Electric Power Grid Interdiction

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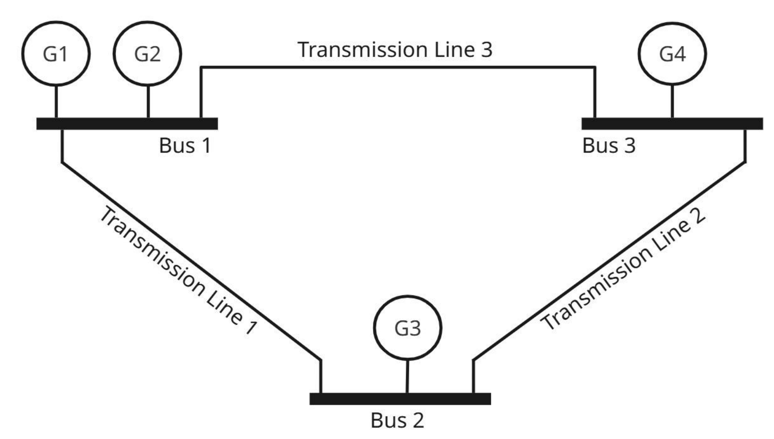
What is Electric Power Grid Interdiction?

An electric power grid system is composed of buses, generators, and transmission lines. A power grid system is concerned with delivering power across generators and transmission lines, denoted as P^{Gen} and P^{Line}, in a manner that minimizes generation costs, c_g and penalties for unmet demand, f_b.



$$\min_{P^{Gen},P^{Line},S} \sum_g c_g P_g^{Gen} + \sum_b S_b f_b$$

$$\text{subject to:} \quad -\overline{P}_l^{Line} \leq P_l^{Line} \leq \overline{P}_l^{Line} \quad \forall l$$

$$0 \leq P_g^{Gen} \leq \overline{P}_g^{Gen} \quad \forall g$$

$$0 \leq S_{ic} \leq d_{ic} \quad \forall i, c.$$

An electric power grid is susceptible to "interdiction", or disruption of grid components leading to unmet demand, denoted as **S**. An "interdictor" makes decisions about how to optimally allocate interdiction resources to maximize disruption based on the decisions a distributor makes to optimally deliver power. This is the essence of a bilevel optimization problem.

Interdiction Variables

$\delta^{Gen}, \delta^{Line}$: binary variables for interdiction

Formulation

$$\max_{\delta^{Gen},\delta^{Line}} \gamma(\delta^{Line}, \delta^{Gen})$$

$$\text{where } \gamma = \min_{P^{Gen},P^{Line},S} \sum_g c_g P_g^{Gen} + \sum_b S_b f_b$$

This bilevel optimization model can be deconstructed using AMPL, an optimization software, to create what is called the "Interdiction Algorithm". The algorithm helps a distributor understand how an interdictor selects optimal attacks, allowing them to identify the most critical grid components for delivery. The distributor can use "hardening" to protect them.

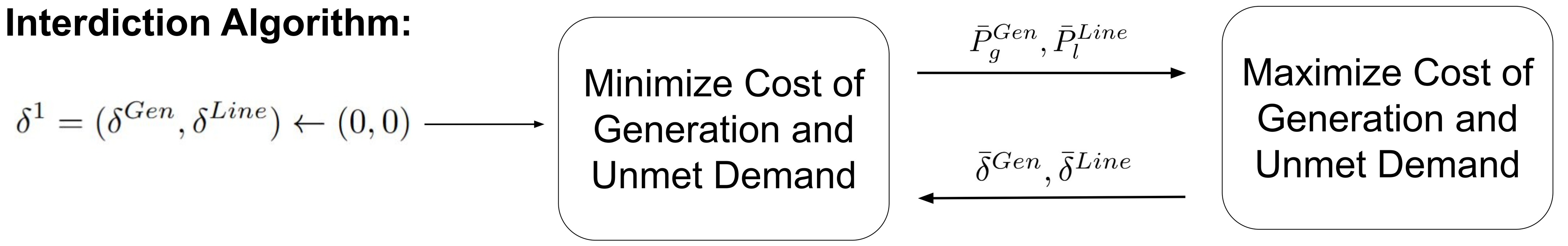
Shifting the focus to delivery patterns, we aim to answer the question: How can a distributor deliver power in a way that is resilient to all possible attacks? The effectiveness of an optimization-based approach is assessed by comparing expected costs to those of a chosen heuristic.

