Torque and Force-Free Swimming at Low Reynolds Number

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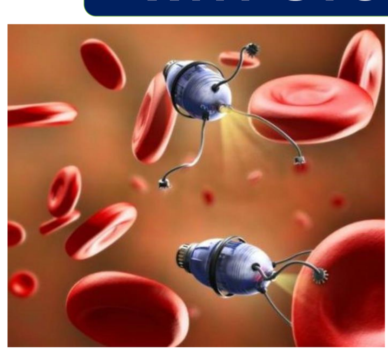
Introduction and Background

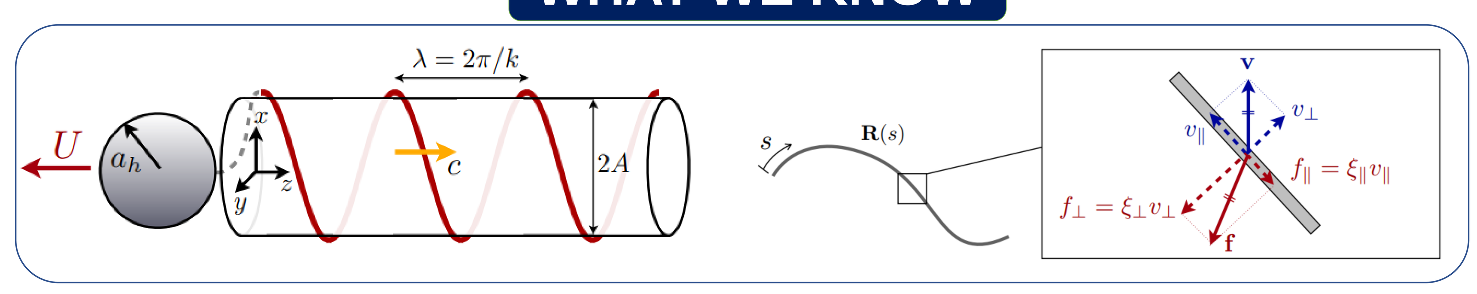
WHY STUDY MICROSWIMMERS?



Artificial bacteria are strong candidates for biomedical applications such as drug delivery and minimally invasive surgery

- ✓ Reduced side effects
- ✓ Faster recovery
- ? **Swimming at small scales & in biological fluids is challenging!**
- $Re = \frac{inertial\ forces}{viscous\ forces} = \frac{\rho U L}{\mu} \rightarrow 0$
- Biological fluids are complex fluids

WHAT WE KNOW



$$\mathbf{f}_{\text{vis}} = -[\xi_\perp \mathbf{nn} + \xi_\parallel \mathbf{tt}] \cdot \mathbf{v}\,, \quad \xi_\perp = \frac{2\mu\pi}{\ln\left(\frac{L}{r}\right) - 1/2}, \ \xi_\parallel = \frac{4\mu\pi}{\ln\left(\frac{L}{r}\right) + 1/2}$$

$$\mathbf{v} = \mathbf{v_d} + \mathbf{U} + \boldsymbol{\Omega} \times \mathbf{r}$$

$$\mathbf{v_d} = \frac{\partial \mathbf{r}}{\partial t} = [A\omega \sin(k\alpha s - \omega t), -A\omega \cos(k\alpha s - \omega t), 0]$$

$$\mathbf{F}_{vis} = \int_0^L \mathbf{f}_{vis} ds\,, \quad \mathbf{M}_{vis} = \int_0^L \mathbf{r} \times \mathbf{f}_{vis} ds$$

$$\mathbf{F}_{vis} \cdot \mathbf{e}_z + F_{head} = 0, \ F_{head} = 6\pi\mu a_h U$$

$$\mathbf{M}_{vis} \cdot \mathbf{e}_z + M_{head} = 0, \ F_{head} = -8\pi\mu a_h^3 \Omega$$

