

**Strategic Capacity Planning in the  
Semiconductor Industry:  
A Stochastic Programming Approach**

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## **STRATEGIC CAPACITY PLANNING IN THE SEMICONDUCTOR INDUSTRY: A STOCHASTIC PROGRAMMING APPROACH**

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### **Abstract**

We study strategic capacity planning in the semiconductor industry. Working with a major US semiconductor manufacturer on the strategic configuration of their worldwide production capacities, we identify two unique characteristics of this problem as follows: (1) wafer demands and manufacturing capacity are *both* main sources of uncertainty, and (2) capacity planning must consider two distinct viewpoints: a *product* perspective concerning marketing and strategic demand management, and a *process* standpoint involving manufacturing, yield, and technology configuration. These two unique characteristics change, in a fundamental way, how strategic capacity planning problem should be approached. To describe this complex problem, we first formulate a multi-stage stochastic program with recourses where demand and capacity uncertainties are incorporated via a scenario structure. To reconcile the *marketing* and *manufacturing* perspectives to the problem, we consider a decomposition of the planning problem resembling decentralized decision-making involving the headquarter, the marketing manager, and the manufacturing manager. To study various trade-offs under this decentralized structure, we develop recourse approximation schemes simulating different decentralization strategies. These schemes vary in information requirements and complexity, while providing insight on the value of information in this environment. We conduct extensive experiments to analyze the characteristics of decisions under different levels of uncertainties, and assess the value of alternative schemes from the standpoint of computational requirements and solution quality. The results indicate that it is possible to arrive at near optimal solutions (within 6.5%) with information decentralization while using a fraction (less than 16.2%) of the computer time.

## 1. Introduction

Production capacity is the most significant portion of capital investment in semiconductor wafer manufacturing. Effective utilization and expansion of production capacity have significant cost implications, and arguably drives the profitability of the operation. Capacity management in the industry typically entails long-term strategic planning and short-term operational planning organized in a hierarchical manner. *Strategic planning* decisions include how much of which aggregate microelectronics technology to produce in what facilities, and which capacity element to expand within what timeframe so as to meet the projected demands. *Operational planning* determines capacity adjustment or reconfigurations among microelectronic technologies when more accurate demand and capacity information becomes available. Operational planning is frequent and dynamic so as to accommodate weekly production wafer "starts" to be released to manufacturing. In this paper we focus on the strategic capacity-planning problem while considering operational planning decisions as the short-term *recourse* of the capacity plan. Our study is based on real planning scenarios at a major US semiconductor manufacturer.

One important characteristic of semiconductor capacity planning is that both product demands and manufacturing capacity are sources of uncertainty. As is the case in most hi-tech industries, the semiconductor market has a demand structure that is intrinsically volatile. A microelectronic chip which faces high demands today may be quickly outdated in a few months with the introduction of a next-generation chip which requires an enhanced manufacturing process. New manufacturing processes create high variability in the yields, and consequently uncertainty on the manufacturing throughput, which in turn lead to uncertainty in capacity estimation. Since the production volumes are typically high (for the interests of achieving economies of scale), extreme outcomes on demand and capacity realizations can lead to very undesirable business consequences. Therefore, it is important to consider different scenarios during long-term capacity planning, and to hedge against extreme outcomes. We propose a scenario based stochastic programming model to the problem, which produces a capacity configuration that hedges against extreme outcomes of demand and capacity fluctuation.

The idea of incorporating uncertainty in mathematical modeling goes back to the early work of Dantzig (1955). Although this modeling view represents real world problems more accurately, it is not attractive for practical applications until recently due to its high computational requirement. With the availability of inexpensive computing power and sophisticated solvers, stochastic programming models are increasingly popular. Numerous stochastic programming models have been suggested for strategic decision making such as capacity planning, medium term production planning, and power generation planning (Takriti et. al. (1996)). Bienstock and Saphiro (1988) model the strategic resource acquisition decisions as a stochastic program with recourse. They apply the model at an electric utility to make fuel contract, and plant construction decisions under demand uncertainty. In another study, Eppen et al. (1989) model the strategic capacity-planning problem of a major automobile manufacturer. Decisions in the model include setting up or shutting down facilities for some of the product lines under consideration. The main source of uncertainty in their model is product demand over a medium term planning period. The authors report actual implementation of their model at the manufacturer. In a more recent study Berman et. al. (1994) apply a stochastic programming model to solve for the capacity expansion problem in service industry with uncertain demand. Their model determines the size, location, and timing of the expansions so as to maximize the total expected profit. Escudero et al. (1993), analyze different modeling approaches to the production and capacity-planning problem, within the stochastic programming framework. The decisions considered are production volume, product inventory and resource acquisition decisions under uncertain demand. A distinguishing feature of the capacity-planning problem we consider is that there is uncertainty in both demand and capacity availability.

For the semiconductor manufacturer in our study, capacity planning is an aggregate planning problem to be considered at the beginning of the planning year. We propose a somewhat unique two-stage stochastic program where the stage-one decisions concern about *capacity expansions and configurations* to be made here-and-now, while the stage-two decisions consider *operational decisions* as recourses carried out by two distinct decision entities: product (or marketing) managers (PMs), and manufacturing managers

(MMs). PMs and MMs take different recuperative actions under each particular scenario realization of demand and capacity.

The literature that studies the effects of multiple decision entities and decentralized information is known as *multidivisional* decision-making. In a typical setting, authority is delegated to different divisions of the firm so as to ease the complexity of information gathering, processing, and hence decision-making. In the decision process, these divisions each consider their own local constraints and private information while trying to maximize their local, often-conflicting objectives under the authority of a headquarter who imposes an implementable final plan. Resolving local conflicts is an important issue for multidivisional decision making since conflict resolution schemes impose strong dependencies between divisions and have a strong influence to solution quality.

Burton and Obel (1984) discuss the rational behind decentralization and review issues in designing decentralization mechanisms in detail. In the earlier literature of decentralized planning, the most widely used approach is to model the overall decision problem as a deterministic linear program, identify the decision entities (divisions) and their subproblems then apply mathematical decomposition techniques to facilitate the information flow between the decision entities and the central coordinator (Christensen and Obel (1978), Burton and Obel (1980), (Burton and Obel, (1995))).

The application of decentralized decision-making has mostly focused on linear and deterministic decision models taking advantage of the well-studied decomposition structures in these models. We are not aware of any work in the literature that studies the effect of decentralization in a stochastic decision making environment. As we will demonstrate in the paper, the fact that each decision entity considers a different set of scenarios in their local decision problem creates challenging issues both analytically and computationally. In the next section, we first describe the capacity-planning problem as a stochastic program; we then consider various forms of recourses from the decision makers' viewpoints, which result in different complexity and information requirements.

## 2. Problem Formulation and Solution Approach

### 2.1. The Strategic Capacity Planning Problem

For the semiconductor manufacturer we studied, strategic capacity planning is considered at the beginning of each fiscal year, with a typical time horizon of five years. Capacity planning can be described as an iterative process between the following two main components: (1) *capacity expansion*, given projected product demands, identify the required manufacturing technologies and their capacity levels to be physically expanded or outsourced through the planning period, and (2) *capacity configuration*, determine which facility is to be configured with which technologies mix. The overall objective is to meet a revenue model based on strategic demand planning (which blends demand forecasting *and* proactive market development strategies). This objective can be viewed as meeting projected demands with minimized total costs. Capacity is expressed in terms of *wafer starts per week* and capacity configuration is the number of wafer starts for each technology at each facility throughout the planning horizon. Physical capacity-expansion requires a lead-time up to two years and these decisions are implemented once determined at the beginning of the planning period, i.e., fixed before actual demand and capacity realize. Outsourcing arrangements require a shorter lead-time and can be delayed till uncertainty is somewhat reduced during the planning period. Capacity configuration decisions are subject to adjustments throughout the year in order to accommodate unforeseen changes in capacity and demand.

