Generalized Economic Life

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Abstract

The economic life of an asset is generally defined as the length of time that results in the minimum annual equivalent cost of owning and operating an asset. Traditionally, this optimal replacement age is determined under the assumption of age-dependent costs. However, the current market value of many assets, such as an automobile, is defined by the age, cumulative utilization (mileage) and condition of the asset, in addition to the specifications and features of the asset itself. Additionally, operating and maintenance costs are dependent on both the age and usage of the asset. We present a stochastic dynamic programming formulation in which an asset is represented by state variables of age, cumulative utilization and condition and generalize the definition of economic life accordingly. Automobile replacement examples are presented to illustrate the model.

Keywords: Economic life, replacement analysis, dynamic programming

1 Introduction

The economic life of an asset is generally defined as the length of time that results in the minimum annual equivalent cost of owning and operating an asset (Park and Sharp-Bette [21]). If costs are strictly dependent on the age of the asset and the horizon is infinite, then the economic life is stationary and the optimal policy to continually replace the asset at its economic life. Preinreich [22] may have been the first to use the term economic life when referring to this chain of replacements.

The application of models which assume strictly age-dependent costs is limited as replacements are generally motivated by deterioration and obsolescence of the current asset, commonly referred to as a defender. Deterioration is an intrinsic effect defined by asset wear that includes the effect of both asset age and cumulative utilization. Obsolescence is an extrinsic effect caused by improvements in the technology of newer assets available for purchase, generally referred to as challengers. Modeling of technological change or obsolescence generally requires time variant costs (see, for example, Alchian [1], Christer [9], Grinyer [14] and Jones and Tanchoco [17]), and thus, stationary replacement cycles may not exist.

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Terborg [26] was the first to consider obsolescence and technological change in replacement decisions. However, it was not until the work of Bellman [5] and the use of dynamic programming that assumptions of repeatability and an infinite horizon were removed. Since then, a variety of finite and infinite horizon generalized replacement models based on dynamic programming have been developed, including Dreyfus [11], Oakford et al. [20], and Bean et al. [3] [4], among others.

Research has also examined relaxing deterministic assumptions common in replacement problems. Derman [10] introduced the replacement problem under uncertain deterioration, modeling the problem as a Markov process. Recently, Hopp and Nair [16] presented a model with both Markovian deterioration and technological change. Various techniques have also been used to model risky cash flows (Lohmann [19], Brown [7] and Fleischer [13], among others).

This paper is motivated by the traditional definition of economic life which refers to the optimal age at which an asset should be replaced. There exist numerous examples in which the costs incurred over the life of an asset and the salvage value of the asset over time are influenced by factors other that age. For example, automobile salvage values are generally defined by the age, cumulative utilization (mileage) and some measure of the condition of the vehicle, in addition to characteristics of the asset itself (engine type, air conditioning, etc.). While Hartman [15] proposed a model including age and cumulative utilization, this paper generalizes the definition of economic life to include the three variables of age, cumulative utilization and asset condition through the use of stochastic dynamic programming. A general model is presented in which an asset may be kept or replaced at the end of each period, unless it reaches its maximum physical age, maximum cumulative utilization or maximum state of deterioration (condition), at which time it must be replaced. After this keep or replace decision is made, the asset is utilized at some level according to a probability. Additionally, the asset deteriorates in condition according to a probability. These probabilistic changes in asset utilization and condition may be independent of, or dependent on, each other. Thus, this model treats the resulting asset state as uncertain as both cumulative utilization and condition are stochastic.

The stochastic dynamic program which generalizes economic life is defined in the following section. Section 3 presents various automobile replacement problems to illustrate the model and describe the meaning of economic life in this context. Conclusions and directions for future research are given in Section 4.

2 Stochastic Dynamic Programming Solution

2.1 Problem Statement

At time zero, an asset is owned which may be kept or replaced by a new asset. If replaced, a salvage value, dependent on the age, cumulative utilization and condition of the asset, is received for the old asset and a purchase cost is paid for the new asset. The retained or acquired asset is then utilized at some probabilistic level in the following period. At the end of the period, the asset has aged one period, accumulated utilization according to the probabilistic level and degraded in condition according to some probability. The asset incurs operating and maintenance costs which are dependent on the age, utilization and condition of the asset during the period. At the end of the period, this decision process repeats in that the asset may be kept or replaced.