WHAT IS MISSING

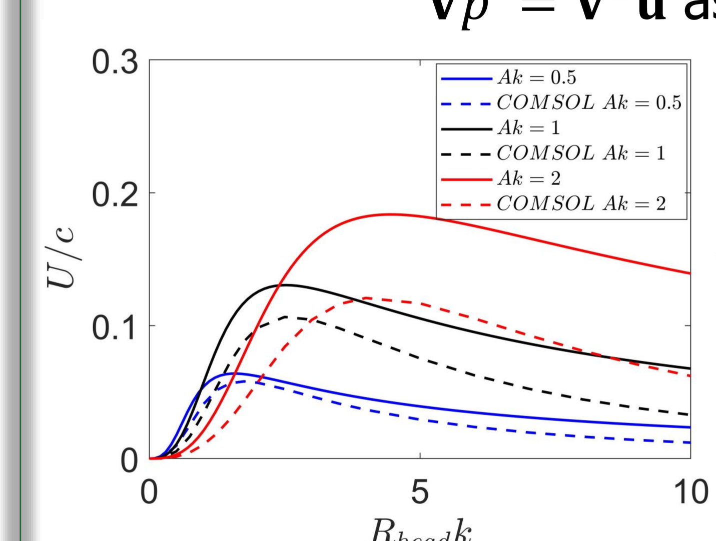
- ? **Hydrodynamic interactions**
- Slender body assumption used in Pak & Lauga [1] does not take hydrodynamic interactions between helical turns, and tail and head into account
- ? Nonlinear effects
  - Slender body theory assumes that force and torque are linear combinations of velocity and angular velocity
- ? Effects of shear-thinning rheology
  - Most bodily fluids are shear-thinning!

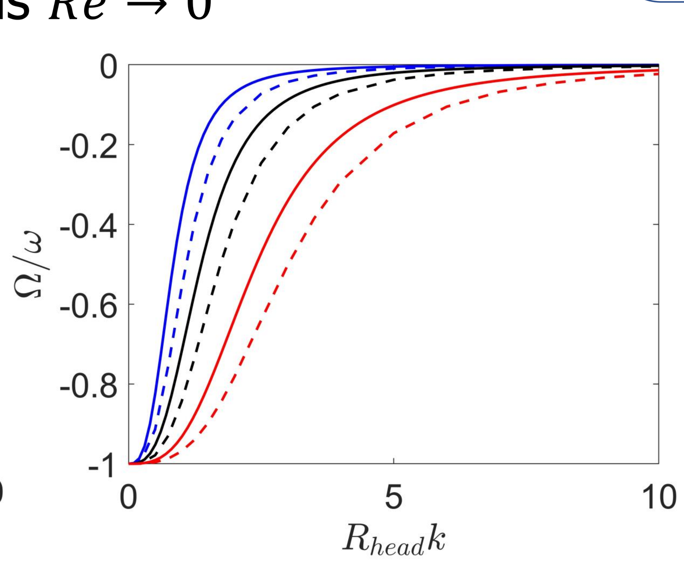
CFD Model: Governing Equations and Validation

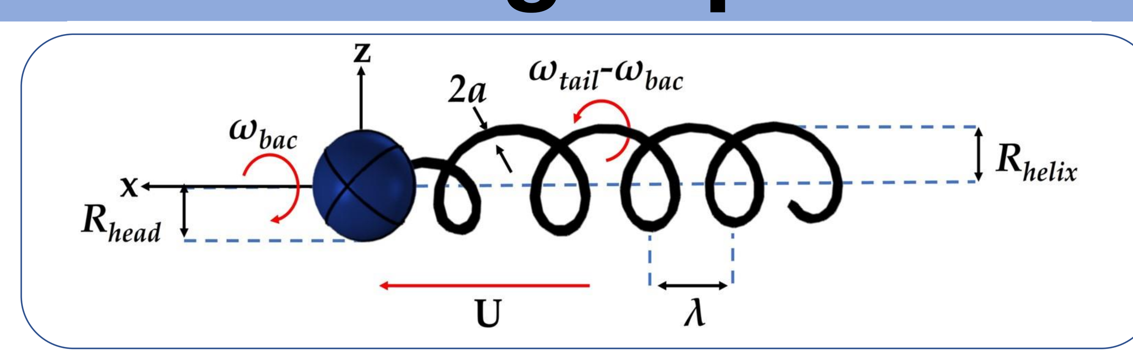
NEWTONIAN

$$Re\left(\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u}\right) = -\nabla p + \nabla^2 \mathbf{u}$$

