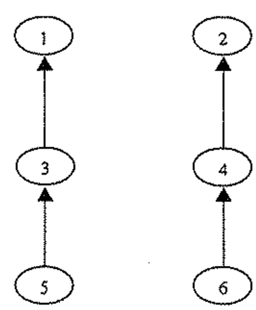
Unt. Trv. Cost	% change in Pro. Pl.	% change in Trn. Pl.	Sequential solution	Best UB	Best LB	% Gap	% Gain
5	3.11	2.775	131486	131271	131271	0	.16
25	7.96	1.79	137606	136531	136531	0	.78
50	7.96	1.79	142256	143106	143106	0	1.48
100	9.85	3.58	160256	156256	156256	0	2.68
200	9.85	3.58	191156	182556	182556	0	4.50

These problems took far less computation time than the problems involving load variables. Optimal solutions are found in three iterations for all the problems. Definitions of the columns are the same as before. In this table, we see similar trends to the load variables experiments.

Based on this small experimental study, we conclude that the saving due to integrated decision making can be substantial depending on the cost structure of the problems.

A sensitivity analysis on cost parameters

In order to see the effect of the three cost parameters of the problem (namely, unit inventory holding cost, setup cost, and unit travel cost) on the integrated solution we ran experiments using two levels of these factors. We set the planning horizon to four periods and assumed that each facility produces a single type item. The iteration limit of the subgradient search is fixed at 60. We set the iteration limit higher than that of previous experiments in order to see the effects of cost parameters better. There are eight problems corresponding to the combinations of the three factors. We solved two sets of eight problems, one set for the case with general product structure and the other one for serial product structure. The serial structure we use in the experiments is the following;



The general product structure we used here is the same as the one we had before in previous section. The problem data is, however, some what different than before. The results for the problems with serial and general product structures are summarized in Tables 3 and 4, respectively.

Table 3. Computational Results for Problems with a Serial Product Structure

Cost Parameters			No. Tran. Lots	No. Prod. Lots	Total Cost			Tran. Cst /Prod. Cst.	% Change		% Gain	% Gap
Inv.	Setup	Tran.			Inv.	Setup	Tran.		Tran. Plan	Prod. Plan.		
L	L	L	6	10	12379.5	28800	11900	.29	140	28	5.08	7.09
L	L	H	6	10	12379	28800	59500	1.44	48	111	12.69	9.92
L	H	L	6	9	14299	137756	11900	.08	140	28	1.7	2.24
L	H	H	6	9	14299	137756	59500	.39	51	29	6.47	8.90
H	L	L	8	12	36300	35353	15450	.21	144	25	2.1	9.10
H	L	H	5	12	36300	42999	59500	.75	48	111	12.58	21.98
H	H	L	6	10	61897	144000	11900	.06	140	29	1.09	1.73
H	H	H	6	10	61897	144000	59500	.29	51	29	5.07	7.12
Serial product structure												

Table 4. Computational Results for Problems with General Product Structure

Cost Parameters			No. Tran. Lots	No. Prod. Lots	Total Cost			Tran. Cst /Prod.Cst.	% Change		% Gain	% Gap
Inv.	Setup	Tran.			Inv.	Setup	Tran.		Tran. Plan	Prod. Plan.		
L	L	L	4	6	15306	16300	9700	.31	100	153	15.80	2.32
L	L	H	6	9	7606	23600	76500	2.45	100	52	10.58	35.43
L	H	L	4	6	15306	81500	9700	.10	100	0	0	0
L	H	H	4	6	15306	81500	48500	.50	100	0	0	0
H	L	L	12	14	4780	34900	26000	.65	137	0	0	29.6
H	L	H	9	12	14755	31900	104250	2.23	100	51	11.06	56.42
H	H	L	4	6	76530	81500	9700	.06	100	153	3.54	1.35
H	H	H	4	6	76530	81500	48500	.31	100	153	15.80	2.32
General product structure												

We make the following observations based on the two result tables above;

1. There exists a consistent proportionality between transportation cost parameter and % gain. This agrees with intuition. One expects more room for improvement over the sequential solution as the transportation part of the problem becomes more important.

2. If we consider the runs LLL and HLL, we see an increase in the number of transportation lots and total transportation cost. This is explainable by the fact that we tend not to carry bulky loads which, in turn, would lead to bulky inventories. Inventory cost in run HLL is much higher than that of run LLL and hence, the result of run HLL shows small lot sizes and small transportation loads compared to run LLL.