Specifically, the decisions that are needed for capacity planning are as follows:  $x_{ijt}$ , the amount of wafer starts per week of technology  $i$  ( $i \in M$ ) at facility  $j$  ( $j \in F$ ) during planning period  $t$  ( $t \in T$ ),  $\sigma_{jt}$  and  $y_{jt}$  represent 0-1 capacity expansion decision and the expansion amount, respectively, at facility  $j$  during period  $t$ .  $z_{ijt}$  represents the technology certification decision, a 0-1 variable indicating whether technology  $i$  should be certified for production at facility  $j$  in period  $t$ . To simplify the notation, we use the set names interchangeable with the set cardinality. The capacity configuration ( $x_{ijt}$ ) and capacity expansion amount ( $y_{jt}$ ) are both continuous nonnegative variables, whereas the capacity expansion ( $\sigma_{jt}$ ) and technology certification ( $z_{ijt}$ ) decisions are binary. These decisions

represent the semiconductor headquarters' strategic plan for medium- to long-term capacity expansion and configuration. The plan is implemented on a rolling horizon basis in that a new five-year plan is generated at the beginning of each year.

We define the two elements of uncertainty, demand and capacity, as scenario sets  $S_1$  and  $S_2$ , respectively. Each element  $s_1 \in S_1$  ( $s_2 \in S_2$ ) identify a demand (capacity) vector with probability of occurrence of  $\pi_{s1}$  ( $\pi_{s2}$ ). A scenario for the capacity-planning problem is fully described by a pair of demand and capacity vectors from  $S_1$  and  $S_2$ . Let  $S$  be the set of all scenarios obtained by all combinations of demand and capacity possibilities and  $\pi_{s1s2}$  ( $(s_1, s_2) \in S$ ) be the probability obtained by  $\pi_{s1}\pi_{s2}$  (demand and capacity scenarios are defined exogenously and independently).

Recall that capacity configuration decisions are subject to adjustments throughout the year to accommodate demand and capacity variations. Modeled as recourse, we define a positive and a negative configuration change under scenario  $(s_1, s_2) \in S$  as  $\delta_{ijt, s1s2}^+$  and  $\delta_{ijt, s1s2}^-$ , respectively. While changing capacity configurations is unavoidable, it is costly because this disrupts regular flow of the manufacturing system, which in turn increases inventory and manufacturing cycle times. Although it is desirable to satisfy all demand from in-house production, for a certain type of capacity shortfall, outsourcing could be a more economical alternative. Preferably outsourcing is considered together with the option of building ahead in-house while carrying inventory through periods. We denote outsourcing and inventory decisions under different scenarios by variables  $O_{mts1s2}$ , and  $I_{mts1s2}$  respectively. Note that holding inventory is highly undesirable in the semiconductor industry due to the volatile nature of demands and the fact that the chance of scraping on-hand inventory is as high as 50%.

Another factor that influences the capacity planning decisions is the *bias* each product manager (PM) has about the supply source. Due to the perceived quality level and delivery performance, some manufacturing facilities are strongly preferred over others. Interestingly, some of these preferences are imposed by the customers, who would go so

far as paying a premium to ensure the manufacturing origin of their products. This factor is capture in the model, where we measure the violation of PMs' supply preferences by a variable  $E_{ijts1s2}$  and assume that there is a profit loss associated with preference violations (an additional cost in the model). On the manufacturing side, due to high capital costs, it is desirable to keep the facilities highly utilized. We measure the amount of capacity usage that is required by MMs' to achieve a target utilization level by variable  $U_{jts1s2}$ . This penalizes capacity under-utilization and ensures that expansions are not considered before existing capacity is utilized.

Let  $c^x_{ijt}$ ,  $c^\sigma_{jt}$ ,  $c^y_{jt}$ ,  $c^z_{ijt}$ ,  $c^U_{jt}$ ,  $c^I_{it}$ ,  $c^O_{it}$ ,  $c^E_{ijt}$  denote the unit costs associated with the variable in the superscript (e.g.  $c^x_{ijt}$  is the unit cost associated with  $x_{ijt}$ ). Capacity expansion costs and technology certification costs are adjusted to reflect the time value of money, however, to simplify notation we do not express this explicitly. We define  $c^+_{ijt}$ ,  $c^-_{ijt}$  as the unit costs for positive and negative configuration changes, respectively. Having laid out all the variables and parameters we can now define the cost minimizing objective function. The objective is to minimize total costs involved in meeting all projected demand, which consists of total capacity expansion and expected production, configuration deviation, capacity under-utilization, inventory carrying, outsourcing and preference violation costs. The objective function is expressed more formally as follows:

$$\begin{aligned}
& \text{Minimize (total costs)} \\
& \sum_{j \in F} \sum_{i \in M} \sum_{t \in T} c^x_{ij} x_{ijt} + \sum_{j \in F} \sum_{t \in T} c^\sigma_{jt} \sigma_{jt} + \sum_{j \in F} \sum_{t \in T} c^y_{jt} y_{jt} + \sum_{(i,j) \in N} \sum_{t \in T} c^z_{ij} z_{ijt} \\
& + \sum_{t \in T} \sum_{(s_1, s_2) \in S} \pi_{s_1 s_2} \sum_{j \in F} \sum_{i \in M} ((c^+_{ijt} + c^x_{ijt}) \delta^+_{ijt, s_1 s_2} + (c^-_{ijt} - c^x_{ijt}) \delta^-_{ijt, s_1 s_2}) \\
& + \sum_{t \in T} \sum_{(s_1, s_2) \in S} \pi_{s_1 s_2} \sum_{j \in F} c^U_{jt} U_{jts_1 s_2} + \sum_{t \in T} \sum_{(s_1, s_2) \in S} \pi_{s_1 s_2} \sum_{i \in M} c^I_{it} I_{its_1 s_2} \\
& + \sum_{t \in T} \sum_{(s_1, s_2) \in S} \pi_{s_1 s_2} \sum_{i \in M} c^O_{it} O_{its_1 s_2} + \sum_{t \in T} \sum_{(s_1, s_2) \in S} \pi_{s_1 s_2} \sum_{(i,j) \in N} c^E_{ijt} E_{ijt, s_1 s_2} \quad (1)
\end{aligned}$$

Let  $p_{jts1s2}$  denote the capacity of facility  $j$  in period  $t$  under scenario  $s_1 s_2$ . We describe the capacity availability constraints as follows. The capacity that will be available through expansion is assumed to be known with certainty, and once a capacity expansion is made



it is available in succeeding periods too. An intermediate variable  $X'_{jts_1s_2}$  is defined in (3) so that the actual configuration can be measured under each scenario.

$$\sum_{i \in M} X'_{ijts_1s_2} \leq p_{jts_1s_2} + \sum_{\tau=1}^t y_{j\tau} \quad \forall j \in F, t \in T, (s_1s_2) \in S \quad (2)$$

$$X'_{ijts_1s_2} = x_{ijt} + \delta_{ijts_1s_2}^+ - \delta_{ijts_1s_2}^- \quad \forall j \in F, i \in M, t \in T, (s_1s_2) \in S \quad (3)$$

Capacity expansion at each period are constrained by the upper and lower bounds ( $u_{jt}$  and  $l_{jt}$  respectively). The upper bound is imposed due to physical limitations such as space availability for installing machines, and the lower bound reflects the fact that once a new machine is installed capacity is increased by at least a fixed amount. The expansion constraints can be described as follows.

$$y_{jt} \geq l_{jt} \sigma_{jt} \quad \forall j \in F, t \in T \quad (4)$$

$$y_{jt} \leq u_{jt} \sigma_{jt} \quad \forall j \in F, t \in T \quad (5)$$

A single facility cannot manufacture all microelectronic products (each require a different technology) offered by the firm. Although there is always room to improve profitability by installing new equipment or implementing new process that expand the capability of a facility. For a facility to qualify for producing a certain microelectronics technology, a rather costly *quality certificate* must be obtained from an independent audit. This typically involves the testing of initial batches of wafers satisfying specified yield and quality requirements. Constraint set (6) enforces this certification constraint, where  $B$  is a sufficiently big number.