These decisions occur at the end of each period over the finite decision horizon. The only deviation from this analysis is that if an asset reaches its maximum limit in age, cumulative utilization or condition, it must be replaced by a new asset. The optimal decision is the sequence of keep/replace decisions over the finite horizon such that the net present value of all costs less salvage values is minimized.

2.2 Assumptions

The periodic usage of an asset, which defines its cumulative utilization at the end of each period, may undergo one of m scenarios in each period. That is, the utilization of an asset may be at any level $u = u_1, u_2, \ldots, u_m$ each period according to a specified probability. These levels correspond to some level of output, such as miles traveled or parts produced. For example, a given period may have five utilization scenarios, where a vehicle is to be utilized at either u_1 = 2,500, u_2 = 5,000, u_3 = 7,500, u_4 =10,000 or u_5 =12,500 miles in a period, each with a given probability. It is assumed that the difference between these levels (2,500 miles) is constant throughout the horizon. This is required to maintain a lattice structure in asset utilization which reduces the growth of the state space in the dynamic programming model. That is, an asset that undergoes 2,500 miles of travel followed by 5,000 in the next period results in an equivalent cumulative utilization state as the same asset that undergoes 5,000 in the first period followed by 2,500 miles of travel in the ensuing period. Additionally, the measure of output can be mapped to a unit of cumulative utilization such that one unit is equivalent to 2,500 miles in this example. Thus, in this example, $u_1 = 1$ and $u_5 = 5$.

A similar lattice structure is assumed to hold with condition. Here, conditions of excellent, good, fair and poor may be represented by the levels $k = 0, 1, \ldots D$, where D is three. As with cumulative utilization, it follows that the resulting condition state for moving from a condition of excellent to fair in one period and remaining fair for the second period is equivalent to the resulting condition state for moving from excellent to good and good to fair in successive periods.

The final asset state variable of age is assumed to be tracked in periods, which are generally years. This is straightforward in that an asset ages one period in each period.

With these lattice structures, the number of new possible states that can be reached in the ensuing period is reduced drastically. Thus, the stochastic dynamic program can be solved quickly, despite having three state variables.

2.3 Notation

An asset is defined by its age i, cumulative utilization j and current condition k. At the end of each period $t = 0, 1, \ldots, T-1$, where T is the horizon, the asset may be kept or replaced with a new asset. If at any time over the horizon an asset reaches its maximum physical age limit N, maximum cumulative utilization level M or maximum degraded condition D, it must be replaced. As this is a finite horizon problem, the asset is sold for its salvage value at time T.

If replaced, the asset is sold for its salvage value $S_t(i,j,k)$ and a new asset is purchased at price P_t. An asset purchased new is assumed to have initial states $i=j=k=0$ at the beginning of the period of the purchase. If kept, the asset is utilized for the period at level u for the cost $C_t(u,i,j,k)$ which is assumed to occur at the end of the period. The asset may be used at any level $u=u_1,u_2,\ldots,u_m$ in each period according to the probability $p_t(u)$. Additionally, the condition of an asset k, may remain at state k or worsen to state k', $k'>k$, in a given period according the probability $p_t(k'|k,u)$, as it is also dependent on the periodic usage. (Note that $p_t(u)\cdot p_t(k'|k,u)$ defines the joint distribution of the periodic usage and change in condition in a given period. Independent probability distributions may also be utilized.)

Purchases and sales of assets are assumed to occur at the beginning of the period while operating and maintenance costs occur at the end of the period. All costs are discounted over time, with α representing the periodic discount factor. Generally, this would equal $1/(1+r)$ where r is the periodic interest rate.

2.4 Formulation

The dynamic programming solution determines the sequence of decisions that minimizes the expected net present value of costs when starting with an asset defined by state (i,j,k) at time zero and making optimal decisions through time T. This is accomplished by solving the recursion defined by Equations (1) and (2) backwards (Dreyfus and Law [12]).