3. % gain and % gap are pretty much proportional. This can be explained as follows. Subgradient search basically converts a sequential initial solution to an integrated one. % gain is high when the transportation cost parameter is high in general. It can be anticipated that as the transportation cost parameter gets higher the sequential initial solution deviates more from the integrated solution since the sequential solution is the solution where there is no regard to transportation cost. As the sequential initial solution deviates more from the integrated solution, % gap gets larger since % gap is simply a measure of the success of the subgradient search with limited number of iterations in converting the sequential initial solution to an integrated one.

4. There is a consistency between transportation cost to production cost ratio and % gain and again between transportation cost to production cost ratio and % gap.

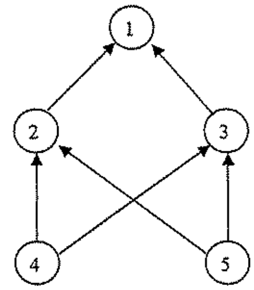
5. Number of production lots is reduced on the average for the obvious reason when we increase the inventory cost parameter.

5. Conclusions

There exist a lack of research and practice in approaching production and transportation planning in an integrated manner. In this paper, we suggest new production and transportation models which include required adjustments and additions to the existing ones in order to simultaneously consider the cost factor of both models. We develop a Lagrangean based approach to solve the integrated model of production and transportation planning. It is pointed out in our experiments that saving in total cost due to integrated decision making could be significant depending on the cost structure of the problems.

Appendix A. Index Conversion example

Suppose the following is the product structure of a supply chain and the length of the planning horizon, $|T| = 3$.



Predecessor sets of this structure are $PR_1 = \{2,3\}$, $PR_2 = \{4,5\}$, $PR_3 = \{4,5\}$. There are six links in this example and each of them corresponds to an item-predecessor pair. In total, we need to have 6x$(|T|-1)=$ 6x2 $= 12$ pickup node indices in the transportation model to represent each load variable. Following are the conversions of this example;

i, j, t □ l

$2,1,1$ $\quad \sum_{k=1}^{1-1} |PR_k|(3-1) + (ord(2)-1)(3-1) + 1$

$= 0 +$ $(1-1)$x2 $+1 = 1$

$2,1,2$ $\quad \sum_{k=1}^{1-1} |PR_k|(3-1) + (ord(2)-1)(3-1) + 2$

$= 0 +$ $(2-1)$x2 $+2 = 2$

$3,1,1$ $\quad \sum_{k=1}^{1-1} |PR_k|(3-1) + (ord(3)-1)(3-1) + 1$

$= 0 +$ $(2-1)$x2 $+1 = 3$

$3,1,2$ $\quad \sum_{k=1}^{1-1} |PR_k|(3-1) + (ord(3)-1)(3-1) + 2$

$= 0 +$ $(2-1)$x2 $+2 = 4$

$4,2,1$ $\quad \sum_{k=1}^{2-1} |PR_k|(3-1) + (ord(4)-1)(3-1) + 1$

$=$ 2x2 $+$ $(1-1)$x2 $+1 = 5$

$4,2,2$ $\quad \sum_{k=1}^{2-1} |PR_k|(3-1) + (ord(4)-1)(3-1) + 2$

$=$ 2x2 $+$ $(1-1)$x2 $+2 = 6$

$5,2,1$ $\quad \sum_{k=1}^{2-1} |PR_k|(3-1) + (ord(5)-1)(3-1) + 1$

$=$ 2x2 $+$ $(2-1)$x2 $+1 = 7$

5, 2, 2 $\sum_{k=1}^{2-1} |PR_k|(3-1) + (ord(5)-1)(3-1)+2$

$= 2\text{x}2 + (2-1)\text{x}2 + 2 = 8$

4, 3, 1 $\sum_{k=1}^{3-1} |PR_k|(3-1) + (ord(4)-1)(3-1)+1$

$= (2\text{x}2 + 2\text{x}2) + (1-1)\text{x}2 + 1 = 9$

4, 3, 2 $\sum_{k=1}^{3-1} |PR_k|(3-1) + (ord(4)-1)(3-1)+2$

$= (2\text{x}2 + 2\text{x}2) + (1-1)\text{x}2 + 2 = 10$

5, 3, 1 $\sum_{k=1}^{3-1} |PR_k|(3-1) + (ord(5)-1)(3-1)+1$

$= (2\text{x}2 + 2\text{x}2) + (2-1)\text{x}2 + 1 = 11$

5, 3, 2 $\sum_{k=1}^{3-1} |PR_k|(3-1) + (ord(5)-1)(3-1)+2$

$= (2\text{x}2 + 2\text{x}2) + (2-1)\text{x}2 + 2 = 12$

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