$$x_{ijt} \leq B \sum_{\tau=1}^t z_{ij\tau} \quad \forall (i, j) \in N, t \in T \quad (6)$$

The total capacity is the maximum capacity available for a product mix. Due to technological restrictions imposed by bottleneck resources in the clean room, a given technology can only use up a part of the total capacity in any given period. Let  $g_{ij}$  be the fraction of capacity of facility  $j$  that is available for technology  $i$ . Constraint set (7) describes these technology-related capacity restrictions.

$$X'_{ijts_1s_2} \leq g_{ij} (p_{jts_1s_2} + \sum_{\tau=1}^t y_{j\tau}) \quad \forall j \in F, i \in M, t \in T, (s_1s_2) \in S \quad (7)$$

As a managerial policy at the semiconductor headquarter, 90% capacity utilization is considered as a target, which forms a capacity upper bound. This is due to the historic

observation that when facilities operate beyond a certain utilization level, throughput drops significantly due to increased equipment failures and congestion in the system. Constraint set (8) imposes the utilization rule.

$$\sum_{i \in M} X'_{ijts_1s_2} + U_{jts_1s_2} \geq 0.9(p_{jts_1s_2} + \sum_{\tau=1}^t y_{j\tau}) \quad \forall j \in F, t \in T, (s_1s_2) \in S \quad (8)$$

At the strategic capacity planning phase, the policy of the firm is to satisfy all demands under every scenario. The demands can be balanced by either inventory from previous periods or outsourcing at that period as described by (9), where  $d_{its_1s_2}$  represents the demand for technology  $i$  during period  $t$  under scenario  $s_1s_2$ .

$$\sum_{j \in F} X'_{ijts_1s_2} + I_{it-1s_1s_2} - I_{its_1s_2} + O_{its_1s_2} = d_{its_1s_2} \quad \forall i \in M, t \in T, (s_1s_2) \in S \quad (9)$$

Let  $h_{ij}$  represent the fraction of total demand for technology  $i$  that is preferred to be supplied by facility  $j$ . Constraint set (10) specifies supply preferences of the PMs, where  $N$  is the set of facility-technology pairs for which certification decisions are considered.

$$X'_{ijts_1s_2} + E_{ijts_1s_2} \geq h_{ij}d_{its_1s_2} \quad \forall (i, j) \in N, t \in T, (s_1s_2) \in S \quad (10)$$

The beginning and ending inventory is desired to be zero for planning purposes. Constraint set (11) implements this decision rule.

$$I_{its_1s_2} = 0 \quad \forall i \in M, (s_1s_2) \in S, t = 0, T \quad (11)$$

Let (SP) denote the strategic capacity planning model described by (1) - (11). While this model captures all important aspects of the problem from the viewpoint of the semiconductor headquarter, it is difficult to gather all demand and capacity related information from the local divisions. Another concern is that with realistic size problems it may be difficult to solve (SP) optimally. In the next section, we develop alternative stochastic programming models which approximate (SP) while reducing the problem size and information requirement considerably. These models also represent different scenario approximation schemes, serving as a basis to study the effects of decentralized planning schemes where PMs and MMs consider their own local decisions while the headquarter consider only global aspect of the problem.

## 2.2. A Decentralized Planning Model

Before starting our exposition, we first make the observation that the strategic capacity planning problem (SP) is a multistage stochastic program with block separable recourse (Louveaux, 1986). Therefore, it can be posed as a two-stage stochastic model as follows:

(SP)

$$\begin{aligned} \text{Minimize} \quad & \sum_{j \in F} \sum_{i \in M} \sum_{t \in T} c_{ij}^x x_{ijt} + \sum_{j \in F} \sum_{t \in T} c_{jt}^\sigma \sigma_{jt} + \sum_{j \in F} \sum_{t \in T} c_{jt}^y y_{jt} + \sum_{(i,j) \in N} \sum_{t \in T} c_{ij}^z z_{ijt} + Q^{SP}(x, y, z, S) \\ \text{subject to} \quad & (4)-(6) \end{aligned}$$

where  $Q^{SP}(x, y, z, S)$  is the recourse function defined by the second stage problem:

$$Q^{SP}(x, y, z, S) = \left\{ \begin{aligned} & \text{minimize} \sum_{t \in T} \sum_{(s_1, s_2) \in S} \pi_{s_1 s_2} \sum_{j \in F} \sum_{i \in M} ((c_{it}^+ + c_{ij}^x) \delta_{ijt, s_1 s_2}^+ + (c_{it}^- - c_{ij}^x) \delta_{ijt, s_1 s_2}^-) \\ & + \sum_{t \in T} \sum_{(s_1, s_2) \in S} \pi_{s_1 s_2} \sum_{j \in F} c_j^U U_{jts_1 s_2} + \sum_{t \in T} \sum_{(s_1, s_2) \in S} \pi_{s_1 s_2} \sum_{i \in M} c_{it}^I I_{its_1 s_2} \\ & + \sum_{t \in T} \sum_{(s_1, s_2) \in S} \pi_{s_1 s_2} \sum_{i \in M} c_{it}^O O_{its_1 s_2} + \sum_{t \in T} \sum_{(s_1, s_2) \in S} \pi_{s_1 s_2} \sum_{(i,j) \in N} c_{ij}^E E_{ijt, s_1 s_2} \\ & \text{s.t.} \quad (2), (3), (7)-(11) \end{aligned} \right\}$$

In this two-stage setting, the first stage problem makes up the capacity expansion decisions of the headquarter (at the strategic level) whereas the second stage recourse problem constitutes operational level adjustments made by the PMs and MMs once a demand and/or capacity scenario is observed. Also observe that the second stage problem is a combined manufacturing and marketing subproblems bundled by the capacity reconfiguration variables for each scenario (i.e.  $\delta_{ijt, s_1 s_2}^+$  and  $\delta_{ijt, s_1 s_2}^-$ ). Although one may quickly observe that the second stage problem is separable by scenarios, this is not the most sensible way to decompose the decisions due to the way information is organized at the firm. From the viewpoint of decentralized decision making where main decision entities or the PMs and MMs keep control over their local information, it is useful to take the following point of view: the recourse problem represent the manufacturing (marketing) subproblem with complete information about capacity (demands) scenarios and *certain* information about the demands (capacity), characterized by scenario set  $S^{MM}$  ( $S^{PM}$ ). When complete and symmetrical information is available to PMs and MMs, the two versions of the recourse problems become one (i.e.,  $S^{MM} = S^{PM} = S$ ).

and constraints (2),(3),(7)-(11)). When partial and/or asymmetric information is available from the other side, PMs (MMs) must approximate the capacity (demands) scenarios from the manufacturing (marketing) side (i.e.,  $S^{MM} \subset S$ ,  $S^{PM} \subset S$ , and  $S^{MM} \neq S^{PM}$ ) while considering a subset of the constraints (e.g., its own local constraints). To analyze this particular model of decentralized planning, we introduce the following notation: define  $\delta_{ijts_1s_2}^{(MM)+}$  and  $\delta_{ijts_1s_2}^{(MM)-}$  as the capacity reconfiguration decisions of the manufacturing managers, and  $\delta_{ijts_1s_2}^{(PM)+}$  and  $\delta_{ijts_1s_2}^{(PM)-}$  as the desired resource adjustment of the product (marketing) managers. Using these new variables, and scenario sets  $S^{MM}$  and  $S^{PM}$ , we defined the manufacturing managers' problem (MMP) and the product managers' problem (PMP) as follows.