Interdiction Algorithm

Sample AMPL Code for Bilevel Problem:

```
minimize minCost: sum {g in NG} genCost[g]*x[g] + sum {b in NB} curtCost[b]*y[b];
maximize maxCost: sum {g in NG} genCost[g]*x[g] + sum {b in NB} curtCost[b]*y[b];
subject to Generation {b in NB}:
        sum{g in NG} Abg[b,g]*x[g] + sum{l in NL} Alb[l,b]*t[l] >= demand[b] - y[b];
subject to CurtailmentLimit{b in NB}: y[b]<= demand[b];
subject to GenerationLimit{g in NG}: x[g] <= maxGen[g]*(1-IG[g]);
subject to linelowerbd{l in NL}: -transCap[l] *(1-IL[l]) <= t[l];
subject to lineupperbd{l in NL}: t[l] <= transCap[l] * (1-IL[l]);
subject to budgetConstraint:
        sum{g in NG} genAttackCost+[g]*IG[g] + sum{l in NL} lineAttackCost[l]*IL[l] <= budget;
```

Interdiction Algorithm:



Resilient Power Delivery Planning

After understanding the vulnerabilities of the grid system, it is natural to develop a delivery strategy that is robust to any attack. formulation.

Formulation

$$\min_{\pi} \mathbb{E}_{\delta^{Gen},\delta^{Line}\sim\pi} \left[\gamma(\delta^{Line}, \delta^{Gen})\right]$$

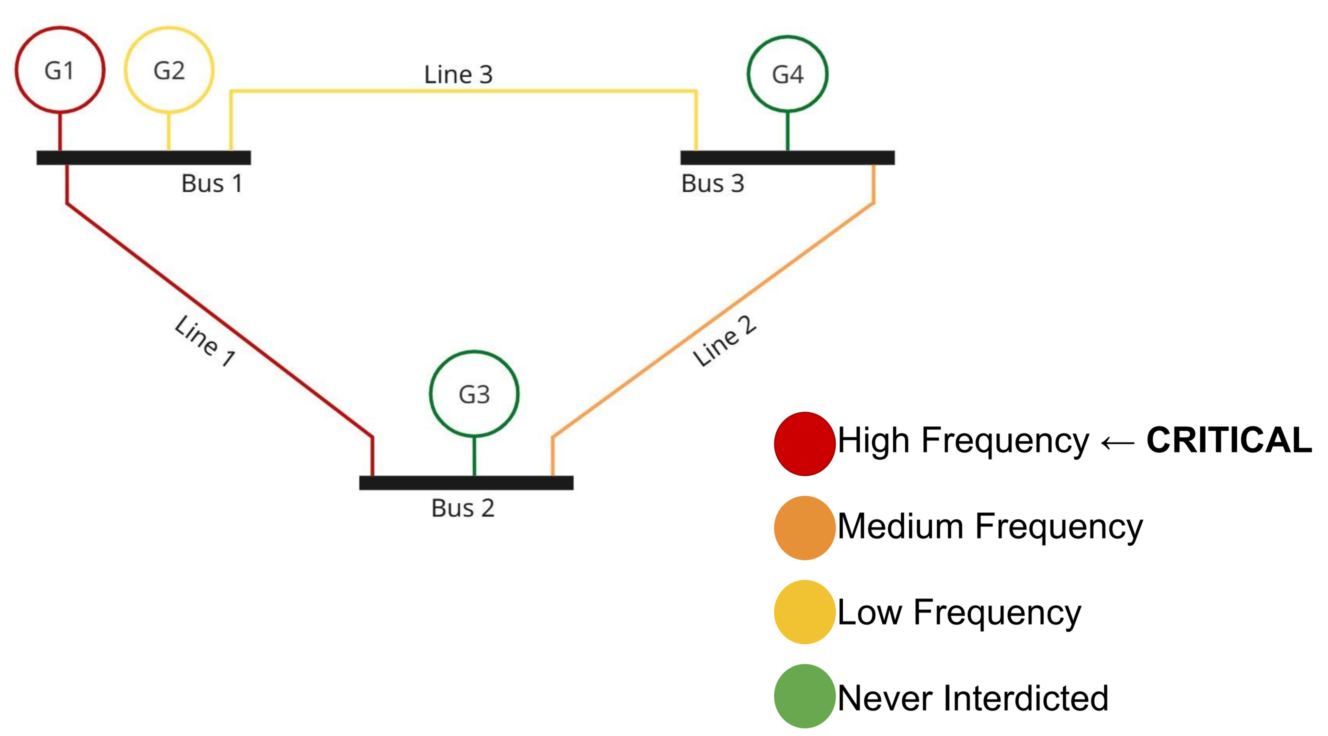
where π represents probability over interdiction plans