$\nabla p = \nabla^2 \mathbf{u}$ as $Re \rightarrow 0$







$$\nabla p = \nabla \cdot \boldsymbol{\tau} \qquad \nabla \cdot \mathbf{u} = 0$$

$$\mathbf{u} = \mathbf{U} + \boldsymbol{\Omega} \times (\mathbf{r} - \mathbf{r}_0), \quad \mathbf{r} \in S$$

$$\mathbf{F}_{net} = \int_S \boldsymbol{\sigma} \cdot \mathbf{n} dA = \mathbf{0}$$

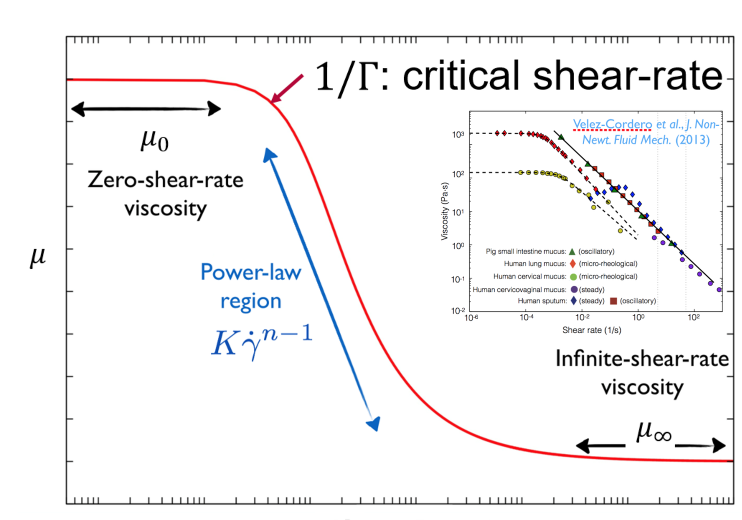
$$\mathbf{T}_{net} = \int_S (\mathbf{r} - \mathbf{r}_0) \times \boldsymbol{\sigma} \cdot \mathbf{n} dA = \mathbf{0}$$

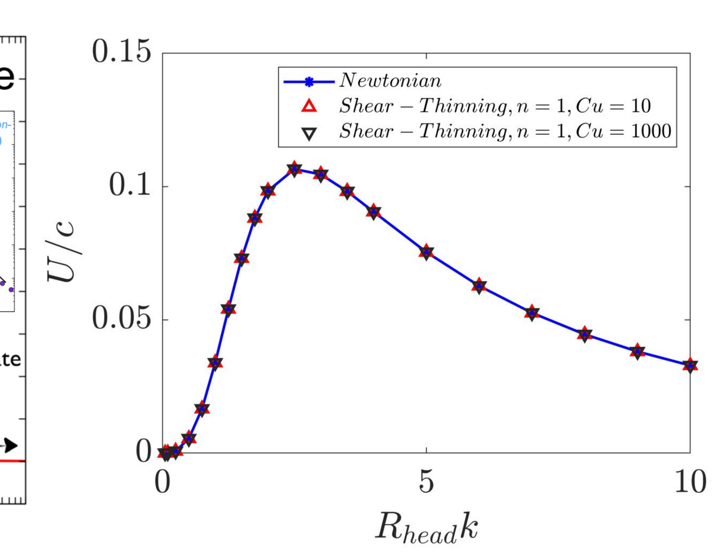
$$\eta = {}^{\eta_{tug}}/_{\eta_{swim}}$$