The boundary condition assigned to period T represents the sale of the asset after the final period, as in (1).

$$f_T(i,j,k) = -S_T(i,j,k) \tag{1}$$

For periods $t=0,1,\ldots,T-1$, the recursion equation is given in (2).

$$f_t(i,j,k) = \min\left\{\begin{array}{ll} \text{Keep:} & \alpha\sum_{u=u_1}^{u_m}\sum_{k'=k}^{D} p_t(u)p_t(k'|k,u)\left[C_t(u,i,j,k)+f_{t+1}(i+1,j+u,k')\right], \\ \text{Replace:} & P_t - S_t(i,j,k) + \alpha\sum_{u=u_1}^{u_m}\sum_{k'=0}^{D} p_t(u)p_t(k'|k,u)\left[C_t(u,0,0,0)+f_{t+1}(1,u,k')\right] \end{array}\right\} \tag{2}$$

The solution of Equations (1) and (2) determines whether it is optimal to keep or replace the asset for each possible asset state (i,j,k) at each time period t over the horizon. The optimal decisions minimize expected purchase and O&M costs less expected salvage values which are discounted to time zero.

While the solution of the dynamic program provides keep and replace decisions for each possible asset state over the horizon, we can also determine optimal policies under the assumption that costs are not dependent on t. Bellman [5] showed that the economic life of an asset could be determined by solving a recursion for each feasible asset age and selecting the minimum. Thus, the optimal policy was to keep the asset until it reaches its economic life and then replace it with an identical asset. Here, the policy is defined by age, cumulative utilization and condition, and the policy is no longer a point (age), but rather a surface. That is, for a stationary problem, a new asset is to be kept until it reaches a combination of (i, j, k) that defines the economic life. This is illustrated in the following section where the model is applied to several vehicle replacement problems.

3 An Application in Vehicle Replacement

This section presents a number of different examples in the context of vehicle replacement. The automobile was chosen for these examples as data is readily available from a number of sources. For instance, salvage value data based on model specifications, age, mileage and condition is available from sources such as Kelley Blue Book [6] and Edmunds [23]. Operating and maintenance cost data is also readily available for automobiles from sources such as Automotive Fleet [2] and Consumer Reports [18]. The purchase price of current automobiles can be determined from current advertisements and probable inflation rates through time may be interpreted from statistical databases (i.e. Standard and Poor's Statistical Service [25]).

For this example, it is assumed that there are five levels of utilization possible in each period, namely u_1 =9,000 through u_5 =15,000 miles incremented by 1,500 miles. These levels are designated as $u_1 = 6$ through $u_5 = 10$, respectively, such that one level of cumulative utilization maps to 1,500 miles. Additionally, there are four levels of condition for the automobile, namely "excellent", "good", "fair" and "poor." These are designated levels $k = 0$ through $k = 3$, respectively. It is assumed that the asset must be replaced if it reaches the age of 12 years ($N = 12$), 150,000 miles ($M = 100$) or the condition of poor ($D = 3$).

All examples were solved using the Visual Basic macro language in Microsoft Excel on a personal computer with a 300 Mhz processor and 128 MB of RAM. For the time-invariant cost examples, the average solution time was 36 seconds while it was 42 seconds for the time-variant cost examples.

3.1 Time-Invariant Cost Examples

Initially, we solve various problems under the assumption of time-invariant data. Although this is unrealistic as it does not allow for the modeling of technological change or inflation, it determines a stationary replacement policy which may be useful for decision making. Also, it allows for the generalization of the definition of economic life.

From the data sources listed above, we make the following assumptions about costs. The purchase price of the sedan being evaluated is \$16,800 for a new asset. The O&M costs are assumed to be influenced by age, current utilization, cumulative utilization and the condition of the automobile. The operating costs are \$0.20 per mile for a new car, increasing 1% every 1,500 miles driven. Maintenance costs for a new car are \$100, rising \$200 for each year of use. Maintenance costs are also expected to increase drastically by condition, in that $k = 1$ adds \$1000 and $k = 2$ adds \$4000. Finally, age based costs such as insurance are estimated at \$800 for a new car, rising \$100 per year.