#### (MMP)

*Minimize (expected operating costs):*

$$\sum_{t \in T} \sum_{(s_1s_2) \in S} \pi_{s_1s_2} \sum_{j \in F} \sum_{i \in M} ((c_{it}^+ + c_{ij}^x) \delta_{ijts_1s_2}^{(MM)+} + (c_{it}^- - c_{ij}^x) \delta_{ijts_1s_2}^{(MM)-}) + \sum_{t \in T} \sum_{(s_1s_2) \in S} \pi_{s_1s_2} \sum_{j \in F} c_j^U U_{jts_1s_2}$$

*subject to*

$$\begin{aligned} \sum_{i \in M} X'_{ijts_1s_2} &\leq p_{jts_1s_2} + \sum_{\tau=1}^t y_{j\tau} & \forall j \in F, t \in T, (s_1s_2) \in S^{MM} \\ X'_{ijts_1s_2} &= x_{ijt} + \delta_{ijts_1s_2}^{(MM)+} - \delta_{ijts_1s_2}^{(MM)-} & \forall j \in F, i \in M, t \in T, (s_1s_2) \in S^{MM} \\ X'_{ijts_1s_2} &\leq g_{ij} (p_{jts_1s_2} + \sum_{\tau=1}^t y_{j\tau}) & \forall j \in F, i \in M, t \in T, (s_1s_2) \in S^{MM} \\ \sum_{j \in M} X'_{ijts_1s_2} + U_{jts_1s_2} &\geq 0.9(p_{jts_1s_2} + \sum_{\tau=1}^t y_{j\tau}) & \forall j \in F, t \in T, (s_1s_2) \in S^{MM} \end{aligned}$$

#### (PMP)

*Minimize (expected order fill costs):*

$$\begin{aligned} \sum_{t \in T} \sum_{(s_1s_2) \in S} \pi_{s_1s_2} \sum_{j \in F} \sum_{i \in M} ((c_{it}^+ + c_{ij}^x) \delta_{ijts_1s_2}^{(PM)+} + (c_{it}^- - c_{ij}^x) \delta_{ijts_1s_2}^{(PM)-}) + \sum_{t \in T} \sum_{(s_1s_2) \in S} \pi_{s_1s_2} \sum_{i \in M} c_{it}^I I_{its_1s_2} \\ + \sum_{t \in T} \sum_{(s_1s_2) \in S} \pi_{s_1s_2} \sum_{i \in M} c_{it}^O O_{its_1s_2} + \sum_{t \in T} \sum_{(s_1s_2) \in S} \pi_{s_1s_2} \sum_{(i,j) \in N} c_{ijt}^E E_{ijts_1s_2} \end{aligned}$$

*subject to*

$$\begin{aligned}
\sum_{j \in F} X'_{ijts_1s_2} + I_{i,t-1,s_1s_2} - I_{it,s_1s_2} + O_{its_1s_2} &= d_{its_1s_2} & \forall i \in M, t \in T, (s_1s_2) \in S^{PM} \\
X'_{ijts_1s_2} &= x_{ijt} + \delta_{ijts_1s_2}^{(PM)+} - \delta_{ijts_1s_2}^{(PM)-} & \forall j \in F, i \in M, t \in T, (s_1s_2) \in S^{PM} \\
X'_{ijts_1s_2} + E_{ijts_1s_2} &\geq h_{ij} d_{its_1s_2} & \forall (i, j) \in N, t \in T, (s_1s_2) \in S^{PM} \\
I_{its_1s_2} &= 0 & \forall i \in M, (s_1s_2) \in S^{PM}, t = 0, T
\end{aligned}$$

If we are to allow local decision makers to solve their own decision problems (MMP) and (PMP) instead of the bundled recourse problem  $Q^{SP}(x, y, z, S)$ , a significant reduction of computational requirement could be expected, as the cardinality of the scenario set may be significantly smaller (i.e., in the extreme case when MM and PM only consider their own scenarios,  $|S| = |S^{MM}| \times |S^{PM}|$ ). However, it is not difficult to see that doing so is equivalent to relaxing the following necessary bundling constraints between the two divisional subproblems when considered under  $Q^{SP}(x, y, z, S)$ :

$$\delta_{ijts_1s_2}^+ = \delta_{ijts_1s_2}^{(MM)+} = \delta_{ijts_1s_2}^{(PM)+} \quad \forall i \in M, j \in F, t \in T, (s_1s_2) \in S \quad (12)$$

$$\delta_{ijts_1s_2}^- = \delta_{ijts_1s_2}^{(MM)-} = \delta_{ijts_1s_2}^{(PM)-} \quad \forall i \in M, j \in F, t \in T, (s_1s_2) \in S \quad (13)$$

Solving (MMP) and (PMP) without constraints (12) and (13) will most likely lead to poor lower bound solution to  $Q^{SP}(x, y, z, S)$ , thus the overall problem (SP). This should be intuitive because each subproblem misses very decisive information about the other side (i.e., demand or capacity), it is likely to generate highly biased decisions to its own favor, which in turn may steer the headquarter's solution away from the overall optimum. Specifically, the manufacturing subproblem (MMP) has no information about demand scenarios and how their reconfiguration decisions may affect the costs of the marketing division. On the other hand, the marketing subproblem (PMP) uses only the configuration deviation costs to adjust local decisions of "how much supply to request" but without any information on capacity availability and utilization costs. The two sides are unlikely to reach any agreement, even less efficient coordination.

From a pure computational point of view, traditional mathematical programming technique such as Lagrangean decomposition would suggest relaxing the bundling constraints while adding a Lagrangean term in the subproblem objectives. Unfortunately, this approach requires excessive communication between the two decision entities (on their recourse variable values,  $\delta^{PM}$  and  $\delta^{MM}$ ). We propose a new form of decomposition

motivated by the information requirements of this decentralized decision problem. The basic idea is that we require the MMs (PMs) to consider a version of the PMs' (MMs') problem as a *second stage recourse* problem of its own local decision problem, i.e., we reformulate each subproblem as a two-stage stochastic program with recourse where the first stage is the local decision problem MMP (PMP) while the recourse problem is defined as a model of it's opponent's subproblem PMP (MMP). The intuition behind this approach is that each decision maker contemplates a "model of others" when making its decisions since the other decision entities represent a *main source of uncertainty* to its decision problem. With this decomposition scheme, we compensate for the loss of information by relaxation of constraints (12) and (13). We expressed the reformulation as subproblems (MMP)' and (PMP)' as follows:

(MMP)'

$$\text{Minimize } (MMP) + Q^{MM}(\delta_{ijts_1s_2}^{(MM)+}, \delta_{ijts_1s_2}^{(MM)-})$$

$$\text{where } Q^{MM}(\cdot) = \{(PMP) \mid \delta_{ijts_1s_2}^{(MM)+} \rightarrow \delta_{ijts_1s_2}^{(PM)+}, \delta_{ijts_1s_2}^{(MM)-} \rightarrow \delta_{ijts_1s_2}^{(PM)-}\}$$

(PMP)'

Minimize

$$(PMP) + Q^{PM}(\delta_{ijts_1s_2}^{(PM)+}, \delta_{ijts_1s_2}^{(PM)-})$$

$$\text{where } Q^{PM}(\cdot) = \{(MMP) \mid \delta_{ijts_1s_2}^{(PM)+} \rightarrow \delta_{ijts_1s_2}^{(MM)+}, \delta_{ijts_1s_2}^{(PM)-} \rightarrow \delta_{ijts_1s_2}^{(MM)-}\}$$

The recourse functions  $Q^{MM}(\cdot)$  and  $Q^{PM}(\cdot)$  correspond to the manufacturing and marketing subproblems, respectively. Notice that both (MMP)' and (PMP)' are *approximations* for the term  $Q^{SP}(x, y, z, S)$ , which is the recourse based on complete information in problem (SP). To establish a basis that would allow us to compare *centralized vs. decentralized* planning strategies, we now define decentralized planning as an *approximation* to the centralized planning problem (SP) as follows using (MMP)' and (PMP)'.

