Computational Risk Averse Optimization in Finance

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Abstract

With a multitude of pandemics, market crashes, global recessions, and wars occurring in the past couple of decades, it can be observed that financial markets are a risky place for asset allocation. The issue with this is that the only way to grow assets outside of entrepreneurship is investing in said markets. The CVaR constraint uses the mean of the beta of the tail of a portfolio return distribution to create an optimized weighted portfolio that minimizes investor's losses. We are improving on this method by using a Long Short Term Memory Neural Network to project the asset prices in a portfolio and predict the optimal portfolio allocations. We have decided to use a portfolio comprised of only stock's in this research. These stocks are distributed across different industries.

What is Portfolio Optimization?

Portfolio Optimization is the process of choosing a group of assets and allocating your money optimally across those assets.

Asset allocation is determined by the investors objectives. This can be maximizing portfolio returns, minimizing losses, only having a certain amount of money allocated to an industry, invest in ESG companies only etc…

General Assets	
Cash	Real Estate
Fixed Income	Cryptocurrency
Stock Market	Funds

CVaR Optimization

Computational Risk Averse Optimization focuses on using the Conditional Value at Risk (CVaR) measure to optimize a portfolio. The CVaR of a distribution is defined as the mean of the end tail of a distribution, starting from the Value at Risk (VaR) or in this case, *alpha*, of the distribution. We can formulate our problem as a linear program, subject to CVaR constraints:

$$\underset{(w,\alpha,u_1,\ldots,u_J)}{\operatorname{argmin}} \quad -w^T\bar{m}$$

$$s.t. \quad w \in W, \alpha \in \mathbb{R}$$

$$\alpha + \frac{1}{J(1-\beta)}\sum_{j=1}^{J} u_j \leq \delta$$

$$u_j \geq 0, \quad j = 1,\ldots,J$$

$$w^T r_j + \alpha + u_j \geq 0, \quad j = 1,\ldots,J$$

The formula above allows for the optimization of the allocation of a portfolio with a maximum return within a certain risk limit, expressed with CVaR.

- w, weights of each stock
- m, expected returns of each stock
- α, the Value at Risk (VaR)
- J, the time series
- β, the confidence level
- δ, CVaR limit
- u_j, vector for auxiliary variables

Note: w and m are vectors

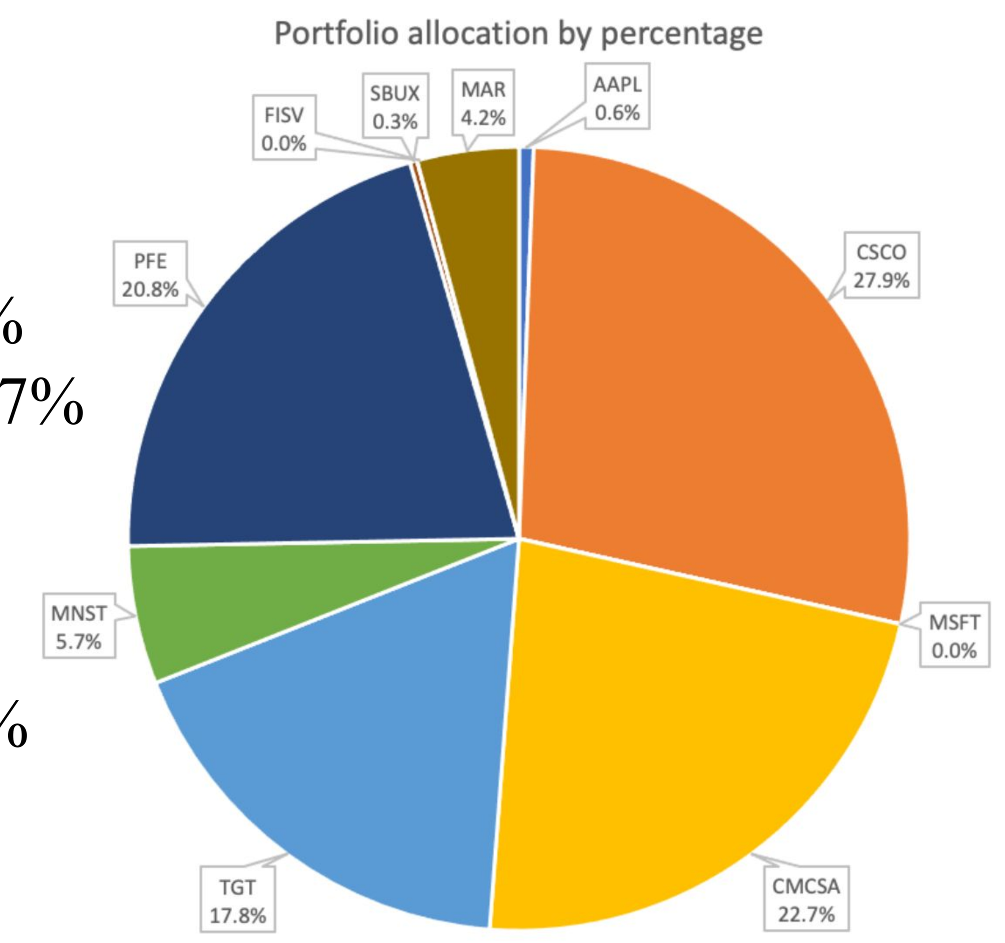
CVaR Optimized Portfolio

We chose 10 companies across multiple industries and used the stock prices that ranged from the 2012 global financial crisis through the 2022 coronavirus peak.