The salvage values are determined from the Kelley Blue Book [6], with the value determined for each feasible combination of age, cumulative mileage and condition. Table 1 provides an example of the data utilized. The data shown is for the 1996 sedan which is assumed to be the salvage value for a three-year old asset. Similar data is available for older models which may be used to approximate salvage values based on age. Interpolation was used to estimate salvage values for other possible levels of cumulative utilization.

Table 1: Salvage value estimates for three-year old vehicle based on cumulative mileage and condition.

Cumulative Mileage	Excellent	Good	Fair	Poor
27000	\$7425	\$6900	\$6010	\$4755
31500	\$7275	\$6750	\$5860	\$4605
36000	\$7175	\$6650	\$5760	\$4505
40500	\$7050	\$6525	\$5635	\$4380
45000	\$6850	\$6325	\$5435	\$4180

The probability of an asset worsening in condition over time is given in Table 2. For example, it is assumed that a new car will stay in "excellent" condition with probability 0.6 while there is a 0.3 probability that it will degrade to a condition of "good." It is assumed that if an asset reaches the state of "poor" that it must be replaced. For these examples, it was assumed that the utilization level probabilities and the probabilities of moving from one condition to another were independent.

Table 2: Probabilities of moving from condition state k to k' in one period.

$p_t(k')$	Excellent ($k'=0$)	Good ($k'=1$)	Fair ($k'=2$)	Poor ($k'=3$)
Excellent ($k=0$)	0.6	0.3	0.09	0.01
Good ($k=1$)	0	0.8	0.18	0.02
Fair ($k=2$)	0	0	0.9	0.1
Poor ($k=3$)	0	0	0	1

With this cost and parameter data, 10 different examples differing according to the probabilities assigned to periodic usage levels are solved using the stochastic dynamic program. This includes five deterministic utilization level examples. These deterministic utilization scenarios are included to illustrate the different solutions that may arise under different utilization scenarios and different stochastic/deterministic assumptions. The probability assigned to each possible utilization level for each example is given in Table 3. This table also provides the optimal keep or replace decision for an asset defined with initial state $(i, j, k) = (5, 42, 2)$ at time zero. This corresponds to a vehicle of age 5 with 63,000 miles in fair condition. The net present value (NPV) cost of making optimal decisions over a 50-year horizon is also provided.

As the solutions in the table show, the optimal time zero decision is highly dependent on the

Table 3: Various solutions with initial asset state $(i, j, k) = (5, 42, 2)$ under time-invariant economics.

Example	$p(u_1)$	$p(u_2)$	$p(u_3)$	$p(u_4)$	$p(u_5)$	NPV Cost	Time Zero Decision
1	1	0	0	0	0	$133,647.32	Keep
2	0	1	0	0	0	$138,410.08	Keep
3	0	0	1	0	0	$143,369.98	Keep
4	0.	0	0	1	0	$147,736.14	Replace
5	0	0	0	0	1	$152,850.22	Replace
6	0.4	0.2	0.2	0.1	0.1	$139,847.20	Keep
7	0.2	0.4	0.2	0.1	0.1	$140,806.38	Keep
8	0.1	0.2	0.4	0.2	0.1	$143,223.73	Keep
9	0.1	0.1	0.2	0.4	0.2	$145,546.01	Replace
10	0.1	0.1	0.2	0.2	0.4	$146,550.93	Replace

expected utilization pattern of the asset. When it is more probable that the asset is to be utilized at a high rate, the decision is more likely to replace the five-year old asset at time zero.

As these examples have time-invariant costs, solutions for all possible states can be determined from one solution of the dynamic program. If we examine the solution for each possible asset state at time zero, they provide the steady state solution for the infinite horizon problem, assuming we have selected a finite horizon that is sufficiently long such that the economic life of an asset can be determined. (The choice of a finite horizon to approximate an infinite horizon is beyond the scope of this article. See, among others, Bean et al. [3], Chand and Sethi [8], Sethi and Chand [24] for more discussion on this subject.) For problems in which costs are strictly defined by the age of the asset, this stationary replacement cycle defines the economic life of an asset. Here, the asset is defined by its age, cumulative utilization and condition. Thus, the economic life is now a function of these three parameters and is no longer a point, but rather a surface.