(ASP)

$$\begin{aligned} \text{Minimize } & \sum_{j \in F} \sum_{i \in M} \sum_{t \in T} c_{ij}^x x_{ijt} + \sum_{j \in F} \sum_{i \in T} c_{jt}^\sigma \sigma_{jt} + \sum_{j \in F} \sum_{t \in T} c_{jt}^y y_{jt} + \sum_{(i,j) \in N} \sum_{t \in T} c_{ij}^z z_{ijt} \\ & + \alpha (MMP)' + (1 - \alpha) (PMP)' \end{aligned}$$

subject to (4)-(6) and  $\alpha \in [0, 1]$ .

Problem (*ASP*) consists of stage 1 of problem (*SP*) plus a convex combination of two recourse approximations as described by (*MMP*)' and (*PMP*)'. The weight  $\alpha$  can be interpreted as the headquarter's way to reconcile operational decisions made by the divisions based on the perceived accuracy and information value of the related division's computations. Throughout this study, we set  $\alpha$  to 0.5. It should be clear that problem (*ASP*) approximates (*SP*) while at the same time attaining a structure suitable for decentralized planning.

Interestingly, (*ASP*) takes the viewpoint that the headquarter makes stage-1 capacity expansion and configuration decisions considering future recourses to be made by manufacturing and marketing. Different from a typical two-stage stochastic program is the fact that decision makers who actually carry out the recourses are different from ones who make the first stage decision. This is in fact the very nature of the semiconductor capacity planning problem in reality. In other words, we assume that the recourse policy used under a particular scenario is not known exactly *a priori* and must be approximated at time zero. Coordination of the three decision entities: the headquarter, the PMs and the MMs ultimately drives the efficiency of the decision. This *strategic* decision structure suggests an *operational* structure for capacity adjustment over the course of the year where the recourse problems (*MMP*)' and (*PMP*)' represent base decision models for the local decision makers in manufacturing and marketing, respectively.

### 2.3 Solution Strategies for (*ASP*) and Their Implications on Information Usage

We now define a few solution strategies for (*ASP*) so as to gain insights on the performance-computation trade-off when different forms of information are used.

**Full Recourse and No Recourse:** Recall that the recourse function in subproblems (*MMP*)' and (*PMP*)' represent a "model of others" in the sense that it captures some aspects of the scenarios and constraints of the other side. Conceptually, the recourse term in the subproblems (*MMP*)' and (*PMP*)' can vary from a full description of the scenario of the other side (complete information, no approximation), to a complete empty set (no information). We refer to the former case as the *full recourse* model which corresponds to

the original problem (SP) (i.e. (ASP) = (SP)), or the centralized model. We refer to the latter extreme as the *no recourse* model.

**Partial Recourse:** Since problems (MMP)' and (PMP)' are both posed as two-stage stochastic programs, we can apply recourse approximation methods to construct recourse functions for subproblems (i.e.  $Q^{MM}(\cdot)$  and  $Q^{PM}(\cdot)$ ). One approach is to include all the constraints and decision variables but approximate the scenarios of the other side. Since the capacity and demand distributions are considered private information for the manufacturing and marketing divisions, respectively, this approximation is consistent with the information availability in real problems. Two most basic recourse approximation methods in the stochastic programming literature are Jensen's lower bound and Edmunsen-Madansky upper bound (Kall and Wallace (1994), and Birge and Louveaux (1996)) approximations for minimization problems. Jensen's lower bound is computed by using the expected value of random parameters in the recourse function. Modified this way the recourse function intentionally underestimates the true value over all values of the first stage variables. On the other hand, Edmunsen-Madansky upper bound is computed using an approximate distribution for the random parameters in the recourse. This distribution is constructed by taking the support points of the random variable distribution and resample the probability density function on these points. Edmunsen and Madansky prove that the recourse function formed this way provides an upper bound to the original over all values of first stage variables. Although simple and use little information about the random variables, the two methods are known to perform well in terms of approximating the original recourse function. We term this approximation as the *partial recourse* model. It is closer to the *full recourse* model in terms of information requirements and the degree of coupling between subproblems. There are several more sophisticated and accurate recourse approximation techniques in the literature (c.f., Birge and Wets (1986)), however, they also require a much increased information transparency to the headquarter, which render them unsuitable for our study.

**Simple Recourse:** The decisions that affect both subproblems are the capacity configuration decisions that are set by the headquarter (the stage 1 decision in (SP)). The



two subproblems must compute their local costs as a function of the configuration decisions. Since both subproblems are expressed as linear programs, their corresponding recourse functions (in SP) are piecewise linear. We can therefore construct a linear approximation of this cost function as follows: first, include in the subproblem a *deviation cost* incurred to the other side for each unit of capacity configuration that deviates from some *base configuration value*. Define  $X^{*(MM)}$  and  $X^{*(PM)}$  as follows.

$$X^{*(MM)} = \{X^{(MM)} \mid \min (MMP)\}$$

$$X^{*(PM)} = \{X^{(PM)} \mid \min (PMP) \text{ s.t. } \delta_{ijts_1s_2}^{(PM)+} = \delta_{ijts_1s_2}^{(PM)-} = 0\}$$

$X^{*(MM)}$  is the capacity configuration vector that optimizes the manufacturing subproblem. Similarly,  $X^{*(PM)}$  minimizes the marketing subproblem which does not consider the capacity configuration deviations. Next, we make the following definitions.

$$C^{*(MM)} = \min (MMP) \text{ s.t. } X = X^{*(MM)}$$

$$C^{*(PM)} = \min (PMP) \text{ s.t. } \delta_{ijts_1s_2}^{(PM)+} = \delta_{ijts_1s_2}^{(PM)-} = 0, X = X^{*(PM)}$$

Consider the manufacturing subproblem (i.e. (MMP)) and the modified marketing subproblem (i.e. (PMP) s.t.  $\delta_{ijts_1s_2}^{(PM)+} = \delta_{ijts_1s_2}^{(PM)-} = 0$ ). If we set  $X = X^*$  in both problems and solve them, the dual variable values provide us with the unit cost of deviating from the configuration given by  $X^*$ . In this setting, deviating from  $X^*$  is the same as perturbing the right hand side of the relevant subproblem constraints. Let  $\beta^{*(MM)}$  ( $\beta^{*(PM)}$ ) be the dual prices associated with unit deviations (both positive and negative) in  $X^{*(MM)}$  ( $X^{*(PM)}$ ), and  $\delta^{*(MM)+}, \delta^{*(MM)-}$  ( $\delta^{*(PM)+}, \delta^{*(PM)-}$ ) be the variables which measure the deviations from the capacity configuration that is optimized for the manufacturing (marketing) subproblem. Thus, we can define  $Q^{MM}(\cdot)$  and  $Q^{PM}(\cdot)$  for simple recourse as follows.

$$Q^{MM}(\cdot) = \min \sum_{i \in T} \sum_{s_1 \in S_1} \pi_{s_1} \sum_{j \in F} \sum_{i \in M} \beta_{ijt}^{*(PM)} (\delta_{ijts_1}^{*(PM)+} + X_{ijts_1}^{*(PM)-}) + C^{*(PM)}$$

s.t.

$$X_{ijts_1}^{(MM)} = X_{ijt}^{*(PM)} + \delta_{ijts_1}^{*(PM)+} - \delta_{ijts_1}^{*(PM)-} \quad \forall j \in F, i \in M, t \in T, s_1 \in S_1, \text{ and}$$

$$Q^{PM}(\cdot) = \min \sum_{i \in T} \sum_{s_2 \in S_2} \pi_{s_2} \sum_{j \in F} \sum_{i \in M} \beta_{ijt}^{*(MM)} (\delta_{ijts_2}^{*(MM)+} + X_{ijts_2}^{*(MM)-}) + C^{*(MM)}$$

s.t.

$$X_{ijts_2}^{(PM)} = X_{ijt}^{*(MM)} + \delta_{ijts_2}^{*(MM)+} - \delta_{ijts_2}^{*(MM)-} \quad \forall j \in F, i \in M, t \in T, s_2 \in S_2$$

Note that the constant terms  $C^{*(MM)}$  and  $C^{*(PM)}$  are shown in the formulation for illustrative purposes. They do not go into the model during optimization. The above procedure is analogous to generating two cuts (around the optimal solution) in Benders' decomposition given the capacity configuration values that optimize each subproblem. From a different perspective, we can say that the “other side's” subproblem is approximated by using a dual basis. Since the formulation of (MMP)' ((PMP)') with the above  $Q^{MM}(\cdot)$  ( $Q^{PM}(\cdot)$ ) definition corresponds to a two-stage stochastic programming model with simple recourse (Kall and Wallace (1994)), we refer to this approach as the *simple recourse* model. The *simple recourse* model is in between *partial recourse* and *no recourse* models in terms of approximation complexity and information decentralization.

