

**A Petri Net Model for Flexible and  
Dynamic Process Planning**

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# A Petri Net Model for Flexible and Dynamic Process Planning

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## Abstract

In this paper a Petri net based modeling approach for flexible process planning is presented. The proposed model can represent alternative operations and operation sequences for a given part. It is possible to observe the precedence constraints on the Petri net and all possible process plans can be generated by applying reachability analysis. Various heuristic criteria can be used to find efficient process plans among high volume of alternatives. Since the proposed model offers a considerable flexibility for process planning task, it can be used as a process planning module in an integrated manufacturing system for making effective real-time decisions.

*Key Words:* CIM, CAPP, process planning, Petri nets, flexibility in manufacturing systems

## 1. Introduction

Rapid progress in the computer technology and the automation technology has increased the flexibility in manufacturing systems. The flexibility has become a very important issue for manufacturing companies since it is nowadays a "must" in order to survive under increasing competition and decreasing profitability of organizations. In order to achieve flexibility effectively, effective integration of the design, analysis and engineering functions of products and processes is inevitable. This integration results in reduced cost, decreased development time, and improved quality (Singh, 1996). Integration of planning functions may be the most important component of overall concurrent engineering implementations. As a planning function process planning has a strategic place since it links the design and production planning tasks. Process planning can be defined as a function that selects the manufacturing processes and parameters to be used to transform a part from its initial form to final state according to design specifications. It is the systematic generation of the detailed methods by which parts can be transformed from raw material to finished product. Another planning function, scheduling, is one of the most important tasks that induce the level of performance of the production. It is concerned with allocated limited resources to tasks to optimize certain objective functions. On-time delivery of jobs has become one of the main factors for customer satisfaction. Scheduling plays the key role in achieving this goal. The recent developments in scheduling techniques are briefly reviewed by Lee *et al.*, 1997. Specifically job shop scheduling problems are very complex and known to NP-hard for almost all kinds of objective functions (Pinedo, 1995). In order to find the optimal schedule for all parts, methods considering only the pre-determined optimal process plan for each part are not efficient in most cases because of the conflicts in machines to be used, variety in the conditions of the shopfloor, the breakdown of machines and so on. These situations mostly cause re-planning and low performance. Responding to dynamic and/or unexpected changes in the manufacturing systems can not be efficiently achieved without flexible process plans. In order to achieve on-line scheduling and control for manufacturing systems rigidity of the process plans containing only a linear sequence of operations, should be overcome. This difficulty can be resolved by introducing flexible process planning.

A flexible process plan provides alternative feasible plans for a part using different routings. Within the computer integrated manufacturing (CIM) system a flexible process plan brings about significant benefits. The first major benefit is that it makes easier for machine shops to adapt a changing environment. Re-routing of the parts on the shopfloor can be accomplished without significant complications when it is necessary. The ability of a process planning system in generating and evaluating alternative plans and operation sequences can be used as an important tool for finding the optimum process plan. Another advantage of employing flexible process planning is its key role in integration. With alternative operations and sequences on-line integration of process planning and scheduling will be possible. As a result more efficient schedules can be generated in real-time. Especially during last decade integration issues have attracted the attention of many researchers. Some of these researches that consider non-linear process planning for the integration with

the job shop scheduling are Nasr and Elsayed (1990), Dong *et al.* (1992), Chen and Khoshnevis (1993), Lee and Jung (1994), Jiang and Hsiao (1994), Kempenaers *et al.* (1996) and Erkoç *et al.* (1998).

In this paper we focus on the process planning problem and a Petri net model for dynamic and alternative process planning is proposed. The goal of the model is to generate flexible process plans for each part in a manufacturing system, to select the optimum process plan, to build a process planning model that will work with the scheduling module concurrently in order to achieve the on-line integration of process planning and scheduling and, to provide an alternate plan to the shopfloor whenever abnormality occurs during execution. Petri nets have been used for modeling many discrete event systems and FMS. The basics of Petri nets and review of their applications in manufacturing are presented by Kamath and Viswanadham (1986), Murata (1989) and, Zurawski and Zhou (1994). Specifically, Cecil *et al.* (1992) give a review of Petri net applications in process planning.