SHEAR-THINNING

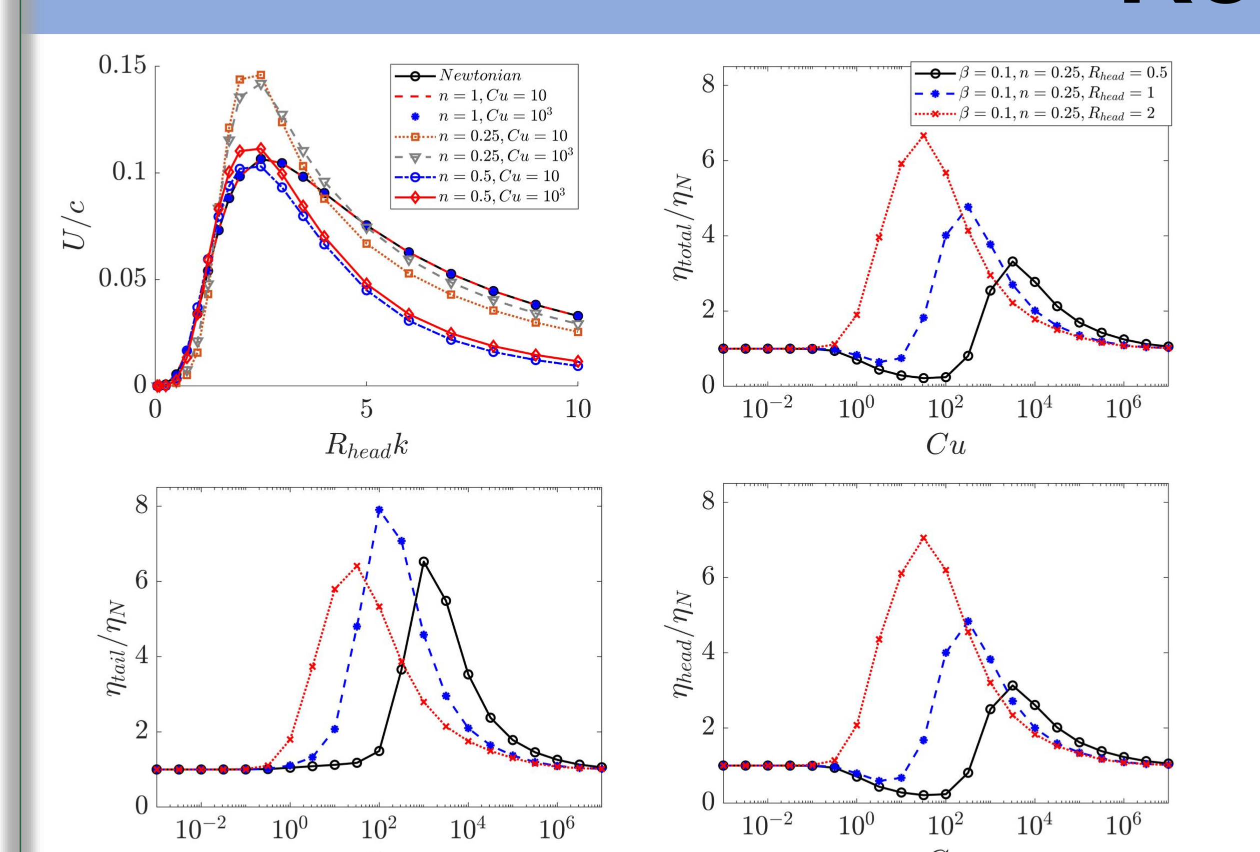
$$\dot{\boldsymbol{\gamma}} = \nabla \mathbf{u} + (\nabla \mathbf{u})^T \quad \mu_{eff}^* = \beta + (1-\beta)(1 + Cu^2|\dot{\boldsymbol{\gamma}}^*|^2)^{\frac{n-1}{2}}$$

$$\boldsymbol{\tau} = [\mu_{eff}]\dot{\boldsymbol{\gamma}} \quad Cu = \omega\Gamma = \frac{\omega}{{}^1/_\Gamma} \sim \frac{shear\ rate}{critical\ shear\ rate}, \ \beta = \frac{\mu_\infty}{\mu_0}$$