In our analysis using the dual prices to determine configuration deviation costs (i.e.  $\beta^{*(MM)}$ ,  $\beta^{*(PM)}$ ), we realized that the solution is quite sensitive at  $X^*$  in both subproblems. That is, for most unit deviations the basis changes, thus invalidates the dual prices. Since the *simple recourse* model is an approximation in itself, we wanted to obtain the deviation costs accurately in order not to cause more deterioration in the approximation. This requires solving the subproblem optimally for each unit deviation from  $X^*$  and recording the objective function difference between the computed one and  $C^*$ . Considering that we are changing the right hand side of a subproblem by one unit it takes only a few more iterations from the basis obtained for computing the  $X^*$ , to find the new optimum. Hence, the computational burden that this approach brings over using the dual prices is negligible, but the accuracy it brings is significant.

In this section we have developed four decentralized planning schemes that approximate the centralized model. In these approximation schemes, as the modeling complexity decreases, the degree of coupling between manufacturing and marketing subproblems also decreases. In the next section, we conduct intensive computational testing on these approximation schemes. We are particularly interested in the tradeoff between solution quality and the degree of “coupling” between subproblems; we use this study as a means

to analyze the cost-tradeoff between centralization and decentralization capacity planning.

### **Further Discussions and Remarks**

There are different decomposition techniques available to solve the stochastic models developed in this section (see Ruszczyński (1997)), among which Benders' decomposition (Benders (1962)) is the most widely used one due to its simplicity and effectiveness. Application of Benders' decomposition to solve the models in a decentralized environment can be interpreted as follows. The center announces a capacity configuration and expansion decision vector. The manufacturing and marketing subproblems solve (MMP)' and (PMP)' respectively and send their bid prices against the announced decisions. In return, the Headquarter takes a convex combination (as in the objective function of (ASP)) of the prices they receive from the subproblems to form an estimation of the cost function associated with their decisions (that is, add an optimality cut to the master problem). This is a typical resource-directive decomposition application, which will converge to the optimal solution of problem (ASP) at the end of iterations.

For computational testing, we will solve each of the above approximation as modified versions of the monolithic model as opposed to implementing the actual decomposition algorithm one-by-one. In this way, we can isolate the merit of the proposed approximation methods without the bias introduced by any particular implementation.

## **3. Computational Experiments**

### **3.1. Data generation**

To test a wide range of demand and capacity scenarios we use characteristics learned from real-world semiconductor data to generate experimental data. To simulate matured technologies with stable demand, and new technologies with highly variable demand over time we generate the demands for a technology over multiple periods from a uniform distribution with mean and range drawn from two uniform distributions. The capacities from fab facilities are drawn from a uniform distribution whose mean is set equal to the

total demand for that facility with the upper and lower supports set at 30% above and below of the mean, respectively. Negative values are truncated to zero. On average, taken over the instances used in the experiment, the total demand exceeds total capacity by about 10%. For each facility (technology), scenarios are generated by discretizing a normal distribution defined by the capacity (demand). The normal distribution is truncated at 3 times the standard deviation from the mean on both ends and divided into 6 equal intervals. The midpoints of the intervals are taken as the scenario values and the probability density contained in an interval is taken as the probability associated with that scenario. All capacity (demand) scenarios are applied to all facilities (products). Table 1 summarizes the parameters that have a common generation method over all the problem instances.

**Table 1.** Distributions used to generate common cost parameters

Parameter	Value distribution	Parameter	Value distribution
$p_{jt}$	$A = \frac{\sum_m d_{mt}}{F}$ $p_{jt} \sim U(a*0.7, a*1.3)$	$d_{it}$	$a \sim U(300, 600)$ $b \sim U(50, 150)$ $d_{it} \sim U(a-b/2, a+b/2)$
$u_{jt}$	$U(p_{jt}*0.30, p_{jt}*0.50)$	$l_{jt}$	$U(p_{jt}*0.10, p_{jt}*0.15)$
$h_{ij}$	$U(d_{it}*0.40, d_{it}*0.80)$	$g_{ij}$	$U(p_{jt}*0.30, p_{jt}*0.70)$
$c^x_{ij}$	$U(50, 100)$	$c^L_{ij}$	$U(100, 200)$
$c^y_{jt}$	$U(25, 50)$	$c^U_{it}$	50

### 3.2. Experimental Design

We identify three important factors that can affect the performance of the proposed models: (i) degree of variability in both capacity and demand scenarios, (ii) the problem size, and (iii) the cost structure. Since all models use scenario approximation in the (second stage) recourse problem, as the variability in the scenario distribution increases the approximation error may also increase. In the experiment, we used coefficient of variation values of 0.1 and 0.3 to represent low and high variability, respectively.

The *simple recourse* model approximates the decision variables and the constraints of the other side's subproblem, hence, as the problem size increases the approximation quality may be affected. We therefore examine two levels of problem sizes. For *small* problems

we set the number of facilities to 3, the number of technologies to 12 and the number of planning periods to 3 with 6 demand scenarios and 6 capacity scenarios defined per period. In addition, we set the cardinality of set  $N$  (technology facility pairs for which certification decisions are to be made) to 40% of the number of technologies and the set is formed by selecting random facility-technology pairs. We implemented all models in AMPL and used CPLEX 6.5 as the solver. The problems are solved on a Pentium Celeron based PC running at 433 Mhz with 128MB of memory. The resulting (SP) model has 21 binary, 15,129 variables with 10,602 linear constraints and on the average it takes 30 CPU seconds to find the optimal solution. For *large* problems, we increased the number of technologies from 12 to 18 and the number of planning periods from 3 to 5. The resulting problem has 50 binary, 37,725 variables with 26,310 linear constraints, which on the average takes 160 CPU seconds to find the optimal solution.

Since there are many cost parameters involved in the model, we considered cost structure an additional experimental factor. Through initial trials, we identify a subset of cost factors that are most influential on the solution quality as follows: capacity expansion fixed cost, technology certification fixed cost, outsourcing cost, inventory carrying cost, and configuration deviation costs. We group these cost factors into two: (1) capacity expansion fixed costs and certification costs which affect the stage 1 costs and capacity expansion amount, and (2) outsourcing, inventory carrying and configuration deviation costs which effect whether the optimum solution will rely more on in-house capacity (including capacity expansions) or more on outsourcing. We consider two levels (high and low) for each parameter group which in turn defines four different cost structures (CS1-CS4) as summarized in Table 2. Given historic information we have in the semiconductor environment under study, we performed pilot experiments to set the distribution parameters such that under *low* expansion cost the total capacity expansions made is approximately 8% of the total expected capacity (over all facilities and periods), whereas for *high* expansion cost this is reduces to 5%. Similarly, the *low* and *high* levels of outsourcing costs is set by pilot experiments as follows: we measured the ratio of total expected *in-house production* and total *expected demand*; at the low level, where outsourcing is relatively cheaper than inventory carrying and deviation costs, the ratio is

set at 80%, whereas it is set at 95% at the high level where outsourcing is relatively more expensive. These conditions are consistent with the insights we have gained from experienced planners. Table 3 further summarizes the experimental factors and their levels.