The model proposed in this study is an extension to the one presented by Kiritsis and Porchet (1996). They first build the machining table where the requirements and data for each operation are listed. Using this table they produce the precedence graph of operations. Then they generate a Petri net model for modeling the machining sequences under the precedence constraints. Although their model is flexible in the sense that the reachability analysis gives all possible sequences of operation, alternative operations are not considered in the model. A Petri net model developed by Lee and Young (1994) represents the alternative operations thus, the alternative process plans. However, their model is not able to model the different sequence alternatives of operations efficiently. Basically, in our study we try to combine the two foregoing approaches and propose a Petri net model that models the flexible dynamic process planning task both with alternate operations and sequences. This model can be used to simulate the system while visually showing non-desirable conflict situations on the net graph and to produce all possible process plans throughout the reachability analysis.

In the next two sections we give the preliminary definitions for our model and explain the two approaches mentioned above in more detail. Based on these definitions and explanations the proposed model is presented in the third section. Process planning generation methods using the model is discussed in the forth section. Section 5 concludes the paper.

## 2. Preliminary Definitions

### 2.1. Petri Nets

A Petri net can be described as a bipartite directed graph whose nodes are a set of places and a set of transitions. Graphically places are represented by circles while rectangles are used for representing transitions. Arcs connect places with transitions and represent the relations between them. The direction of the arc indicates the flow of information or entities (tokens) through the net. Detailed descriptions of Petri nets can be found in Desrochers and Al-Jaar (1995). In general, we can define the ordinary Petri net (OPN),  $N$ , as a quintuple  $N = (P, T, I, O, M)$  where

- $P = \{p_1, \dots, p_n\}$ ,  $n > 0$ , is a finite set of places;
- $T = \{t_1, \dots, t_m\}$ ,  $m > 0$ , and is a finite set of transitions with  $P \cup T \neq \emptyset$ ,  $P \cap T = \emptyset$ ;
- $I: P * T \rightarrow \{0, 1\}$  and is an input function that defines the set of directed arcs from  $T$  to  $P$ ;
- $O: P * T \rightarrow \{0, 1\}$  and is an output function that defines the set of directed arcs from  $T$  to  $P$ ;
- $M: P \rightarrow \{0, 1, 2, \dots\}$  and is a marking whose the  $i$ th component represents the number of tokens in the  $i$ th place.

In an OPN, a transition is enabled for firing if there is at least one token in each of its input places. When a transition fires the tokens in the input places are removed and a token is added to each output place. A firing of transition yields a new marking.

A marking,  $m_r$ , is said to be reachable from a marking  $m_o$  if there exists a firing sequence that will yield  $m_r$ . The set of all reachable markings from  $m_o$  is called a reachability set and can be represented by a reachability graph. A Petri net is  $k$ -bounded with respect to an initial marking  $m_o$  if each place in the net gets at most  $k$  tokens for all markings in the reachability set.

### 2.2. SPLIT-AND and SPLIT-OR Sets

In this paper we use SPLIT-AND and SPLIT-OR concepts, which are introduced by Cho and Wysk (1995) so as to generate a form feature graph using an AND/OR graph concept to represent process plans. Operations in a SPLIT-AND set must be processed in any sequence and from the operations in a SPLIT-OR set only one of them must be selected for machining. Basically all the SPLIT-AND and SPLIT-OR sets provide the information of precedence relationships of the operations. After giving these basic definitions we can discuss the Petri net models developed for alternate process plan generation and dynamic process planning.

### 3. Petri Net Models for Flexible Process Planning

As mentioned above the proposed model in our study based on combining and extending two different approaches to Petri net modeling of flexible process planning. In order to represent the alternative process plans for a part Lee and Jung (1994) use a so-called *R-Net*, which is a special form of an OPN. In this model places denote operations such as end milling, boring, reaming etc. and firing sequences of the transitions correspond to the operation sequences. Each transition can be fired at most one time. After the last firing the process plan is determined according to the firing sequences from the initial marking until the final marking. R-Net is defined as a seven-tuple where  $R\text{-Net} = (P, T, I, O, O_p, M_{op \rightarrow P}, M)$ . The additional sets are described as follows;

- $O_p$  denotes a set of operations;
- $M_{op \rightarrow P}$  is an assignment function from operations to operation places;