Consider Example 8 from Table 3. Figure 3.1 illustrates the keep and replace decisions for each of the possible asset states. Again, these are the optimal decisions in any time period as costs are not dependent on time. The graph illustrates all states in which it is optimal to keep the asset in gray and all states in which it is optimal to replace an asset in black.

Examining the figure more closely, the black states can be used to define the economic life of this asset in this context. For an asset it excellent condition ($k = 0$), an asset should be kept if it is of age 6 or younger, regardless of cumulative utilization. If the asset is age 7, it should be retained if its cumulative utilization is less than or equal to 55 and if the age is 8, it should be kept if its cumulative utilization is less than or equal to 50. This is seem more explicitly in Figure 3.2 which shows just the $k = 0$ states of the three dimensional figure.

Examining decisions with $k = 1$, the asset should be kept if it is age 6 or less, but only retained at age 7 if the cumulative utilization is less than 52. For the case where $k = 2$, the asset should be

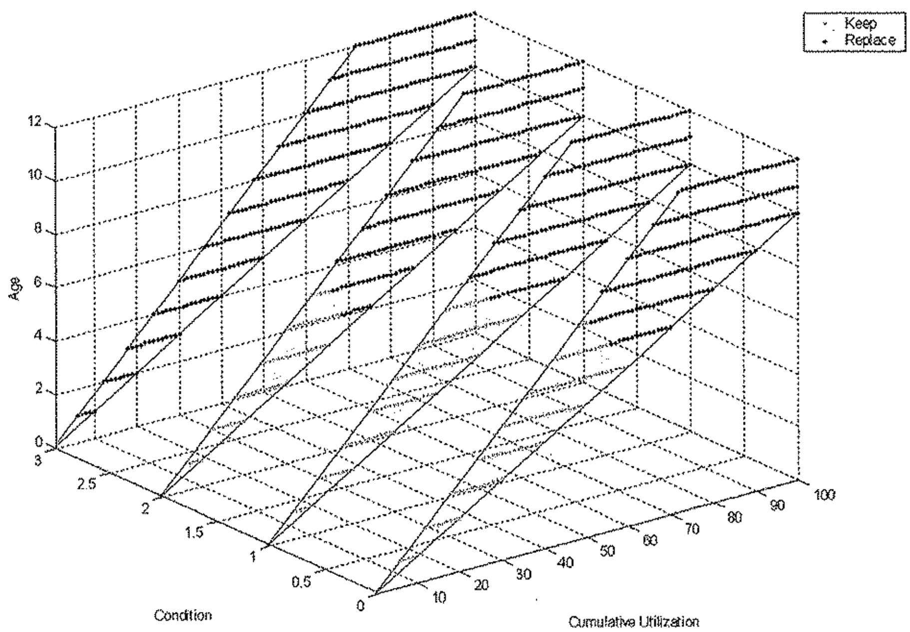

Figure 3.1: Plotted decisions for Example 8 of time invariant economics examples.

replaced if it is of age 5 with cumulative utilization greater than 42, age 6 if utilization is greater than 40 and age 7 and greater (regardless of usage). These states in which the asset should be replaced define the economic life of the asset in terms of age, cumulative utilization and condition.

The actual surface defined in Figure 3.1 is a function of the estimated costs and salvage values. As shown in this example, the transition from keep decisions to replace decisions is smooth, which is also a function of the costs. If operating and maintenance costs, salvage values, and their summation move consistently, then there is a smooth transition from keep to replace decisions.

3.2 Time-Variant Cost Examples

While the time-invariant economics examples are good for determining a stationary economic life for an asset, they are not realistic in that inflation must be ignored and the modeling of technological change with costs that change over time cannot be included. In the following examples (Table 4), the above examples are solved again with time-variant data.