**Table 2.** Levels of the cost structure factor and their descriptions

	CS1	CS2	CS3	CS4
Expansion Cost	Low	Low	High	High
Outsourcing Cost	High	Low	High	Low
$c_{jt}^s$	$U(30000,40000)$	$U(30000,40000)$	$U(70000,80000)$	$U(70000,80000)$
$c_{ij}^z$	$U(30000,40000)$	$U(30000,40000)$	$U(70000,80000)$	$U(70000,80000)$
$c_{it}^o$	$U(400,500)$	$U(200,300)$	$U(400,500)$	$U(200,300)$
$c_{it}^f$	$U(200,300)$	$U(400,500)$	$U(200,300)$	$U(400,500)$
$c_{it}^+, c_{it}^-$	$U(200,300)$	$U(400,500)$	$U(200,300)$	$U(400,500)$

**Table 3.** Experimental factors and their levels

Experimental Factor		Levels	
1.	Variability in capacity scenarios (coefficient of variation) - Cap_CV	0.1	0.3
2.	Variability in demand scenarios (coefficient of variation) - Dem_CV	0.1	0.3
3.	Problem size (S=small, L=large)	S	L
4.	Cost Structure	CS1 - CS4	

As discussed earlier, the strategic capacity planning decisions include capacity configuration, capacity expansion and certification, which make up the stage 1 of the (SP) model. The operational decisions correspond to stage 2 recourses and is computed only to determine the optimum stage 1 decisions rather than actual implementation. We therefore evaluate the four alternative recourse models (solution strategies for ASP) as follows: we first solve problem (ASP) using one of the three strategies (i.e. *partial recourse (PR)*, *simple recourse (SR)*, *no recourse (NR)*, and *full recourse (OPT)*), then set the stage 1 decisions variables in (SP) to the solution obtained in (ASP), we then resolve (SP) optimally. The resulting solution is the *simulated stochastic solution* for problem (ASP), which we consider in the evaluations. We do not consider the CPU time used for finding the *simulated* solution value in the time comparison. Neither do we include the *preprocessing* time (i.e., CPU time involved to find the base configuration values and deviation cost coefficients) needed for the simple recourse approach.

A common approach used to measure the benefits of stochastic analysis versus a deterministic one is to compute the *Value of Stochastic Solution* (VSS) (Birge and Louveaux, 1997). Since all the models developed in this study are different forms of Stochastic Programming models, we included the simulated stochastic solution of the deterministic model, which we denote by EEV, in the comparisons as an absolute performance minimum that any stochastic model must achieve.

We applied performance analysis on the proposed models using two main criteria: percent deviation from the overall stochastic optimal and computation time. We present the experimental results in the next section.

### 3.3. Experimental Results

We identified two recourse approximation methods that can be used for recourse approximation under the *partial recourse* approach: Jensen's lower bound (LB) and Edmunsen-Madansky upper bound (UB) approximations. Our pilot experiments reveal no significant difference between these two methods when combined with other factor combinations. We therefore use the Jensen's lower bound approach for the *partial recourse*. We further determine the number of replications for each factor combination after some pilot runs. We tried up to 30 replications and found that the relative ranking does not change after 10 replications. Therefore, we used 10 replications throughout the experiment.

A first look at the results showed that the problem size and capacity expansion cost level have virtually no effect on the performance of the proposed models in relation to the optimum. This indicates that the performance of the methods may scale well with the problem size. It is not surprising that the level of capacity expansion fixed cost and certification cost does not have a substantial effect because these costs constitute a small fraction of the total expected cost in the optimal solution (on the average 14% to 18% of the total at their low and high levels, respectively). Therefore, in the rest of the analysis we present the results for large problems using cost structures CS1 and CS2.

We first analyzed the effects of the scenario variability on the optimal solution. We examine several different combinations of capacity and demand CV values and record the *OPT*, the optimal solution value,  $t_a$ , the ratio between expected capacity usage and expected available capacity (including expansions), and  $t_e$ , the ratio of total capacity expansion and total expected available capacity. The results are summarized in Table 4.

**Table 4.** Analysis of the optimal solution under different levels of scenario variability

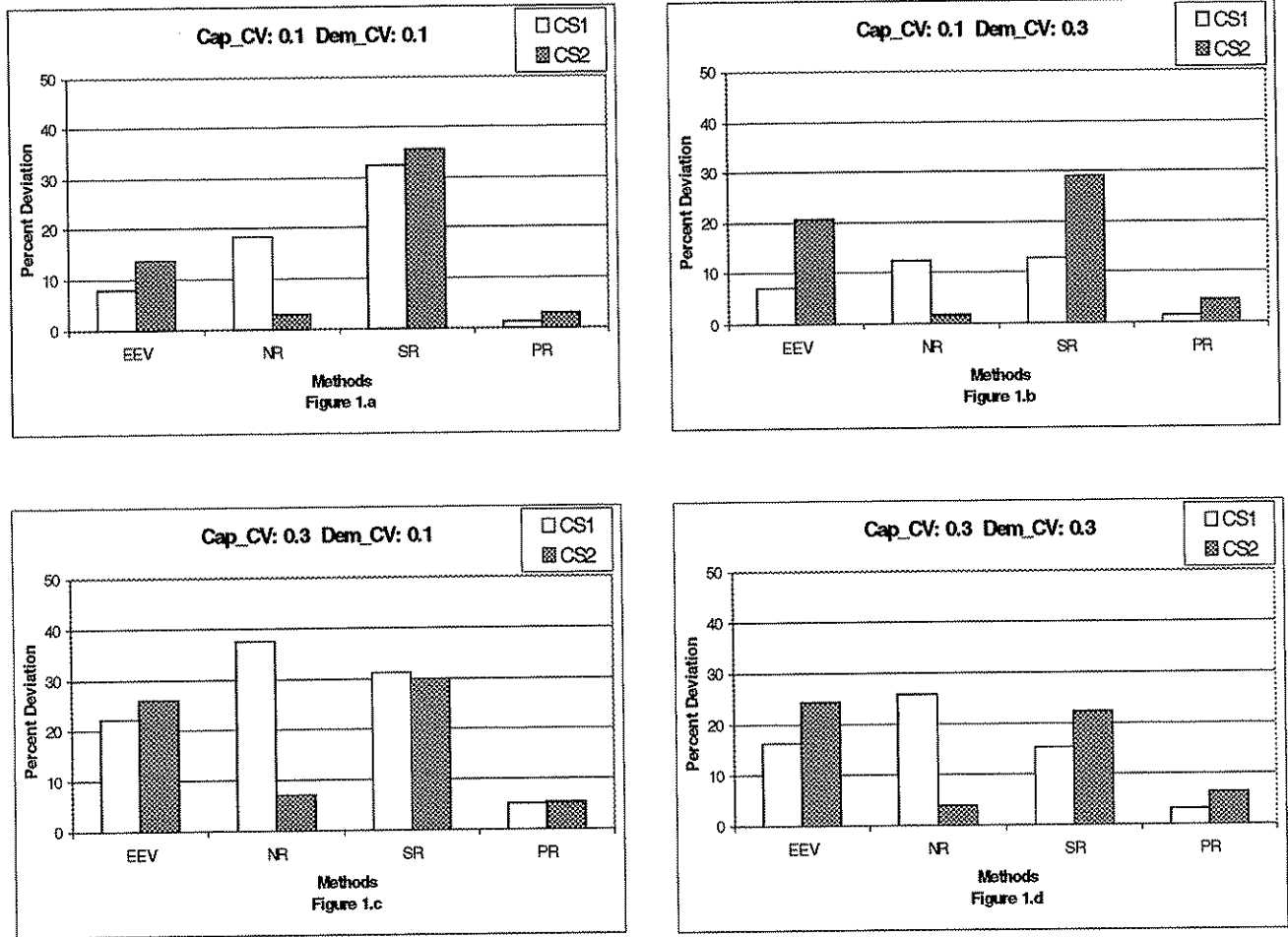
Fac_CV	Dem_CV	CS1			CS2		
		<i>OPT</i>	$t_a$	$t_e$	<i>OPT</i>	$t_a$	$t_e$
0.1	0.1	4434266	97.8	5.8	4295228	91.6	5.0
0.1	0.3	6520839	91.0	6.1	5851379	76.8	2.3
0.3	0.1	5377170	97.2	10.9	5336760	90.9	10.6
0.3	0.3	7133114	92.8	9.8	6854442	76.8	8.1

As one would expect, an increase in demand and capacity variability increases the expected total costs regardless of the cost structure. However, the cost increase is more sensitive to demand variability than it is to capacity variability. Another observation is that high variability in capacity leads to capacity expansions (notice the increase in  $t_e$ ) whereas demand variability is smoothed out by outsourcing (notice the decrease in  $t_a$ ). This suggests for the interest of reducing overall costs, spending effort on reducing demand variability is more fruitful than reducing capacity variability (by expansion).