An example of an R-Net is given in figure 1. In this example there are three alternative process plans for a given part. The place  $i (p_i)$  in the net represents operation  $i (O_i)$ , except the sink place, which indicates the completion of the process plan. Each firing sequence corresponds to a process plan. Also the precedence relations of operations can be observed. With this model a flexible process planning architecture is provided. The model can represent all SPLIT-OR sets. However, the flexibility in this model is limited in the sense that alternative sequences of operations are not modeled that is, it can not model SPLIT-AND operation sets. Suppose that it is possible that  $O_4$  can precede  $O_2$ . In order to consider the case where  $O_4$  precedes  $O_2$  we need to generate a different model as seen in figure 2. In this graph two extra places are introduced to model the possible sequences of two operations. The obvious disadvantage of this net is the introduction of dummy places for each operation pairs, which do not have any precedence constraint with respect to each other. For a complex part with relatively more operations and flexibility in sequences, a huge net with multiple places representing only one operation will be obtained. This is a case that many modelers are avoided, since it gets harder to analyze and interpret the nets as they get bigger and more complex. At some point the information flow in the net becomes intractable especially under the existence of dummy places.

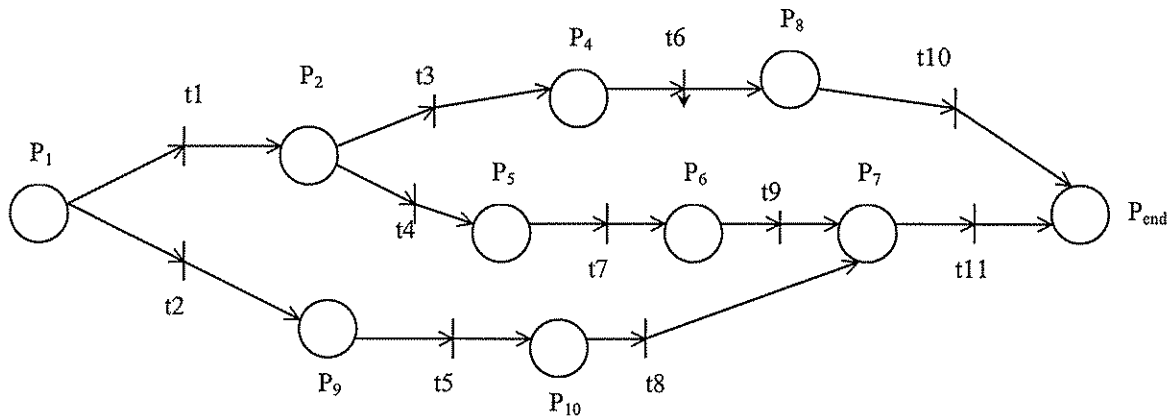


Figure 1. R-Net model for flexible process planning

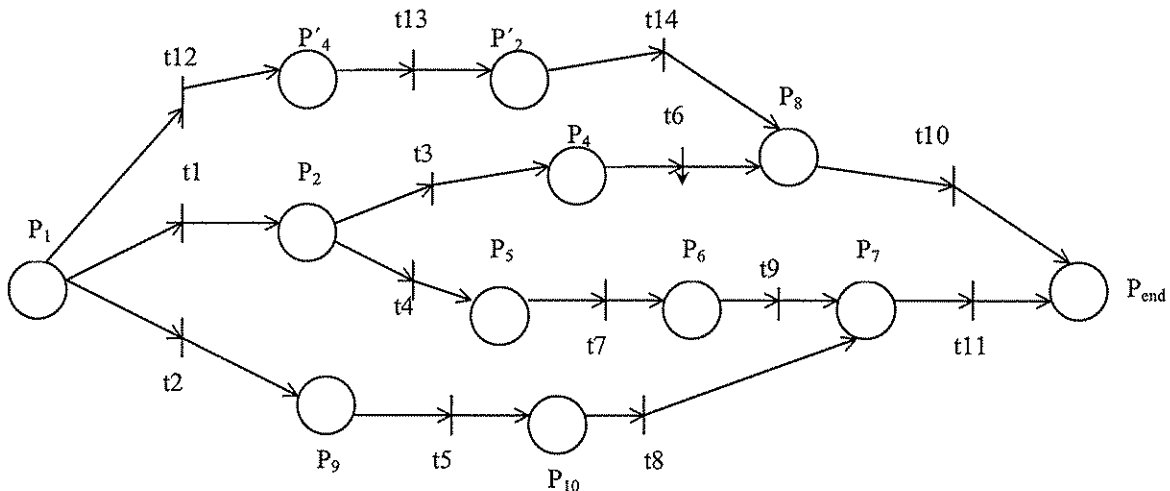


Figure 2. R-Net model for flexible process planning with alternative operation sequences