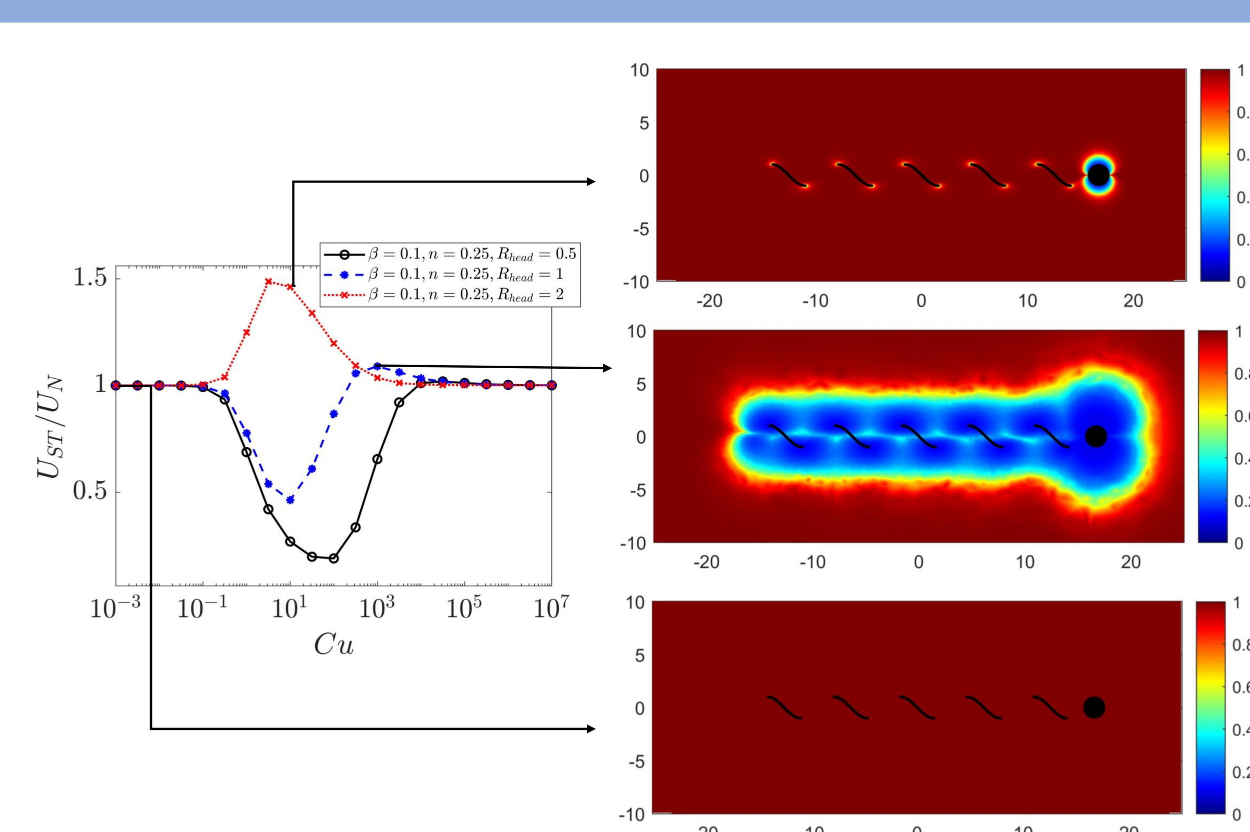




Results



- ➢ Carreau model converges to Newtonian results at n=1 as expected
- ➢ There is an optimum head size that maximizes velocity. $\mathbf{U} = \mathbf{U}(R_{head}, Cu, n)$
- ➢ Lower n (more shear-thinning) yields higher swimming velocities
- ➢ Tail efficiency is enhanced overall consistent with literature [2], but head efficiency (and total efficiency) enhancement is size dependent



- ➢ Both enhanced and reduced **U** may be obtained compared to swimming in Newtonian fluids
- ➢ Soft confinement effect is observed and contributes to enhanced velocity
- ➢ **Swimmer geometry and actuation angular velocity can be tuned to obtain desired results**

Future work

- ? **Effect of boundaries**
- Circulatory system and GI tract are networks of cylindrical channels which will contribute to hydrodynamic interactions
- ? **Effect of finite tail length**
- Studies focusing on tail geometry consider infinite helical tails
- ? **Multiple swimmers**
  - Interactions between multiple swimmers may affect the swimming trajectories

References

[1] Pak, O. S., & Lauga, E. (2012). Theoretical models in low-Reynolds-number-locomotion. In *Fluid-Structure Interactions in Low-Reynolds-Number Flows* (1st ed.). Royal Society of Chemistry.

[2] Demir, E., Lordi, N., Ding, Y., & Pak, O. S. (2020). Nonlocal shear-thinning effects substantially enhance helical propulsion. *Physical Review Fluids*, *5*(11), 111301

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