Computational Risk Averse
Optimization in Finance

What is Portfolio Optimization?
Portfolio Optimization is the process of choosing a group of assets and allocating your money optimally across those assets. Asset allocation is determined by the investors objectives. This can be maximizing portfolio returns, minimizing losses, only having a certain amount of money allocated to an industry, invest in ESG companies only etc…

General Assets
- Cash
- Real Estate
- Fixed Income
- Cryptocurrency
- Stock Market
- Funds

CVaR Optimization
Computational Risk Averse Optimization focuses on using the Conditional Value at Risk (CVaR) measure to optimize a portfolio. The CVaR of a distribution is defined as the mean of the end tail of a distribution, starting from the Value at Risk (VaR) or in this case, \( \alpha \), of the distribution. We can formulate our problem as a linear program, subject to CVaR constraints:

\[
\min \quad -u^T m
\]
\[
\text{s.t.} \quad w \in \mathbb{W}, \alpha \in \mathbb{R}
\]
\[
\alpha = \frac{1}{\beta} \sum_{j \in J} u_j \leq \delta
\]
\[
u_j \geq 0, \quad j = 1, \ldots, J
\]
\[
u_j \leq m_i, \quad i = 1, \ldots, I
\]

The formula above allows for the optimization of the allocation of a portfolio with a maximum return within a certain risk limit, expressed with CVaR.

- \( w \), weights of each stock
- \( m \), expected returns of each stock
- \( \alpha \), the Value at Risk (VaR)
- \( J \), the time series
- \( \beta \), the confidence level
- \( \delta \), CVaR limit
- \( u_j \), vector for auxiliary variables

Note: \( w \) and \( m \) are vectors

CVaR Optimized Portfolio
We chose 10 companies across multiple industries and used the stock prices that ranged from the 2012 global financial crisis through the 2022 coronavirus peak. After solving the CVaR linear optimization problem (without LSTM projections) we have this portfolio composition:

- Company Portfolio Allocations
  - Apple - .6%
  - Cisco - 27.9%
  - Pfizer - 20.8%
  - Microsoft - 0.0%
  - Starbucks - 0.3%
  - Comcast - 22.7%
  - Fiserv - 0.0%
  - Target - 17.8%
  - Marriott - 4.2%

LSTM Future Research
Long Short Term Memory Neural Networks are a type of machine learning tool suited for time series forecasting. LSTM’s are capable of learning long term dependencies and are trained using back propagation. We use this method to project asset prices and create an optimized portfolio that can be realized at the projection date.

Symbols
- Sigmoid - \( \sigma = 1/(1 + e^{-x}) \)
- Tanh
- Weight - \( W \)
- bias - \( b \)
- Cell state - \( C \)
- Input Content - \( X \)
- Previous State - \( h \)

LSTM Basic Implementation
AAPL Stock
Trading Days: 1/1/22 - 1/19/22
Actual Price = $165.018
Predicted Price = $168.26

Trading Days: 1/19/2012 - 1/20/22