Another Petri net model proposed for modeling dynamic process planning by Kiritsis and Porchet (1996), can represent this relationship in a more efficient way. The Petri net model in Figure 3 models process planning task by using operations 1,2,4 and 8. Also in this net one transition can be fired at a time. The net is safe, that is, the number of tokens in every place does not exceed one. In this net instead of places, transitions represent the operations. There is a common input-output place (control place) with an initial token. For each transition one output place with no successor transition is created. Such a place is called as end-place ( $EP_i$ ). If there is a token in an end-place it indicates that its input transition has already been fired and can not be fired again. For each successor transition  $t_m$  of a transition  $t_i$  one output  $CP_{im}$  of  $t_i$  which is an input place for  $t_i$  is defined. These places model the precedence relationship constraints. In this net a process plan can be obtained after a complete simulation cycle by tracing the information carried by the token of the control place during its traveling through all transitions. In order to obtain a process plan all transitions should be fired exactly one time during the simulation cycle.

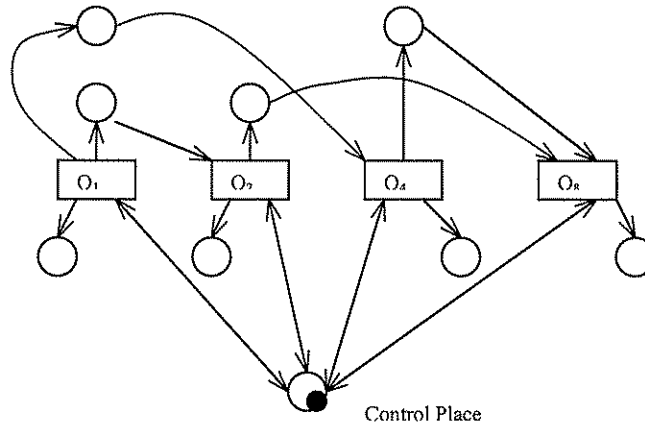


Figure 3. Petri net model for operations 1,2,4 and 8.

Now the question arises: “Can alternative operation sets be included in this net?” The answer is no to this question. Although precedence relationships are well captured in this model, with the present characteristics alternative operations can not be represented. In the present model all transitions will be fired in any simulation cycle. However in a case where OR relations are in question between operations it is not possible to produce a process plan. Firing of a transition  $t_i$  should disable other transitions that are alternatives of the operation corresponding to  $t_i$ . Consequently, we can say that this model well captures the SPLIT-AND relationships but can not model SPLIT-OR relationships.

Now we have two models each has an important function but can not carry both functions concurrently. In the next section we introduce an extended Petri net model which models both SPLIT-AND and SPLIT-OR relationships.

#### 4. Extended Petri Net for Flexible Process Planning

The Petri net model proposed in this section has similar characteristics with one proposed by Kiritsis and Porchet (1996) and explained in the previous section. However in order to generate a more flexible Petri net model we make some changes in the original net and define additional components and functions for the new net. New arcs are introduced in the model. As explained above each  $EP_i$  in the original net is an output place for the corresponding transition,  $t_i$ . The token in such a place should disable the corresponding transition as it has already been fired. So that it will be more convenient to assign an inhibitor arc, which is emanating from  $EP_i$  and is an input function to  $t_i$ . If there is token in an end place then the inhibitor arc will inhibit the firing of the corresponding transition permanently since there is no output transition for this place. We can use these inhibitor arcs for modeling the SPLIT-OR relationships. If an operation  $O_i$  in a specific SPLIT-OR is selected, in other words  $t_i$  is fired, then the other transitions representing the other operations in the set should be disabled permanently. In order to achieve this an input arc to the corresponding end places emanating from  $t_i$  can be added. Thus, whenever  $t_i$  fires the end place of transitions of operations, which have SPLIT-OR relationships with  $O_i$  receives a token. And this will, obviously, prevent these transitions from being fired. Now, consider again the example used in the previous section. Suppose that besides operations 2 and 4, operations 9 and 10 also have a SPLIT-AND relationship. We can define the SPLIT-AND and SPLIT-OR sets as follows:

$$\begin{aligned}
 SPLIT-AND_1 &= \{O_2, O_4\} \\
 SPLIT-AND_2 &= \{O_9, O_{10}\} \\
 SPLIT-OR_1 &= \{O_2, O_3\}
 \end{aligned}$$

$$SPLIT-OR_2 = \{O_4, O_5\}$$

The related flexible process planning model is depicted in Figure 4. This model can be used to represent accurately and dynamically the process planning procedure for a given part. The precedence relations and constraints can be easily provided from the model. It provides a powerful simulation tool for process planning and gives all possible process plans by simulation tracing or reachability analysis. Since its construction is based on a set of standard generic rules and similar components are used to represent any part, this model is generic and can be used for any mechanical part.