Figure 1.(a-d) depicts the relative performance of four strategies under different factor combinations measured in percentage from optimum (*OPT*). Each factor combination is repeated for 40 replications, which means the statistics collected in the figure is the result of  $(40 \times 5 \times 2 \times 4)$  1600 runs. As shown in the Figure, the *PR* model performs consistently well under all factor combinations, producing solutions within 1.2-6.5% of the optimum. No factor seems to have a substantial effect of the performance of the *PR* method. The *SR* model, on the other hand has rather disappointing performance. It seems to be performing relatively better under high variability case (Figure 1.d) at which it is almost the same as the EEV when the cost structure is CS1. Although the *SR* model uses more information in the subproblems this information is apparently distorted as the configuration decisions of the local problem ( $X^{(MM)}$  ( $X^{(PM)}$ )) deviates from the optimum configuration for the other



subproblem  $((X^{*(PM)} (X^{*(MM)})))$ . This may have led to wrong decisions with costly consequences.



**Figure 1.** Relative performance of the four solution strategies (in percentage from the optimum)

Another observation about the *SR* strategy is that it performs relatively better at the highest variability case (Figure 1.d). We offer the following explanations: recall that the recourse in subproblems of the *SR* model uses a basis around the optimum solution of the other subproblem. Two factors that may lead to a basis change (therefore invalidate the information in the *SR* model) are the configuration deviations and the realizations of the stochastic parameters (capacity and demand). When variability is high, the change in the stochastic parameters is the dominant factor, when the opposite is true, configuration deviations dominates. Therefore, in high variability cases the cost coefficients computed

in the simple recourse represent correct values within a large range of configuration deviations, which in turn leads to relatively better solutions.

Observe the Figure further; we see that the *NR* strategy is significantly affected by the cost structure. Under cost structure *CS2*, its performance is comparable to *PR*, stay within 1.8-7.0% of the optimal solution. However, under *CS1*, *NR* performs the worst in all but one case. This can be explained by the fact that under *CS1* the outsourcing costs is high which makes it more damaging to ignore the other side's subproblem. In *PR* and *SR*, the recourse is approximated by taking average over the recourse approximations that come from *both* subproblems. In *NR*, the average is taken only over the configuration deviation costs (the only common recourse cost component between the subproblems).

EEV is also effected by the cost structure factor and performs relatively better at *CS1*. This is quite predictable because at *CS2* the recourse costs are higher due to high outsourcing levels and the EEV does not consider the recourse costs.

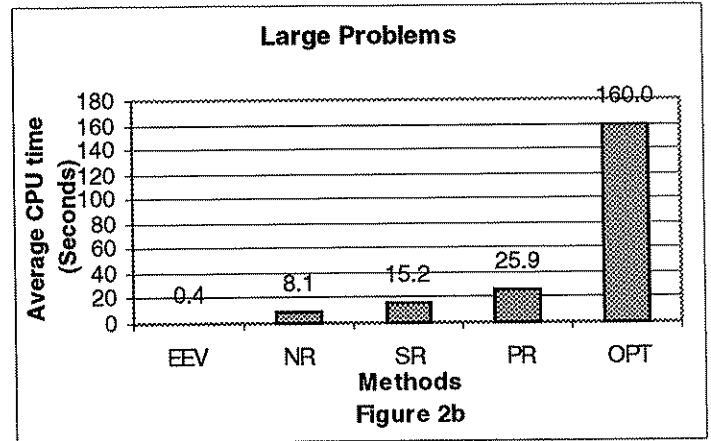
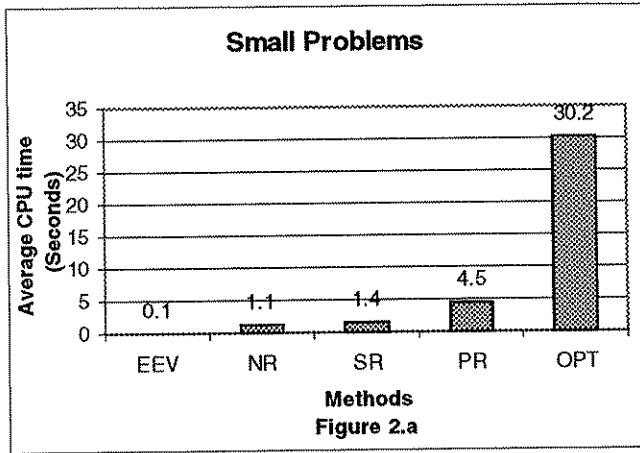
Figure 2 shows the overall computational requirements across all approximation schemes. This comparison reveals a dramatic difference between computing the global optimal solution and the decentralized models. Table 5 summarize the computer time required by each approximation schemes as a percentage of the computer time required by the optimum. The problems we studied in the semiconductor firm typically have around 55 technologies and between 5-10 planning periods with 5 facilities. In these cases, even the *PR* model will require a significant amount of computer time.

**Table 5.** Computational time of approximation schemes relative to optimum

%	EEV	NR	SR	PR
Small Problems	0.28	3.81	4.57	14.78
Large Problems	0.23	5.03	9.48	16.17

#### 4. Conclusion

In this study, we develop a strategic capacity-planning model for a major semiconductor manufacturer. Motivated by the needs to ease the computational requirements of a large stochastic program and to provide a framework for decentralized decision making under



**Figure 2.** Average computational requirements for the approximation schemes

uncertainty, we proposed alternative scenario approximation schemes based on various assumptions of information usage. These schemes vary in complexity and information requirements, thus providing unique insights on the trade-off between decentralization and solution quality. We conduct extensive experiments (1,600 runs not including the pilot experiments) to gain insight on the solution characteristics under different levels of scenario variability, cost structures, and decision environments; and at the same time evaluate the performance of different scenario approximation schemes.

We observe that demand and capacity variability has distinct effects on the optimum stochastic solution. Specifically, capacity uncertainty induces more capacity expansions whereas demand uncertainty induces a higher level of outsourcing. This observation provides insight on strategic capacity planning decisions in semiconductor industry as capacity expansion and outsourcing present distinctly different cost implications.

Our experiment shows that substantial benefits can be achieved by using a stochastic programming model for strategic capacity-planning problem (as demonstrated by the comparison with EEV), and that decentralization of decisions to the manufacturing and marketing managers can be achieved with a somewhat minor tradeoff in total cost. The partial recourse (*PR*) scheme, although the most demanding in terms of information and computational requirements produces near-optimal solutions (within 6.5%) with a small fraction of the computer time (at 16.17%). On the other hand the *NR* scheme performs

well under a specific cost structure where the outsourcing costs dominate. Considering the low information requirements of *NR* and its low computational requirements, it may be worthwhile to investigate more special cases where it performs well. The simple recourse scheme falling between *NR* and *PR* (in terms of complexity) produces rather disappointing results.

### **Acknowledgement**

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