After solving the CVaR linear optimization problem (without LSTM projections) we have this portfolio composition:

Company Stock Tickers

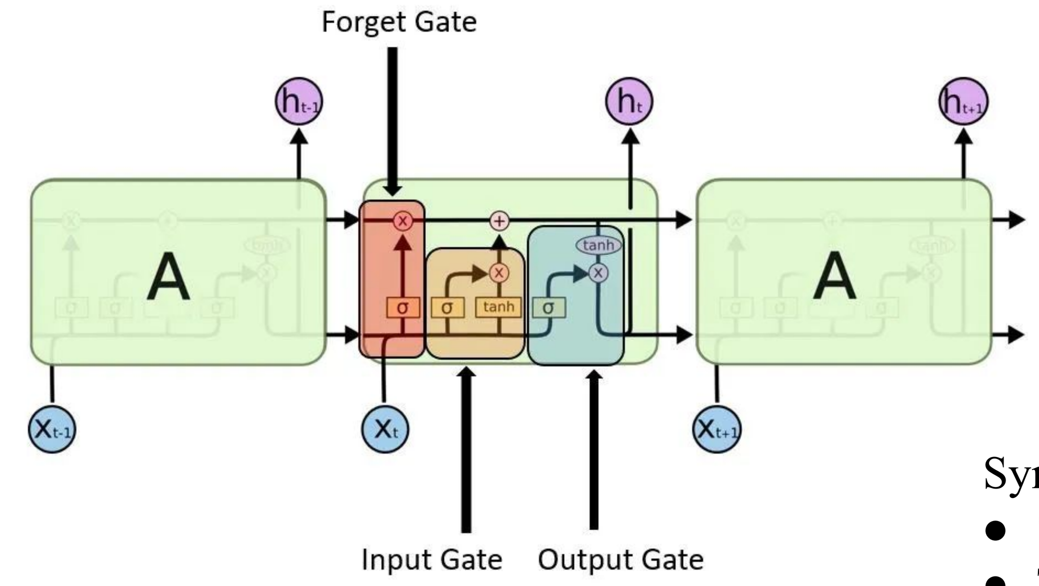
1. AAPL - Apple - .6%
2. CSCO - Cisco - 27.9%
3. MSFT - Microsoft - 0.0%
4. CMSCA - Comcast - 22.7%
5. TGT - Target - 17.8%
6. MNST - Monster - 5.7%
7. PFE - Pfizer - 20.8%
8. SBUX - Starbucks - 0.3%
9. FISV - Fiserv - 0.0%
10. MAR - Marriott - 4.2%



Trading Days: 1/19/2012 - 1/20/22

LSTM Future Research

Long Short Term Memory Neural Networks are a type of machine learning tool suited for time series forecasting. LSTM's are capable of learning long term dependencies and are trained using back propagation. We use this method to project asset prices and create an optimized portfolio that can be realized at the projection date.



There are 3 "gates"

Symbols

- Sigmoid - $\sigma = 1/(1 + e^{-x})$
- Tanh
- Weight - W
- bias - b
- Cell state - C
- Input Content - X
- Previous State - h

Input

$$i_t = \sigma\left(W_i \cdot [h_{t-1}, x_t] \ + \ b_i\right)$$

$$\tilde{C}_t = \tanh(W_C \cdot [h_{t-1}, x_t] \ + \ b_C)$$

Output

$$o_t = \sigma\left(W_o \ [h_{t-1}, x_t] \ + \ b_o\right)$$

$$h_t = o_t * \tanh\left(C_t\right)$$

Forget

$$f_t = \sigma\left(W_f \cdot [h_{t-1}, x_t] \ + \ b_f\right)$$

LSTM Basic Implementation

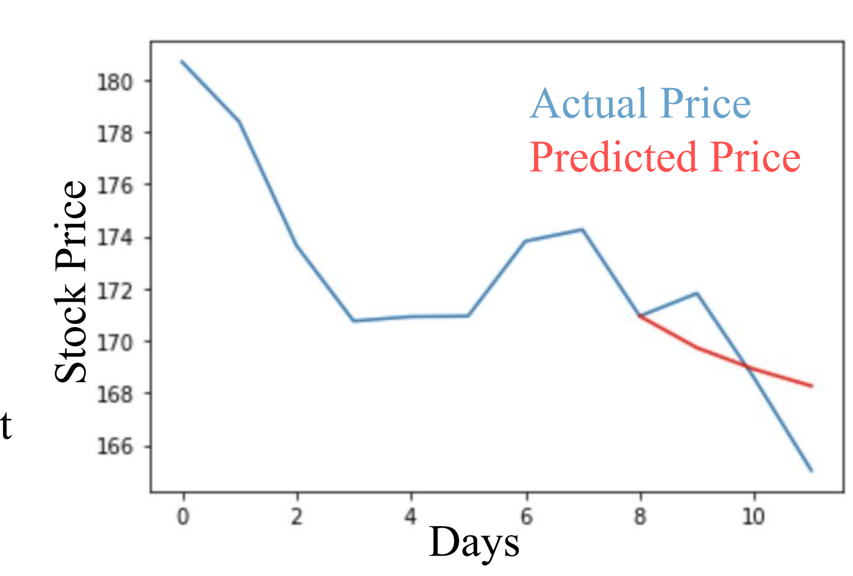
AAPL Stock

Trading Days: 1/1/22 - 1/19/22

Actual Price = $165.018

Predicted Price = $168.26

Note: This is a basic single node LSTM that has not been optimally parameterized.



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