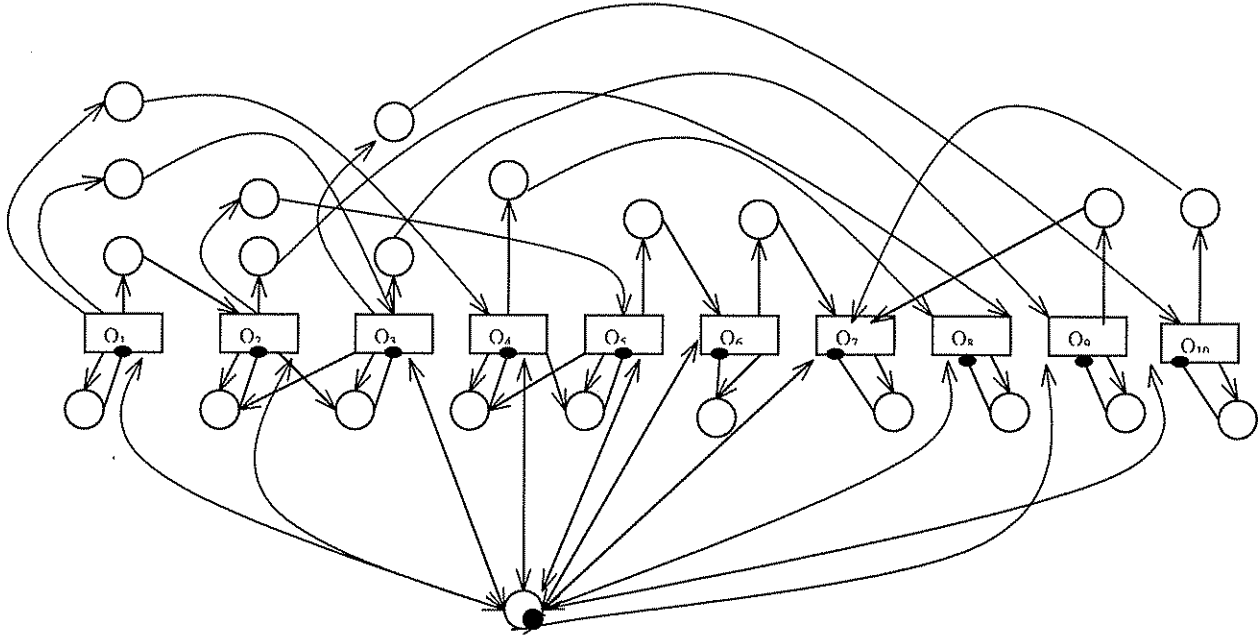


Figure 4. Proposed Petri net model for the example part

## 5. Functionality of the Extended Petri Net

The underlying characteristic of the proposed net is that it can model process plans with alternative operations and precedence relationships. From this point of view it is a powerful graphical tool for representing the knowledge. For a specific part our net is capable of representing accurately and dynamically a higher variety of process plans thus, yields a higher level of flexibility in modeling and in integration with other functions as well. It can also be used as a simulation tool for process planning. Various operation sequences and alternative routings on the net can be traced by simulation. A final process plan is obtained from the net by firing it and tracing token of the control place from beginning to end.

The reachability analysis gives all alternative process plans for the Petri net model. By performing reachability analysis a reachability tree of the Petri net is generated. In a reachability tree the initial marking is at the top of the tree and the future markings are leaves at the end of a branch. Hence, the computation of all possible future markings starting from the initial marking can be drawn in the form of a reachability tree (Desrochers and Al-Jaar, 1995). In other words this tree represents all possible transition sequences. Since each transition sequence represents a process plan we can conclude that all alternative process plans are given by the reachability tree of the net.