Results

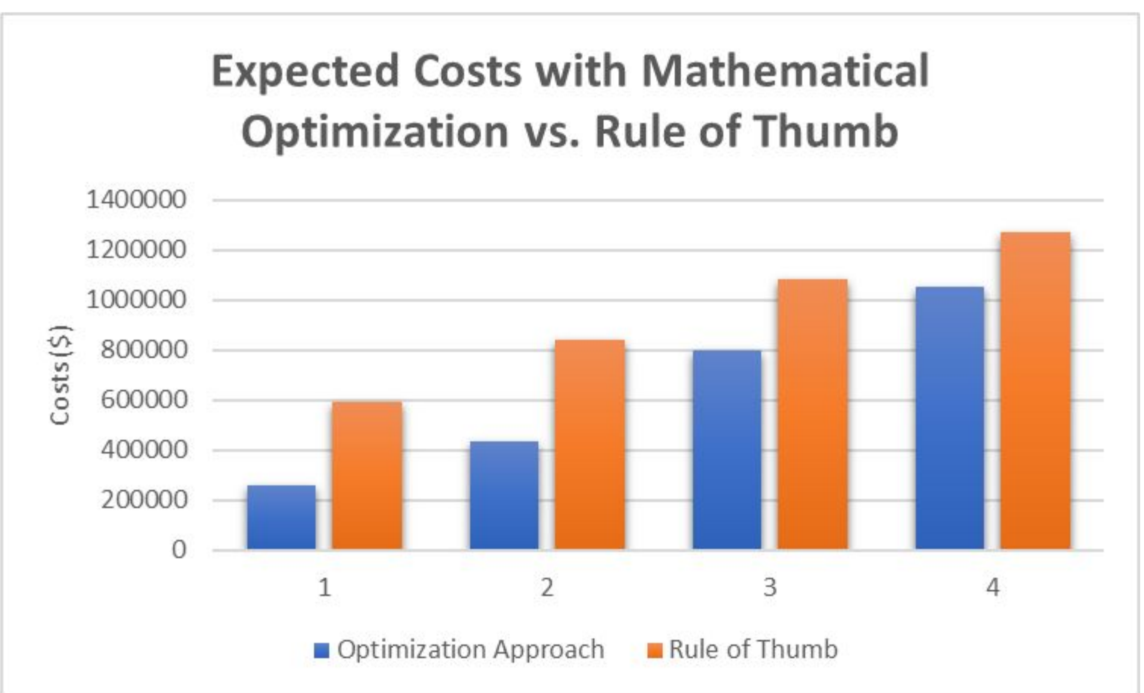
After repeatedly running the algorithm with increasing interdiction budget levels, components that frequently appear in the optimal interdiction solutions are considered critical and thus prime candidates for "hardening".

Budget Level	Gen 1	Gen 2	Gen 3	Gen 4	Line 1	Line 2	Line 3
1	0	0	0	0	1	0	0
2	1	0	0	0	1	0	0
3	1	0	0	0	1	1	0
4	1	1	0	0	1	1	0
5	1	1	0	0	1	1	1

1 if interdicted in optimal solution
0 if not interdicted



The Expectation Formulation is compared to a rule of thumb, chosen as equal distribution of demand across generators, for four grid instances.



Conclusions

The interdiction algorithm identifies critical grid components that should be prioritized for hardening. Hardening strategies, such as building physical barriers around generators, increase system resilience by making interdiction more costly and less effective. An expected cost formulation achieves lower average costs compared to a general heuristic.This demonstrates the value of an optimization-driven strategy in improving grid reliability and cost-effectiveness.

References

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