The purchase price of the sedan is assumed to increase by 5 percent each period, which in turn raises the ensuing salvage values. Also, O&M costs are assumed to change in two manners. First, the fixed charge of $900 experienced by all assets is assumed to decrease by 4 percent each period to signify a technological improvement in assets. Second, all O&M costs are subject to a 3 percent rate of inflation. Under these new cost assumptions, the results to similar examples are given in Table 4 for the 50-year horizon problem. Specific time zero decisions are provided for an initial asset defined by state $(i, j, k) = (4, 32, 2)$ which corresponds to a four-year old vehicle with 48,000

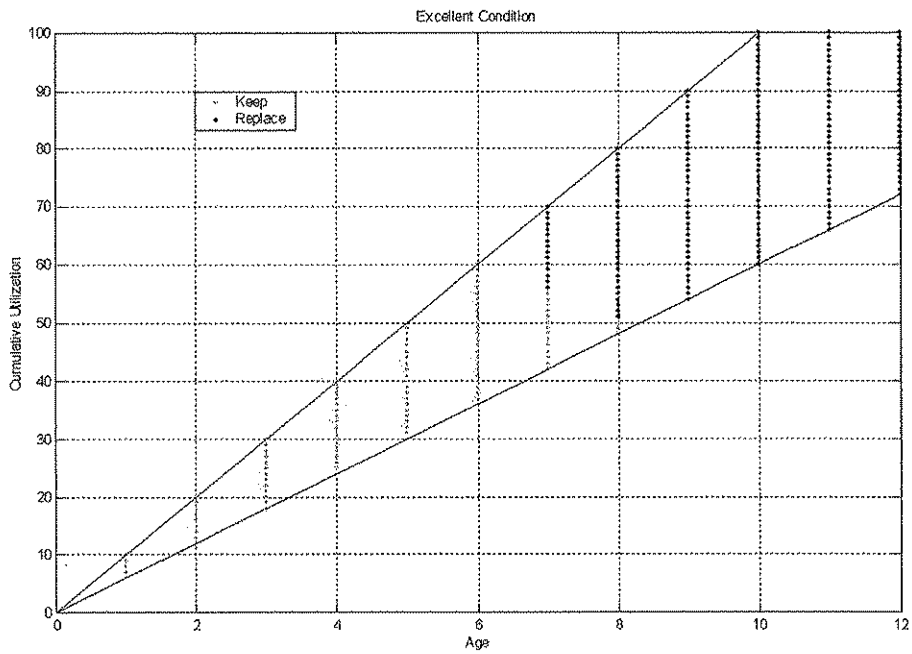

Figure 3.2: Plotted decisions for Example 8 of time invariant economics examples with $k = 0$.

miles in fair condition.

As shown in Table 4, the optimal time zero decisions are highly dependent on the expected utilization of the asset, as with the time invariant economics examples. If it is only expected that the asset will be used for 9,500 or 12,000 miles in year one, the optimal decision is to keep the asset while the optimal decision is to replace it for any greater expected mileage. When probabilities are assigned to all possible utilization levels, as are given in Examples 6 through 10 in Table 4, the optimal decision is to replace the initial asset at time zero.

For these examples, there is no stationary replacement policy over time. However, we can still develop a plot of state dependent keep and replace decisions for each time period. Thus, for each time period, this surface represents the economic life of the asset, but only in the given time period. The optimal time zero decisions for Example 8 are plotted in Figure 3.3.

Examining this figure, it is optimal to keep an asset of age 3 or less, regardless of cumulative utilization or condition. If the asset condition is 0 or 1, a 4-year old asset is also retained. For an asset in excellent condition, a replacement occurs at age 5 if cumulative utilization is greater than 46 and age 6 if it is greater than 44. For an asset is good condition, a replacement occurs at age 5 if cumulative utilization is greater than 44 and age 6 if it is greater than 42. In both of these cases, an asset of age 7 or higher is replaced. Finally, for an asset in fair condition, the asset is replaced if it has an age of 3 or 4 and cumulative utilization greater than 31 or its age is greater than 4. Again, these decisions only hold for the first period as the economic life may shift over time due to technological changes and/or inflation. Thus, Figure 3.3 must be replicated for each period to

Table 4: Various solutions with initial asset state $(i, j, k) = (4, 32, 2)$ under time-variant economics.