Although we can find all possible process plans can be generated by reachability tree to find an optimum or most convenient process plan from that tree becomes almost impossible when there are big numbers of alternative solutions. The Petri nets for process planning can also be utilized for finding optimum plans by using some heuristic criteria. Different criteria such as minimum number cutting tool change, minimum number of position change for the tool axis, minimum number of position change for the rotary table, minimum operation cost, shortest processing time etc. can be employed each time a conflict on the net is encountered. We just simply determine the next transition to be fired accordingly. Kiritsis and Porchet (1996) discuss a method and a software implementation for finding an optimum process plan using the first three criteria mentioned above. Although it can be argued that some advance algorithms can beat the simple heuristic rules for combinatorial problems such as process planning and scheduling, if these rules are relatively

efficient and reasonable they can be applied by mapping these problems on to Petri nets and using standard Petri net tools without developing new software (van der Aalst, 1996).

Other search algorithms derived from the well known graph search algorithm A\* (Nilsson, 1980). In such a search algorithm basically a cost estimation function  $f(m)$  can be used. The cost can be defined as any criteria depending on the desired quality of the process plan to be generated. Here  $m$  represents the present marking on the Petri net.  $f(m)$  consists of two parts. The first part is the lowest cost from the initial marking to the current marking obtained so far by the algorithm. The other part is an estimate of the minimum cost from the marking  $m$  to the final marking. Various functions for the second part can be employed such as depth of the marking  $m$ , number of expected number of remaining operations, lower or upper bounds for the expected processing times of the downstream operations etc. This approach has been used in Petri net based scheduling by Lee and DiCesare (1992) and Sun *et al.* (1994).

The flexibility in manufacturing has become a very important issue in recent years since it plays a key role in integrated manufacturing. During the last decade many researchers has focused on the integration of process planning with design and scheduling functions. Since the Petri net model proposed in this paper can be used to give decisions by using the advantages of flexible process planning, it can be used as modeling the process planning module in an integrated manufacturing system. More efficient production for job shops and flexible manufacturing systems can be achieved by giving the process planning and scheduling decisions concurrently. Overall system can be modeled by using hierarchical Petri nets. In such a model process planning module and scheduling module can function in coordination and in cooperation as well. Each time an operation of a job is completed new decisions related to determining and scheduling the next operation can be made by using the both modules. The process planning module can support decision making activities on conflict resolutions during the scheduling task. Lin and Lee (1997) propose an integrated control and scheduling model based on Petri nets where conflicts such as shared-resource, alternative activity, selectable-resource and selectable entity conflicts are considered. Integrating a process planning module represented by our model will bring about a great deal of flexibility to solve the conflicts by providing competent alternatives. Based on the current conditions of the shopfloor most convenient operation or operation sequence is selected by the process planning module so that the resulting performance of the system will be high and the disruptions due to breakdowns or other causes can be recovered in efficient ways. Thus an integrated on-line process planning and scheduling system can be developed and controlled. This necessitates connecting the process planning to the scheduling nets. The connection can be achieved by developing a multi input multi output net (MIMO). Ramaswamy *et al.* (1997) propose some useful extensions for Petri nets that can be used for development of MIMO net models.

## 6. Conclusions

A Petri net modeling approach to flexible process planning has been proposed in this paper and discribed by using an example. In the proposed model process planning knowledge and flexibility are represented. We have extended the Petri net model developed by Kritsis and Porchet (1996) by including the alternative operation representation in to the model. In order to find efficient process plans for a part the planner should not only generate the process plans according to the precedence relationships between operations but also he should observe the opportunities that can be obtained by considering alternate operations. The flexibility then can be accomplished completely. The proposed model provides a significant flexibility to process planning function by letting the process planner to evaluate a wide range of possible process plans. The reachability analysis of the Petri net gives all possible process plans for a given part based on the precedence constraints and process capabilities. The best process plan can be found by using heuristic rules or search algorithms without developing new software. Application and development of such algorithms on the proposed model can be focused on as a future work.

The proposed model can be used in integration of functions in a manufacturing system by developing it in a hierarchical system. This can be achieved by building MIMO nets, which include the Petri net for process planning. Thus, an on-line integrated decision system is obtained. In such a system search for efficient process plans in real time according to the current conditions on the shopfloor will be possible. The future study will be the development of an integrated higher level Petri net model that includes the proposed representation of process plans.

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