Example	$p(u_1)$	$p(u_2)$	$p(u_3)$	$p(u_4)$	$p(u_5)$	NPV Cost	Time Zero Decision
1	1	0	0	0	0	\$230,430.20	Keep
2	0	1	0	0	0	\$237,419.74	Keep
3	0	0	1	0	0	\$244,837.47	Replace
4	0.	0	0	1	0	\$250,801.44	Replace
5	0	0	0	0	1	\$258,458.46	Replace
6	0.4	0.2	0.2	0.1	0.1	\$239,496.24	Replace
7	0.2	0.4	0.2	0.1	0.1	\$240,904.74	Replace
8	0.1	0.2	0.4	0.2	0.1	\$244,432.63	Replace
9	0.1	0.1	0.2	0.4	0.2	\$247,720.19	Replace
10	0.1	0.1	0.2	0.2	0.4	\$249,239.83	Replace

make decisions over time. This can be accomplished by solving the dynamic program for various initial asset states.

4 Conclusions and Directions for Future Research

This paper presents a stochastic dynamic programming formulation which generalizes the definition of economic life to include parameters of age, cumulative utilization and condition. While the traditional definition of economic life is the age at which an asset should be replaced to minimize asset operating and ownership costs over an infinite horizon, the solution to the dynamic program defines the optimal keep or replace decision for each feasible asset state defined by age, cumulative utilization and condition. Examples illustrate that the dynamic program can be solved quickly despite the use of three state variables over a finite horizon.

As this paper has included utilization and condition parameters in addition to age as variables to determine optimal asset life, it highlights a very important line of research: combined replacement and utilization analysis. Traditional replacement analysis literature assumes some level of utilization over time and therefore, decisions can be made based on the age of the asset. However, due to the uncertainty associated with operations, these expected utilization patterns may not be realized, thus negating expecting utilization scenarios and the resulting replacement decisions. To minimize both ownership (purchase and salvage) and operating (operating and maintenance) costs, some measure of asset utilization must be included in the analysis. With multiple assets, the utilization of the assets may also be optimized.

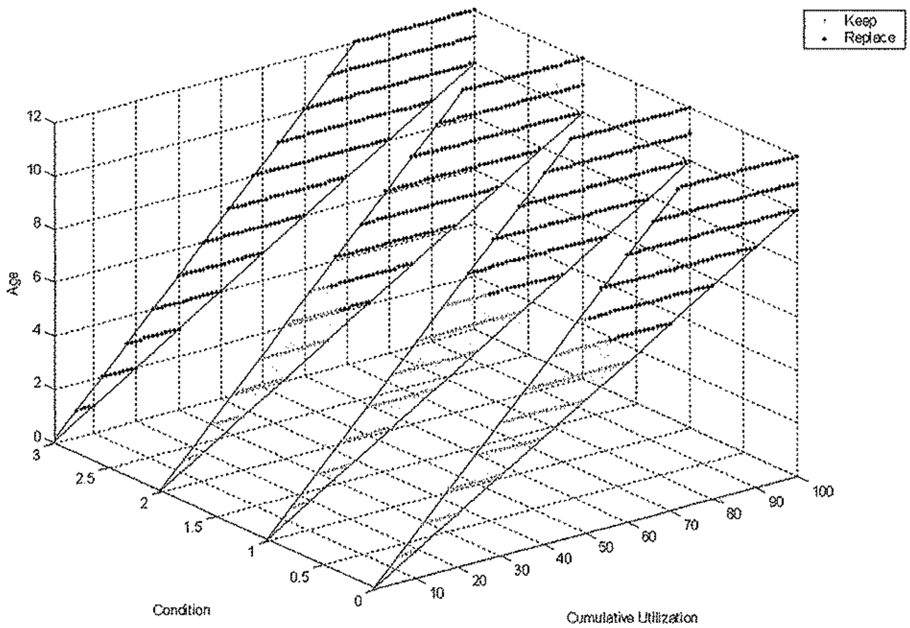

Figure 3.3: Plotted decisions for Example 8 of time variant economics examples.

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